Optimal taxation of polluting goods... and also clean ones?

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Abstract
The paper studies the changes in the optimal tax system when an externality is “discovered”, in a Mirlees setting with heterogeneous agents. We consider a world in which people differ in their (separable) preferences and ability. We study how the tax system should be modified when taking into account an externality, compared to the initial optimal system (with no externalities). We introduce three types of goods, a dirty good (polluting transport and energy), a clean substitute good (non polluting transport and energy), and a composite good that includes other types of goods. We assume that people have to pay a transaction or access cost in order to consume the clean substitute. We find that if ability and the access cost to the clean (perfect) substitute are negatively correlated, then no good should be taxed if there is no externality. With externality, we find that the optimal way to redistribute amongst agents is, in addition to set a non linear income tax, to tax the dirty good less than the pigovian tax and to levy a positive tax on the clean good as well.

1 Introduction
In recent years, there have been debates, for instance in France, on the desirability of a carbon tax. The main criticism of its detractors was that such a tax reform is regressive, because the energy part is larger in the expenses of the poorer. In addition, some argued that poorer people have less substitution possibilities, in France, because they live far from city centers and thus do not have access to public transportation or city gaz. Fig.1 illustrates the fact that CO2 emissions are relatively larger in areas where GDP per inhabitant is low.

If a carbon tax reform is to be done with a constant budget constraint, the way the tax proceeds are redistributed is of foremost importance for equity reasons. To offset the potential regressive bias of environmental taxes, the french government suggested to give lump sum transfers (“chèques verts”) to

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people, based on criteria such as income and localization\(^1\). The idea was to tag those people who have less substitution possibilities, based on where they live, and to transfer lump sum some of the tax proceeds to them. However, this solution was not very popular and its implementation seemed complicated, as it required accurate information on agents. If tagging people who have less substitution possibilities, and thus pollute relatively more with the same income, is not possible, then redistribution may be achieved by modifying the income tax schedule and indirect taxes.

One of the main results of the optimal taxation literature is that if agents are “indistinguishable”, that is to say if their consumption choices depend only on their disposable income, the Atkinson Stiglitz theorem (Atkinson & Stiglitz (1976)) holds even when there is an externality: it is unnecessary to use indirect taxes, apart from the tax on the commodity that generates the externality (Gauthier & Laroque (2008), Kaplow (2004), Kaplow (2006)). As stated in Cremer et al. (1998)\(^2\) “Any Given commodity tax and income-tax system, differential commodity taxation can be eliminated in a manner that results in a Pareto improvement”. The optimal carbon tax is then equal to the Pigouvian tax, i.e. the marginal damage from consumption of polluting goods divided by the marginal cost of public funds. In particular, this is the case if

\(^1\) « Pour nos concitoyens les plus défavorisés et des habitants des territoires ruraux, nous étudions en outre des mesures financières d’accompagnement. Il est hors de question d’appliquer uniformément ce dispositif à des Français qui ont le choix et à d’autres qui ne l’ont pas » François Fillon (08/2009).
agents differ only in their productivity and if utility function (common to all) is separable between leisure and other consumption goods. The Atkinson-Stiglitz theorem is no longer valid, even in the case of a separable utility function, when agents differ in another characteristic than their productivity (Boadway & Pestieau (2002), Kaplow (2008) Saez (2002), Cremer, Pestieau & Rochet (2001)). In this case, it is difficult to obtain general results (see Rochet & Chone (1998)). In this paper, we assume that agents differ not only in their productivity but also in their access cost to clean substitutes for polluting goods. Indirect taxes on goods can be useful because they allow to relax the incentive constraints, and thus allow better redistribution.

When combining separable heterogeneous preferences with externalities, one finds that the tax on the polluting good is the sum of the Pigouvian tax and the usual formula for commodity taxation (see Cremer et al. (1998), Cremer, Gahvari & Ladoux (2001)). Specifically, as explained by Kopczuk (2003), “one can build the following policy prescription: correct the externality directly, using the pigouvian tax (imposed on the dirty commodity), then find and apply the optimal taxes while ignoring the externality by using prices corrected by the pigouvian tax and taking into account that some income is collected by it.” The general formulation corresponds to the classical calculation rule, to which is added a tax equal to marginal damage only on the good that causes the externality, the principle of “targeting” of Dixit (1985) is generalized (Kopczuk (2003)): we must correct the externality by targeting the source of the externality directly.

This result does not imply, however, that taking into account the externality does not change the existing tax system, calculation rules remain the same, but the externality can change the relative level of all indirect taxes. In their applied paper, Cremer et al. (2002) look at the optimal tax system when externality is taken into account. They consider heterogeneous agents who consume two types of goods: a polluting good and a composite good that puts together all the other types of goods. The only room for redistribution through indirect taxes, in their paper, is then to set up a tax on the dirty good which is less than the pigouvian tax, and to change the direct tax system in order to make it more redistributive. We account here for the fact that the inequality between people has to do with the differences in the ease of access to clean substitute for the dirty good. We introduce a third type of good, which is a substitute for the polluting good and which can be consumed easily only by some agents. We focus on changes in the system of direct and indirect taxes when an externality is taken into account (in comparison with cases where there is no attempt to correct it), and when people differ along two dimensions. Individuals do not all have the same productivity nor the same preferences: some have the choice between meeting their demand for energy and transport with clean technologies (city gas, transportation) or dirty (car, home heating oil) while others only have access to dirty energy and transport. We ask what is the optimal redistribution scheme when a carbon tax is introduced. In
particular, with a constant budget constraint, should other commodity taxes be changed because of the introduction of a carbon tax? Or in other terms, are the same incentive compatibility constraints binding as without externality?

The aim of the paper is thus to study how the tax system should be modified when an externality is “discovered”, in order to maximize social welfare, and thus taking into account equity considerations. When a carbon tax is introduced, we find that redistribution could be achieved by modifying the relative level of other indirect taxes. In particular, if ability and the access cost to the clean (perfect) substitute are negatively correlated, then the tax on the clean good should increase (relative to the composite good) when the externality is “discovered”. In section 2 of the paper, we present the model. In section 3, we look for the optimal tax system in this model when there is no externality, we find that there should be no indirect taxation at all. Then, in section 4 of the paper, we study the optimal tax system when the externality is taken into account. We find that introducing carbon tax on the dirty good leads to introduce also a positive tax on the clean good, compared to the composite good.

2 The model

People do not all have the same ease of access to non polluting goods that could substitute for their consumption of polluting goods. This is the case for instance of public transportation, which is mainly accessible in large cities, as well as city gas. Because of this differences in “access cost”, the project of a carbon tax, proposed in France in 2009, was considered unfair and eventually abandoned.

In order to take into account the heterogeneity of people with respect to their access to a clean substitute of a polluting good, we adopt the following modeling. We assume that people can consume three types of goods, a dirty good $e_1$ (polluting transport and energy), a clean good $e_2$ (non polluting transport and energy), and a composite good $x$ that includes other types of goods. The utility achieved by an individual consuming $x, e_1$ (dirty), $e_2$ (clean), and working to generate the rough income $z$ is:

$$u(x, e_1 + e_2) - \theta \psi(e_2) - \sigma \varphi(z)$$

where $u$ is homogenous with degree one, $\psi$ a convex function measuring a potential transaction or access cost to the clean energy, with $\psi'(0) = 0$, and $\varphi$ is the disutility of effort (labor), with $\varphi' > 0$ and $\varphi'' > 0$. $\theta \in \Theta \subseteq \mathbb{R}_+$ and $\sigma \in \Sigma \subseteq \mathbb{R}_+$, are two individual parameters measuring the intensity of the transaction cost and disutility of effort. We denote by $f(\sigma, \theta)$ the joint density of the two parameters. We assume that the distribution of types is continuous on its domain $\Sigma \times \Theta$, which is a convex open bounded subset of $\mathbb{R}_+^2$. The marginal cost of the consumption good is $c$ and the common marginal cost of
both dirty and clean energy is $\alpha$.

Let us comment the main features of this utility function: 1) it is separable between labor and consumption, 2) $u$ is homogeneous with degree one, 3) utility depends on some access cost to the clean good $e_2$ and 4) clean and dirty energy are perfect substitute, except for the access cost.

The fact that the utility function is separable between labor and consumption is a classical assumption in the optimal taxation literature. If there were no other sources of heterogeneity than the differences in productivity, it would allow that people with the same income, regardless of their productivity, choose the same consumption bundle. In other terms, people would be indistinguishable and two people with the same productivity would necessarily work the same number of hours. Here, however, people also differ in another characteristic: their access cost to clean energy. As a result, people with the same revenue do not necessarily consume the same bundle. Thus, if the utility function $u$ was strictly concave, people with the same productivity would not have the same marginal utility of revenue and thus would not work the same time. Assuming that $u$ is homogeneous with degree one hence ensures that, conditional on each income level, the behavioral responses (the change in hours worked) following a change in the tax system do not depend on the consumption pattern. We make this assumption because, as stated in Saez (2002), “for most goods, there are no reasons to think that conditional on income, consumption patterns should be related systematically to substitution or income effects parameters”. The fact that, conditional on each income level, behavioral responses are independent of consumption patterns is one of the three necessary conditions that make indirect taxation superfluous, as demonstrated by Saez (2002). However, because people differ in another parameter than their productivity, the two other necessary conditions of Saez (2002) that make indirect taxation superfluous are not necessarily satisfied.

The originality of this utility function is the way the clean good and the dirty good appear. The two goods are perfect substitute in $u$ and they have the same cost, so that another interpretation of this utility function is possible. Let $t, \tau_1, \tau_2$ be the linear taxes on goods $(x, e_1, e_2)$. One can consider instead energy as a whole: $e = e_1 + e_2$ and $e_2 = e - e_1$ as a de-pollution effort. In this case, the tax on energy $e$ is equal to $\tau_1$ and the subsidy for de-pollution is $\tau_1 - \tau_2$. One can check easily that the two representation are equivalent. We can think of many examples, for which the access cost varies among people. Transportation is a typical example. In large cities, one can use the metro, one’s own car or even (in Paris for instance) electric cars that are at one’s disposal at many power terminal points in the city, on payment of an annual subscription. If access cost is zero, the utility is the same whatever the mode of transport. In the countryside on the other hand, people have to use their personal car. Recycling is another example. If $e$ is one agent total consumption of waste, she will recycle $e_2$, but only if there are
recycling points not too far. We also made the assumption that the two goods have the same cost, so that, absent any taxation, nobody consumes the clean good, except those people who do not incur any access cost and who are then indifferent between the clean and the dirty good\(^2\). The idea is that if there is no subsidy to de-pollution, people do not recycle their waste, except if they do not incur any transaction cost. Those assumptions are strong assumptions but allow us to have analytical results on optimal taxation, to be able to compare the optimal tax systems with externality and without externality and to understand the intuition behind the differences in the tax systems. The results remain the same if the producer price of the clean good is higher than the production price of the dirty good. In this case, if there are no indirect taxes at all, nobody consumes the clean good (even people with zero access cost).

We chose a parametric description for access cost and disutility of labor. We find, however, the same results with more general functional forms: the access cost could be more generally a function \(\psi(\theta, e_2)\) such that \(\frac{\partial \psi}{\partial e_2}(\theta, 0) = 0\). We could also have considered a positive access cost only above some level of \(e_2\), i.e. \(\psi(\theta, e_2 - \bar{e}_2)\) with \(\bar{e}_2\) independent of \(\theta\). In this case, without indirect taxation, agents would consume \(\bar{e}_2\) even if they have positive access cost. Similarly the disutility of labor could be a more general function \(\varphi(\sigma, z)\). Note that the differences in the disutility of labor can thus be interpreted as differences in productivities instead (e.g. \(\varphi(\sigma z)\) where productivity is \(\frac{1}{\sigma}\)), with all agents having the same disutility of the number of hours worked (e.g. \(\varphi(\cdot)\)).

We denote \(T(\cdot)\) the non linear income tax and the indirect tax system \(t, \tau_1, \tau_2\). Denote \(N(z) = z - T(z)\) the net income. Facing this tax structure, the consumer solves (for \(N(z)\) given) :

\[
\max_{x, e_1, e_2} \{ u(x, e_1 + e_2) - \theta \psi(e_2) - \sigma \varphi(z), (c + t)x + (\alpha + \tau_1)e_1 + (\alpha + \tau_2)e_2 = N(z) \}
\]

Let \(U(\theta, \sigma)\) be the achieved utility.

**Lemma 1.** For all \(N(\cdot)\) \(t, \tau_1, \tau_2\), let \(z^*, x^*, e_1^*, e_2^*\) functions of \((\theta, \sigma)\) solution of the program,

1. \(U\) is a decreasing convex function of \((\theta, \sigma)\),
2. \(\partial \theta U = -\psi(e_2^*), \partial \sigma U = -\varphi(z^*), u(x^*, e^*) = U - \theta \partial \theta U - \sigma \partial \sigma U, (c + t)x^* + (\alpha + \tau_1)e_1^* + (\alpha + \tau_2)e_2^* = N(z)\)

**Proof.** \(U\) is convex as maximum of linear functions. Envelope theorem insures the two first equalities, the other two equalities are obtained by computing the level of achieved utility and the budget constraint. \(\square\)

Define the expenditure function :

\(^2\)All the results of the paper remain true with a Dirac point mass at \(\theta = 0\).
Definition 1. For $c$ and $\alpha$ given, let the function $D$:

$$D(v) = \min \{ cx + \alpha e, u(x, e) = v \}$$

Since $u$ is homogeneous with degree one, $D$ is linear increasing.

Lemma 2. $cx^* + \alpha(e_1^* + e_2^*) \geq D(U - \theta \partial_{\theta} U - \sigma \partial_{\sigma} U)$

Proof. straightforward since we have $u(x^*, e^*) = U - \theta \partial_{\theta} U - \sigma \partial_{\sigma} U$, \hfill \Box

3 Taxation with no externality

In this section, we are interested in finding the optimal direct and indirect tax system if agents have the utility function described in the previous section and if there is no externality arising from the consumption of the dirty good. It is the second best solution, given the informational structure, if the government does not know that consuming the dirty good generates some externality. As we have already noticed in the previous section, if there are no indirect taxes on goods, then agents are indistinguishable i.e. people with the same revenue consume the same bundle regardless of their type. This is the case because, with no indirect taxes, nobody consumes the clean good, except those people with zero access cost who are indifferent between the clean and the dirty good. However, if there is indirect taxation and if the indirect tax on the dirty good is higher than the indirect tax on the clean good, then agents are not indistinguishable, as the consumption bundle of people with different types (here $\theta$) is not the same.

We show in this section that, if agents with high productivity also have low access cost to the clean substitute, then indirect taxation is superfluous. The intuition is rather simple. The government would like to redistribute toward low productivity agents. He cannot redistribute much with only direct taxation because otherwise high productivity agents would mimic low productivity agents. In order to deter these agents from mimicking, the government can tax relatively more the goods that are preferred by high productivity agents. In our case, the government may want to tax the clean good, but then nobody would consume it, so that indirect taxation is superfluous.

Even if the intuition looks clear, the simplest proof of the Atkinson-Stiglitz theorem (see Laroque (2005) and Gauthier & Laroque (2008)) does not work here. Indeed, let $(t, \tau_1, \tau_2, N())$ be any government policy so that the achieved utility $U(\theta, \sigma)$ is well defined. From the point of view of the agent, the government policy is equivalent to another government policy, where he derives $U(\theta, \sigma) - \sigma \partial_{\sigma} U$ from consumption and works $z(\sigma, \theta)$. The new allocation of Laroque (2005) is obtained by achieving the same levels $U(\theta, \sigma) - \sigma \partial_{\sigma} U$ and $z(\sigma, \theta)$ . He argues that this new allocation can be supported with a more efficient choice of prices and income. He takes:

$$(\hat{x}, \hat{e}_1, \hat{e}_2) = \arg\min_{x, e_1, e_2} \{ cx + \alpha(e_1 + e_2) \mid u(x, e_1 + e_2) - \theta \psi(e_2) \geq U(\theta, \sigma) - \sigma \partial_{\sigma} U \}$$
and $\tilde{N}(z(\sigma, \theta)) = c\tilde{x} + \alpha(\tilde{e}_1 + \tilde{e}_2)$ as after tax income. Remark that this is not feasible if two people with the same $z(\sigma, \theta)$ have different “optimal” bundle $(\tilde{x}, \tilde{e}_1, \tilde{e}_2)$ which leads to different after-tax income. Indeed, in this case $\tilde{N}(.)$ is not defined, as the same $z$ can be mapped to several after tax incomes $c\tilde{x} + \alpha(\tilde{e}_1 + \tilde{e}_2)$. Take, in our model, two agents with the same $\sigma$ but different $\theta$. Then, they have the same $z$ (because $u$ is homogeneous with degree one). However, they do not necessarily have the same utility level $U(\theta, \sigma) - \sigma \partial_\sigma U$ if, initially, $\tau_1 > \tau_2$. Then they do not necessarily have the same optimal consumption bundle $(\tilde{x}, \tilde{e}_1, \tilde{e}_2)$. As a result, one cannot use Laroque (2005) proof in our model to show the uselessness of indirect taxation.

However, it is fruitful to use the same method as Laroque (2005) and Gauthier & Laroque (2008) to prove the results. Assume that the government uses a concave Social Welfare function $W$. The Social surplus achieved when taxes are $N(,) t, \tau_1, \tau_2$, writes:

$$\hat{\Sigma} \int \int W(U(\theta, \sigma)) f(\theta, \sigma) d\theta d\sigma$$

$$+ \lambda \left( \int \int [z^*(\theta, \sigma) - c x^*(\theta, \sigma) - \alpha e^*(\theta, \sigma)] f(\theta, \sigma) d\theta d\Sigma - G \right)$$

Where $G$ stands for public expenditures; $z^*(\theta, \sigma), x^*(\theta, \sigma)$ and $e^*(\theta, \sigma) = e^*_1(\theta, \sigma) + e^*_2(\theta, \sigma)$ are the before tax income and the consumption bundle chosen by agent $(\theta, \sigma)$ under the tax system $N(,) t, \tau_1, \tau_2$. We are going now to show that there exists another tax structure involving no indirect taxation that gives a larger social surplus. Given $U$ we build a function $V$ of $\sigma$ only such that the weighted sum of agents welfare is at least as large as with the tax system $(N(,) t, \tau_1, \tau_2)$ if all agents with type $\sigma$ reach utility $V(\sigma)$, and such that the government revenues can be increased. Define $V$ in the following way:

$$V(\sigma) = E(U/\sigma) - \int \int U(\theta, s) \partial_\sigma f(\theta/s) d\theta ds + v$$

Where $v$ is some constant.

Next, we choose $v$ such that:

$$\int \int U(\theta, \sigma) f(\theta, \sigma) d\theta = \int \int E(U/\sigma) f(\sigma) d\theta = \int \int V(\sigma) f(\sigma) d\sigma$$

The value of $v$ must be:

$$v = \int \int \int \left[ \int U(\theta, s) \partial_\sigma f(\theta/s) d\theta \right] f(\sigma) ds d\sigma$$
As the social planner has concave preferences, the social welfare is higher under $V$ than under $U$ if the values of $V$ are more concentrated than the values of $E(U|\sigma)$ around their common mean value, that is if $E(U|\sigma)$ is a mean preserving spread of $V$. We prove this is true if the probability to have a transaction cost smaller than $\theta$ is decreasing with $\sigma$, that is if low productivity people have high transaction costs. We use Lemma 3 to prove the result.

**Lemma 3.** Let $F(\theta/\sigma)$ be the probability that the energy transaction cost is smaller than $\theta$ for given cost of effort $\sigma : F(\theta/\sigma) = \int_{0}^{\theta} f(t/\sigma) dt$. Then:

$$\partial_{\sigma}(F(\theta/\sigma) \leq 0 \Rightarrow 0 \geq V'(\sigma) \geq \partial_{\sigma}[E(U/\sigma)]$$

**Proof.** It is straightforward that $V'(\sigma) = E(\partial_{\sigma}U/\sigma) \leq 0$.

$$\partial_{\sigma}E(U/\sigma) = E(\partial_{\sigma}U/\sigma) + \int_{\Theta} U(\theta, \sigma) \partial_{\sigma}f(\theta/\sigma)d\theta$$

$$= V'(\sigma) + \int_{\Theta} U(\theta, \sigma) \partial_{\sigma}f(\theta/\sigma)d\theta$$

As $f(\theta/\sigma)$ is a density w.r. $\theta$, $\int_{\Theta} \partial_{\sigma}f(\theta/\sigma)d\theta = 0$. An integration by part gives:

$$\int_{\Theta} U(\theta, \sigma) \partial_{\sigma}f d\theta = -\int_{\Theta} \partial_{\theta}U(\theta, \sigma) \partial_{\sigma}F(\theta/\sigma)d\theta$$

As $\partial_{\theta}U(\theta, \sigma) = -\psi(e^2) < 0$, $\int_{\Theta} U(\theta, \sigma) \partial_{\sigma}f(\theta/\sigma)d\theta$ has the sign of $\partial_{\sigma}F(\theta/\sigma)$.

Hence, if low productivity is correlated with high access cost, the (negative) slope of $V$ is less steep than $\partial_{\sigma}[E(U/\sigma)]$ one. We show that this implies that $E(U|\sigma)$ is a mean preserving spread of $V$, and hence that the social welfare is higher under $V$ than under $U$. Indeed compute the Social welfare when agents with type $(\theta, \sigma)$ reach $U(\theta, \sigma)$:

$$W_{U} = \int_{\Sigma} \int_{\Theta} W(U(\theta, \sigma)) f(\theta, \sigma) d\theta d\sigma$$

As $W$ is concave:

$$W_{U} \leq \int_{\Sigma} W(E(U/\sigma)) f(\sigma) d\sigma$$

By definition, $E(U/\sigma)$ and $V(\sigma)$ have the same mean. But if $V$ is steeper than $E(U/\sigma)$, the values of $V$ are more concentrated than those of $E(U/\sigma)$. Indeed, let $F(\sigma)$ the cumulative distribution function (CDF) of $\sigma$ Note $\tilde{U}(\sigma) = E(U/\sigma)$. Call $F_{\tilde{U}}$ the CDF of $\tilde{U}$ and $F_{V}$ the CDF of $V$. Random variable $\tilde{U}$ is a
mean-preserving spread of $V$ (or, equivalently, $V$ is second-order stochastically dominant over $\tilde{U}$) if, for all $v$:

$$\int_{v_{\min}}^{v} (F_{\tilde{U}}(t) - F_{V}(t)) dt \geq 0$$ \hspace{1cm} (1)

But $F_{V}(t) = P(V(\sigma) \leq t) = P(\sigma \geq V^{-1}(t)) = 1 - F(V^{-1}(t))$. Similarly, $F_{\tilde{U}}(t) = 1 - F(\tilde{U}^{-1}(t))$. So that Eq.1 is equivalent to, for all $v$:

$$\int_{v_{\min}}^{v} F\left(\tilde{U}^{-1}(t)\right) - F\left(V^{-1}(t)\right) dt \leq 0$$ \hspace{1cm} (2)

Consider :

$$F\left(\tilde{U}^{-1}(v)\right) - F\left(V^{-1}(v)\right)$$

As $0 > V' > \tilde{U}'$ and $E(U') = E(V)$ the curves $V$ and $\tilde{U}$ cross once and once only. Then for low values of $v$: $\tilde{U}^{-1}(v) \leq V^{-1}(v)$ and for large ones $\tilde{U}^{-1}(v) \geq V^{-1}(v)$, and so $F\left(\tilde{U}^{-1}(v)\right) - F\left(V^{-1}(v)\right)$ is first negative and then positive. But as $E\left(\tilde{U}\right) = E(V)$, $\int_{v_{\min}}^{v_{\max}} (F\left(U^{-1}(v)\right) - F\left(V^{-1}(v)\right)) dv = 0$. So we have :

$$\forall v, \int_{v_{\min}}^{v} \left[F\left(U^{-1}(t)\right) - F\left(V^{-1}(t)\right)\right] dt \leq 0$$

which implies that $E\left(U/\sigma\right)$ is a mean-preserving spread of $V$ and then that for all concave function $W$,

$$\int_{\Sigma} W\left(E(U/\sigma)\right) f(\sigma) d\sigma \leq \int_{\Sigma} W\left(V(\sigma)\right) f(\sigma) d\sigma$$

So that the social welfare is higher if all agents with type $\sigma$ have utility $V(\sigma)$, than if agents with types $(\theta, \sigma)$ have utility $U(\theta, \sigma)$.

Let us comment these results. As $\sigma$ increases, there are two effects that make $E(U|\sigma)$ decrease. The first effect comes from the decrease in utility entailed by an increase in $\sigma$: take all people with type $\sigma$ and increase a little their $\sigma$, keeping their $\theta$ constant. Their utility will decrease as their disutility of work increases. This decrease is equal to $E(\partial_{\sigma}U|\sigma) = V'(\sigma)$. The second effect has to do with the correlation between $\theta$ and $\sigma$: as $\partial_{\sigma}(F(\theta/\sigma) \leq 0$, the people with higher $\sigma$ also have higher $\theta$. So that $E(U|\sigma)$ decreases with $\sigma$ not only because of the increase in the disutility of work but also because, among people with higher $\sigma$, there are more people with higher $\theta$, that is with higher access cost and lower utility level. So that the decrease in $E(U|\sigma)$ is larger than the decrease in $V$, as $E(\partial_{\sigma}U|\sigma) = V'(\sigma)$, and, as a result, the values that take $V$ are more concentrated than the values that take $E(U|\sigma)$. 

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We have showed that social welfare is higher under $V$ than under $U$. The next step is to show that $V$ can be obtained as the solution of the first problem with some tax levels such that $\tau_1 = \tau_2 = t = 0$. In this case, the problem of agent $(\theta, \sigma)$ amounts to find $(x, e, z)$ that solve:

$$\max \{ u(x, e) - \sigma \varphi(z), cx + \alpha e = N(z) \}$$

We seek a function $\tilde{N}$ of $z$ such that:

$$V(\sigma) = \max \{ u(x, e) - \sigma \varphi(z), cx + \alpha e = \tilde{N}(z) \}$$

Let $\tilde{z}(\sigma) = \varphi^{-1}(-V'(\sigma))$ and define:

$$D(v) = \min \{ cx + \alpha e, u(x, e) = v \}$$

Assume that $V'' > 0$. Then $V(\sigma) - \sigma V'(\sigma)$ is decreasing, as $\tilde{z}(\sigma)$ is also decreasing, each value of $\tilde{z}$ is mapped to a unique $\sigma$, which is mapped to a unique value of $V(\sigma) - \sigma V'(\sigma)$. We can define $\tilde{N}$ such that $\tilde{N}(\tilde{z}(\sigma)) = D(V(\sigma) - \sigma V'(\sigma))$. It is easy then to check that:

$$V(\sigma) = \max \{ u(x, e) - \sigma \varphi(z), cx + \alpha e = \tilde{N}(z) \}$$

Call $\tilde{e}(\sigma)$ and $\tilde{x}(\sigma)$ the achieved consumptions. We have:

$$\{ \tilde{x}(\sigma), \tilde{e}(\sigma) \} = \arg \min \{ cx + \alpha e, u(x, e) = V(\sigma) - \sigma V'(\sigma) \}$$

$$= \arg \max \{ u(x, e), cx + \alpha e = \tilde{N}(\tilde{z}(\sigma)) \}$$

So that, if $V'' > 0$, it is possible to find an income tax such that the utility level of all agents with type $\sigma$ is $V(\sigma)$. We prove in appendix (Lemma 4) that $V$ is convex.

The last step is to prove that the government revenues are higher with the new tax system than with the former. The intuition is the following: by definition of $\tilde{z}$ and $V(.)$, it is straightforward that the labor offer, and thus the before tax income, is the same with the new tax system as with the initial tax system. Moreover, $V(.)$ has been built such that the mean level of utility is also the same. However, with the new tax system, there is no access cost that decreases the level of utility. As a result, people, in average, consume relatively less than with the initial tax system so that the total level of utility (the utility from consumption minus the access cost), in average, remains the same. Then the tax revenues, equal to the before tax revenues minus the value of consumption, are higher with the new tax system. More formally, collected taxes by the government, $G_V$, are equal to:

$$G_V = \int_{\Sigma} [\tilde{z}(\sigma) - c\tilde{x}(\sigma) - \alpha \tilde{e}(\sigma)] f(\sigma) d\sigma$$
That is:

\[ G_V = \int_{\Sigma} \left[ \varphi^{-1} (-V' (\sigma)) - D (V (\sigma) - \sigma V' (\sigma)) \right] f (\sigma) d\sigma \]

Consider now the Government revenues with the former tax system \((\tau_1, \tau_2, N())\):

\[ G_U = \int_{\Sigma} \int_{\Theta} \left[ z^* (\theta, \sigma) - cx^* (\theta, \sigma) - \alpha e^* (\theta, \sigma) \right] f (\theta, \sigma) d\theta d\sigma \]

Using the lemmas in section 2 gives:

\[ G_U \leq \int_{\Sigma} \int_{\Theta} \left[ \varphi^{-1} (-\partial_\sigma U) - D (U - \theta \partial_\theta U - \sigma \partial_\sigma U) \right] f (\theta, \sigma) d\theta d\sigma \]

By linearity of \(D\):

\[ G_U \leq \int_{\Sigma} \int_{\Theta} \varphi^{-1} (-\partial_\sigma U) f (\theta, \sigma) d\theta d\sigma - \int_{\Sigma} D \left[ \int_{\Theta} (U - \theta \partial_\theta U - \sigma \partial_\sigma U) f (\theta, \sigma) d\theta \right] d\sigma \]

That we can rewrite, since \(\partial_\theta U \leq 0\):

\[ G_U \leq \int_{\Sigma} \int_{\Theta} \varphi^{-1} (-\partial_\sigma U) f (\theta, \sigma) d\theta d\sigma - \int_{\Sigma} D \left[ \int_{\Theta} (U - \sigma \partial_\sigma U) f (\theta, \sigma) d\theta \right] d\sigma \]

And then:

\[ G_U \leq \int_{\Sigma} \int_{\Theta} \varphi^{-1} (-\partial_\sigma U) f (\theta, \sigma) d\theta d\sigma - \int_{\Sigma} D \left[ \int_{\Theta} (E(U/\sigma) - \sigma E(\partial_\sigma U/\sigma)) f(\sigma) d\sigma \right] \]

But as \(\varphi^{-1}\) is concave:

\[ G_U \leq \int_{\Sigma} \varphi^{-1} (E (-\partial_\sigma U/\sigma)) f (\sigma) d\sigma - \int_{\Sigma} D \left[ (E(U/\sigma) - \sigma E (\partial_\sigma U/\sigma)) \right] f(\sigma) d\Sigma \]

Then replacing \(E[\partial_\sigma U/\sigma]\) by \(V' (\sigma)\) in Eq.3 and using the linearity of \(D\) gives:

\[ G_U \leq \int_{\Sigma} \varphi^{-1} (-V' (\sigma)) f (\sigma) d\sigma - \int_{\Sigma} D \left[ (V (\sigma) - \sigma V' (\sigma)) \right] f(\sigma) d\sigma \]

We have then found a function of \(\sigma\) only that is such that the total surplus is better than the one for \(U\). The following theorem holds:
Theorem 1. Let \( F(\theta/\sigma) \) the probability that the energy transaction cost be smaller than \( \theta \) for given cost of effort \( \sigma \). If this probability is decreasing with \( \sigma \) (that is if low productivity people have high transaction costs) then optimal taxation requires only income taxation and no indirect taxes.

Note that this theorem comes from the fact that agents are indistinguishable if there is no indirect taxation. However, the solution of the maximization of any agent’s program, given that there is no indirect taxation, in not a corner solution as \( \psi'(e_2 = 0) = 0 \).

4 Optimal taxation with externality

We move now to the case where there is an externality. We assume now that consuming a quantity \( E_1 = \int_{\Theta, \Sigma} e_1(\theta, \sigma)f(\theta, \sigma)d\theta d\sigma \) of the polluting good generates the environmental damage:

\[-aE_1\]

We choose a constant marginal damage of pollution because emissions from a country at some date only impact marginally the worldwide stock of pollutant in the atmosphere.

We want to show that the standard Atkinson-Stiglitz result does not hold in general. The tax on the dirty good is not equal, in general, to the pigovian tax, and the tax on the clean good is not equal, in general, to zero. As before, we assume that \( u(x,e) \) is homogeneous of degree one. We also assume that the government has redistributive tastes, so that he maximizes a sum of \( W(U - aE_1) \) where \( W \) is increasing and concave.

Denoting \( E_1 \) the sum of emissions, the social surplus is thus:

\[
\int_{\Sigma} \int_{\Theta} W(U(\theta, \sigma, \tau, N())) - aE_1)f(\theta, \sigma)d\theta d\sigma
+ \lambda \int_{\Sigma} \int_{\Theta} [z^*(\theta, \sigma) - cx^*(\theta, \sigma) - \alpha e^*(\theta, \sigma)]f(\theta, \sigma)d\theta d\sigma
\]

If the Atkinson Stiglitz theorem was to hold, then indirect taxes \( \tau_1 \) on good 1 would be equal to the social marginal externality cost of consuming good \( e_1 \), and the tax \( \tau_2 \) on good 2 would be zero. Assume that the optimal tax system indeed satisfies:

\[
\tau_1 = \left( \int_{\Theta} \int_{\Sigma} W'(U(\theta, \sigma) - aE_1)f(\theta, \sigma)d\theta d\sigma \right) \frac{a}{\lambda} \quad (4)
\]

\[
\tau_2 = 0
\]

Where \( \lambda \) is the marginal cost of public funds. Assume also that the income tax \( T(z) \) is optimal given this indirect tax system.
reform \((0, dτ₁, dτ₂)\) on goods \((x, e₁, e₂)\). To study whether this tax reform is profitable, we proceed as in Saez (2002). The indirect utility of consumption is again denoted \(w(θ, N)\) where \(N\) is disposable income:

\[
w(θ, N) = \max_{x, e₁, e₂} \{u(x, e₁ + e₂) - θψ(e₂)\}
\]

s.t\[
(α + τ₁)e₁ + αe₂ + cx ≤ N
\]

The social planner seeks \((τ₁, τ₂, N())\) which maximize:

\[
\hat{Σ} \hat{Θ} W(U(θ, σ) − aE₁)(∂N w) e₁ f(θ, σ)dθdσ dτ₁
\]

**Tax reform \(dτ₁\)** We start from a case where \(τ₁\) satisfies Eq.4 and \(τ₂ = 0\). We look at the effect of a tax reform \(dτ₁\).

\[
U(θ, σ) = \max_z \{w(θ, N(z)) − σϕ(z)\}
\]

So that:

\[
∂_{τ₁} U(θ, σ) = ∂_{τ₁} w(θ, N)
\]

\[
= −(∂N w)e₁ dτ₁
\]

So that the total effect on welfare can be written:

\[
− \left( ∫_Σ ∫_Θ W'(U(θ, σ) − aE₁)(∂N w) e₁ f(θ, σ)dθdσ \right) dτ₁
\]

\[
−a \left( ∫_Σ ∫_Θ W'(U(θ, σ) − aE₁) f(θ, σ)dθdσ \right) dE₁
\]

The tax change has two effects on public fund:

- It raises mechanically the revenue:

\[
λ(E₁ dτ₁ + τ₁ dE₁)
\]

- It has a behavioral effect \(dz_{τ₁}(θ, σ)\), via change in \(z(θ, σ)\):

\[
λ T'(z(θ, σ)) dz_{τ₁}(θ, σ) f(θ, σ)dθdσ
\]

We see that, given the value of \(τ₁\) defined in Eq.4, the second term in Eq.6 simplifies with the second term in Eq.7. So that the total effect is the sum of the tree terms:

\[
− \left( ∫_Σ ∫_Θ W'(U(θ, σ) − aE₁)(∂N w) e₁ f(θ, σ)dθdσ \right) dτ₁
\]
\[ \lambda E_1 d\tau_1 \quad (10) \]

\[ \lambda \int \int \Sigma \int_{\Theta} T'(z(\theta, \sigma)) dz_{\tau_1} (\theta, \sigma) f(\theta, \sigma) d\theta d\sigma \quad (11) \]

To check the sign of the sum of Eq.9 + Eq.10 + Eq.11, we use the fact that any small income tax reform has no first order effect on welfare because the income tax is optimal. We look at the effect of a small income tax \( dT \), such that \( dT(z) = E_1(z) d\tau_1 \), where \( E_1(z) \) denotes average consumption of \( e_1 \) for individuals earning \( z \). The effect of this tax change can also be decomposed into mechanical, welfare and behavioral effect.

**Tax reform \( dT(z) \)** Denoting \( f(\theta, \sigma|z) \) the probability density of types \( (\theta, \sigma) \) knowing \( z \), and \( f(z) \) the probability density of income \( z \), the effect on welfare of this income tax reform is the following:

\[ - \int_{\Sigma} \int_{\Theta} W'(U(\theta, \sigma) - aE_1) (\partial_N w) E_1(z) f(\theta, \sigma|z) d\theta d\sigma f(z) dz d\tau_1 \]

\[ - a \left( \int_{\Sigma} \int_{\Theta} W'(U(\theta, \sigma) - aE_1) f(\theta, \sigma) d\theta d\sigma \right) dE^T_1 \quad (12) \]

Where \( dE^T_1 \) is the decrease in \( E_1 \) due to this tax reform. As before, there is no first order effect via a change in \( z^* \) because the agent had chosen \( z^* \) optimally (given the income tax schedule and the indirect taxes).

The reform raises revenues:

\[ \lambda \int_{\Sigma} \left( \int_{\Theta} (dT(z) + \tau_1 dE^T_1(z)) f(\theta, \sigma|z) d\theta d\sigma \right) f(z) dz \quad (13) \]

And has behavioral effect:

\[ \lambda \int_{\Sigma} \int_{\Theta} T'(z(\theta, \sigma)) dz_{\tau}(\theta, \sigma) f(\theta, \sigma|z) d\theta d\sigma f(z) dz \quad (14) \]

We see that the second term in Eq.12 simplifies with the second term in Eq.13. So that the total effect is the sum of the three terms:

\[ - \int_{\Sigma} \left( \int_{\Theta} W'(U(\theta, \sigma) - aE_1) (\partial_N w) E_1(z) f(\theta, \sigma|z) d\theta d\sigma \right) f(z) dz d\tau_1 \quad (15) \]

\[ \lambda \int_{\Sigma} \left( \int_{\Theta} dT(z) f(\theta, \sigma|z) d\theta d\sigma \right) f(z) dz \quad (16) \]

\[ \lambda \int_{\Sigma} \left( \int_{\Theta} T'(z(\theta, \sigma)) dz_{\tau}(\theta, \sigma) f(\theta, \sigma|z) d\theta d\sigma \right) f(z) dz \quad (17) \]
Comparison between the two reforms  We compare the effects, term by term, of the two tax reforms. The difference in the effect on welfare (Eq.9-Eq.15) is the following:

\[-\int_z^\infty \left( \int_{\theta,\sigma} W'(U(\theta, \sigma) - aE_1(\theta, \sigma))(\partial Nw)(e_1(\theta, \sigma) - E_1(z))f(\theta, \sigma|z)d\theta d\sigma \right) f(z)dzd\tau_1 (18)\]

If \( \partial Nw \) does not depend on \( \theta \), conditionnal on \( z \), then this term can be rewritten:

\[-\int_z^\infty (\partial Nw) \left( \int_{\theta,\sigma} W'(U(\theta, \sigma) - aE_1(\theta, \sigma))(e_1(\theta, \sigma) - E_1(z))f(\theta, \sigma|z)d\theta d\sigma \right) f(z)dzd\tau_1 (19)\]

This is the case if \( u \) is homogeneous of degree one (proof in appendix). Let look at \( U(\theta, \sigma) \), conditional on \( z \). It is easy to see that if \( u \) is homogeneous of degree one, all agents with the same \( z \) also have the same \( \sigma \), as \( \partial_z \sigma < 0 \) (because \( \partial_z^2 U > 0 \) and \( \partial_z \sigma = 0 \) (because \( \partial_w^2 \sigma w = 0 \), see lemma 5 in appendix), and the same disutility of labor \( \sigma \varphi(z) \). For a given level \( z \), and the associated after tax income \( N \), agents of type \( \theta \) maximize:

\[w(\theta, N) = \max_{x, e_1, e_2} \{u(x, e_1 + e_2) - \theta\psi(e_2)\} \quad \text{s.t} \quad \alpha e_1 + \sigma e_2 + cx \leq N\]

A revealed preference argument gives immediately that, at \( z \) given (and thus at \( \sigma \) given), higher \( \theta \) have lower \( U(\theta, \sigma) \) and thus higher \( W'(U - aE_1) \). Moreover, amongst agents earning \( z \), the consumption of \( e_1 \) increases with \( \theta \) (see proof in lemma 5 in appendix), and then increases with \( W'(U - aE_1) \). As a result, \( \int_{\theta,\sigma} W'(U(\theta, \sigma) - aE_1)(e_1(\theta, \sigma) - E_1(z))f(\theta, \sigma|z)d\theta d\sigma > 0 \), and thus

\[-\int_z^\infty (\partial Nw) \left( \int_{\theta,\sigma} W'(U(\theta, \sigma) - aE_1(\theta, \sigma))(e_1(\theta, \sigma) - E_1(z))f(\theta, \sigma|z)d\theta d\sigma \right) f(z)dzd\tau_1 < 0\]

The two remaining terms are:

\[\lambda \int_z^\infty \int_{\theta,\sigma} (e_1(z)d\tau_1 - dT(z))f(\theta, \sigma|z)d\theta d\sigma f(z)dz = 0\]

by definition and:

\[\lambda \int_z^\infty \int_{\theta,\sigma} T'(z(\theta, \sigma))(dz_1(\theta, \sigma)- dz_T(\theta, \sigma))f(\theta, \sigma|z)d\theta d\sigma f(z)dz (20)\]

In order to compare \( dz_1(\theta, \sigma) \) and \( dz_T(\theta, \sigma) \), we use the fact that the reform \( d\tau_1 \) induces the same behavioral response as a reform of the income tax schedule, specific to each agent of type \( (\theta, \sigma) \), with \( dT(\theta, \sigma) = e_1^2(\theta, \sigma)d\tau_1 \) (see Saez (2002)).
Agent \((\theta, \sigma)\) chooses his amount of hours worked as if he was subject to a linear budget constraint \(R + (1 - t)z\), where \(t\) is the marginal tax rate at \(z = z^*(\theta, \sigma)\). He chooses thus \(z^*\) that maximizes: \(U(\theta, \sigma, \tau, R + (1 - t)z, z)\) (with \(R = z - T(z) - (1 - t)z\) and \(t = T' (z)\)). Using Slutsky and denoting \(-z^*\) the compensated demand for leisure, we get that:

\[
\frac{\partial z(\theta, \sigma)}{\partial (1 - t)} = \frac{\partial z^*(\theta, \sigma)}{\partial (1 - t)} + \frac{\partial z(\theta, \sigma)}{\partial R} z
\]

The behavioral effect of the reform \(dT(\theta, \sigma)\) is then:

\[
dz_{\tau_1} = \frac{\partial z(\theta, \sigma)}{\partial R} dR - \frac{\partial z(\theta, \sigma)}{\partial (1 - t)} dt
\]

Using that \(R = z - T(z) - (1 - t)z\):

\[
dR = (1 - T' (z))dz - (1 - t)dz - dT(z) + zdt = -dT(z) + zdt
\]

and

\[
dt = dT'(z) + T''(z)dz = \frac{\partial e_1}{\partial (\theta, \sigma)} d\tau_1 + T''(z)dz
\]

\[
dz_{\tau_1} = -\frac{\partial z(\theta, \sigma)}{\partial R} d\tau_1 + \left(\frac{\partial z}{\partial R}(\theta, \sigma) z(\theta, \sigma) - \frac{\partial z}{\partial (1 - t)} (\theta, \sigma) \right) dt
\]

\[
= -\frac{\partial z}{\partial R}(\theta, \sigma) e_1(\theta, \sigma, \tau_1, z - T(z)) d\tau_1 - \frac{\partial \tilde{z}}{\partial (1 - t)} (\theta, \sigma) dt
\]

So that the behavioral effect of this reform is:

\[
dz_{\tau_1} (1 + \frac{\partial \tilde{z}}{\partial (1 - t)} T''(z)) = -\frac{\partial z}{\partial R} E_1(z) d\tau_1 - \frac{\partial \tilde{z}}{\partial (1 - t)} \frac{\partial e_1}{\partial R} d\tau_1
\]

On the other hand, the reform \(dT(z)\) yields to a behavioral effect on agent \((\theta, \sigma)\)

\[
dz_T (1 + \frac{\partial \tilde{z}}{\partial (1 - t)} T''(z)) = -\frac{\partial z}{\partial R} E_1(z) d\tau_1 - \frac{\partial \tilde{z}}{\partial (1 - t)} \frac{\partial e_1}{\partial z} d\tau_1
\]

So that the overall difference between the behavioral terms (Eq.20) is:

\[
\int z \int_{(\theta, \sigma)} \frac{T'(z)}{(1 + \frac{\partial z}{\partial (1 - t)} T''(z))} \left(-\frac{\partial z}{\partial R} E_1(z) d\tau_1 - \frac{\partial \tilde{z}}{\partial (1 - t)} \frac{\partial e_1}{\partial R} d\tau_1\right) f(\theta, \sigma | z) d\theta d\sigma f(z) dz
\]

\[
- \left(-\frac{\partial z}{\partial R} E_1 d\tau_1 - \frac{\partial \tilde{z}}{\partial (1 - t)} \frac{\partial E_1}{\partial z} d\tau_1\right) f(\theta, \sigma | z) d\theta d\sigma f(z) dz
\]

\[
= d\tau_1 \int z \int_{(\theta, \sigma)} \frac{T'(z)}{(1 + \frac{\partial z}{\partial (1 - t)} T''(z))} \left(\frac{\partial z}{\partial R} (E_1 - e_1) + \frac{\partial \tilde{z}}{\partial (1 - t)} \left(\frac{\partial E_1}{\partial z} - \frac{\partial e_1}{\partial z}\right)\right) f(\theta, \sigma | z) d\theta d\sigma f(z) dz
\]

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If \( \frac{\partial \tilde{z}}{\partial (1-t)} \) and \( \frac{\partial z}{\partial R} \) do not depend on the agent’s type, conditionnal on \( z \), then Eq.20 boils down to:

\[
\int_{\tilde{z}} T'(z) \left( \frac{\partial \tilde{z}}{\partial (1-t)} \right) \frac{\partial z}{\partial (1-t)} \int_{(\theta,\sigma), \tilde{z}=z} \left( \frac{\partial E_1}{\partial z} - \frac{\partial e_1}{\partial z} \right) f(\theta, \sigma|z) d\theta d\sigma f(z)dz (21)
\]

In particular, this is the case if \( u(x, e_1 + e_2) \) is homogenous of degree one (see the proof in appendix, lemma 6).

Let look at the remaining term (from Eq.21):

\[
\int_{\tilde{z}} T'(z) \left( \frac{\partial \tilde{z}}{\partial (1-t)} \right) \frac{\partial z}{\partial (1-t)} \int_{(\theta,\sigma), \tilde{z}=z} \left( \frac{\partial E_1}{\partial z} - \frac{\partial e_1}{\partial z} \right) f(\theta, \sigma|z) d\theta d\sigma f(z)dz
\]

This term is negative if agents who earn \( z + dz \) would consume relatively less \( e_1 \) than agent earning \( z \) if they were forced to earn \( z \). We need the distribution of \( (\theta|z) \). But if \( u \) is homogeneous of degree one, then all agents choosing the same \( z \) have the same \( \sigma \) (as \( \partial^2 \sigma, U > 0 \)). So that the distribution \( f(\theta|\sigma) \) can be rewritten \( f(\theta|\sigma = g(z)) \) with \( g' < 0 \).

\[
E_1(z) = \int_{\theta} e_1(\theta, g(z)) f(\theta|\sigma = g(z)) d\theta
\]

\[
dE_1(z) = \int_{\theta} \frac{\partial e_1(\theta, g(z))}{\partial \theta} f(\theta|\sigma = g(z)) d\theta + \int_{\theta} e_1(\theta, g(z)) \frac{\partial \sigma f(\theta|\sigma = g(z))}{\partial \theta} g'(z) d\theta
\]

So that:

\[
\int_{(\theta,\sigma), \tilde{z}=z} \left( \frac{\partial E_1}{\partial z} - \frac{\partial e_1}{\partial z} \right) f(\theta, \sigma|z) d\theta d\sigma
\]

\[
= - \int_{\theta} \frac{\partial e_1(\theta, g(z))}{\partial \theta} \frac{\partial \sigma F(\theta|\sigma = g(z))}{\partial \theta} g'(z) d\theta
\]

But, we know that \( F_\sigma(\theta|\sigma = g(z)) \leq 0 \) and \( g' < 0 \). Moreover \( \frac{\partial e_1(\theta, g(z))}{\partial \theta} > 0 \) (see proof in appendix, lemma 5). We assume here that \( T'(z) \geq 0 \) for every \( z \) (this result is valid when leisure is a non-inferior good and when the government has redistributive tastes, see Seade (1982)). So that increasing \( \tau_1 \) yields negative marginal surplus.

Exactly the same reasoning applies with \( \tau_2 \), we can show that increasing \( \tau_2 \) from 0 to \( d\tau_2 \) increases total welfare (because, at \( z \) given, \( \partial \theta e_2 < 0 \)).
Theorem 2. In the presence of externality, with redistributive tastes and with heterogeneous preferences:

1. In general, the tax on the clean good should not be zero and the tax on the dirty good should not be the pigovian tax.

2. Let $F(\theta/\sigma)$ the probability that the energy transaction cost be smaller than $\theta$ for given cost of effort $\sigma$. If this probability is decreasing with $\sigma$ (that is if low productivity people have high transaction costs) then the tax on the dirty good should be less than the pigovian tax and the tax on the clean good should be strictly positive.

5 Conclusion

We have studied how the indirect tax system should be modified when an externality is taken into account. We have assumed that the initial tax system would be optimal if there were no externality. We chose to do this hypothesis because, even if the initial tax system is actually non optimal without externality, it is difficult to know which taxes are too low or too high. If the tax on the dirty good was originally too low, than the carbon tax would entail a double dividend. As the occurrence of such a double dividend depends crucially on the characteristics of the non optimal original tax system, we prefer to focus on the differences between two optimal situations: the first one when there is no externality and the second one when the externality is discovered. We find that when a carbon tax is introduces, the whole indirect tax system should be modified, for equity reasons.

If agents with high productivity also have low access cost to the clean substitute, then, if there is no externality, indirect taxation is superfluous. With externality, on the other hand, it is optimal to set up a carbon tax on the dirty good and a positive tax on the clean good as well. This comes from the fact that the carbon tax reveals a difference between agents that was not visible in the original situation: their ease of access to a clean substitute. In the original situation, even people with low access cost consume the dirty good. But as soon as the dirty good becomes more expensive, because of the carbon tax, they consume the clean good which becomes relatively cheaper. The consumption bundle varies among agents earning the same income.

This paper of course relies on strong assumptions. The main one is that the clean and the dirty good are perfect substitutes and have the same producer price. This assumption makes the comparison between the original situation and the externality situation easier. Thanks to this assumption, nobody consumes the clean good in the original situation (except maybe a Dirac point mass). What would be interesting is to take instead a constant elasticity of substitution function between the clean and the dirty good. Then, even in the initial situation, there would be differences in consumption bundles of agents.
with different access costs. However, it is plausible to think that, as the carbon tax increases the difference in price of the dirty and the clean good and then discrepancies between consumption bundles, introducing a carbon tax might lead to increase the tax on the clean good as well. However, it is very difficult to obtain analytical results in such a setting. It would be interesting to have at least numerical insight.
A Proofs

Lemma 4. $V''(\sigma) > 0$

Proof.

$$V''(\sigma) = \partial_\sigma \left[ \int_{\Theta} \partial_\sigma U f(\theta/\sigma) d\theta \right]$$

That is:

$$V''(\sigma) = \int_{\Theta} \partial_\sigma^2 U f(\theta/\sigma) d\theta + \int_{\Theta} \partial_\sigma U \partial_\sigma f(\theta/\sigma) d\theta$$

An integration by part gives:

$$V'' = \int_{\Theta} (\partial_\sigma^2 U f(\theta/\sigma) d\theta - \partial_\sigma^2 U \partial_\sigma F(\theta/\sigma)) d\theta$$

The first term is positive. As $f(\theta/\sigma)$ is a density w.r. $\theta$, $\partial_\sigma F(\theta/\sigma) = 0$.

Let $F(\theta/\sigma) = \int_{\Theta} f(\theta/\sigma) d\theta$, we have $\partial_\sigma F(\theta/\sigma) = \int_{\Theta} \partial_\sigma f(\theta/\sigma) d\theta$. An integration by part gives:

$$\hat{\theta} \partial_\sigma U \partial_\sigma f(\theta/\sigma) d\theta = - \int \partial_\sigma^2 U \partial_\sigma F d\theta$$

If $\partial_\sigma^2 U \geq 0$, a sufficient condition for $V'' > 0$ is $\partial_\sigma F \leq 0$. We check under that condition $\partial_\sigma^2 U$ is greater than 0. We denote:

$$w(\theta, N) = \max_{x,e_1,e_2} \{ u(x, e_1 + e_2) - \theta \psi(e_2), (c + t)x + (\alpha + \tau_1)e_1 + (\alpha + \tau_2)e_2 = N \}$$

Take the optimal income tax schedule $z \rightarrow \hat{\theta}(z)$ for some given indirect tax system $(t, \tau_1, \tau_2)$, with $\hat{\theta}' > 0$. We have that:

$$U(\theta, \sigma) = \max_z \left\{ w(\theta, \hat{\theta}(z)) - \sigma \varphi(z) \right\}$$

$$z(\theta, \sigma) = \arg\max_z \left\{ w(\theta, \hat{\theta}(z)) - \sigma \varphi(z) \right\}$$

$$\partial_\sigma^2 U = -\varphi' \partial_\theta z = \partial_\sigma^2 U = \frac{\partial_\sigma^2 w \hat{\theta}' \partial_\sigma z}{w \hat{\theta}' \partial_\sigma z}$$

We have that $\partial_\sigma z < 0$ (as $\partial_\sigma^2 U > 0$). In order to determine whether $\partial_\sigma^2 U > 0$, we study the sign of $\partial_\sigma^2 w$. Recall that:

$$w(\theta, N) = \max_{x,e_1,e_2} \{ u(x, e_1 + e_2) - \theta \psi(e_2), (c + t)x + (\alpha + \tau_1)e_1 + (\alpha + \tau_2)e_2 = N \}$$

So that

$$\partial_{\theta}^2 w = \partial_{\sigma}^2 w = -\psi'(e_2) \partial_{\sigma} e_2$$

$$\partial_{\theta}^2 w \leq 0 \iff \partial_{\sigma} e_2 \geq 0$$

So that $\partial_{\theta}^2 w \leq 0$ if and only if $e_2$ is a normal good (i.e. if the consumption of $e_2$ increases with disposable income $N$). If $u$ is homogeneous of degree one, we have that $\partial_{\sigma}^2 w = 0$, so that $V'' > 0$. \qed
Lemma 5. if $u$ is homogeneous of degree one, then

1. $\partial_{\theta,N}^2 w = 0$
2. At $N$ given, $\partial_{\theta} e_1 \geq 0$
3. At $N$ given, $\partial_{\theta} e_2 \leq 0$

Proof. The program of agent $(\theta, \sigma)$ reads:

$$u(x, e_1 + e_2) - \theta \psi(e_2) + (\partial_N w)(N - cx - (\alpha + \tau)e_1 - \alpha e_2)$$

Where $(\partial_N w)$ is the lagrangier multiplier for type $(\theta, \sigma)$ associated with his budget constraint. The solution reads:

$$u_x = (\partial_N w)c$$
$$u_e = (\partial_N w)(\alpha + \tau)$$
$$\theta \psi'(e_2) = (\partial_N w)\tau$$
$$cx + (\alpha + \tau_1)e_1 + \alpha e_2 = N$$

We look for $\partial_{\theta,N}^2 w$. At $N$ given, we compute the variation of $w_N$ induced by $d\theta$. We get:

$$u_{xx} dx + u_{xe} de = d(\partial_N w)c$$
$$u_{ee} de + u_{xe} dx = d(\partial_N w)(\alpha + \tau_1)$$
$$\theta \psi''(e_2) de_2 + d\theta \psi'(e_2) = d(\partial_N w)\tau_1$$
$$cdx + (\alpha + \tau)de - \tau de_2 = 0$$

As $u$ is homogeneous of degree one, $u_x$ and $u_y$ are homogeneous of degree zero and using Euler theorem:

$$u_{x,x}x + u_{x,e}e = 0$$
$$u_{y,x}x + u_{y,e}e = 0$$

So that $(x, y)$ is a eigenvector of matrix $\left( \begin{array}{cc} u_{x,x} & u_{x,y} \\ u_{y,x} & u_{y,y} \end{array} \right)$ So that the Hessian $H(u)$ of $u$ is zero. We find that:

$$d(\partial_N w) = 0$$
$$de_2 = -\frac{\psi'(e_2)}{\theta \psi''(e_2)} d\theta < 0$$
$$de_1 = \frac{-c^2 u_{ee} - (\alpha + \tau_1)\alpha u_{xx} + (2\alpha + \tau_1)\alpha u_{ee} + \psi'(e_2)}{-c^2 u_{ee} - (\alpha + \tau_1)^2 u_{xx} + 2(\alpha + \tau_1)\alpha u_{ee} + \theta \psi''(e_2)} d\theta$$

From Euler theorem $u_{ex}$ and $u_{xe}$, have opposite signs, so that $\partial_{\theta} e_1 > 0$
Lemma 6. if \( u \) is homogeneous of degree one, then \( \frac{\partial z}{\partial (1 - t)} \) and \( \frac{\partial z}{\partial R} \) do not depend on the agent’s type, conditionnal on \( z \).

Proof. The program of agent \((\theta, \sigma)\) reads:

\[
u(x, e_1 + e_2) - \theta \psi(e_2) - \sigma \varphi(z) + (\partial_N w)(N(z) - cx - (\alpha + \tau)e_1 + \alpha e_2)\]

Where \((\partial_N w)\) is the lagrangier multiplier for type \((\theta, \sigma)\) associated with his budget constraint. The solution reads:

\[
u_x = (\partial_N w)c \\
u_e = (\partial_N w)(\alpha + \tau) \\
\theta \psi'(e_2) = (\partial_N w)\tau \\
\sigma \varphi'(z) = (\partial_N w)(1 - t) \\
cx + (\alpha + \tau_1)e_1 + \alpha e_2 = R + (1 - t)z
\]

We look for \( \frac{\partial z}{\partial (1 - t)} \). We compute first orders effect of a compensated variation of \((1 - t)\), we get

\[
u_{xx} dx + u_{xe} de = d(\partial_N w)c \\
u_{ee} de + u_{xe} dx = d(\partial_N w)(\alpha + \tau) \\
\theta \psi''(e_2) de_2 = d(\partial_N w)\tau \\
\sigma \varphi''(z) dz = d(\partial_N w)(1 - t) + (\partial_N w)d(1 - t) \\
cdx + (\alpha + \tau)de - \tau de_2 = (1 - t)dz (U remains constant)
\]

We find that

\[
dz = \frac{(\partial_N w)(\theta \psi''(e_2)(-\alpha^2 u_{xx} - (\alpha + \tau)u_{xx} + 2(\alpha + \tau)cu_{xe}) + \tau^2 H(u)) d(1 - t)}{\sigma \varphi''(z) \theta \psi''(e_2)(-\alpha^2 u_{xx} - (\alpha + \tau)u_{xx} + 2(\alpha + \tau)cu_{xe}) + ((1 - t)^2 \theta \psi''(e_2) + \sigma \varphi''(z)\tau^2) H(u)}
\]

\[
dz = \frac{(\partial_N w)(\theta \psi''(e_2)(-\alpha^2 u_{xx} - (\alpha + \tau)u_{xx} + 2(\alpha + \tau)cu_{xe}) + \tau^2 H(u)) d(1 - t)}{\sigma \varphi''(z) \theta \psi''(e_2)(-\alpha^2 u_{xx} - (\alpha + \tau)u_{xx} + 2(\alpha + \tau)cu_{xe})}
\]

\[
= \frac{(\partial_N w)(\sigma \varphi''(z)) d(1 - t)}{\sigma \varphi''(z)(1 - t) \varphi''(z)} d(1 - t)
\]

Similarly, we compute \( \frac{\partial z}{\partial R} \). We compute first orders effect of a variation of \( R \), we get

\[
u_{xx} dx + u_{xe} de = d(\partial_N w)c \\
u_{ee} de + u_{xe} dx = d(\partial_N w)(\alpha + \tau) \\
\theta \psi''(e_2) de_2 = d(\partial_N w)\tau \\
\sigma \varphi''(z) dz = d(\partial_N w)(1 - t) \\
cdx + (\alpha + \tau)de - \tau de_2 = dR + (1 - t)dz
\]
We get that:

\[ dz = \frac{(\partial N_w) \left[ -(1-t)\theta \psi'' (e_2) H(u) \right] d(1-t)}{\sigma \varphi''(z) \theta \psi'' (e_2)(-c^2 u_{ee} - (\alpha + \tau) u_{xx} + 2(\alpha + \tau) c u_{xe}) + ((1-t)^2\theta \psi'' (e_2) + \sigma \varphi''(z) \tau^2) H(u)} \]

There is no first order effect of the variation of \( R \) on \( z \). \( \square \)

References


**URL:** [http://ideas.repec.org/a/eee/ecolet/v87y2005i1p141-144.html](http://ideas.repec.org/a/eee/ecolet/v87y2005i1p141-144.html)


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