Interpreting Europe and US labor markets differences: the specificity of human capital investments∗

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Abstract: We show a fundamental property of human capital investments: they are not independent of the aggregate state of labor markets. In particular, frictions and slackness of the labor market raise the returns to specific human capital investments relative to general investments. This is a property that Becker’s seminal contribution in the context of perfect labor markets did not consider. We then build a macroeconomic model where in equilibrium emerge different regimes. In the G-regime, workers invest in general skills. This occurs when they face high turnover labor markets and in the absence of employment protection. The S-regime in which workers invest in skills specific to their job appears when employment protection is high enough. Low job turnover is both a cause and a consequence of specific investments in human capital.

This paper then re-interprets Europe-US differences in arguing that the US are closer to the G-regime and Continental Europe to the S-regime. This conjecture provides, among other things, a rationale for differences in labor mobility and reallocation costs, which are typically ignored in American ‘International Trade’ textbooks while considered as extremely large in the public debate in Europe. In a S-regime, mobility costs are high and transitions between steady-states have especially strong adverse effects. On the other hand, in the steady-state, workers in the S-regimes are very productive. Each regime has thus its own coherence, although the European type incurs higher transition costs when macroeconomic conditions change.

• Key Words: Training, Specific Human Capital, General Human Capital, Unemployment, Matching
• JEL classification: J63, J30

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Differences in the labor markets of Continental Europe and the US, either in terms of institutions or in terms of performance, are very important. The dark side of Europe has been widely discussed. The rates of unemployment have diverged from US rates since the late 70’s, and have been much larger and more persistent in countries such as Spain, France, Italy or Belgium. Unemployment benefits, in level and in duration, are typically larger except perhaps in Italy, encouraging low job search activity. Employment protection legislation is tougher in Europe than in the US, discouraging firms from taking risks in hiring workers. Another well known fact is the lack of occupational and regional mobility in several European countries, especially in Southern Europe, as compared to the US. According to the OECD and Layard and Nickell (2000), mobility rates in terms of the fraction of the population moving from one region to another is between twice and five times lower in France, Germany and Italy (between 0.6 and 1%) than in the US (around 3%). Several authors already discussed low mobility of workers as a perhaps important cause of diverging experience of US and Europe. For Bertola and Ichino (1996) notably, this is a key factor of lower unemployment and higher wage dispersion in the US.

High mobility in the US has some influence on the way economic theory is exposed to students. Indeed, mobility costs are ignored in the most used American textbooks, such as the 5th International edition of International Economics, Theory and Policy (Krugman and Obstfeld, 2000). International trade theory usually discusses welfare gains and losses of free-trade in adopting most of the time a long-run perspective. Short-term reallocation costs of trade-openness are at best briefly evoked, and more typically, totally ignored, and the conclusion focuses on discussing who are the long-run winners and losers. In a European context, the fact that (older) workers in traditional industries need to leave their sector/region of origin, due to reallocation of activities induced by country-specialization, may sound much more critical, as well as neglecting the associated short-run mobility costs. European students typically react to these features of the model. Finally, what is sometimes less emphasized is that, despite these differences, productivity gaps between Europe and the US were close to zero in mid 1990’s or even larger in Europe when hourly productivity is concerned (Layard and Nickell 2000). The growth of output per-worker was also significantly larger in Europe between 1976 and the early 1990’s.

The goal of this paper is to propose an interpretation for these Europe/US differences in a general framework that encompasses all these aspects of labor markets. The interpretation is the following: European workers invest more at the margin in specific human capital, while US workers invest more in general human capital. Accordingly, sectorial, occupational and geographical mobility
costs are much higher in Europe than in the US. We then discuss the predictions of the model to offer an alternative view of the transatlantic comparisons between labor markets.

Why would there be such differences in the nature of human capital investments? The answer is in two steps. First, this result is connected with an ignored but important property of human capital investments: the return of general skills relative to specific skills is higher, the higher the exit rate from unemployment and more generally, the lower labor market frictions. Said otherwise, in an economy made of very distant islands, it is better to learn the technology of the island in which one lives. In contrast, the more connected the island are, the more profitable it is to learn the common technology to all islands. Second, continental Europe and the US differ in the magnitude of employment protection. It has been made clear in a recent literature (e.g. Bertola and Rogerson 1997) that employment protection is a key factor of difference between those labor markets. It reduces inflows from employment into unemployment but also the outflows from unemployment into employment. By reducing turnover, we will contribute to this literature and show that employment protection modifies the structure human capital investments, towards specific human capital investments.

The model thus features the intuition that when workers feel secure in their jobs, i.e. when the expected duration of employment is important, they can afford investing a lot in the knowledge of their firm and job. This specialization improves their productivity and they receive rents, i.e. wages fairly above their outside opportunities: indeed, the specificity of their investments makes their outside options in case of displacement quite low, ceteris paribus i.e. controlling for unemployment benefits. In contrast, when workers live permanently with the idea of mobility as is the case in absence of employment protection, they tend to invest smaller amounts in firm’s specific knowledge, and much more in recyclable skills, i.e. in general human capital. As a result, their outside options are high, although their efficiency on the job is lower than when they are more specialized. Mobility from job to job is high too, since there is little surplus to share with employers.

Differences in human capital investments are an a priori plausible story of the divergence between continental Europe and the US, but there are few models allowing for a rigorous investigation of these issues. The task of this paper is to provide a simple, tractable model to organize thinking and to identify a few relevant policy parameters that account well for the facts described above. Overall, a quantification of the European and the US economy shows the crucial role of firing costs
in the shape of human capital investments.

In addition, we recover in a macroeconomic context some results emphasized in the contract literature regarding the role of several labor market institutions. For instance, a higher bargaining power of workers promote skill acquisition as workers capture a higher share of their investments, which is a version of the hold-up problem. We extend this result to the type of skills, showing that workers’ bargaining power raise the amount invested in specific skills. We show that unemployment benefits also raise the specificity of human capital investments, not directly but by a labor market feedback effect: higher benefits reduce job creation which reduces job turnover in the labor market. Employers, adversely hurt by these institutions (unemployment benefits and employment protection), are however compensated by the fact that employees have specific skills, which reduces turnover, maintain constant outside option of workers and yields high profits. On the employee side, we also explore some political economy features: in a world where employees have general skills, they do not really care about unemployment benefits or employment protection, since they are trained to obtain new jobs at low investment costs. On the contrary, if they have specific skills and are thus in a monopsony situation, they require in exchange some protection against the large welfare loss in case of labor reallocation shocks.

Overall, we find that it is difficult to Pareto-rank a world with large employment protection and specific skills on one hand, and low employment protection and general skills on the other hand, as both equilibria have pros and cons in a steady-state. What we can say however is that when workers have general skills, the economy has a better ability to cope with large job reallocation waves and macroeconomic turbulence, as workers suffer less from displacement.

The first section introduces the model. The second section summarizes the basic trade-off of skill investments which in a frictional labor market is basically a choice between being better off inside or outside the firm. We then show that this trade-off depends on market frictions. The third section derives the labor market equilibrium in which labor demand is endogenous and identify the conditions under which different skill regimes emerge. Section 4 examines the robustness of the model and checks whether various extensions alter the main argument by examining the case of i) continuous investments ; ii) the provision of firms’ incentives to affect skill investments ; iii) technology choices ; iv) education choices. Section 5 quantifies different economies and reinterprets the experience of the last decades from both sides of the Atlantic along several dimensions. Section 6 discusses more formal empirical evidence and concludes.
1. The set-up

1.1. Workers

Time is continuous; the discount rate of all (risk-neutral) agents is \( r \). Workers die with a Poisson death rate of \( \delta \) and are replaced by new-born workers with no human capital. These new born workers look for jobs and are randomly matched with employers looking for workers to fill vacant units of production. At the entry into a job, workers decide about which type of skills they learn. Following the bulk of human capital theory, investments in human capital are worker’s decisions under a set of incentives provided by the firm.

The human capital of the workers is denoted by \( h \). It can be either general or specific. As Becker (1964) himself noted, no skill is purely general nor specific. It does not imply that the theoretical distinction is not useful. Both types of capital are imperfect substitutes. As an extreme form of complementarity, we postulate that \( h \) can take only three values: \( h^0 \), \( h^* \) or \( h^9 \) where \( h^* \) (resp. \( h^9 \)) represents the level of specific (resp. general) skills. \( h^0 \) can be interpreted as schooling investments although this margin is not introduced in our benchmark model. The effort cost of each investment is respectively \( 0 \), \( C^* \) or \( C^9 \). We assume to simplify that investments in human capital are made instantaneously at the entry into the firm.\(^2\) Gains from specialization imply that \( h^9 < h^* \). One will soon realize that this assumption is necessary, since \( h^9 \geq h^* \) would imply that no rational worker would ever invest in specific skills. As a pure normalization, we assume that \( h^0 \) is so low that a job is not productive if the worker does not invest in skills.

The problem of the choice of skills is intertemporal.\(^3\) It is described at the entry into the firm by the variable \( i = 0, s, g \). Thus, \( i \) is a control variable. Once a new born worker obtains a job, he must chose \( i = s \) or \( g \), otherwise the match is immediately dissolved. We denote by \( k(i) = s, g \) the state variable of the employed worker corresponding to the choice \( i \) that has been made which thus

\(^2\) The fact that workers pay human capital investments in terms of effort has no direct implication on the choice of skills: as shown in Becker (1964) and in a search context by Acemoglu and Pischke (1999a) and (1999b), wages partly or fully internalize the cost structure. The assumption is however important in that the firm cannot easily write a contract to induce an efficient level of effort if effort is imperfectly observable. We extend the model to firms paying part of the costs in sub-section 4.1. Sub-section 4.2 investigates how technology choices by firms may act as a substitute for an efficient contract. Finally, sub-section 4.3.1 relaxes the assumption of discrete human capital investments (see notably footnote 10 to justify our modelling choice). We further introduce and discuss education choices in sub-section 4.3.2. These extensions introduce no qualitative differences with the benchmark model.

\(^3\) In absence of risk aversion, one cannot discuss interesting aspects of skill investments such as the insurance components of general skills. I thank a referee, Eric Gould and Ed Lazear for pointing this out. See also Gould (2002) and Gould et al. (2000) who focus on general education and wage inequality instead of training as here. More generally, specific skills are more risky which justifies the assumption of a higher return. Also, the risk associated with specific skills may require a higher demand by workers for employment protection or alternatively for employment protection, in absence of insurance market, an assumption often done (e.g. Rogerson and Schindler 2002).
also reflects the human capital of the worker. Let us denote by \( p \) the transition intensity of workers from unemployment to employment and by \( b \) the level of unemployment benefits. Finally, \( W_{0}^{k(i)} \) is the corresponding asset value of the choice. The asset value of the new born unemployed \( U^{0} \) writes recursively as

\[
(r + \delta)U^{0} = b + p \max_{i=s,g}(W_{0}^{k(i)} - C^{i}) - pU^{0}
\]

\[\text{s.t. } W^{k(i)} - C^{i} \geq U^{0}\]

The constraint (1.2) verifies that human capital investment costs do not discourage workers from learning.

If the worker subsequently leaves the firm, specific skills are lost but general skills are retained. In symbols, at the time of separation \( k(i) \) becomes \( k'(i) \), where the state variable \( k'(i) \) is such that \( k'(s) = 0 \) and \( k'(g) = g \). Implicitly, it is assumed that workers have a zero-mass probability of obtaining subsequently a job in the same firm (there is no recall or temporary layoff). We then denote by \( U^{k'(i)} \) the value of unemployment for a worker having already invested in skills. Unless there is an ambiguity, we describe choices and arbitrage conditions with the state variable \( k \) and \( k' \), thus skipping the control variable \( i \).

### 1.2. Firms

A productive unit is the association of one worker and one firm. As such, the productivity of a worker is the sum of a firm’s component and the human capital of the workers. The firm’s component is random and denoted by \( \varepsilon \). It can be interpreted as a firm fixed effect. It evolves according to a Poisson process with intensity \( \lambda \) and is drawn from a density function \( f(\varepsilon) \) with c.d.f. \( F(\varepsilon) \). The density has support \( (\varepsilon^{-}, \varepsilon_{0}) \) and \( \varepsilon_{0} \) is also the initial value of \( \varepsilon \) at the time of match formation. The marginal productivity of workers \( y \) with human capital \( k = s, g \)

\[
y = \varepsilon + h^{k}
\]

The framework thus extends Mortensen-Pissarides (1994) to a case in which the human capital of workers is endogenous.

The timing of events and of decisions is as follows. First, at the entry into the firm, the worker decides about the investment of type \( i \) yielding human capital \( k(i) \). He/she correctly anticipates the outcome in terms of wages and job duration. The firm and its worker then bargain an entry wage \( w_{0}^{k} \). Second, at a random time, a new value of \( \varepsilon \) is drawn from its distribution. The worker
is now protected from separation by a firing tax $T$ paid by the firm in case of separation. The worker and the firm bargain a new wage $w^k$ if there is a positive surplus to share between the firm and the worker. In the opposite case, they optimally separate. We assume, following the literature (Mortensen and Pissarides 1999) that $T$ is paid by the firm and is not transferable to the worker.4 Last, any subsequent change in $\varepsilon$ leads to either a destruction of the match or a new negotiation.

1.3. Stationarity assumptions

We will further restrict the analysis to stationary aggregate state. It follows that the human capital choice of workers is necessarily as follows. First, if new born workers strictly preferred specific skills in their first job, they never invest later on in general skills: workers with specific skills, upon separation, behave exactly as new born workers since they have no skills outside the firm and in equations, $W^g_0 - C^s > W^g_0 - C^g$ throughout workers’s career. Second, if new born workers strictly preferred general skills in their first job, they never invest later on in specific skills: workers with general skills don’t have to repay the cost of general skills in the next job. In equations, if at a time $W^g_0 - C^g > W^g_0 - C^s$, a fortiori $W^g_0 > W^s_0 - C^s$. And third, if new born workers are indifferent between general and specific skills in their first job, they randomize according to a mixed strategy. Those having chosen general skills never invest later on in specific skills since they save on investment costs and keep general skills. Those who invested in specific skills will randomize again in the second job. In equations, if new born workers are such that $W^g_0 - C^g = W^g_0 - C^s$, the same is true in the next job for those who have invested in specific skills. For those who have chosen general skills, the next job will be such that $W^g_0 > W^g_0 - C^s$.

1.4. Wages and surplus sharing

In a job of type $k = s, g$, the present discounted value of employment at the entry stage is $W^k_0$ and his/her outside option in case of separation is the asset value of being unemployed upon separation which is denoted by $U^{k'}$. Recall that $k' = 0$ if $k = s, k' = g$ otherwise. Similarly, the firm’s value is denoted by $J^k_0$, the outside option of the firm is $V$. The entry wage is negotiated according to a Nash-bargaining rule. The outside options of agents are equal to their threat point in bargaining over the wage. The total surplus of a match between a worker of type $k$ and a firm where the value of the total surplus is

\[
S^k_0 = W^k_0(\varepsilon) - U^{k'} + J^k_0(\varepsilon) - V
\]  

4 The properties of pure transfers in case of separation are well known: such transfers reduce wages and do not affect destruction and creation decisions (Burda 1992, Lazear 1990).
the value of which is derived in Appendix A as well as the arbitrage equations of employed workers and firms. This quantity is split in shares $\beta$ and $1-\beta$ where $\beta$ is an index of the bargaining power of the worker with $0 \leq \beta \leq 1$:

$$W_0^k - U^{k'} = \beta S_0^k$$

(1.5)

This determines the entry wage $w_0^k$.

After the realization of the first transition of the firm’s component $\varepsilon$, a new wage is negotiated. The continuing asset value of holding a job for an employee is denoted by $W^k(\varepsilon)$, while the continuing value of the firm is denoted by $J^k(\varepsilon)$. Firm’s outside option is now $V + T$. The surplus after the first random change in $\varepsilon$ is denoted by $S^k(\varepsilon)$. It is now defined as

$$S^k(\varepsilon) = W^k(\varepsilon) - U^{k'} + J^k(\varepsilon) - V + T$$

(1.6)

The value of the surplus $S^k(\varepsilon)$ is also derived in Appendix A. It is also shared in parts $\beta$ and $1-\beta$:

$$W^k(\varepsilon) - U^{k'} = \beta S^k(\varepsilon)$$

(1.7)

which determines the continuation wage $w^k(\varepsilon)$. All wage equations are derived in Appendix A in equations (A10) and (A11)

### 1.5. Job destruction

After an idiosyncratic shock on $\varepsilon$, if the new value is such that $S^k(\varepsilon) < 0$, it is jointly profitable for both the worker and the firm to dissolve the match and to separate: indeed, in such a case, no wage renegotiation can satisfy both bargaining parts. The reservation strategy is easy to derive, following Mortensen and Pissarides (1994): one can first observe that $\frac{dS}{d\varepsilon} = \frac{1}{r + \delta + \lambda}$: the surplus is a linear function of $\varepsilon$ with a positive slope. So, we define by $R^k$ the unique reservation productivity defined by $S^k(R^k) = 0$. Thanks to equation (1.7) in Appendix A, the strategy is joint to both workers and firms. The job destruction reservation rule is obtained after an integration by part using (A5):

$$R^k + h^k + \frac{\lambda}{r + \lambda + \delta} \int_{R^k}^{\varepsilon_0} (1 - F(\varepsilon'))d\varepsilon' = (r + \delta)(U^{k'} + V - T)$$

(R)

The left hand side is increasing in $R^k, h^k$ and in $\lambda$ and decreasing in $\delta$ and $r$, and the right hand side is increasing in $U^{k'}$ and $V$, and decreasing in $T$. 

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2. Human capital decisions in partial equilibrium

2.1. Partial equilibrium

We first ignore the determination of \( V \) and thus \( p \). We thus fully describe the partial equilibrium. The reason for this is that introducing labor demand too early makes it impossible to separate the supply of skills from its demand. To get around this problem, we chose to separate the steps at the cost of being less conventional than in the matching literature. This will in fact reveal interesting and usually hidden aspects of the Mortensen-Pissarides framework.\(^5\)

Given the stationarity assumption made above, the partial equilibrium is simply characterized by the choice being made by the new born workers. Using equation (A9) in Appendix A, the choice can be represented by the following set of equations:

\[
\begin{align*}
  i &= \text{ArgMax } [W^{k(i)}_0 - C^i] \\
  \text{with } W^g_0 &= \beta \left( \frac{\varepsilon^0 - R^g}{r + \lambda + \delta} - T \right) + U^g \\
  W^s_0 &= \beta \left( \frac{\varepsilon^0 - R^s}{r + \lambda + \delta} - T \right) + U^0 \\
  \text{if } i &= s, (r + \delta)U^0 = b + p\beta \left( \frac{\varepsilon^0 - R^s}{r + \lambda + \delta} - T \right) - pC^g \\
  \text{if } i &= g, (r + \delta)U^g = b + p\beta \left( \frac{\varepsilon^0 - R^g}{r + \lambda + \delta} - T \right)
\end{align*}
\]

Equations \((U^0)\) and \((U^g)\) are obtained in replacing the worker’s capital gain when hired by the value of the surplus calculated earlier.

**Definition:** the partial equilibrium for each pure regime is characterized by the decision \( i \) made by each type of worker (new born workers and workers having already hold a job) and the corresponding value of \( R^k \) and \( U^k \), solving equations \((R)\) and \((U^g)\) or \((U^0)\) under inequality \((1.2)\). \( V \) and \( p \) are taken as parameters by workers and firms.

2.2. The basic trade-off of specific skills

We can now rigorously explore the determinants of the human capital decisions. The bottom line is that specific skills increase productivity more than general skills and thus improves workers’ wages and job duration. On the other hand, general skills raise the bargaining point of workers thus both

\(^5\) In the conventional Mortensen-Pissarides analysis, the value of \( V \) is equal to zero and \( p \) is determined accordingly. The problem here arises from the fact that when \( V = 0 \), \( p \) will itself directly depend on the choice made by workers as the overall amount of job creations will depend on the type of investments made by workers. Our method is no different from conventional microeconomic analysis where one first derives a supply curve at all prices, and then determine an equilibrium price through aggregate demand.
their wage and the future utility derived from unemployment. In other words, the trade-off is between improving prospects of the worker within the firm or outside the firm. We can formalize this trade-off in Proposition 1, assuming that $C^s = C^g = C > 0$ as a normalization, i.e. to get rid of unimportant constants.

**Proposition 1. Choice of skills.** *Ceteris paribus, i.e. for the same level of p and V,*

**a) Outside option of workers.** Unemployed workers are better off if they have previously invested in general skills than in specific skills, i.e. $U^g \geq U^0$ with equality when $p = 0$ with $U^g = U^0 = b/(r + \delta)$.

**b) Duration of jobs.** Jobs last longer when workers have invested in specific human capital instead of general human capital, i.e. $R^g > R^s$.

**c) Trade-off.** The choice of specific skills occurs when the effect of the second mechanism dominates the effect of the first one.

**Proof.** On a), unemployed workers with $k' = g$ have an additional option compared to unemployed workers with $k' = 0$: they may decide not to pay any skill investment cost and just retain their skill level $h^g > h^0$. Thus $U^g \geq U^0$. When $p = 0$, no worker obtain a job and the unemployed simply obtain the present discounted value of unemployment compensation. On b), denote by $\Delta R = R^g - R^s$, $\Delta h = h^g - h^s < 0$ and $\Delta U = U^g - U^0 \geq 0$. By difference of equation (R) for $k = s, g$ we have

$$\Delta R + \frac{\lambda}{r + \lambda + \delta} \int_{R^s}^{R^g} (1 - F(\varepsilon')) d\varepsilon' = (r + \delta) \Delta U - \Delta h$$

(2.2)

The proof is by contradiction. Suppose that $\Delta R \leq 0$. From (2.2), the right hand side is strictly positive since $\Delta h < 0$. This implies that at least one of the two terms in the left hand side is strictly positive, which violates the starting assumption $\Delta R \leq 0$. Thus $\Delta R$ is necessarily positive. The expected duration of a job is negatively related to $R^k$, i.e. is given by $1/\lambda F(R^k)$. On c), the decision to invest in specific skills occurs when $W^s_0 = \beta \frac{\delta - r - R^g}{r + \lambda + \delta} + U^0 > W^g_0 = \beta \frac{\delta - R^g}{r + \lambda + \delta} + U^g$ i.e. when $\Delta U < \frac{\beta \Delta R}{r + \lambda + \delta}$. ■

The interpretation of Part a) above is straightforward. We can interpret b) in using equation (2.2) in the proof above. This equation indicates that $\Delta R$ is all the more important, the larger $\Delta U$ and $-\Delta h$ are. The fact that workers with general skills have a high outside option (higher by an amount $\Delta U$) reduces the size of the surplus and make the match more fragile, meaning that it is more frequent to have idiosyncratic shocks to $\varepsilon$ leading to job destruction for those workers. The productivity effect $h^s > h^g$ reinforces this gap between $R^g$ and $R^s$, since specific skills bring a higher productivity than general skills. Note however that $\Delta R$ is still strictly positive when $h^s = h^g$ since $\Delta U > 0$ for all positive $p$. The interpretation of c) is also easy. As equation (2.1) shows, the choice of skills depends on their gross returns in $W^0_0$ which themselves depend on the partial equilibrium reservation strategy $R^k$. Since $R^s > R^g$, the higher duration of jobs of type $s$ favors specific skills,
while since $U^s < U^g$, the better outside option of workers with general human capital investment tends to discourage specific skills accumulation. There is thus a non-trivial trade-off between the two types of skills.

### 2.3. Skills and frictions

The trade-off that was described above is basically affected by two key variables: the first one is the job finding rate $p$ of workers. The second variable is $V$, the component of the threat point of firms $V$. We could interpreted it here as a friction in the good market, i.e. a barrier to entry when $V > 0$. Note that $p$ and $V$ may depend on each other in various ways: for instance, a higher number of firms may for instance raise $p$ but reduce $V$. The exact link depends on the arbitrage forward value of $V$ and of the equilibrium determination of $p$. For the sake of clarity, we will nevertheless decompose the impact of variation of each variable on the human capital choice of workers, and combine their impact in next section.

**Theorem 1. The impact of frictions on the choice of skills.**

a) **Effect of the job finding rate** $p$. At a fixed $V$:

   a1) For low values of $p$, workers invest in specific skills. For large values of $p$, workers invest in general skills.

   a2) Define by $p^*(V)$ the values of $p$ such that workers are indifferent between general and specific skills. For any $V$ there is at least one such $p^*(V)$. Further, under the sufficient condition that job destruction rates are as elastic in $g$-jobs as in $s$-jobs for $p$ close to $p^*(V)$, there is a unique cutoff point $p^*(V)$. For all $p$ above it, workers prefer general skills, for all $p$ below it they prefer specific skills.

   a3) $p^*$ declines with $V$ and has a finite limit for large $V$.

b) **Effect of the threat point of firms in bargaining** $V$. For a given level of $p$, there are cut-off points for $V$ above which workers don’t invest in human capital and below which they invest in either general or specific skills. These cut-off points decrease with $p$.

**Proof.** All this is represented in Figure 2.1. Part a2) is just a generalization of a1), where the sufficient condition rules out uninteresting cases. The intuition of the proof is given in a static case in Appendix B.1 and the full proof is developed in Appendix B.2.

The first part shows the effect of $p$ on human capital investments. When labor market prospects are better for workers, they tend to favor general human capital investments, while, when the labor
Figure 2.1: Partial equilibrium: effect of frictions $V$ (threat point of firms) and $p$ (job finding intensity) on worker’s skill choice.

market is bad for workers, they invest preferentially in specific human capital. The reason is twofold: first, specific skills require to repay an investment cost at the end of a spell of unemployment, which occurs less frequently when the exit rate $p$ is small. Second, a low exit rate $p$ is associated with a higher surplus of a job, thus workers tend to prefer longer duration jobs (which is what specific skills bring).

The parameter $p$ also quantifies the amounts of frictions in the labor markets. Part a) of Theorem 1 above thus indicates that labor market frictions encourage specific skills. This is one of the important properties of human capital choices which in turns will be the corner stone of the paper. Note that this result could not be anticipated by Becker (1964) when he introduced the fruitful distinction between general and specific human capital. In Becker’s frictionless world, the job finding process is infinitely fast. In fact, the initially closest result to our paper was obtained by Acemoglu and Pischke (1999a and b) who considered the role of frictions and labor market institutions on the investments decisions in general human capital and showed, among other important results, that frictions lead to sharing the costs of those investments, contrary to Becker’s implications that these costs are paid by workers through a lower entry wage.

The second part of the theorem states that a larger threat point of firms $V$ reduces the value of surplus of workers and reduce their incentives to invest in skills.

3. Labor demand and labor market equilibrium

To make the exposition easier, one can introduce the notion of regimes, which exactly match the choice $i = s, g, 0$ of new born workers given this stationarity of decisions. When $i = s$, the economy
is in a (pure) regime $S$, when $i = g$ we have a (pure) regime $G$. When workers are indifferent, there is a third case which is a (mixed) regime $M$. The no-investment case is called regime 0. As we will see later on regime $M$ is more than a zero-mass subset of the parameter space and must thus be studied as well as pure regimes. Hereafter, we will thus describe regimes $S$, $M$ and $G$. We simply check ex-post under which conditions on parameters the regime 0 is reached, using inequality (1.2).

In the pure regimes $S$ (respectively $G$), $\kappa^s = 1$ ($\kappa^s = 0$ respectively).

### 3.1. The value of a vacant position and matching

At each contact generated through the matching process, firms face a relative fraction $\kappa^s$, $\kappa^g$ and $\kappa^0$ of workers making the investment $s$, $g$ or no investment at the entry level (with $\kappa^s + \kappa^g + \kappa^0 = 1$). Under inequality (1.2) which is assumed to hold throughout, $\kappa^0 = 0$.

Denote by $\gamma$ the cost of posting a vacancy and by $q$ the rate at which firms recruit a suitable worker. The value of a vacant position $V$ is then given by

$$ (r + q)V = -\gamma + q \sum_{k=s,g} (1 - \beta)\kappa^k \left( \frac{\beta^0 - R^k}{r + \lambda + \delta} - T \right) $$

This equation thus brings a positive link between $V$ and $q$ for given values of $R^k$. We can now relate $V$ to $p$. For that, one can first remark that there is an explicit dependence between $q$ and $p$: this link arises through the existence of an aggregate matching process between workers and firms. The labor force is normalized to 1, so that $u$ is both the unemployment rate and the number of unemployed workers. We assume the existence of a constant returns to scale matching function $x(u, V)$ between those $u$ unemployed workers and $V$ recruiting firms$^6$, such that, posing $\theta = V/u$ the labor market tightness and denoting by $q$ the transition intensity of firms from a vacancy to a filled position,

$$ q = q(\theta) = \frac{x(u, V)}{V} \text{ with } -1 < \theta q'/q = -\eta(\theta) < 0 $$

$$ p = p(\theta) = \frac{\theta q(\theta)}{u} \text{ with } p' > 0 $$

where $-\eta(\theta)$ is the elasticity of $q$. Alternatively, we can represent $q$ as a function of $p$ by $q = p/\theta = p/\theta^{-1}(p)$ with $dq/dp < 0$. For instance, with a Cobb-Douglas matching function $x(u, V) = x_0 u^\eta V^{1-\eta}$, we have $q = x_0^{1/(1-\eta)} p^{-\eta/(1-\eta)}$. This allows to derive a last partial equilibrium result which leads straight away to the labor market equilibrium. Indeed, we prove in Appendix B.3 that

$$ \frac{\partial V}{\partial p} < 0 $$

(3.1)

$^6$ On-the-job-search is not allowed.
Figure 3.1: Towards the general equilibrium: dependence of the threat point of the firm $V$ with respect to the job finding rate $p$ and determination of equilibrium $p^*$ at $V = 0$ (free-entry). In this example, $p^*$ is such that the $G$-regime is reached.

In Figure 3.1, we thus add up to the representation of Figure 2.1 the path $V(p)$ along with this choice can be made. We also replace the workers’ individual choice $i = s, g, 0$ by the regimes $K = S, G, 0$. As the value of $p$ increases, we move from regime 0 to regime $S$ and then regime $G$.

3.2. Entry of firms and equilibrium labor market tightness

Free-entry of firms means that, as long as there are profit opportunities from opening a job vacancy, firms enter the market. In equilibrium, job creation opportunities are exhausted so that the value of a job vacancy is:

$$V(p^*) = 0 \quad \text{(FE)}$$

In Figure 3.1, the regime is characterized by the intersection of $V(p)$ with the horizontal axis at a point denoted by $p^*$. We have here that $p^* > p^M$, suggesting that regime $G$ holds. If $V(p)$ was shifted to the left, one would instead have regime $M$ ($p^* = p^M$) or regime $S$ ($p^* < p^M$).

There is another intuitive characterization of the labor market equilibrium, which compares the present-discounted value of hiring costs and of expected profits for the firm, as a function of labor market tightness, as represented in Figure 3.2. Indeed, in equilibrium, labor market tightness is such that the expected value of search costs $\gamma/(r + q(\theta))$ must equal the expected discounted value of profits at the time of vacancy opening $q(\theta)/(r + q(\theta))EJ_0$ where $EJ_0 = \sum_{k=\epsilon, \theta} (1 - \beta)\kappa^k \left( \frac{\theta - \bar{R}_k}{r + \lambda + \delta} - T \right)$ is the expected value of profits at the time of match creation. The search costs are an increasing concave function of $\theta$. The profit curve is downward sloping and discontinuous. Indeed, the vertical
Figure 3.2: General equilibrium: recruitment costs and expected profits as a function of labor market tightness $\theta$. In this example, the profit curve and the search cost intersect in the G-regime.

part in $\theta = \theta^M$ defined by $p(\theta^M) = p^M$ represents the mixed regime. When $\theta$ is larger than $\theta^M$, workers invest in general skills (Theorem 1a) and firms’ profits $(1 - \beta) \frac{\varepsilon_0 - R^s}{r + \lambda + \delta} - T$ are lower than in the S-regime as $R^s < R^g$ (Proposition 1). Within each regime, profits continuously decline with $\theta$: the larger $\theta$, the higher wages and thus the lower firms’ profits. Finally, very low values of $\theta$ correspond to large values of $V$ and according to Theorem 1b, workers prefer not to invest. This is accounted for by the lower bound $\theta^{\min}$ in 3.2 below which no market equilibrium is viable.

3.3. The pure regimes

Denote by $\theta^K$ the equilibrium value of $\theta$ in regime $K = S, G$. The pure regimes are described by a couple $(R^k, \theta^K)$ solving the conventional equations (JD) and (JC) with $k = s, g$ if $K = S, G$ respectively:

$$R^k + h^k + \frac{\lambda}{r + \lambda + \delta} \int_{R^k}^{\varepsilon_0} (1 - F(\varepsilon'))d\varepsilon' = b + \frac{\beta}{1 - \beta} \gamma \theta^K - p(\theta^K) I_k C^a - (r + \delta)T$$ (JD)

$$\frac{\gamma}{q(\theta^K)} = (1 - \beta) \left( \frac{\varepsilon_0 - R^k}{r + \lambda + \delta} - T \right)$$ (JC)

with $I_k$ is equal to zero if $k = g$ and 1 if $k = s$. The first equation is obtained in combining (R) and $(U^{k'})$ for $k' = 0, g$ when $V = 0$ and in replacing the surplus of the worker in $(U^{k'})$ by its equilibrium value $\beta/(1 - \beta)\gamma/q(\theta)$. This equation represents the job destruction margin. The binary variable $I_k$ indicates that specific skills have to be paid by workers at each new job which reduces their threat point and thus raises the stability of matches $k = s$. The second equation is simply the vacancy equation (V) combined with free-entry (FE). It represents the job creation margin. The proof of existence and uniqueness of $R^k$ and $\theta^K$ within a regime would follow closely
in Mortensen-Pissarides (1999): (JC) and (JD) have opposite slopes and viability of the market is insured by a large enough value of $\varepsilon_0$. Finally, $T$ reduces $R^k$ in (JD) at a given $\theta^K$ but reduces $\theta^K$ in (JC) at a given $R^k$ which is the standard effect of employment protection: ambiguous effect on $\theta^K$ and decrease of $R^k$. As we will see, it also has an impact on the choice of skills.

### 3.4. The mixed equilibrium

The equilibrium is characterized by $W^g_0 = W^s_0$, and by the job creation and job destruction margins, i.e.

$$
\beta \frac{R^g - R^s}{r + \lambda + \delta} = \frac{p(\theta^M)C}{r + \delta + p(\theta^M)} \quad (3.2)
$$

$$
R^k + h^k + \frac{\lambda}{r + \lambda + \delta} \int_{R^k}^{\varepsilon_0} (1 - F(z'))dz' = b + p(\theta^M)\beta \left( \frac{\varepsilon_0 - R^k}{r + \lambda + \delta} - T \right) - p(\theta^M)I_k C^s - (r + \delta)\ln T
$$

$$
\frac{\gamma}{q(\theta^M)} = (1 - \beta) \left( \frac{\varepsilon_0 - (1 - \kappa^g)R^g - \kappa^g R^g}{r + \lambda + \delta} - T \right) \quad (3.4)
$$

Note that the second equation defines two equalities as it has to be satisfied for both $k = s, g$. The uniqueness of $\theta^M$ comes from Theorem 1 applied to $V = 0$. Appendix C then shows the existence of a steady-state mixed strategy equilibrium which solves for $\theta^M$, $R^s$, $R^g$ and $\kappa^g$. It notably establishes a one-to-one correspondence between $\kappa^g$ and the ex-ante fraction of unemployed workers without general skills and randomly choosing general skills at the entry into the firm (denoted by $\alpha^g$).

### 3.5. Parameter space and the regimes

Consider the following limit cases. When $\beta$ is very close to zero, we have regime 0 as workers have little incentives to invest in skills. When $\beta$ is positive but small, equation (JC) implies that tightness is large and thus from Theorem 1 at $V = 0$, we have regime $G$. When $\beta \to 1$, equation (JC) implies that $\theta \to 0$ while $\theta^M$ remains finite, implying regime $S$. A larger $T$ reduces the surplus, profits and labor market tightness, so for a given $\beta$ raise the likelihood of $S$. At a given low $\beta$ it can even imply regime 0. In the space parameter $(T, \beta)$, we can thus illustrate the different regimes we obtain on Figure 3.3.

### 3.6. Closing the model

Unemployment is determined by steady-state conditions on workers’ flows. Details of calculations are in Appendix C). In regime C, the total unemployment rate is the sum of unemployment of the
new born and of the ‘older’ workers and is given by

\[ u_C = \frac{\delta + \lambda F(R^\psi)}{p + \delta + \lambda F(R^\psi)} \]  (3.5)

In regime $S$ in which every one invests in specific skills, the derivation of steady-state unemployment is straightforward and we obtain the usual value for the unemployment rate

\[ u^S = \frac{\delta + \lambda F(R^s)}{p + \delta + \lambda F(R^s)} \]  (3.6)

In the mixed regime, unemployment is more complex but is the sum of $u^0$ and $u^\psi$ solving the system (C10) and (C11) in Appendix (C).

**3.7. Welfare**

Since we don’t have multiple equilibria we cannot Pareto-rank the regimes. It is only possible to discuss costs and benefits of each regime. In the $S$-regime, national output is reduced by the cost of re-investing in human capital at each job destruction, but turnover is low in this regime; further, matches are more productive. The $G$-regime has the opposite characteristics: turnover is higher and jobs are less productive, but workers do not repay training costs each time they are displaced. Higher turnover hurts firms which pay recruiting costs more frequently. Each regime has thus some well identified pros and cons, and there is a trade-off between low turnover and high aggregate training costs.
4. Robustness

4.1. Firm’s sponsored training

In Becker (1964), workers pay for general human capital as they obtain all the benefits from it, but he presumed that the costs of specific human capital would be shared with the firm. One can improve reasonably the predictive power of our model in assuming that this is the case. The reasons for improvement will be discussed with more details next Section. Here, let us simply extend the model and let the cost $C^s$ being shared equally by workers and firms. We keep the assumption that workers pay the full cost $C^g$ and $C^s = C^g = C$. The model is basically unchanged, except that (JC) and (JD) curves are now

$$
\gamma/q(\theta^S) = (1 - \beta) \left( \frac{\bar{\varepsilon}_0 - R^s}{r + \lambda + \delta} - T \right) - C/2
$$

$$
R^s + h^s + \frac{\lambda}{r + \lambda + \delta} \int_{r_0}^{\varepsilon_0} (1 - F(\varepsilon'))d\varepsilon' = b + \frac{\beta}{1 - \beta} \gamma \theta^S - p(\theta^S)C/2 - (r + \delta)T
$$

while it is unchanged in the G-regime. The main insight here is that firms gain from specific skills by having more productive and more stable workers, but need to share the training cost and anticipate it which shifts down the (JC) curve above (a lower $\theta^S$ at a given $R^s$). As workers have only half of the cost to repay in the next job, they have a better outside option and the reservation productivity $R^k$ becomes higher. All properties of Theorem 1 and Proposition 1 remain true, but the frontier between general and specific skills in equation (3.2) is also affected: $pC$ has to be replaced by $(p - r - \delta)C/2$.

Note finally that the profit curve in Figure (3.2) is shifted down by $C/2$. This implies that the area of mixed regime in the space parameters is smaller. If firms paid a larger share of the training costs, one could even have multiple equilibria, a property that we have not explored further.

We have assumed that firms were technologically constrained to share the costs. One could push the logic even further and allow firms to discretionary chose to compensate workers to make specific investments even in the $G$-regime. As we argued earlier on that, given the timing of events, this ex-post transfer to the workers is difficult to implement since workers may anticipate a hold-up given the nature of these investments costs which are sunk. We nevertheless explore this possibility in

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7 Sharing of training costs may be a technological constraint (e.g. the firm faces a decline in the intensity of utilization of capital to let workers train). One could also argue that there will be some bargaining over the cost with $\beta = 0.5$.

8 The additional term $-(r + \delta)C/2$ here simply accounts for the fact that new born workers investing in general human capital are not sponsored by the firm. This is the equity value of $C^g - C^g/2 = C/2$ showing up in this equation.

9 There are several situations in the labor markets in which incontractibility issues arise. In such cases, simple
Appendix D and conclude that this does not alter our conclusions since firms cannot always induce specific investments.

4.2. Technological choices by firms

The firms have another option to induce workers to choose specific investments. They might for instance invest at costs say \( j^s \) or \( j^g \) to raise the productivity of the workers \( h^s \) or \( h^g \). Here again, the extreme parts in the parameter space are going to be unaffected by the firm’s investment strategy: indeed, if workers are biased against general investments in human capital, the firm is not going to spend any resource on raising the productivity of the specific technology. Instead, it prefers to raise the productivity of the general technology. The same is true when workers are biased towards investments in specific human capital: firms wish to invest in specific technologies. There is thus a strong complementarity between firms’ and workers’ behavior. This reinforces the differences between \( S \) and \( G \) economies as well as the stability of each equilibrium with respect to changes in the macroeconomic environment. *A contrario*, in the middle of the space parameter and more precisely, in the neighborhood of the mix-regime, firms are going to consider seriously the possibility of changing workers’ investments decisions, and in the specific case, of turning workers’ decisions into specific human capital to increase stability. This can be done by raising \( h^g \) so that workers obtain a higher bargained wage and face a longer duration of jobs. This situation characterizes well the regime \( S \): ex-ante, workers may be relatively indifferent between investing in general or specific human capital. But employers manipulate workers’ investments by increasing the productivity of specific technologies, which is in the joint interest of workers and firms.

4.3. Different patterns of investments

4.3.1. Continuous investments in skills

So far we have restricted human capital investments to be discrete and instantaneous at the entry into the job. These assumptions are mostly for convenience, to avoid taking care of a too complex distribution of workers. The former assumption can be however relaxed to obtain a result similar to Theorem 1 in partial equilibrium. Assume that investments in human capital are made continuously with intensity \( i^s \) and \( i^g \), at a cost \( C(i^s,i^g) \) where \( C \) is increasing, convex in each variable. Workers obtain a human capital level \( h = f(h^g(i^g), h^s(i^s)) \) where \( f(\cdot), h^g(i^g) \) and \( h^s(i^s) \) are increasing schemes such as commitments, issuing bonds or long-term contracts may solve this problem and restore efficiency, but are not often observed in practice. This is why the modelling strategy of this paper has so far taken this inefficiency issue as given.
functions. Denote by $\Phi = \frac{\partial U}{\partial \bar{y}}$ the slope of the outside option of employed workers with respect to their general human capital investments. One can easily derive that:

$$\frac{\partial W_0}{\partial \bar{y}} = B \frac{\partial W_0}{\partial i^s} + A \Phi$$

where $A$ is a positive function of parameters and $B$ is simply $\frac{\partial f}{\partial h^g \partial h^g \partial i^g} > 0$, a corrective term for possible differences in gross returns to skill investment. As a normalization, we could assume that technology is such that $B \equiv 1$ i.e. when $f = i^s + i^g$. One can prove fairly easily, in the spirit of Appendix B.1, that the larger the job finding rate $p$, the larger $\Phi$ and thus the larger the investment in general skills and the lower the investment in specific skills. We do not derive the demand side of the model with this continuous investment specification.\(^{10}\)

### 4.3.2. Education choices

Theorem 1 can easily be extended to education choices. Assume that general skills are obtained in the beginning of workers’ life, while specific skills are obtained at the entry into the first job. Krueger and Kumar (2003) introduced a similar set-up and focus on growth differentials between Europe and the US. They do not deal with the choice of skills, taking them exogenous and policy-driven.

In our model, educated workers may also decide to invest in specific skills. Thus $h$ can take three values ($h^g$, $h^s$, $h^{s+g}$ and $h^0$). The program of the new born worker is

$$(r + \delta)U^0 = b + \max_{i=0, g} \left[-C^i + p \max_{j=0, s} (W_{i+j}^0 - C^j - U^i)\right]$$

where $i$ characterizes the education choice ($0$ or $g$) and $j$ characterizes the choice of specific skills. The important thing to observe is that, provided that $h^{s+g}$ is not too large compared to $h^g$, we obtain the same properties as in Theorem 1. The argument runs as follows. In some cases, workers with general education may find it too costly to reinvest in specific skills if $p$ and more generally job turnover is large, as specific skills have to be re-invested every new job. In such a case, the program would thus simplify to

$$(r + \delta)U^0 = b + \max \left[-C^g + p (W_{i+j}^0 - C^j - U^g); p (W_{i}^s - C^s - U^0)\right]$$

\(^{10}\) More details can be found in an older version of this paper, e.g. in CEPR dp 3780. With continuous investments in skills, one however faces additional complications. First, as workers hold several successive jobs, one need to impose some concavity of returns or convexity of costs to avoid a divergence of value functions. Further, the individual history of workers determines their skills, so that one need to take care of the distribution of human capital investments of workers in the cross-section to derive the firms’ job openings decision. This is possible but rather involving and our benchmark model conveys most insights without too much complexity.
Table 5.1: Summary Statistics, Europe and US

<table>
<thead>
<tr>
<th></th>
<th>Eu (6)</th>
<th>Us</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate, 1990-94</td>
<td>9.2</td>
<td>6.4</td>
</tr>
<tr>
<td>GDP/capita, 1994</td>
<td>73</td>
<td>100(^{(a)})</td>
</tr>
<tr>
<td>GDP/worker, 1994</td>
<td>90</td>
<td>100(^{(a)})</td>
</tr>
<tr>
<td>GDP/hour worked, 1994</td>
<td>104</td>
<td>100(^{(a)})</td>
</tr>
<tr>
<td>Emp. Protection Ranking, 1985-93</td>
<td>16</td>
<td>1(^{(b)})</td>
</tr>
</tbody>
</table>

Eu(6): Belgium, France, West Germany, Italy, Spain, and the Netherlands. Eu data weighted by employment size. a) Index 100 for the US; b) 1 is the lowest degree of protection, see OECD 1999 for details. Country ranking is respectively 17, 14, 15, 21 and 9 in order.

In this case, it appears even more clearly than in Section 2 that as the job finding rate $p$ goes to zero, the costs of general investments dominate the benefits and the economy ends-up in a $S$-regime. We thus obtain the same impact of frictions on the nature of investments.

Interestingly, the system of equations characterizing this economy are exactly the same as (JD) and (JC): the benchmark 'on-the-job training' and the 'education model' have exactly the same steady-state. They would differ in response to a transition after a change in macroeconomic conditions. If the new dominant regime is the G-regime, the speed of transition to this new regime is governed by the job matching process in the benchmark model, and by the demographic turnover in the education model.

5. Europe vs. the US: quantification and interpretations

The goal of this section is to provide a feel of how one can interpret Europe and US differences in light of our model of skill investment. In the first part, we calibrate a benchmark G-economy, and show that reasonable values of parameters characterizing employment protection and unemployment compensation can lead to distinct regimes of skill accumulation. In a second part, we discuss the implications of the model and notably how economies face higher turbulence.

5.1. A calibration

To get a sense of the order of magnitude of the phenomena we want to measure and possibly replicate, we compile summary statistics illustrating productivity, unemployment and legislation differences using various data sources from the OECD data and Layard and Nickell (2000). We compare weighted averages for six European countries in the early 1990’s with the US. The OECD country ranking for employment protection is also shown in the last row.

Table 5.1 confirms that in Europe, output per capita tends to be fairly low compared to the
US, but deflating by employment or hours reverses the comparison: the productivity per worker is about 5% in favor of the six European countries. These countries also rank high in the severity of employment legislation (in average 16 compared to 1 in the US).

In our calibrations of Table 5.2, we want to match these statistics. We use two specifications concerning the costs $C^w$. Specification I corresponds to the model developed in sections 2 and 3 in which workers pay the cost of training. Specification II corresponds to the model developed in sub-section 4.1 where specific training costs are shared equally between the firm and the worker. The time period is quarterly. Using a uniform distribution for $\varepsilon$ with lower support $\varepsilon^\text{min}$, we first calibrate a pure G regime in specification I and II. The parameter $\lambda$ characterizing the frequency of idiosyncratic shocks is set to 0.07, so as to have 15% of the firms it by a shock a quarter. We follow Cole and Rogerson (1999) to fit Davis, Haltiwanger and Schuh (1996) figures for job destruction, i.e. a 5.5% quarterly rate. The unemployment rate is targeted to be at 6% and the implied average number of weeks in unemployment is around 17 weeks, close to Cole and Rogerson. We set $\beta = \eta = 1/2$ where $\eta$ is the elasticity of the matching process represented by a Cobb-Douglas function. As a normalization, we set $h^g = 0$ and $\varepsilon_0 = 1$. We let the program chose the values of the parameters $(x_0, \varepsilon_\text{min}, \text{unemployment benefits } b$ and recruitment costs $\gamma )$ so that the average productivity of firms in the cross-section net of unemployment benefits is, as in Mortensen and Pissarides (1994) equal to 0.75. This implies that the replacement ratio, $b$ over the average wage is then 0.23, while labor market tightness is 1.57. To compute the average wage and average productivity, we determine the share of jobs in the entry stage $\varepsilon_0$ which is denoted by $\chi = \frac{\delta + \lambda F(R^e)}{\delta + \lambda + \lambda F(R^e)}$ and second account for the truncation of productivity given the reservation point $R^e$. In the benchmark simulation as in the other economies, $\chi$ is around 0.45. We then check that in this benchmark economy, workers do not wish to invest in specific skills. This requires training costs to be 1.0, i.e. slightly less than the entry quarterly production and $h^x = 0.045$ which means that workers with specific skills are permanently five percent more productive given the average productivity in the cross-section. This is a very conservative estimate as the 5% gap in the data is an average across several sectors, but 5% is sufficient to obtain most results.

We then use these parameters values to fit a S-economy with $u = 9.1\%$. This requires a value for $T$ equal to 1.5 the quarterly entry level of productivity and a higher levels of unemployment benefits. As the second column in specification I indicates, the job destruction rate is 4.6% a quarter, i.e. 21% smaller than for the G-economy. One also finds that the replacement rate is above 80% which is too large. The reason is simple: given that workers with specific skills are more
stable and slightly more productive, firms create many jobs in this regime. A higher $T$ reduces the total surplus but it also reduces job destruction and it is difficult to obtain high unemployment unless one changes other parameters such as unemployment benefits. The last two columns in the table explore an alternative parametrization of a $S$-economy and show that a change in the value of parameter $\beta$ can bring back to a more plausible replacement rate. With specification I and with $\beta = 0.66$, we indeed obtain a replacement ratio around $2/3$.

The third column, $k = M$, solves for the model in the mixed regime with intermediate values of $T$ and $b$. Here, the share of new born workers choosing general skills $\alpha_G$ is 7.9% but given the life-cycle aspects, the share of workers with general skills hired by the firms reaches a 48%.\footnote{This interplay between entry investments and the stock of workers gives a feel of the unexplored dynamic properties. There is some persistence in skill choices generated by the low demographic turnover and longer job tenures in the S-model. We discuss some dynamic issues and the transitions since the 1960’s next sub-section.} In the mix regime, the job destruction rate depends on the type of skills and each rate is very close from the corresponding rate in each pure regime.

Some improvement regarding the dimension of the replacement ratio can be offered in specification II. We set a value of $C = 1$ for training costs, meaning that firms pay $C/2$ to train a worker with specific skills but workers with general skills pay the full among $C = 1$. The $G$-regime is exactly identical to the previous one since only new born workers are affected by the larger value of $C$ but this has no incidence on wage determination as the cost is sunk. In the $S$-regime, firms pay $C/2$ per match and thus reduce job creation. Lower values of $T$ and $b$ are thus needed, respectively 0.75 and 0.7, leading to a replacement ratio of 0.74, still a bit large. The last column of the table shows that a small increase in $\beta$ up to 0.55 brings back to the conventional replacement rate of $2/3$.

The column $k = S^*$ in specification II shows the effect of a policy reform reducing employment protection ($T$ is reduced to 0.5) and the replacement ratio (55%). This yields to a significant reduction in the unemployment rate, to 6.5%. But this is not the end of the story: in this new institutional environment, this regime is never achieved in a steady-state: a comparison of $W_{0}^{*} - C/2$ and of $W_{0}^{G} - C$ shows that the latter dominates now, contrary to the column next to the left. As a result, workers in the new steady-state (next to the right, column $k = G^*$) all invest in general skills despite the fact that firms are ready to pay for half of $C^*$. The outcome is a 1.2 percentage point increase in the unemployment rate which goes back to 7.6%, in between the G and the S regimes. It is possible to parametrize the model such that in the column $k = G^*$ the steady-state unemployment rate is above 9%, provided that $h^*$ is sufficiently large.
Table 5.2: Calibration statistics

<table>
<thead>
<tr>
<th>Variant</th>
<th>I: C^\text{a} paid by workers</th>
<th>II: C^\text{a} shared with the firm</th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime k</td>
<td>k=G</td>
<td>k=S</td>
<td>k=M</td>
<td>k=G</td>
</tr>
<tr>
<td>Fixed parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount rate r</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Death rate $\delta$</td>
<td>0.005</td>
<td>0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity of matching $\eta$</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Workers' share $\beta$</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entry productivity $\varepsilon_0$</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arrival rate of shocks $\lambda$</td>
<td>0.07</td>
<td>0.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skill gap $h^* - h^g$</td>
<td>0.045</td>
<td>0.045</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Training cost $C$</td>
<td>0.40</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calibrated parameters</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Min. productivity $\varepsilon_{\text{min}}$</td>
<td>0.428</td>
<td>0.428</td>
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<tr>
<td>Recruitment cost $\gamma$</td>
<td>0.433</td>
<td>0.433</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scale matching $x^0$</td>
<td>0.750</td>
<td>0.750</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Policy parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U. benefits $b$</td>
<td>-</td>
<td>0.91</td>
<td>0.74</td>
<td>-</td>
</tr>
<tr>
<td>Mean $\varepsilon$ minus b: $\varepsilon - b$</td>
<td>0.75</td>
<td>-</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>Firing cost $T$</td>
<td>0</td>
<td>1.50</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>Endogenous variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market tightness $\theta$</td>
<td>1.57</td>
<td>0.46</td>
<td>0.69</td>
<td>1.57</td>
</tr>
<tr>
<td>Unemployment rate $u$</td>
<td>0.060*</td>
<td>0.091</td>
<td>0.080</td>
<td>0.060*</td>
</tr>
<tr>
<td>Job destruction $\lambda F(R)$</td>
<td>0.055*</td>
<td>0.046</td>
<td>(a)</td>
<td>0.055*</td>
</tr>
<tr>
<td>Replacement rate $b/\pi$</td>
<td>0.234</td>
<td>0.855</td>
<td>0.713</td>
<td>0.234</td>
</tr>
<tr>
<td>Share $\chi$ of jobs at $\varepsilon_0$</td>
<td>0.461</td>
<td>0.422</td>
<td>0.431</td>
<td>0.461</td>
</tr>
<tr>
<td>New born share of $g\alpha g$</td>
<td>1</td>
<td>0</td>
<td>0.08(b)</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Job destruction rates differ according to the type of job and are respectively 0.42 and 4.46%. (b) The implied value of the share of new hires with general skills is $K_g=0.48$.  

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5.2. Implications

5.2.1. Declining sectors in Europe and job specific vs. industry specific human capital

The specific skills sectors feature well traditional declining industries. Layoffs in large firms operating in such sectors attract a lot of media attention in Continental Europe\textsuperscript{12} but also in the US. At the time of announcement of displacements, workers realize their incur a huge welfare loss. What seems typical of the situations described above is a 'misperception' (meaning forecast error) by workers of the duration of the job they occupy. If someone invests in skills specific to a job in thinking that it will last forever, and forgets about investing in general human capital skills, its market productivity in and out of the firm will drastically differ. Note here that it does not matter, in declining sectors, whether skills are industry or job specific skills: the probability to obtain a job in the same sector (textile, coal mining, etc...) for laid-off workers is close zero, and \textit{de facto}, industry specific human capital becomes job specific. Low employability of laid-off workers is reinforced by three factors: the failure of on-the-job 'training' to provide sufficiently general skills (sub-section 4.1); the choice of technologies by employers requiring specific rather than general skills (sub-section 4.2); and low levels of schooling for those workers due to under-investment into higher education in the past (sub-section 4.3.2).

5.2.2. Unemployment compensation and wage setting institutions

Interestingly, unemployment benefits $b$ have a similar impact as employment protection. As $b$ enter in the same way into $U^0$ and $U^9$, they do not directly affect the training decisions in partial equilibrium, summarized by $p^M$.\textsuperscript{13} They however raise wages and thus reduce profits and thus job creations, thus reducing $\theta^K$, $K = S,G$ and thus reinforcing the likelihood of regime $S$.

The role of unemployment insurance is central in Hassler et al. (2002) which, in their model, is endogenously determined. The authors notably investigate the links between geographical mobility and endogenous collective preference and notably derive a theory of multiple steady-states (low mobility, high unemployment insurance). Although this important property can also be embedded in our framework as emphasized in the discussion of political economy complementarity, we prefer to center the discussion on laid-off costs, rather than focusing only on unemployment compensation.

In Hassler et al. (2000), there is a very clear discussion on the choices of investments in human

\textsuperscript{12} See e.g. the typical case of Moulinex (part of SEB) in Basse-Normandie and an example of media coverage in http://www.lexpress.fr/info/economie/dossier/moulinex/dossier.asp.  
\textsuperscript{13} This would however not be the case if benefits were proportional to the previous wage. This would act as an increase in the effective bargaining power $\beta$ and thus raise the occurrence of regime $S$.  

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capital, being more ‘versatile’ in the US than in Europe. The model in Section 5 of their work is rather an illustration of the empirical facts discussed earlier in this paper, and clearly share some properties of ours. They notably identify a condition for choosing flexible skills in relation with the level of unemployment insurance, but tend to concentrate on the determination of the level of insurance. They however ignore the main fact of our analysis, namely that the structure of investments depends on aggregate labor market conditions, and also don’t focus on other welfare state institutions.

Our paper is also in the spirit of Acemoglu and Pischke (1999a) who focus on general human capital and institutions such as minimum wage. A key difference is that our paper is centered on the decisions to invest in general and specific skills, the latter being not central in their paper. 14

In our model, a minimum wage may prevent investments in human capital if returns are too low, as is the case of general skills. A minimum wage thus potentially distorts human capital investments in the same way as unemployment insurance.

There is also some scope for wage bargaining institutions in the model. Notably, a larger bargaining power of workers may be in the interest of firms as it provides incentives to workers to accumulate specific knowledge, as a sort of commitment for firms to compensate for training. This is however only the case when the ex-post (post hiring) contractual freedom of agents give rise to a hold-up problem.

5.2.3. Political economy of employment protection

As discussed right above, the implicit contract in traditional activities in the 70’s may have been the acceptance by workers to acquire specific skills in exchange of job protection. The model suggests a complementarity between institutions and the specificity of human capital investments, leading to the persistence of such institutions. This is close to Saint-Paul’s (2000) discussion of complementarity between institutions and labor market slackness generating persistence or multiplicity of equilibria. The arrival of reallocation shocks may have lead to the breakdown of this contract, explaining the social tensions surrounding the emblematic displacement of workers as discussed next.

14 There is on page 559 of their paper a section devoted to specific human capital, including a choice by firms about the type of investments. Their focus is on the complementarity between general and specific investments. Here we mostly focus on workers’ investments, for we assume that workers have a fair amount of discretion in their unobservable learning efforts.
5.2.4. Turbulence and transitions between steady-states

An interesting implication of the model concerns the interaction between regime choices and the arrival rate of shocks $\lambda$ which can feature the macroeconomic environment. Notably, $\lambda$ might rise with higher turbulence, a large unexpected wave of innovations, or declines in international transportation costs and trade barriers. In both regimes, an increase in $\lambda$ raises unemployment, reduces tightness of the labor market and increases the duration of unemployment. To illustrate, we investigate what happens in our economy when $\lambda$ increases from 7% to 9 and 10% a quarter. See Table 5.3. In the $G$-economy, workers have no employment protection and face a 20 to 30% increase in the job destruction rate while the unemployment rate increases by 1 to 2 percentage points. However, the ratio of training costs to GDP is not affected (here, the slight increase is due to a decrease in GDP), as training costs are paid by the new born workers only. In the $S$-economy, the job destruction rate increases but thanks to employment protection, this is to a lesser extent than in the $G$-economy, by 12 to 15%. This also limits the rise in unemployment, contained within one percentage point. Nevertheless, the share of training costs, which was initially large, 4.8% (half of it is paid by workers, half of it by firms), increases by 0.6 percentage point which is the order of magnitude of the total amount of training costs in the $G$-economy. Workers thus suffer from larger welfare and human capital losses. At $\lambda = 0.1$, there is a regime change (from $S$ to $G$ indicated by column $k = S^*$) as and the economy needs to reinvest massively in skills to reach the new optimal steady-state $G$ in the last column. The total cost of this transition is of the order of magnitude of $C/\varepsilon_0$ i.e. 25% of annual GDP, spread over several years.

The bottom line is that the two regimes have pros and cons, in a steady-state, without obvious dominance of one over the other, but this symmetry breaks down in that short-run cost of adjustment to a new macroeconomic environment in the $S$-economy are larger. They are so large that they can’t be disregarded in international trade theory applied to $S$-type economies. These results are in line with a recent paper by Blanchard and Wolfers (2001) who show the strong complementarity between shocks and institutions to account for the European experience. This paper is also connected with the work of Ljungqvist and Sargent (1998, 2002) who also introduce human capital losses and labor market institutions to explain transatlantic comparisons. Ljungqvist and Sargent similarly see turbulence as negatively interacting with institutions. These views are similar to ours: in our terminology, a $G$-economy (resp. a $S$-economy) is the same as the LS (laissez-faire) (resp. WS, Welfare-State) economy in Ljungqvist and Sargent (2002). In their 2002 model notably, human capital is general, but at the separation time, workers lose some of this capital. They model the
search margin, contrary to the present paper, but do not investigate the determinants of human capital investments: in their paper, individual human capital follows a random process depending only on the employment status. More importantly, we develop the labor demand side of the model, while they consider an exogenous distribution of wage offers.

6. Concluding comments

This paper has derived a usually ignored property of human capital: specific human capital investments have a higher relative return when the job finding rate is low as compared to general human capital investments. It has matched this result with the fact that job protection reduces turnover in the labor market and thus raises incentives to invest in specific skills. The result is an alternative way of describing Europe-US differences in the labor market, that is linked to the nature of human capital decisions, and at the same time consistent with the interaction of shocks and institutions (Blanchard and Wolfers 2001, Ljunqvist and Sargent, 1998, 2002). It has been argued that the relevant policy parameter is the magnitude of employment protection. With reasonable values of parameters (lay-off costs about a quarter of output, a training cost about 10% of quarterly output and workers about 10% more efficient with specific skills than with general skills), we obtain radically different economies with a 30% gap in job destructions and a ratio of 1 to 3 in job creations.

In an important paper, Bertola and Ichino (1995) had claimed that mobility costs are a central factor to the diverging development of both US. and Europe. In their set-up, mobility interacts with wage institutions. Our model indicates the same direction: labor mobility costs are higher in Europe, due here to the different nature of human capital investments. Further documentation

---

Table 5.3: Interaction between turbulence and regimes

<table>
<thead>
<tr>
<th>Variant</th>
<th>Specification II: $C^\alpha$ shared with the firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime $k$</td>
<td>$k=G$</td>
</tr>
<tr>
<td>Arrival rate of shocks $\lambda$</td>
<td>0.07</td>
</tr>
<tr>
<td>Endogenous variables</td>
<td></td>
</tr>
<tr>
<td>Unemployment rate $u$</td>
<td>0.060*</td>
</tr>
<tr>
<td>Job destruction $\lambda F(R)$</td>
<td>0.055*</td>
</tr>
<tr>
<td>Training costs over GDP (%)</td>
<td>0.57</td>
</tr>
</tbody>
</table>

(a) Specification for the $S$-regime: bargaining power of workers = 0.55 (last column of table 5.2).
of low mobility rates within European countries has however to be undertaken. For instance, the replication of Blanchard and Katz (1992) to different European countries has shown that the high persistence of regional unemployment was due to low migration rates between regions (see e.g. Decressin and Fatás, 1995).

More direct evidence could be obtained by a careful investigation of returns to seniority. In an older paper, Hashimoto and Raisian (1985) had provocatively demonstrated that Japanese and American workers deeply diverged in terms of number of jobs occupied throughout their career. For instance, in the late 70’s, they showed that in the US., 15% of male workers aged 40-49 had more than 20 years of tenure, while this was the case of 31% of Japanese workers. At the age 65, US workers had occupied in average slightly more than 10 jobs, while the average Japanese workers had occupied only four jobs. Perhaps as a cause, perhaps as a consequence, they found that, in large firms, returns to tenure were 7% a year (with a quadratic term of -0.03%) in Japan, and only 1.2% a year in the US (with an identical quadratic coefficient), controlling for total experience, schooling and several interaction terms. This has been a much controversial result, subsequent studies for the US oscillated between larger and significant returns to tenure (Topel 1991, Kletzer 1989, see also Kletzer’s survey 1998 based on US displaced workers) and zero returns (Abraham and Farber 1987, Altonji and Shakotko 1987), while for Japan, Clark and Ogawa (1992) indicated that strong returns to tenure were less true in the mid 1980’s, due to changes in the demographic structure of Japan. In France, Lefranc (2003) found evidence of large wage losses after displacement, but not that larger than in the US. Interestingly, he argues that wages losses in France are a loss of accumulated human capital while in the US this is mostly due to downgrading of occupation. The first finding fits our story while the second would require an extension to job search strategy as in Ljunqvist and Sargent (2002). All these studies are actually very sensitive to measurement issues (Lefranc 2004) and identification strategies.

An alternative test is to look at the content of firms’ sponsored training. Loewenstein and Spletzer (1999) argue that it is mostly general in the US, which confirms our story. I am not aware of similar studies for Europe, but theory suggest more balanced results as firms expect their workers to remain attached for a while. A final test would be to use evidence of job-to-job mobility, as indirect evidence of specific skills investments. In a recent paper by Jolivet et al. (2003), one sees

G. In Bertola and Rogerson (1997) there is an emphasis on the relation between employment protection and the wage structure, while in Rosén and Wasmer (2002) we found that higher T reduces wage dispersion when skills are ex-ante heterogenous. This is a possible extension to unify the two approaches cited above and this paper. Gould (2002) and Gould et al. (2001) study wage inequality growth as driven by technology shocks and general education. An older paper by Galor and Tsiddon (1997) studies growth and inequality in connection with the life-cycle of technologies and their accessibility, a notion inversely related to skills specificity.
a perfect correlation between the ranking of the OECD for employment protection and job-to-job turnover. The negative correlation is as strong for skilled workers (-0.81) as for the unskilled (-0.77). Mechanisms other than skill specific investments, such as lower job creation by firms (reducing job-to-job mobility) in countries with high employment protection, would affect unskilled more than the skilled. It can thus not be the primary explanation of these two correlations. These findings are thus rather reassuring for our story of transatlantic differences.
Appendix

A. Bellman equations and wages

We have then for each \( k = g, s \) (and \( k’ = 0, g \)) the respective values of workers’ surplus and firms’ surplus in the continuation stage and the entry stage:

\[
(r + \delta + \lambda) [W^k(\varepsilon) - U^{k'}] = w^k(\varepsilon) - (r + \delta)U^{k'} + \lambda \int \max[0, W^k(z) - U^{k'}] dF(z)
\]

(A1)

\[
(r + \delta + \lambda) [J^k(\varepsilon) - V + T] = \varepsilon + h^k - w^k(\varepsilon) - (r + \delta)(V - T) + \lambda \int \max[0, J^k(z) - V + T] dF(z)
\]

(A2)

\[
(r + \delta + \lambda) [W^k_0 - U^{k'}] = w^k_0 - (r + \delta)U^{k'} + \lambda \int \max[0, W^k_0(z) - U^{k'}] dF(z)
\]

(A3)

\[
(r + \delta + \lambda) [J^k_0 - V] = \varepsilon_0 + h^k - w^k_0 - (r + \delta)V - \lambda T + \lambda \int \max[0, J^k_0(z) - V + T] dF(z)
\]

(A4)

The total surplus in the continuation stage thus follow the following rule:

\[
(r + \lambda + \delta) S^k(\varepsilon) = \varepsilon + h^k - (r + \delta)(U^{k'} + V - T) + \lambda \int \max[0, S(z)] dF(z)
\]

(A5)

\[
(r + \lambda + \delta) S^k_0 = \varepsilon_0 + h^k - (r + \delta)(U^{k'}) - \lambda T + \lambda \int \max[0, S(z)] dF(z)
\]

(A6)

and notably : \( S^k_0 = S^k(\varepsilon_0) - T \)

(A7)

The latter equality determines that

\[
S^k_0 = \frac{\varepsilon_0 - R^k}{r + \lambda + \delta} - T
\]

and \( W^k_0 = \beta \left( \frac{\varepsilon_0 - R^k}{r + \lambda + \delta} - T \right) + U^{k'}
\]

(A8)

(A9)

After Nash-bargaining, wages are given by

\[
w^k_0 = (1 - \beta)(\varepsilon) + \beta h^k - \beta \varepsilon - \beta h^k - (r + \delta) V - (r + \lambda) T
\]

(A10)

\[
w^k(\varepsilon) = (1 - \beta)(\varepsilon) + \beta h^k - (r + \delta) V + (r + \lambda) T
\]

(A11)

Initial bargaining over \( w^k_0 \) is not directly affected by firing tax \( T \), because the initial threat point of the firm is simply \( V \) and not \( V - T \) but \( T \) has still an impact because it will strike after the first shock. Indeed, the entry wage \( w^k_0 \) is negatively affected by the term \( -\lambda T \). The continuation wage raises with \( T \) because the same negative surplus effect \( -\lambda T \) is compensated by the increase in the relative bargaining position of workers which adds \( (r + \lambda + \delta) T \) to the wage.

B. Proofs

B.1. Proof of Theorem 1 in a static context

Theorem 1 concerns a model with a full intertemporal structure, with endogenous destruction and with firing costs which themselves generate a two-tier wage structure. The full proof will thus require a fair amount of calculation. It indeed requires to derive the sign of \( W^g_0 - W^g_0 \) (if positive, workers chose general skills by workers) and of \( W^k_0 - C^k - U^{k'} \) (if negative, workers prefer not to investing at all). Those value functions are not explicit but depend on \( R^k \) which is itself only implicitly defined through the job destruction equation (R).

However, the main idea can be captured in a very simple way. Let us assume away any dynamics and consider a simpler economy where agents live two periods, do not discount the future, lose their job after the end of the first period and obtain a new job with probability \( p \leq 1 \). Workers obtain a share \( \beta \) of the marginal
Figure B.1: Partial equilibrium in a simpler, static context: effect of frictions on worker’s skill choice. A thinner line indicates that the border between areas still exists but is irrelevant in this region.

product $h^k$ net of the threat point of firms $V$. There are no unemployment benefits. Then, denoting with a hat the value functions in this static economy, we have that workers’ valuation of a job with general and specific skills is

$$
\hat{W}^g = \beta(h^g - V) + pMax[\beta(h^g - V), \beta(h^s - V) - C]
$$
$$
\hat{W}^s = \beta(h^s - V) + pMax[\beta(h^g - V), \beta(h^s - V) - C]
$$

where $-C$ in both equation means that workers re-pay the training cost in specific skills. It follows that if

$$
\text{if } g > s, \quad \hat{W}^g = \beta(h^g - V) + p\beta(h^g - V))
$$
$$
\text{if } s > g, \quad \hat{W}^s = \beta(h^s - V) + p\beta(h^g - V) - pC
$$

Denote by $\Delta h = h^g - h^s < 0$. By difference of the two functions, the frontier is defined by:

$$
g \succ s \Leftrightarrow \hat{W}^g > \hat{W}^s \Leftrightarrow \beta\Delta h + p(\beta\Delta h + C) > 0
$$

Denote by $\hat{A}(p) = \beta\Delta h + p(\beta\Delta h + C)$ the corresponding function. Its intercept is $\beta\Delta h < 0$. If $C$ is small compared to the intercept, the function is negative: workers always prefer specific skills as retraining is cheap. If $C$ is larger, the function is increasing and above $p^M = (1 - \frac{C}{\beta\Delta h})^{-1} < 1$, workers prefer general skills. Note that $p^M$ is independent on $V$. The participation constraints define non-increasing curves in $(p, V)$ and we have

$$
0 \succ g \Leftrightarrow \hat{W}^g - C > 0 \Leftrightarrow V > h^g - \frac{C}{\beta(1 + p)}
$$
$$
0 \succ s \Leftrightarrow \hat{W}^s - C > 0 \Leftrightarrow V > h^s - \frac{C}{\beta}
$$

In other words, we obtain a partition of the space summarized in Figure B.1.

**B.2. Full proof of Theorem 1**

The strategy of the proof is to express all quantities involved in workers choices ($W_0^g - W_0^s$ and $W_0^h - C^h - U^h$) as functions of $R^k$, $p$ and $V$, and then to show that $R^k$ are themselves a function of $p$ and $V$. We can replicate the proof above and define three frontiers in the space $(p, V)$. The indifference between $s$ and $g$ skills defines a curve $p^M(V)$. We will show that the choice of skills depends on whether $p$ is smaller or larger
than \( p^M(V) \). For that, let us denote by a tilde the functions which are defined by the arbitrage equations and extended out of the range, i.e. for instance \( \tilde{U}^g(p, V) \equiv U^g(p, V) \) even when \( p \) and \( V \) are such that workers prefer specific skills. The variables with a tilde thus do not coincide with workers’ optimal behavior for all \( p, V \), they are simply mathematical objects used to precisely express workers’ choice.

**Step 1.** The quantities \( \tilde{R}^k \) and \( \tilde{U}^k \) are calculated from equations (R) and (U\(^k\)) in the text. Using the difference operator \( \Delta X \) as the difference between \( X^g \) and \( X^s \) (or \( X^0 \) when relevant), we have

\[
\tilde{W}_{0}^g - \tilde{W}_0^s = \Delta \tilde{W}_0 = \Delta \tilde{U} - \beta \Delta \tilde{R}(r + \lambda + \delta)^{-1} \tag{B1}
\]

and

\[
\Delta \tilde{U} = \frac{p}{r + \delta} \left[ C - \frac{\beta \Delta \tilde{R}}{r + \lambda + \delta} \right] \tag{B2}
\]

Using this equation in (B1), we have that

\[
\Delta \tilde{W}_0 > 0 \iff pC - \beta \Delta \tilde{R}^k \frac{r + \delta + p}{r + \lambda + \delta} > 0
\]

Let us from now denote by \( \tilde{R}^k(V, p) \) the function obtained implicitly from the job destruction rule (R) and (U\(^k\)) and by \( \Delta \tilde{R}(V, p) \) the difference. The properties of \( \tilde{R}^k(V, p) \) and \( \Delta \tilde{R}(V, p) \) are stated in Lemma A1.

**Step 2.** Lemma A1 (fragility of the match). The separation between worker and the firm is more frequent if \( p \) and \( V \) are higher, i.e. the reservation rule \( \tilde{R}^k \) is increasing in both \( p \) and \( V \). When \( p \to \infty \), both reservation rules converge to a common value, \( \Delta \tilde{R} \to 0 \).

Proof : Combining (R) and (U\(^k\)), one obtains

\[
\tilde{R}^k \left( 1 + \frac{p\beta}{r + \lambda + \delta} \right) + \frac{\lambda}{r + \lambda + \delta} \int_{\tilde{R}^k}^\infty (1 - F(z'))dz' = H_k(V, p) \tag{B3}
\]

where

\[
H_k(V) = h^k + (r + \delta)(V - T) + p\beta \left( \frac{\varepsilon_0}{r + \lambda + \delta} - T \right) + \mathcal{I}_k pC
\]

\( \mathcal{I}_k \) equal to zero if \( k = g \) and 1 if \( k = s \). Differentiating equation (B3) with respect to \( p \) and \( V \), we obtain

\[
d\tilde{R}^k \left( 1 - \frac{\lambda}{r + \lambda + \delta} + \frac{p\beta}{r + \lambda + \delta} \right) = (\tilde{W}_{0}^g - \mathcal{I}_k C)dp + (r + \delta)dv \tag{B4}
\]

The quantity \( \tilde{W}_{0}^k - \mathcal{I}_k C \) is positive in a viable economy. For this to be true, one simply requires \( \varepsilon_0 \) to be large enough compared to \( C \). \( \tilde{W}_{0}^k - \mathcal{I}_k C \) is of course positive when \( k = g \). Further, one can verify after a few calculations from equation (B4) that

\[
\partial(\Delta \tilde{R})/\partial V < 0 \tag{B5}
\]

given that \( \tilde{R}^g > \tilde{R}^s \). The second line is only a sufficient condition, and the inequality holds when workers prefer general skills as long as \( \tilde{W}_{0}^g - \mathcal{I}_k < C \).

The second part of Lemma A1 comes from the value of \( \Delta \tilde{R} \):  

\[
\Delta \tilde{R} = \frac{\lambda}{r + \lambda + \delta} \int_{\tilde{R}^k}^\infty (1 - F(z))dz - \Delta h \tag{B6}
\]

As \( p \) goes to infinity, one can verify that \( \Delta \tilde{R} \) goes to zero. ■

**Step 3.** Coming back to the sign of \( \tilde{W}_{0}^g - \tilde{W}_0^s \), we now express it as a function of \( p \) and \( V \). Denoting by \( \Lambda(V, p) = (r + \delta)(\tilde{W}_{0}^g - \tilde{W}_0^s) \), we have

\[
\Lambda(V, p) = pC - \beta \Delta \tilde{R}(V, p) \frac{r + \delta + p}{r + \delta + \lambda} \tag{B7}
\]

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Assume that $f$ has no mass point, so that $\tilde{R}^s$ and $\tilde{R}^g$ are continuous and differentiable with respect to $p$. Thus $\Lambda$ is also continuous and differentiable. We can see that

$$\Lambda(V,0) = \frac{-(r + \delta) \lambda}{(r + \delta + \lambda)^2} \left( \int_{R^s(V,0)} (1 - F(z))dz - \Delta h \right)$$

Remembering that $\Delta h < 0$, $\Lambda(V,0) < 0$. Further, replacing the function $\Delta \tilde{R}$ from equation (B6) into $\Lambda(V, p)$, we have

$$\Lambda(V, p) = pC - \beta \frac{r + \delta + p}{r + \lambda + \delta + \beta p} \frac{\lambda}{r + \lambda + \delta} \left( \int_{R^s} (1 - F(z))dz - \Delta h \right)$$

Then we see that

$$\lim_{p \to \infty} \Lambda(V, p) = \infty$$

since $\tilde{R}^s$ and $\tilde{R}^g$ converge to each other. By continuity of $\Lambda$ with respect to $p$, there exists thus at least one $p^M(V)$ such that $\Lambda(V, p^M) = 0$. Further, for low values of $p$, workers prefer specific skills. For high values of $p$ they prefer general skills. The question is to be sure that there is a unique $p^M(V)$. Following the logic of the proof in the static case described above, it would thus be sufficient to prove that $\partial \Lambda / \partial p > 0$.

**Step 4.** Let us calculate this slope taking the derivative of equation (B7) with respect to $p$: we have

$$\partial \Lambda / \partial p = C - \beta \frac{\Delta \tilde{R}}{r + \lambda + \delta} - \beta \frac{d \Delta \tilde{R}}{dp} \frac{r + \delta + p}{r + \lambda + \delta + \lambda}$$

which has a priori no particular sign. Nevertheless, as in the static case, we can see that a large enough $C$ may imply the monotonicity of $\Lambda$ and thus the uniqueness of $p^M$ for each $V$. Let us however make no assumption on $C$.

One strategy to show uniqueness is instead to compute the slope of $\Lambda$ in points $p^M$ where $\Lambda = 0$. Indeed, if the slope is locally positive in such points, this means that there could be a decreasing part on $\Lambda$ (although simulations never found any) but in any case only for values of $\Lambda$ distant from zero. In other words, the first intersection between $\Lambda$ and the origin which necessarily exist, is also necessarily unique. So, exploiting that in points $p^M$, we have

$$\frac{\beta \Delta \tilde{R}}{r + \delta + \lambda} = \frac{p^M}{r + \delta + p^M C}$$

the slope of $\Lambda$ is locally equal to

$$\partial \Lambda / \partial p(p^M) = C \frac{r + \delta}{r + \delta + p^M} - \beta \frac{d \Delta \tilde{R}}{dp} \frac{p^M}{r + \delta + \lambda} = C \left( \frac{r + \delta}{r + \delta + p^M} - \frac{d \ln \Delta \tilde{R}}{d \ln p} \frac{p^M}{(p^M)} \right)$$

A sufficient condition for the uniqueness of $p^M$ is thus that locally in $p^M$ the quantity $\Delta \tilde{R}$ does not vary with respect to $p$ (which would be the case with exogenous destruction rates) or as shown in the second line, that the elasticity of $\Delta \tilde{R}$ with respect to $p$ is smaller than 1. We can actually have some intuition that, for sufficiently large values of $p^M$, $d \Delta \tilde{R} / dp$ is actually negative, since when $p$ goes to infinity, $\tilde{R}^s$ converge towards $\tilde{R}^g$ from below. The same intuition carries through by inspection of equation (B6): the denominator is clearly driving a negative sign of $d \Delta \tilde{R} / dp$. For the numerator to generate an opposite slope of $\Delta \tilde{R}(p)$, this numerator has to vary very fast. So, under the sufficient condition that, when workers are indifferent between general and specific skills, job destruction rates are either: i) inelastic to $p$ or ii) the cut-off point $\tilde{R}$ is not too elastic in $g$-jobs compared to $s$-jobs, there is uniqueness of $p^M$.

**Step 5.** Given that $\Delta \tilde{R}$ decreases with $V$ from inequality (B5), it must be that $\Lambda(V, p)$ increases with $V$ and thus that $p^M(V)$ decreases with $V$.

**Step 6.** The shape of the frontiers between regimes $S$ and 0 (resp. $G$ and 0) is the part b of Theorem 1. We have:
Lemma A2 (regime 0). Workers chose 0 rather than specific skills when \( \tilde{R}^s(V, p) > \varepsilon^0 - (r + \delta + \lambda)(C^s + T) \). Workers chose 0 rather than general skills when \( \tilde{R}^g(V, p) > \varepsilon^0 - (r + \delta + \lambda)(C^g - \Delta U)/\beta + T \).

Proof: New born workers do not invest in skill \( k \) if \( \tilde{W}^k - \tilde{U}^k - C + \tilde{U}^k < U^0 \), i.e. given Lemma A1, if

\[
\beta \frac{\varepsilon^0 - \tilde{R}^s(p, V)}{r + \lambda + \delta} - T > C + \tilde{U}^0 - \tilde{U}^k.
\]

B.3. Proof of inequality (3.1)

The derivative of equation (V) with respect to \( p \) yields

\[
r \frac{\partial V}{\partial p} = \frac{d \alpha}{dp} (1 - \beta) S_0^k - \frac{q_p}{r + \lambda + \delta} \frac{d R^k}{dp}.
\]

Then, \( \frac{d R^k}{dp} = \frac{\partial R^k}{\partial p} + \frac{\partial R^k}{\partial V} \frac{\partial V}{\partial p} \). Substituting, we have

\[
\left( r + \frac{\partial R^k}{\partial V} \frac{q_p}{r + \lambda + \delta} \right) \frac{\partial V}{\partial p} = \frac{d \alpha}{dp} (1 - \beta) S_0^k - \frac{q_p}{r + \lambda + \delta} \frac{d R^k}{dp}.
\]

Lemma A1 above states that \( \frac{\partial R^k}{\partial p} > 0 \) and \( \frac{\partial R^k}{\partial V} > 0 \). Thus, since \( \frac{d \alpha}{dp} < 0 \), we have that \( \frac{\partial V}{\partial p} < 0 \).

C. Stocks and Flows

In a steady-state, there is a number of new jobs, i.e. of jobs having not faced any transition, denoted by \( \varepsilon^0 \).

The total number of jobs is denoted by \( e \). In the text, the fraction \( e^0/e \) is denoted by \( \chi \). We have

\[
d e^0/dt = (1 - e)p - (\delta + \lambda)e^0 = 0
\]

(C1)

In regime G, denoting by \( u^g \) and \( u^0 \) the stocks of unemployed workers with general or no human capital, we have

\[
da^g/dt = \delta - (p + \delta)u^0
\]

(C2)

\[
da^g/dt = (1 - u^0 - u^g)\lambda F(R^g) - (p + \delta)u^g
\]

(C3)

where the first line shows that changes in \( u^0 \) are the difference between the arrival of new born workers (the total active population is normalized to unity) and the successful job seekers in that pool (remembering that a fraction \( \delta \) of them dies), and the second line states that changes in \( u^g \) are the difference between the arrival of laid-off workers and of successful job seekers in that pool (still taking care of deaths in the pool). In a steady-state,

\[
u^0 = \frac{\delta}{\delta + p}
\]

(C4)

\[
u^g = \frac{p}{\delta + p} \frac{\lambda F(R^g)}{\delta + p + \lambda F(R^g)}
\]

(C5)

The sum of the two rates trivially yields \( u^g \) in the text. In regime S in which every one invests in specific skills, the derivation of steady-state unemployment is straightforward and we obtain the usual value for the unemployment rate \( u^g \) in the text.

In the mixed regime, we need to calculate the steady-state stocks of workers in each state: \( u^0, u^g, e^s, e^g \) with \( e^k \) is the number of ‘employed’ workers with human capital \( k, k = s, g \). We have

\[
da^0/dt = \delta + e^s \lambda F(R^s) - (p + \delta)u^0
\]

(C6)

\[
da^g/dt = \lambda F(R^g) e^g - (p + \delta)u^g
\]

(C7)

\[
de^g/dt = \alpha^g pu^0 + u^g p - (\delta + \lambda F(R^g))e^g
\]

(C8)

\[
de^s/dt = (1 - \alpha^g)pu^0 - (\delta + \lambda F(R^g))e^s
\]

(C9)
It follows that, in a steady-state, \( u^0 \) and \( u^\theta \) solve

\[
\begin{align*}
u^0 &= \frac{\delta}{\delta + p} \left( 1 - \frac{\lambda F(R^\theta)}{\delta + \lambda F(R^\theta)} \frac{p}{\delta + \alpha^\theta (1 - \alpha^\theta)} \right)^{-1} \\
u^\theta &= \frac{\lambda F(R^\theta)}{\delta + \lambda F(R^\theta)} \frac{p}{\delta + \alpha^\theta (1 - \alpha^\theta)} - \frac{\lambda^2 F(R^\theta)}{\delta + \lambda F(R^\theta)} \frac{p^2}{\delta + \alpha^\theta (1 - \alpha^\theta) \delta + \lambda F(R^\theta)} \frac{1}{\delta + \alpha^\theta (1 - \alpha^\theta)}
\end{align*}
\]

(C10)  
(C11)

It is easy to verify that \( \alpha^\theta = 0 \) and \( \alpha^\theta = 1 \) brings back to the pure regimes (\( u^S \) and \( u^G \) in the text). Employers recruit workers with general skills coming from both the pool of workers with general skills \( u^\theta \) and a fraction \( \alpha^\theta \) of workers coming from the pool of workers without skills \( u^0 \), i.e.

\[
\kappa^\theta = \frac{\alpha^\theta u^0 + u^\theta}{\alpha^\theta u^0} = \frac{\alpha^\theta + u^0 / u^\theta}{\alpha^\theta}
\]

(C12)

Together with (C11) expressing the ratio of \( u^0 \) to \( u^\theta \), we obtain an expression relating \( \alpha^\theta \) to \( \kappa^\theta \) is then

\[
\frac{\kappa^\theta}{1 - \kappa^\theta} = \frac{\alpha^\theta}{1 - \alpha^\theta} \left( 1 + \frac{\lambda F(R^\theta)}{\delta + \alpha^\theta (1 - \alpha^\theta) \delta + \lambda F(R^\theta)} \frac{p}{\delta + \alpha^\theta (1 - \alpha^\theta)} \right)
\]

(C13)

Naturally, \( \kappa^\theta \geq \alpha^\theta \). Further, \( \delta = 0 \) implies that \( \kappa^\theta = 1 \) whatever \( \alpha^\theta \) (i.e. all the labor force would become skilled with general human capital in steady-state). It is possible to describe the full set of \( \kappa^\theta \) in \( [0, 1] \) with any mixed strategy \( 0 < \alpha^\theta < 1 \) of new-born workers. Notably, it is possible to find the value of \( \alpha^\theta \) corresponding to the equilibrium value \( \theta^M \) as represented on Figure 3.2. This proves the existence of a steady-state mixed strategy equilibrium.

D. Efficient firm’s sponsored training

Let us return to the benchmark assumption that workers pay the full cost \( C \) but that firms can compensate them by a transfer. Suppose indeed that we start from a situation in which \( \theta = \theta^G > \theta^M \). We are thus in the regime \( G \). An individual firm could find it profitable to induce new workers to make a specific investment, by totally compensating the worker by a transfer \( F = W^G_t - W^G_0 \geq 0 \), where the inequality is binding at the firm’s optimum. In partial equilibrium, this possibility is nevertheless limited by the fact that workers may find it too costly to invest in specific skills, which would mean that firms cannot compensate them, because the gain in profits is not large enough. Expressed formally, this means that the gain in profits has to be larger than the transfer, i.e. that \( F < J^G_0 - J^G_0 \). Given equation (B1), this implies the following constraint for such a compensation scheme to be possible:

\[
\Delta R > (r + \delta + \lambda) \Delta U
\]

(D1)

This inequality separates the space \((\Delta R, \Delta U)\) in two parts. Let’s call \( F \) the associated frontier (a straight line with intercept 0 and slope \( r + \lambda + \delta \)). Equation \((U^F)\) defines another curve with higher slope and negative intercept. This implies that, in partial equilibrium, specific investments need to increase sufficiently the duration of jobs for firms to be willing to compensate workers. In other words, in the parameter space, the subset of parameters leading to a pure regime \( G \) is smaller, but not empty since (D1) cannot be satisfied for all values of parameters. The subset of parameters leading to a pure regime \( S \) is larger and the mix regime subset is affected ambiguously by these changes. In equilibrium, the efficient transfer possibility is even more limited: indeed, firms can make higher profits than in the pure \( G \) regime by paying \( F \) to workers. This will generate more entries of firms, thus raising \( \theta \) above \( \theta^G \). This further increases the cost of the transfer \( F \) as workers are all the more induced to choose general skills.

References


