Does City Structure Affect Job Search and Welfare?¹

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We develop a model in which workers’ search efficiency is negatively affected by access to jobs. Workers’ location in a city is endogenous and reflects a trade-off between commuting costs and the surplus associated with search. Different configurations emerge in equilibrium; notably, the unemployed workers may reside far away (segregated city) or close to jobs (integrated city). We prove that there exists a unique and stable market equilibrium in which both land and labor markets are solved for simultaneously. We find that, despite inefficient search in the segregated city equilibrium, the welfare difference between the two equilibria is not so large due to differences in commuting costs. We also show how a social planner can manipulate wages by subsidizing/taxing the transport costs and can accordingly restore the efficiency. © 2002 Elsevier Science (USA)

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1. INTRODUCTION

The urban economics literature has often focused on the existence of areas of high poverty and high criminality, namely the ghettos. The geographic position of these areas within cities coincides in general with high unemployment and, more precisely, with the absence of jobs in the areas surrounding the ghettos. The labor market is thus a very important channel of the transmission and persistence of poverty across city tracts. In the United States, there has been an

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important empirical debate revolving around this issue. The spatial mismatch hypothesis, first developed by Kain [16], stipulates that the increasing distance between residential location and workplace is very harmful to black workers and, with labor discrimination, constitutes one of the main explanation of their adverse labor market outcomes.

Since the study of Kain, dozens of empirical studies have been carried out trying to test this hypothesis (surveyed by Holzer [9], Kain [17], and Ihlanfeldt and Sjoquist [13]). The usual approach is to relate a measure of labor-market outcomes, based on either individual or aggregate data, to another measure of job access, typically some index that captures the distance from residences to centers of employment. The weight of the evidence suggests that bad job access indeed worsens labor-market outcomes, confirming the spatial mismatch hypothesis.

The economic mechanism behind this hypothesis is, however, unclear. Some tend to argue that black workers refuse to take jobs involving excessively long commuting trips (Zax and Kain [32]). Others think that firms do not recruit workers who live too far away from them because their productivity is lower than those residing closer (see, e.g., Zenou [34]). In the present paper, we propose an alternative approach to explain the spatial mismatch hypothesis: we develop a model based on job search in which distance to jobs is harmful because it negatively affects workers’ search efficiency. It is indeed our contention that search activities are less intense for those living further away from jobs because the quality of information decreases with the distance to jobs. On the contrary, individuals who reside close to jobs have good access to information about these jobs and are in general more successful in their job search activities.

This view is consistent with empirical studies. Indeed, Barron and Gilley [1] and Chirinko [4] have shown that there are diminishing returns to search when people live far away from jobs whereas Van Ommeren et al. [28] have found that people who expect to receive more job offers will generally not have to accept a long commute. Rogers [22] have also demonstrated that access to employment is a significant variable in explaining the probability of leaving unemployment. Finally, Seater [24] have shown that workers searching further away from their residences are less productive in their search activities than those who search closer to where they live.

Our first task is thus to analyze the interaction between job search and the location of workers vis-à-vis the job centers, in a framework where two urban configurations can emerge: a “segregated city” equilibrium in which the unemployed workers reside far away from jobs and an “integrated city” equilibrium in which the unemployed workers are close to jobs. The predominance of one equilibrium over the other strongly depends on the differential in commuting costs between the employed and the unemployed, and on the expected return of being more efficient in search. We show that there exists, for each urban
configuration, a unique market equilibrium in which the land and the labor market are solved for simultaneously.

We then discuss welfare issues. We show that, despite higher inefficiency of the search process in the segregated city, the resulting welfare loss is small compared to the other equilibrium, because of higher commuting costs paid by the employed in the integrated city. We also find that, in both equilibria, the market outcome is in general not efficient because of search externalities. In addition, due to the spatial structure of the model, the social planner has one more set of tools to restore the constrained efficiency, namely through the design of transportation policies. Indeed, the social planner can raise wages by subsidizing the commuting costs of the unemployed and reduce them by subsidizing the commuting costs of the employed workers. As a matter of fact, the standard non-spatial Hosios–Pissarides condition of efficiency of the search matching equilibrium is affected by commuting costs. Interestingly, the policy recommendation does not depend on the urban configuration, but the magnitude of the impact of the policy parameter does. Indeed, in the segregated city, the impact of policy is much larger, by a factor of 10 to 15. Stated differently, such transportation policies may have very little impact in an integrated city, and much more in a segregated city.

Our model is related to other theoretical studies that combine search and urban models. The first papers that dealt with these issues (Jayet [14, 15], Seater [24], and Simpson [25]) had a shallow modelling of the urban space. In particular, these papers did not explicitly model the intra-urban equilibrium in which the location of unemployed and employed workers is endogenously determined and influences the labor market equilibrium. More recently, a more explicit modelling of the urban space has been incorporated in search-matching models. In a companion paper (Wasmer and Zenou [30]), we focus on the labor market equilibrium and more precisely on how search theory is affected by the introduction of a spatial dimension. We deal in particular with endogenous search effort. Coulson et al. [5] and Sato [23] focus on inter-urban equilibria and thus on wage and unemployment differences between different cities. Wheeler [31] investigates the link between urban agglomeration and the firm–worker matching process. Smith and Zenou [26] deal with endogenous search effort. Our present paper is quite different from these approaches since we develop an intra-urban search model and we focus on the impact of city structure (where do workers of different employment status reside?) on the labor market outcomes. In particular, contrary to all the papers mentioned above, we evaluate the impact of different urban equilibria on the search-matching equilibrium.

The remainder of the paper is as follows. The next section presents the basic model. Section 3 focuses on the two urban equilibrium configurations whereas the existence and uniqueness of the market equilibrium (solving for both the land and labor markets) is carried out in Section 4. Section 5 is devoted to the welfare analysis. Finally, Section 6 concludes.
2. THE MODEL AND GENERAL NOTATIONS

There is a continuum of firms and workers that are all \((\text{ex ante})\) identical. For simplicity, we normalize the size of the population to 1. This implies that the unemployment rate in the economy is equal to the unemployment level and will be denoted by \(u\).

A firm is a unit of production that can either be filled by a worker whose production is \(y\) units of output or be unfilled and thus unproductive. In order to find a worker, a firm posts a vacancy. We denote by \(v\) the number of vacancies in the economy. A vacancy can be filled according to a random Poisson process. Similarly, workers searching for a job will find one according to a random Poisson process. In aggregate, these processes imply that there is a number of contacts per unit of time between the two sides of the market that are determined by the standard matching function\(^2\)

\[
x(\tilde{s}u, v),
\]

where \(\tilde{s}\) is the average search efficiency of the unemployed workers (a worker \(i\) has an efficiency of search equal to \(s_i\)). Since this term represents the aggregate search frictions, it is also an index of aggregate information about economic opportunities. How \(s_i\) depends on the location of workers is specified below. We simply assume here that \(x(.)\) is increasing both in its arguments, concave, and homogeneous of degree 1 (or equivalently has constant return to scale).

In this context, the probability for a vacancy to be filled per unit of time is \(x(\tilde{s}u, v)/v\). By constant return to scale, it can be rewritten as \(x(1/\theta, 1) \equiv q(\theta)\) where \(\theta = v/\tilde{s}u\) is a measure of labor market tightness in efficiency units and \(q(\theta)\) is a Poisson intensity. By using the properties of \(x(.)\), it is easily verified that \(q(\theta) \leq 0\): the greater the labor market tightness, the lower the probability for a firm of filling a vacancy. Observe that we assume that firms have no impact on their own search efficiency and consider \(\tilde{s}, u,\) and \(v\) as given. Similarly, for a worker \(i\) with efficiency \(s_i\), the probability of obtaining a job per unit of time is \(x(\tilde{s}u, v)/u\). By constant return to scale, it can be rewritten as \(x(1/\theta, 1) \equiv q(\theta)s_i \equiv p(s_i)\), where \(p(s_i)\) is defined as the intensity of the exit rate from unemployment for this worker.

In contrast to the standard model of job matching, where space is absent (Mortensen and Pissarides [18], Pissarides [20]), we make here the important assumption that \(s\) strongly depends on the location of the unemployed workers in the city: the closer the location to the workplace, the greater the efficiency and the more likely is a contact \(s = s(d)\), where \(d\) is the distance between

\(^2\)This matching function is written under the assumption that the city is monocentric; i.e., all firms are located in one fixed location. See the next section for the description of the spatial structure of the city.
residence and workplace, with \( s'(d) < 0 \)). In other words, the probability to find a job now negatively depends on \( d \) and is defined by \( p(d) = \theta q(\theta) s(d) \). Think for example of firms that place help-wanted signs in their windows or that place ads in local newspapers. Then, obviously, workers living further away from these firms have less information on these jobs than those residing closer. For example, Ihlanfeldt [12] has shown that, in Atlanta, inner-city residents were less able to identify the location of suburban employment centers than suburbanites and thus had less information on those jobs. In addition, Turner [27] demonstrated that, in Detroit, suburban firms using local recruitment methods (such as local newspapers or help-wanted signs in their windows) had few inner-city applicants whereas those using general formal methods (such as city newspapers) had much more inner-city applicants. Finally, Holzer and Reaser [10] have found that, in four major metropolitan areas (Atlanta, Boston, Detroit, and Los Angeles), inner-city workers apply less frequently for jobs in the suburbs than in central cities because of higher costs of applying and/or lower information flows.

Once the match is made, the wage is determined by the standard generalized Nash bargaining solution. In each period, there is also a probability \( \delta \) that the match is destroyed. In order to determine the (general) equilibrium, we will proceed as follows. We first determine the partial urban equilibrium configurations. Then, depending on the location of workers and thus on the aggregate search efficiency \( \bar{s} \), we determine the partial labor market equilibria. Hereafter, the labor (respectively urban) equilibrium will refer to the partial equilibrium. The general equilibrium will be denoted a “market equilibrium.” Thus, by denoting by \( R(d) \) the land market price at a distance \( d \) from the city-center, by \( w \) the wage earned by workers, and by \( u \) the unemployment rate, we have the following definition.

**Definition 1.** A market equilibrium consists of a land rent function, wage, labor market tightness, and unemployment level \((R(d), w, \theta, u)\) such that the urban land use equilibrium and the labor market equilibrium are solved for simultaneously.

Thus, a market equilibrium requires solving *simultaneously* two problems:

(i) a location and rental price outcome (referred to as an urban land use equilibrium)

(ii) a (steady state) matching equilibrium with determines \( w, \theta, \) and \( u \) (referred to as a labor market equilibrium)

We will give below more precise definitions of these two markets.

\(^3\)This relation \( s'(d) < 0 \) can be derived endogenously by showing that workers living further away from jobs put less effort in search than those residing closer. See Wasmer and Zenou [29].
3. EQUILIBRIUM URBAN CONFIGURATIONS

The city is monocentric (i.e., all firms are assumed to be exogenously located in the central business district (CBD hereafter)), is linear, is closed, and has absentee landlords. There is a continuum of workers uniformly distributed along the linear city who endogenously decide their optimal residence between the CBD and the city fringe. The utility function of workers is defined such that they are risk neutral, live infinitely, and discount the future at a rate $r$. Workers all consume the same amount of land (normalized to 1) and the density of residential land parcels is taken to be unity so that there are exactly $d$ units of housing within a distance $d$ of the CBD.

Employed workers go to the CBD to work and to shop while unemployed workers go to the CBD to be interviewed and to shop. Let us denote by $t_w d$ and $t_u d$ the transportation cost at a distance $d$ from the CBD for respectively working and unemployed specific activities (interviews, registration), with $t_w > t_u > 0$. For instance, if the unemployed commute once a week to the CBD and the employed commute five times a week, $t_w / t_u$ could be considered as a constant, equal to 5. All workers bear land rent costs at the market price $R(d)$ and receive a wage $w$ when employed and unemployment benefits $b$ if unemployed. We denote by $U$ and $W$ the discounted expected lifetime utilities (or net intertemporal incomes since workers are risk neutral) of the unemployed and the employed, respectively. We assume that location changes are costless. With the Poisson probabilities defined above, infinite lived workers have the intertemporal utility functions (given by the Bellman equations determined at the steady state)

$$rU(d) = b - t_u d - R(d) + p(d) \left( \max_{d'} W(d') - U(d) \right)$$  \hspace{1cm} (1)

$$rW(d) = w - t_w d - R(d) + \delta \left( \max_{d'} U(d') - W(d) \right),$$  \hspace{1cm} (2)

where $r$ is the exogenous discount rate. Let us comment on (1). When a worker is unemployed today, he/she resides in $d$ and his/her net income is $b - t_u d - R(d)$. Then, he/she can obtain a job with a probability $p(d)$ and if so, he/she relocates optimally in $d'$ and obtains an increase in income of $W(d') - U(d)$. The interpretation of (2) is similar.

Since there are no relocation costs, the urban equilibrium is such that all the unemployed enjoy the same level of utility $rU = r\overline{U}$ as well as the employed $rW = r\overline{W}$. Indeed, any utility differential within the city would lead to the relocation of some workers up to the point where all differences in utility disappear.

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4All these assumptions are very standard in urban economics (see, e.g., Brueckner [3] or Fujita [7]). In particular, the extension to a circular symmetric city is straightforward since any ray through the center looks like any other ray. So examining a single ray is almost the same as looking at the whole city.
We now have to determine the optimal location of all workers in the city. The standard way of doing it is to use the concept of bid rents (Fujita [7]) which are defined as the maximum land rent at a distance \( d \) that each type of worker is ready to pay in order to reach its respective equilibrium utility level. Therefore, the bid rents of the unemployed and employed are respectively equal to

\[
\Psi_u(d, \bar{U}, \bar{W}) = b - t_u d + p(d) \bar{W} - (r + p(d)) \bar{U}
\]

(3)

\[
\Psi_e(d, \bar{U}, \bar{W}) = w - t_e d + \delta \bar{U} - (r + \delta) \bar{W}.
\]

(4)

We have now to determine the location of the unemployed and of the employed workers in the city. For that, we need to calculate the bid rent slopes for each type of worker since a steeper bid rent corresponds to an equilibrium location closer to the CBD. They are respectively given by

\[
\frac{\partial \Psi_u(d, \bar{U}, \bar{W})}{\partial d} = -t_u + p'(d)(\bar{W} - \bar{U}) < 0
\]

(5)

\[
\frac{\partial \Psi_e(d, \bar{U}, \bar{W})}{\partial d} = -t_e < 0,
\]

(6)

where \( p'(d) = \theta q(\theta) s'(d) < 0 \). These slopes (in absolute values) can be interpreted as the marginal cost that a worker is ready to pay in order to be marginally closer to the CBD.\(^5\) For the unemployed, this marginal cost is the sum of the marginal commuting cost, \( t_u \), and the marginal probability of finding a job, \( p'(d) \), times the (intertemporal) surplus of being employed, \( \bar{W} - \bar{U} \). On the other hand, the employed workers bear only marginal the commuting cost \( t_e \) since the probability of losing a job \( \delta \) is exogenous and does not depend on the location of workers. If \( d_f \) denotes the city fringe, we have the following definition.

**Definition 2.** The urban land rent function \( R(d) \) is the upper envelope of all workers’ bid rents and of the agricultural land rent \( R_A \); i.e.,

\[
R(d) = \max \{ \Psi_u(d, \bar{U}, \bar{W}), \Psi_e(d, \bar{U}, \bar{W}), R_A \} \text{ at each } d \in [0, d_f].
\]

With this general definition, a large number of urban equilibria can arise. However, for analytical simplicity, we here focus on linear bid rent slopes, in which case only two urban equilibria can emerge. In the first one (Equilibrium 1), the unemployed reside in the vicinity of the CBD and the employed at the outskirts of the city. We call such a city the “integrated city” because the unemployed have a good access to jobs. By contrast, in the other

\(^5\)The set of parameters for which the slope of the two bid rents is equal is of zero measure. Accordingly, we exclude this possibility and thus mixed configurations where both employed and unemployed workers are located in the same place. We thus only consider perfectly separated configurations where only one type of worker locates within the same segment.
one (Equilibrium 2), the unemployed locate at the outskirts of the city and the employed close to the city-center. This city is called the “segregated city” because the unemployed have a bad access to jobs.

In order to have linear bid rents, we assume that \( s(d) = s_0 - ad \), with \( s_0 > 0 \) and \( a > 0 \). To guarantee that search efficiency is always positive, we impose that \( s(d) > 0 \) for all \( d \leq 1 \). This linear formulation implies that \( p''(d) = 0 \) and thus

\[
\frac{\partial^2 \Psi_u(d, \bar{U}, \bar{W})}{\partial d^2} = \frac{\partial^2 \Psi_e(d, \bar{U}, \bar{W})}{\partial d^2} = 0.
\]

Thus, according to (5) and (6), the resulting urban equilibrium is determined by a trade-off between the difference in commuting costs, \( t_e - t_u \), and the marginal probability of getting a job. This is due to the fact that both the employed and the unemployed want to be as close as possible to the CBD because the former want to save on commuting costs and the latter desire to increase their probability of getting a job.

The important condition for having one equilibrium or the other is given by an inequality comparing the slopes of the bid rents of employed and unemployed workers. Equilibrium 1 occurs if and only if:6

\[ t_e - t_u < a_{\theta} q(\theta_1) (\bar{W}_1 - \bar{U}_1). \]

Equation (7) means that the differential in commuting costs per unit of distance between the employed and the unemployed is lower than the marginal expected return of search for the unemployed if he/she moves closer to the center by one unit of distance. Indeed, since \( a = \frac{s_0}{\partial d} \), \( a_{\theta} q(\theta_1) \) is the marginal increase in the probability of obtaining a job for an unemployed worker. Thus if (7) holds, the unemployed occupy the core of the city and the employed the outskirts of the city. In the other equilibrium (Equilibrium 2), we have the opposite inequality:

\[ t_e - t_u > a_{\theta} q(\theta_2) (\bar{W}_2 - \bar{U}_2). \]

and the employed bid away the unemployed.

Observe that anything that may improve the return to search for unemployed workers \( (\theta q(\theta) \) and \( \bar{W} - \bar{U} \) \) can help to switch from Equilibrium 2 to Equilibrium 1. Observe also that when the employed have lower commuting costs than the unemployed, the only possible equilibrium is integrated since the unemployed have stronger incentives to reside closer to the CBD.

---

6All variables determined at the equilibrium \( k(= 1, 2) \) are indexed by \( k \). At each urban equilibrium \( k \) corresponds a labor market equilibrium \( k \).
By using Definition 2 and the two conditions, (7) and (8), we are now able to define the two urban equilibria. If we denote the distance of the border between the unemployed and the employed by $d_k$ ($k = 1, 2$), we have the following definition:

**DEFINITION 3.** The urban land use equilibrium $k = 1, 2$ consists of intertemporal utility functions of being unemployed and employed, a border between the employed and the unemployed, and a city fringe $(U_k, W_k, d_k, d_f)$ such that

\[
d_1 = u_1 \quad \text{or} \quad d_2 = 1 - u_2
\]

\[
d_f = 1
\]

\[
\Psi_e(d_k, U_k, W_k) = \Psi_{e}(d_k, U_k, W_k)
\]

\[
\Psi_e(d_f, U_k, W_k) = R_A \quad \text{or} \quad \Psi_u(d_f, U_k, W_k) = R_A.
\]

These two equilibria are illustrated in Figs. 1 and 2. By using (3), (4), and (9) and replacing them in (11) and (12), we obtain

\[
W_k - U_k = \frac{w_k - b - (t_u - t_u) d_k}{r + \delta + p(d_k)} \quad k = 1, 2,
\]

where $w_k, u_k, \theta_k$ will be determined at the labor market equilibrium $k$. The average search intensity in equilibrium $k$ is equal to

\[
\bar{s}_k = s_0 - a \bar{d}_k \quad k = 1, 2,
\]

where $\bar{d}_k$ is the average location of the unemployed in equilibrium $k$. It is easy to verify that $\bar{d}_1 = u_1/2$ and $\bar{d}_2 = 1 - u_2/2$. 

**FIG. 1.** Urban equilibrium 1 (the integrated city).
FIG. 2. Urban equilibrium 2 (the segregated city).

Since \( \bar{d}_1 < \bar{d}_2 \) (this is always true because \( u_1 + u_2 < 2 \)), the average search efficiency in urban equilibrium 1 is higher than in urban equilibrium 2; i.e., \( \bar{s}_1 > \bar{s}_2 \). This result is quite intuitive. Indeed, in the integrated city, the unemployed reside closer to the CBD than in the segregated city and thus their probability of finding a job is higher.

4. THE MARKET EQUILIBRIUM

4.1. Free-Entry Condition and Labor Demand

Let us denote respectively by \( J_k \) and \( V_k \) the intertemporal profit of a job and of a vacancy in equilibrium \( k = 1, 2 \). If \( \gamma \) is the search cost for the firm per unit of time and \( y \) is the product of the match, then \( J_k \) and \( V_k \) can be written as

\[
\begin{align*}
    r J_k &= y - w_k + \delta (V_k - J_k) \\
    r V_k &= -\gamma + q(\theta_k)(J_k - V_k).
\end{align*}
\]  

(15)  

(16)

Following Pissarides [20], we assume that firms post vacancies up to a point where

\[ V_k = 0 \]

(17)

which is a free entry condition. From (16) and using (17), the value of a job is now equal to

\[ J_k = \frac{\gamma}{q(\theta_k)}. \]

(18)
Finally, plugging (18) into (15), we obtain the following decreasing relation between labor market tightness and wages in equilibrium:

$$\frac{y}{q(\theta_k)} = \frac{y - w_k}{r + \delta} \quad k = 1, 2.$$  \hspace{1cm} (19)

In words, the value of a job is equal to the expected search cost, i.e. the cost per unit of time multiplied by the average duration of search for the firm.

Equation (19) defines in the space $(\theta_k, w_k)$ a curve (see Fig. 3) representing the supply of vacancies. This curve is in fact a labor demand curve $LD$ since it determines the number of vacancies posted per period as a function of wages. This curve is independent of the type of urban equilibrium since, independently of workers’ location, firms post vacancies up to $V_k = 0$. We show in Lemma A.1$^7$ that it is downward sloping in the plane $(\theta_k, w_k)$.

4.2. Wage Determination

The usual assumption about wage determination is that, at each period, the total intertemporal surplus is shared through a generalized Nash-bargaining process between the firm and the worker.$^8$ The total surplus is the sum of the surplus of the workers, $\overline{W}_k - \overline{U}_k$, and the surplus of the firms $J_k - V_k$. At each period, the wage is determined by

$$w_k = \arg\max (\overline{W}_k - \overline{U}_k)^{\beta}(J_k - V_k)^{1 - \beta} \quad k = 1, 2,$$  \hspace{1cm} (20)

$^7$All lemmas are stated and proved in the Appendix.

$^8$All negotiations take place between two agents: the worker and the firm. In particular, no agent is allowed to make offers simultaneously to more than one other agent.
where \( k \) denotes the urban equilibrium and \( 0 \leq \beta \leq 1 \) is the bargaining power of workers. Observe that \( \bar{U}_k \) is the value of continued search in equilibrium \( k \), i.e., the value of workers’ outside option whereas \( V_k \) equals the employers’ value of holding the job vacant in equilibrium \( k \), i.e., the value of firms’ outside option. Observe also that \( \bar{U}_k \), the threat point (or the outside option) of the worker in the urban equilibrium \( k \), does not depend on the current location of the worker, who will relocate if there is a transition in his/her employment status. Using (2) and (15), the first-order maximization condition derived from (20) satisfies

\[
(1 - \beta)(\bar{W}_k - \bar{U}_k) = \beta(J_k - V_k) \quad k = 1, 2. \tag{21}
\]

Now, observing that \( V_k = 0 \) (free-entry condition) and \( p(d_k) \equiv \theta_k q(\theta_k)s(d_k) \), and using (13) and (18), we can write (21) as

\[
(1 - \beta)[w_k - b - (t_e - t_u)u_1] = [r + \delta] \beta \frac{\gamma}{q(\theta_k)} + \theta_k s(d_k)\beta\gamma \quad k = 1, 2.
\]

Finally, since by (19) \( \gamma(r + \delta)/q(\theta_k) = y - w_k \), we obtain, in urban equilibrium \( k = 1, 2 \), the wage

\[
w_k = (1 - \beta)[b + (t_e - t_u)u_1] + \beta[y + s(d_k)\theta_k\gamma], \tag{22}
\]

where \( d_k \) is defined by (9).

Let us first interpret the wage equation (22) for city 1, with a focus on how wages depend on unemployment \( u_1 \). The first part of the RHS of (22), \( (1 - \beta)[b + (t_e - t_u)u_1] \), is what firms must pay to induce workers to accept the job offer: firms must exactly compensate the transportation cost difference (between the employed and the unemployed) of the unemployed worker who is the furthest away from the CBD, i.e., located at \( d_1 = u_1 \) (remember that here unemployment is also a distance). Therefore, when \( u_1 \) increases, this marginal unemployed worker is further away from the CBD and wages must compensate the transportation cost difference. This is referred to as the “compensation effect.” The second part of the RHS of (22), \( \beta[y + s(u_1)\theta_1\gamma] \), implies that distance \( (u_1) \) has the opposite effect. When \( u_1 \) rises, the marginal unemployed (i.e., the one located in \( d_1 = u_1 \)) is further away from the city-center and thus his/her outside option (i.e., the value of continued search) decreases, leading to a reduction in the wage. This is referred to as the “outside option effect.” The first effect is a pure spatial cost since \( (t_e - t_u)u_1 \) represents the space cost differential between employed and unemployed workers paying the same bid rent whereas the second effect is a mixed labor-spatial cost one. The interpretation of Eq. (22) for city 2 is similar, except that one needs to replace \( d_1 = u_1 \) by \( d_2 = 1 - u_2 \).

\[9\text{For empirical evidence of links between local wages and unemployment, see Blanchflower and Oswald [2].}
\]

\[10\text{Using an efficiency wage model, Zenou and Smith [33] found a similar effect.} \]
In search-matching models, the wage-setting curve (a relation between wages and the state of the labor market, here $\theta_k$, $k = 1, 2$) replaces the traditional labor-supply curve. Equations (22) are the two wage-setting curves ($WS_1$ and $WS_2$) and are both positively sloped in the plan ($\theta_k, w_k$). Lemma A.2 states that the curve $WS_1$ is steeper than $WS_2$ but the intercept of $WS_1$ is lower (see Fig. 3). The reason $WS_1$ is steeper than $WS_2$ is the following. The slope of these curves represents the ability of workers to exert wage pressures when the labor market is tighter. Thus, for a given labor market tightness, a more efficient labor market, i.e., $s(d_1) > s(d_2)$, leads to a higher wage pressure per unit of $\theta$, thus a steeper $WS_1$ curve. The latter inequality on search efficiencies is true for reasonable values of the unemployment rates (a sufficient condition being that both $u$’s are less than a half) and implies that the marginal worker is closer and thus more efficient in his/her search in Equilibrium 1. The intercept is, however, higher in Equilibrium 2 since when $\theta = 0$, the outside option effect vanishes and only the compensation effect remains. Since the same marginal worker is further away in Equilibrium 2, he/she must have a higher compensation for his/her transportation costs.

As in the standard labor economics approach, for each equilibrium $k$, the equilibrium value of $w_k$ and $\theta_k$ will be determined by the intersection between the wage-setting curve (22) and the labor demand curve (19), defined above. In fact, two situations may arise: either the labor demand curve cuts the wage setting curve before the intersection point $(\hat{w}, \hat{\theta})$ (see Fig. 3 for an illustration of this case) or after it.\(^{11}\) The value of $\hat{\theta}$, determined in Lemma A.2, is given by

$$\hat{\theta} = \frac{1 - \beta (t_e - t_u)}{\beta \gamma}.$$  \hspace{1cm} (23)

In fact, it turns out that $\hat{\theta}$ also defines a critical value that determines which urban equilibrium prevails. Indeed, it is easy to see that, at the labor market equilibrium, urban equilibrium 1 prevails if and only if $\theta^*_1 > \hat{\theta}$ and urban equilibrium 2 prevails if and only if $\theta^*_2 < \hat{\theta}$.\(^{12}\) Using this result, the following proposition is straightforward to establish.

**Proposition 1.** When $\theta^*_1 < \theta^*_2$:

- if $\theta^*_1 < \theta^*_2 < \hat{\theta}$, urban equilibrium 2 exists and is unique;

\(^{11}\)Because it is too peculiar, we exclude the case when the labor demand curve intersects the two wage setting curves at exactly $(\hat{w}, \hat{\theta})$.

\(^{12}\)This result is derived as follows. Using (17) and (18) in (21), we obtain

$$W_k - U_k = \frac{\beta \gamma}{1 - \frac{\beta}{\gamma} q(\theta_k)}.$$  

Now plugging this value in (7) (respectively (8)) gives the result.
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- if $\hat{\theta} < \theta_1^* < \theta_2^*$, urban equilibrium 1 exists and is unique;
- if $\theta_1^* < \hat{\theta} < \theta_2^*$, there is no urban equilibrium.

When $\theta_2^* < \theta_1^*$:

- if $\theta_2^* < \theta_1^* < \hat{\theta}$, urban equilibrium 2 exists and is unique;
- if $\hat{\theta} < \theta_2^* < \theta_1^*$, urban equilibrium 1 exists and is unique;
- if $\theta_2^* < \hat{\theta} < \theta_1^*$, both urban equilibria exist.

Therefore, when the labor demand curve cuts the wage setting curve before $(\hat{w}, \hat{\theta})$, we have $\theta_2^* < \theta_1^* < \hat{\theta}$, and then according to Proposition 1, there is only one unique urban equilibrium, which is Equilibrium 2. When the labor demand curve cuts the wage setting curve after $(\hat{w}, \hat{\theta})$, we have $\hat{\theta} < \theta_2^* < \theta_1^*$, and, according to Proposition 1, only Equilibrium 1 exists and is unique. One can thus see that several of the above possibilities, including multiple equilibria and no equilibrium, are not possible.

4.3. Steady-State Labor Market Equilibrium

In order to derive the market equilibrium, we have first to define a labor market equilibrium for each urban equilibrium $k = 1, 2$. We have the following definition:

**Definition 4.** A (steady-state) labor market equilibrium consists of a labor market tightness, wage, and unemployment level $(\theta_k, w_k, u_k)$ such that, given the matching technology, all agents (workers and firms) maximize their respective objective function; i.e. this triple is determined by a free-entry condition for firms, a wage-setting mechanism, and a steady-state condition on unemployment.

Since each job is destroyed according to a Poisson process with arrival rate $\delta$, in equilibrium $k = 1, 2$, the number of workers who enter unemployment is $\delta(1 - u_k)$ and the number who leave unemployment is $\theta_k q(\theta_k) \bar{s} u_k$. The evolution of unemployment is thus given by the difference between these two flows,

$$\dot{u}_k = \delta(1 - u_k) - \theta_k q(\theta_k) \bar{s} u_k, \quad k = 1, 2,$$

where, in equilibrium $k$, $\dot{u}_k$ is the variation of unemployment with respect to time. In steady state, the rate of unemployment is constant and therefore these two flows are equal (flows out of unemployment equal flows into unemployment). We thus have:

$$u_k = \frac{\delta}{\delta + \theta_k q(\theta_k) \bar{s} u_k}, \quad k = 1, 2.$$  

(25)

It is easy to see that, when the matching technology exhibits constant returns, Eq. (24) has a unique stable steady-state solution for every vacancy rate. This solution is given by (25).
In the search-matching literature, the above equation defines in the \((u, v)\) space a downward sloping curve referred to as the Beveridge curve. In our model, this curve would be further away from the origin in Equilibrium 2. See Wasmer and Zenou \[29\] for a formal exposition of these results.

4.4. Market Equilibrium: Existence and Uniqueness

We have three equations, (19), (22), and (25), and three unknowns, \(w_k, \theta_k\), and \(u_k\) \((k = 1, 2)\). By combining (19) and (22), the market solution that defines the equilibrium \(\theta_k\) is given by

\[
y - b = \frac{y}{q(\theta_k)} \left[ \frac{\delta + r + \theta_k q(\theta_k) s(d_k) \beta}{1 - \beta} \right] + d_k (t_e - t_u) \quad (26)
\]

and the equilibrium value of \(w_k\) is determined by plugging this \(\theta_k\) in (22).

For each equilibrium \(k = 1, 2\), Eq. (26) defines a first relation between the labor market tightness \(\theta_k\) and the unemployment rate \(u_k\), since \(s(d_k)\) depends on unemployment. Equation (25) defines a second relation between these two variables. These two equations are central to the following result:\[14\]

**Proposition 2.** There exists a unique and stable market equilibrium \((R^*(d), w_k^*, \theta_k^*, u_k^*), k = 1, 2\), and only the two following cases are possible:

- If \(\hat{\theta} < \theta_1^* < \theta_2^*\), urban equilibrium 1 prevails.
- If \(\theta_2^* < \theta_1^* < \hat{\theta}\), urban equilibrium 2 prevails.

**Proof.** See the Appendix.

Observe that multiple urban equilibria can never happen (i.e., having both a segregated and an integrated city) because, according to Proposition 1, the condition for multiple equilibria is \(\theta_2^* < \hat{\theta} < \theta_1^*\) and it can never be verified since the intersection of the labor demand curve with the wage setting curve never satisfies this condition. This is because \(\hat{\theta}\), defined initially as the intersection point between the two wage-setting curves (see Fig. 3), is also the critical value that determines which urban equilibrium prevails.

A stimulating insight of the model is as follows. Suppose that aggregate macroeconomic conditions are summarized by the productivity parameter \(y\). Then, by totally differentiating (26), it is easy to see that \(y\) and \(\theta\) are positively linked. Given that \(\hat{\theta}\) is independent of \(y\) according to Eq. (23), this implies that, using conditions (7) and (8), the higher \(y\), the higher \(\hat{\theta}\) and thus the more likely an integrated city is to be the equilibrium outcome. As a result, in periods of booms, an integrated city is likely to emerge as an equilibrium outcome of our model. Conversely, in periods of slumps (or similarly, when the job matching rate is low due to lower \(s_0\) for instance), segregated cities are more likely

\[14\]A variable with a star as a superscript refers to the market equilibrium.
to occur. This prediction of the model is consistent with the higher attention paid to segregation and ghetto issues in the last decades. It is also consistent with the empirical observations (e.g., Zax and Kain [32]) that adverse shocks of the mid 1970s have lead to the relocation of firms outside the CBDs and thus to an increasing distance between residences and jobs for the unemployed (black) workers. Moreover, a shift from an integrated city to a segregated city due to such a change in aggregate conditions will have a dramatic effect on unemployment (but, as will become clear in the next section, not necessarily on welfare). Indeed, because of a lower average search efficiency, for a given level of vacancy, the unemployment rate will be higher.

5. WELFARE ANALYSIS

Let us now proceed to the welfare analysis. The first issue we address is the respective efficiency of the two types of land market configurations. Since there are no multiple equilibria in the land market, we cannot Pareto-rank the equilibria and it is difficult to compare them. Nevertheless, we are able to investigate what happens to the welfare difference in the range of parameters around the frontier separating the two types of land market equilibria. We show in fact that the welfare difference between the two cities is very marginal so that a social planner does not necessarily want to impose the integrated city. We then focus on the efficiency and the welfare analysis of each equilibrium. We show that each equilibrium is in general not efficient and that a social planner can restore the efficiency using transportation policies.

5.1. Comparison of the Two Cities

We first investigate the welfare of the city and study how it varies in each urban configuration. In the context of our matching model, the welfare is given by the sum of the utilities of the employed and the unemployed, the production of the firms net of search costs, and the land rents received by the (absentee) landlords. The wage $w$ as well as the land rent $R(d)$, being pure transfers, are thus excluded in the social welfare function. The latter is therefore given by

$$\Omega_k = \int_0^{+\infty} e^{-rt} \left\{ \int_{\text{employed}} (y - tz)dz + \int_{\text{unemployed}} (b - tz)dz - \gamma \theta_k \delta_k u_k \right\} dt,$$

where $z$ is used for distance.

To see how this quantity varies and if it can be compared across cities, let us proceed to a simple numerical resolution of the model. We use the following Cobb–Douglas function for the matching function: $x(\delta_k u_k, v_k) = (\delta_k u_k)^{0.5} v_k^{0.5}$. This implies that $q(\theta_k) = \theta_k^{-0.5}$, $p(d_k) = \theta_k^{0.5}$ and, whatever the prevailing urban equilibrium, the elasticity of the matching rate (defined as $\eta(\theta_k) = -q'(\theta_k)\theta_k/q(\theta_k)$) is equal to 0.5. The values of the parameters (in yearly terms) are the following: the output $y$ is normalized to unity, as is the
scale parameter of the matching function. The relative bargaining power of workers is equal to \( \eta(\theta) \); i.e., \( \beta = \eta(\theta) = 0.5 \). Unemployment benefits have a value of 0.2 and the costs of maintaining a vacancy \( \gamma \) are equal to 0.3 per unit of time. Commuting costs \( t_e \) are equal to 0.4 for the employed, and \( t_u = 0.1 \) for the unemployed. The discount rate \( r = 0.05 \), whereas the job destruction rate \( \delta = 0.1 \), which means that jobs last on average 10 years. Finally, \( s_0 \) is normalized to 1, implying that \( 0 \leq a \leq 1 \).

In order to single out the spatial effects from the non-spatial ones, we have decomposed the total unemployment rate given by (25) by using a Taylor first-order expansion for small \( a/s_0 \), i.e.,

\[
\frac{\delta}{\delta + \theta_k q(\theta_k)[s_0 - a(1 - u_k/2)]},
\]

in two parts. The first one is the part of unemployment that is independent of spatial frictions, i.e., when \( a = 0 \) so that \( \tilde{s} = s_0 \). This unemployment rate is thus given by

\[
\frac{\delta}{\delta + \theta_0 q(\theta_0)s_0}
\]

(where \( \theta_0 \) is the labor market tightness when \( a = 0 \) and corresponds to the standard non-spatial unemployment rate in Pissarides [20]). The second one is the part of unemployment that is only due to additional spatial frictions, denoted by \( u_k^s \), and is defined by

\[
u_k^s = u_k^* - u_0.
\]

We have the following results:

<table>
<thead>
<tr>
<th>( a )</th>
<th>City</th>
<th>( u_k^*(%) )</th>
<th>( u_0(%) )</th>
<th>( u_k^s(%) )</th>
<th>( \theta_k )</th>
<th>( d_k )</th>
<th>( \tilde{s} )</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>6.86</td>
<td>6.64</td>
<td>0.22</td>
<td>0.03</td>
<td>1.98</td>
<td>0.069</td>
<td>0.966</td>
</tr>
<tr>
<td>0.75</td>
<td>1</td>
<td>6.85</td>
<td>6.69</td>
<td>0.16</td>
<td>0.02</td>
<td>1.95</td>
<td>0.069</td>
<td>0.974</td>
</tr>
<tr>
<td>0.6</td>
<td>1</td>
<td>6.85</td>
<td>6.72</td>
<td>0.13</td>
<td>0.02</td>
<td>1.93</td>
<td>0.069</td>
<td>0.979</td>
</tr>
<tr>
<td>0.55</td>
<td>1</td>
<td>6.85</td>
<td>6.73</td>
<td>0.12</td>
<td>0.02</td>
<td>1.92</td>
<td>0.069</td>
<td>0.981</td>
</tr>
<tr>
<td>0.525</td>
<td>1</td>
<td>6.85</td>
<td>6.73</td>
<td>0.15</td>
<td>0.02</td>
<td>1.92</td>
<td>0.069</td>
<td>0.982</td>
</tr>
<tr>
<td>522+</td>
<td>1</td>
<td>6.85</td>
<td>6.73</td>
<td>0.12</td>
<td>0.02</td>
<td>1.92</td>
<td>0.069</td>
<td>0.982</td>
</tr>
<tr>
<td>0.522−</td>
<td>2</td>
<td>12.4</td>
<td>6.73</td>
<td>5.65</td>
<td>0.46</td>
<td>1.92</td>
<td>0.876</td>
<td>0.511</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>12.4</td>
<td>6.74</td>
<td>5.63</td>
<td>0.45</td>
<td>1.91</td>
<td>0.876</td>
<td>0.512</td>
</tr>
<tr>
<td>0.25</td>
<td>2</td>
<td>12.1</td>
<td>6.82</td>
<td>5.31</td>
<td>0.44</td>
<td>1.86</td>
<td>0.879</td>
<td>0.530</td>
</tr>
<tr>
<td>0.1</td>
<td>2</td>
<td>10.0</td>
<td>7.81</td>
<td>2.20</td>
<td>0.22</td>
<td>1.39</td>
<td>0.909</td>
<td>0.763</td>
</tr>
</tbody>
</table>

In this table, we have chosen to vary a key parameter \( a \), the loss of information per unit of distance (remember that workers’ search intensity is defined by \( s(d_k) = s_0 - ad_k \)). This parameter \( a \) varies from a very large value 1 (where city 1 is the prevailing equilibrium) to a very small value 0.1 (where city 2 is the prevailing equilibrium). The cutoff point is equal to \( a = 0.522 \). The sign
“−” indicates the “limit to the left,” whereas the sign “+” indicates the “limit to the right.”

The first interesting result of this table is that, when we switch from an integrated city (Equilibrium 1) to a segregated city (Equilibrium 2), for values very close to the cutoff point \( a = 0.522 \), the unemployment rate \( u_k \) nearly doubles (from 6.85% to 12.4%). However, it is clear that this result is due to the spatial part of unemployment \( u_k \) since the non-spatial one \( u_0 \) is not at all affected. Indeed, when we switch from Equilibrium 1, where the unemployed are close to jobs and are very efficient in their job search (\( \bar{s} = 0.982 \)), to Equilibrium 2, where the unemployed reside far away from jobs and are on average not very active in their search activity (\( \bar{s} = 0.511 \)), the spatial part of unemployment changes values from 0.12 to 5.65. Another way to see this is to consider column 6 (\( u_k^s/u_k^t \)): the part of unemployment due to space varies from 2% to 46%. So the main effect from switching from one equilibrium to another is that search frictions are amplified by space and consequently unemployment rates sharply increase. So here the spatial access to jobs is crucial to understanding the formation of unemployment. This result is in accordance with the findings of Cutler and Glaeser [6] who find that residential segregation is very harmful to (black) workers. They show that (black) workers living in segregated areas experience worse labor market outcomes than those residing in less segregated areas.

The last column of the table shows the value of the welfare \( \Omega_k \) when \( a \) varies. The result is very striking: even though unemployment rates are higher in Equilibrium 1 than in Equilibrium 2, this does not imply that the general welfare of the economy is higher in the first equilibrium. Indeed, even though the unemployed are better off in Equilibrium 1 (lower unemployment spells and lower commuting costs), the employed can in fact be worse off because of much higher commuting costs in Equilibrium 1. In the above table, it is interesting to see that at the vicinity of \( a = 0.522 \), switching from Equilibrium 1 to Equilibrium 2 does not involve much change in the welfare level (from 0.714 to 0.712).

### 5.2. Welfare and Policy Implications within Each City

The shape of the city thus has little impact on welfare, since in the segregated city, what is lost from lower search efficiency is gained through lower commuting costs. We now investigate the issue of the optimality of the decentralized equilibrium within each land market equilibrium.

In the standard search-matching literature (Mortensen and Pissarides [18], Pissarides [20]), market failures are caused by search externalities. Indeed, the job-acquisition rate is positively related to \( v_k \) and negatively related to \( u_k \) whereas the job-filling rate has exactly the opposite sign. For example, negative search externalities arise because of the congestion that firms and workers impose on each other during their search process. Therefore, two types of externalities must be considered: negative intra-group externalities (more searching
workers reduces the job-acquisition rate) and positive inter group externalities (more searching firms increases the job-acquisition rate). For a class of related search-matching models, Hosios [11] and Pissarides [20] have established that these two externalities just offset one another in the sense that search equilibrium is socially efficient if and only if the matching function is homogenous of degree one and the worker’s share of surplus $\beta$ is equal to $\eta(\theta)$, the elasticity of the matching function with respect to unemployment (this is referred to as the Hosios–Pissarides condition). Of course, there is no reason for $\beta$ to be equal to $\eta(\theta)$ since these two variables are not related at all and, therefore, the search-matching equilibrium is in general inefficient. However, when $\beta$ is larger than $\eta(\theta)$, there is too much unemployment, creating congestion in the matching process for the unemployed. When $\beta$ is lower, there is too little unemployment, creating congestion for firms.

In our present model, we have exactly the same externalities (intra- and inter-group externalities). The spatial dimension does not entail any inefficiency so that it is easily verified that, for each equilibrium $k = 1, 2$, the Hosios–Pissarides condition still holds; i.e., $\beta = \eta(\theta_k)$.\(^\text{15}\)

However, because of the spatial dimension of our model, the government has more instruments than in the spaceless matching model (Pissarides [20, Chap. 9]). Let us thus be more specific about the structure of transport cost. Let us introduce a difference between the transportation cost faced by agents ($t_e$ and $t_u$) and the (exogenous) marginal cost of transportation of respectively the employed and the unemployed ($c_e$ and $c_u$). Denote by $\sigma$ the subvention rate of these costs. We thus have

$$t_e = (1 - \sigma)c_e \quad \text{and} \quad t_u = (1 - \sigma)c_u.$$

We assume that $c_e > c_u$ (recall that $t_e > t_u$). Note that taxation of the transport cost would imply a negative $\sigma$, which we do not rule out at this stage. We assume that, in each case, there is a lump-sum transfer to the workers, negative if $\sigma > 0$ and positive otherwise.

We would first like to compare the model with no subsidy with the one with subsidy, treating $\sigma$ as exogenous and not as a policy parameter that can be adjusted to achieve efficiency. This will allow us to analyze the discrepancy between the two outcomes. By solving the same problem as in Hosios [11] and Pissarides [20], we obtain the following result:

**Proposition 3.** In Equilibrium $k = 1, 2$, we have the spatial Hosios–Pissarides condition that ensures that the private and the social outcomes

\(^{15}\)To see this, observe that, as in Hosios [11] and Pissarides [20, Chap. 8], the social planner solves the following problem: he/she chooses $\theta_k$ and $u_k$ that maximize (27) under the constraint (24). Then, by comparing the solution of this problem and the market solution (26), it is easy to verify that the private and social solutions coincide if and only if $\beta = \eta(\theta_k)$.\)
coincide,

\[ \beta = \eta(\theta_k) + \Lambda_k, \]  

(28)

where

\[ \Lambda_k = (1 - \beta)(1 - \eta(\theta_k)) \frac{q(\theta_k)/\gamma}{\delta + r + \theta_kq(\theta_k)s(d_k)}(c_e - c_u)d_k \geq 0. \]  

(29)

Proof. See the Appendix.

The following comments are in order. First, the existence of spatial terms modifies the standard Hosios–Pissarides condition. Here, the condition \( \beta = \eta(\theta) \) that guarantees that the market solution is efficient in a non-spatial matching framework is augmented by a term incorporating some spatial aspects. Efficiency requires that: (i) \( \beta < \eta(\theta) \) when transport prices are above their marginal cost, i.e., \( \sigma < 0 \); (ii) \( \beta > \eta(\theta) \) when transport prices are below their marginal cost, i.e., \( \sigma > 0 \); (iii) and finally \( \beta = \eta(\theta) \) when transport prices equal their marginal cost, i.e., \( \sigma = 0 \). The intuition of these results is straightforward. The social planner cares about transport costs (\( c_e \) and \( c_u \)) whereas the decentralized equilibrium only involves transport prices (\( t_e \) and \( t_u \) faced by workers). When \( \sigma = 0 \) (case (iii)), both are equal and the social planner only cares about the internalization of search-matching externalities. This precisely corresponds to the competitive search equilibrium.\(^{16}\) Note also that when the spatial ingredients of the model disappear, \( a = 0 \), and \( c_e = c_u \) to 0, one naturally obtains the standard efficiency condition \( \beta = \eta(\theta) \).

Second, if \( \sigma \neq 0 \), transport expenditure in the decentralized equilibrium is not optimal. Notably, when \( \sigma < 0 \), the transport costs in the decentralized economy are below the optimum whereas when \( \sigma > 0 \), the transport costs in the decentralized economy are above the optimum.

We would now like to focus on transportation policies and treat \( \sigma \) as a policy parameter that can be adjusted to achieve efficiency. For that, one can first notice that a larger number of employed workers in the economy is associated with larger commuting costs. Thus, a higher \( \beta \), which implies higher wages and lower employment, also reduces transport expenditure. One can formally establish the following result:

**Proposition 4.** In a city where the bargaining power of workers \( \beta \) is above \( \eta \), the social planner can bring the economy closer to constrained efficiency by subsiding transport costs. In a city where the bargaining power of workers \( \beta \)

\(^{16}\)Note that case (iii) may also hold in a degenerate case, when the transport cost differential between the employed and the unemployed is zero, i.e., \( c_e = c_u \), which, given inequality (8), is possible only if \( a = 0 \). In such a case, the inequality (8) is binding and the slopes of bid rents of the employed and unemployed are equal: the location of the agents is thus undeterminate.
is below $\eta$, the social planner can bring the economy closer from constrained efficiency by taxing transport costs.

From this proposition, it is easy to see that, in the segregated city (Equilibrium 2), the policy influence of $\sigma$ is much more important than in the integrated city (Equilibrium 2). This deserves some more comments: the type of the city has no impact on the sign of $\Lambda_k$; it has, however, an impact on its magnitude. Notably, given that $\Lambda_k$ is proportional to $d_k$, a policy intervention through $\sigma$ will have much more impact in land market equilibrium 2 than in land market equilibrium 1. Indeed, *ceteris paribus*, a change in $\sigma$ would be $1 - \frac{u_2}{u_1}$ more powerful in configuration 2, i.e., an order of magnitude of 10 to 15 for a rate of unemployment between 6 and 10%. The reason is simply that all the emphasis is put on the marginal worker located at the border $d_k$. For the other workers, the rents’ differential adjusts so that their utility is the same as the one reached by the marginal worker located in $d_k$. The impact of $\sigma$ is thus much larger when the marginal worker has a longer commute. The interpretation is easy to figure out: to make an unemployed worker to accept a job, the firm has to partly compensate for transportation costs. In city 1 (the integrated city), the unemployed are so close to jobs so that the compensation is very small. In city 2 (the segregated city), we have exactly the opposite result: high distance to jobs implies that the compensation is very large. It follows that any change in $\sigma$ has a huge impact on wages and on the welfare condition.

Finally, it is easy to see that if the policymaker can subsidize and tax the employed and the unemployed workers differently, then he/she would subsidize the employed and tax the unemployed when $\beta$ is larger than $\eta$, and tax the employed and subsidize the unemployed when $\beta$ is lower than $\eta$.

To summarize, adding a spatial dimension in the standard matching model does not change the efficiency results (i.e., the Hosios–Pissarides condition) but allows the government to implement transportation policies that can restore social efficiency.

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17 Of course, there are also second order effects of a change in $\sigma$ on the equilibrium values of $\theta_j$. Throughout this discussion, we ignore them for clarity reasons.

18 Observe that the policy recommendation on transport costs of the employed do not consider their effect on search intensity since we have not made the search effort of workers endogenous, or very indirectly, through location choices.

19 In such a case, it is trivial to show that

$$\Lambda_k = (1 - \beta)(1 - \eta(\theta_j)) \frac{q(\theta_j)/\gamma}{\delta + r + \theta_j q(\theta_j) s(d_k)}(\sigma_e c_e - \sigma_u c_u) d_k,$$

where $\sigma_e$ and $\sigma_u$ are the “net” subsidy to transport costs of the employed and the unemployed workers, respectively.
6. CONCLUDING COMMENTS

We would like to discuss some implications of our model for real-world cities. In fact, our two urban equilibria roughly correspond to the two typical types of cities that one observes in the United States (see in particular Glaeser et al. [8]). The first ones, “new or edge cities,”20 are cities where most jobs are created in the suburbs and where most unemployed (especially blacks) reside close to the city center. The second ones, “old cities,”21 are cities where most jobs are still created in the city center and where most unemployed (especially blacks) live close to jobs. Our integrated-city equilibrium (Equilibrium 1) corresponds to an old city since the unemployed reside close to jobs whereas our segregated-city equilibrium (Equilibrium 2) corresponds to an edge city since the unemployed are far away from jobs. The distinction between the two types of cities may thus be quite crucial. In particular, our model predicts that the unemployed living in cities in which they are poorly connected to jobs (“new cities”) should experience worse labor-market outcomes than those residing in cities in which they have better spatial connections to jobs (“old cities”). However, our model also suggests that welfare differences between the two cities are not that large because of higher commuting costs of the employed.

Our policy results suggested that transportation policies should differ from one city to another. This is in accordance with Pugh [21] who categorizes different metropolitan areas according to the severity of the mismatch between the location of jobs and residence. In particular, she advocates different transportation policies depending on the degree of this mismatch. For instance, Atlanta (a “new city”) has been shown to present spatial features that are strongly consistent with the assumptions underlying the spatial mismatch hypothesis: a majority black central city, high rates of segregation, and high rates of entry-level job decentralization. By contrast, Chicago (an “old city”) has less spatial mismatch because of job opportunities within city limits. We have here a validation of Pugh’s insight. Indeed, as explained above, the marginal impact of transportation policies on efficiency crucially depends on which urban equilibrium prevails; it is much higher if it concerns a marginal worker located further away from the center.

To conclude, we call for a better modelling of the labor market structure in urban equilibria. We believe that several other dimensions of the macroeconomic search literature could be investigated. In particular, incorporating labor discrimination and/or endogenous job destruction through quits or technological shocks into our framework could lead to very interesting and policy relevant results.

20New cities, like, e.g., Atlanta, Houston, Los Angeles, or Phoenix, are MSAs that did not rank in terms of population among the 12 largest MSAs in 1900, but do rank among them in 1990.

21Old cities,” like, e.g., Boston, Chicago, New York, or Philadelphia, are MSAs which were already among the 12 most populated agglomerations in 1900 and are still among them in 1990.
APPENDIX

**Lemma A.1** (Labor demand). In the plane \((\theta_k, w_k)\), the labor demand curve (19) is downward sloping. It cuts the axes at \(w_k(\theta_k = 0) = y\) and \(\theta_k(w_k = 0) = \bar{\theta} > 0\) where \(\bar{\theta}\) is defined by the following equation: \(q(\bar{\theta}) = (r + \delta)\gamma/y\).

**Proof.** By differentiating (19) at constant \(u_k\), we obtain
\[
\frac{\partial w_k}{\partial \theta_k} = (r + \delta)\gamma \frac{q'(\theta_k)}{q(\theta_k)} < 0.
\]

Then by using (19), we easily obtain
\[
w_k(\theta_k = 0) = y_k \quad \text{and} \quad \theta_k(w_k = 0) = \bar{\theta} > 0.
\]

**Lemma A.2** (Wage setting curves). Assume that both \(u_1\) and \(u_2\) are less than a half. Then, in the plane \((\theta_k, w_k), k = 1, 2\), the line defined for each \(k\) by (22) is upward sloping. The line for \(k = 1\) is steeper than for \(k = 2\) but the intercept in case \(k = 1\) is lower than for \(k = 2\). The intersection point between these two lines is equal to
\[
\hat{w} = \left(1 - \beta\right) \left[b + \frac{1}{a} (t_e - t_u) s_0\right] + \beta y
\]
and
\[
\hat{\theta} = \left(1 - \beta\right) \frac{(t_e - t_u)}{a \gamma}.
\]

**Proof.** Along (22) for \(k = 1\), we have \(\frac{\partial w_1}{\partial \theta_1} = \beta(s_0 - au_1)\gamma\). Along (22), for \(k = 2\), we have \(\frac{\partial w_2}{\partial \theta_2} = \beta(s_0 - a(1 - u_2))\gamma\). Then we have \(\frac{\partial w_1}{\partial \theta_1} > \frac{\partial w_2}{\partial \theta_2} \iff u_1 + u_2 < 1\). This is the realistic assumption stated in the lemma. The intercepts of (22) are calculated as
\[
w_1(\theta_1 = 0) = (1 - \beta)[b + (t_e - t_u) u_1] + \beta y
\]
\[
w_2(\theta_2 = 0) = (1 - \beta)[b + (t_e - t_u)(1 - u_2)] + \beta y
\]
\(\iff u_1 + u_2 < 1\).

To find the intersection point, one has to solve \((22)_{k=1} = (22)_{k=2}\).
Existence and Uniqueness

The proof is in two steps. First (i) we show that \( \hat{\theta} \) determines the uniqueness of the land market equilibrium. Then (ii) we show that, for a given land market equilibrium, there is a unique equilibrium labor market \((\theta_k^*, u_k^*)\). The combination of the two elements shows the uniqueness of the market equilibrium.

(i) This follows immediately from Proposition 1 and the discussion just after it.

(ii) We have two equations, (26) and (25), and two unknowns, \( \theta_k \) and \( u_k \). Let us first study (26) in the plane \((u_k, \theta_k)\). By differentiating Eq. (26), it is easy to see that the relation between \( \theta_k \) and \( u_k \) is positive if and only if \( \theta_k > \hat{\theta} \) in Equilibrium 1 and \( \theta_k < \hat{\theta} \) in Equilibrium 2. These inequalities are always verified in each respective equilibrium (see above) so that \( \partial u_k / \partial \theta_k > 0, \forall k = 1, 2 \). Moreover, it is easy to verify that the intercept of the curve corresponding to (26) is a finite positive value as long as \( y - b - (t_c - t_u)\bar{d}_k(u = 0) > 0 \), which is the usual viability condition.

Let us now differentiate (25). We obtain

\[
\frac{\partial u_k}{\partial \theta_k} = \frac{[1 - \eta(\theta_k)]\theta_k q(\theta_k)\bar{s}_ku_k}{\delta + \theta_k q(\theta_k)(\bar{s}_k - au_k\bar{x}_k)},
\]

where \( \eta(\theta_k) \) is the elasticity of \( q(\theta_k) \) with respect to \( \theta_k \) in equilibrium \( k = 1, 2 \) and \( \bar{s}_1 = 1 \) (Equilibrium 1) and \( \bar{s}_2 = -1 \) (Equilibrium 2). In Equilibrium 2, the denominator is clearly positive (since it reduces to \( \delta + \theta_k q(\theta_k)(\bar{s}_2 + au_2) > 0 \)), whereas in Equilibrium 1, a sufficient condition for a positive denominator is \( s_0 - 3au/2 > 0 \). Since by construction, \( s(d) > 0 \) for all \( d \leq 1 \), a sufficient condition is \( 3u/2 < 1 \) or \( u < 2/3 \). Thus, except for parameters leading to an implausible large value of unemployment rate, \( \partial u_k / \partial \theta_k < 0, \forall k = 1, 2 \). Furthermore, since \( \lim_{\theta_k \to \infty} \theta_k q(\theta_k) = +\infty \) and \( \lim_{\theta_k \to 0} \theta_k q(\theta_k) = 0 \), it is easy to show that

\[
\lim_{\theta_k \to \infty} u_k = 0 \quad \text{and} \quad \lim_{\theta_k \to 0} u_k = 1.
\]

We have thus shown that the curve corresponding to (26) is upward sloping whereas the curve corresponding to (25) is downward sloping. Therefore, since the intercept of the curve corresponding to (26) has a finite positive value whereas the intercept of the curve corresponding to (25) has an infinite value, the market equilibrium exists and is unique.

Stability

We use the same argument as in Pissarides [19, Chap. 3, p. 47]; see graph 3.2 for instance.
Proof of Proposition 3

The welfare function $\Omega_{\ell k}$ is now given by

$$\Omega_{\ell k} = \int_0^{\infty} e^{-rt} \left\{ \int_{\text{employed}} (y - c_v z) dz + \int_{\text{unemployed}} (b - c_u z) dz - \gamma \theta_k \bar{s}_k u_k \right\} dt. \quad (32)$$

The social planner chooses $\theta_k$ and $u_k$ that maximize (32) under the constraint (24). In this problem, the control variable is $\theta_k$ and the state variable is $u_k$. Let $\lambda$ be the co-state variable. The Hamiltonian is thus given by

$$H = e^{-rt} \left\{ (1 - u_k) [y - c_v d_k] + u_k [b - c_u d_k] - \gamma \theta_k \bar{s}_k u_k \right\} + \lambda \left\{ \delta (1 - u_k) - \theta_k q(\theta_k) \bar{s}_k u_k \right\}. \quad (33)$$

The Euler conditions are $\frac{\partial H}{\partial \theta_k} = 0$ and $\frac{\partial H}{\partial u_k} = -\dot{\lambda}_k$. They are thus given by

$$\gamma e^{-rt} + \lambda q(\theta_k) [1 - \eta(\theta_k)] = 0 \quad (33)$$

$$\left\{ y - b - (c_v - c_u) d_k + \frac{\partial d_k}{\partial u_k} [u_k t_u + (1 - u_k) t_v] + \gamma \theta_k \bar{s}_k + \frac{\partial \bar{s}_k}{\partial u_k} u_k \right\} e^{-rt} + \lambda \left\{ \delta + \theta_k q(\theta_k) \bar{s}_k \right\} = \dot{\lambda}_k, \quad (34)$$

where $\eta(\theta_k) = -q'(\theta_k)q(\theta_k)/q(\theta_k)$. Let us focus on the steady state equilibrium in which $\dot{\theta}_k = 0$. By differentiating (33), we easily obtain that $\dot{\lambda}_k = -r \lambda_k$. By plugging this value and the value of $\lambda_k$ from (33) in (34), and by observing that $\bar{s}_k + (\partial \bar{s}_k/\partial u_k) u_k = s(d_k)$, we obtain

$$y - b = \frac{\gamma}{q(\theta_k)} \left[ \delta + r + \theta_k q(\theta_k) s(d_k) \eta(\theta_k) \right] + (c_v - c_u) d_k. \quad (35)$$

In order to see if the private and social solutions coincide, we compare (26) and (35). It is easy to verify that the two solutions coincide if and only if

$$\frac{\gamma}{q(\theta_k)} \left[ \delta + r + \theta_k q(\theta_k) s(d_k) \beta \right] + (c_v - c_u) d_k$$

$$= \frac{\gamma}{q(\theta_k)} \left[ \delta + r + \theta_k q(\theta_k) s(d_k) \eta(\theta_k) \right] + (c_v - c_u) d_k \quad (36)$$

which leads to (28).

23The transversality condition is given by

$$\lim_{t \to +\infty} \lambda_k u_k = 0$$

and is obviously verified.
REFERENCES

34. Y. Zenou, “How Do Firms Redline Workers?,” unpublished manuscript, University of Southampton (2000).