PROBLEM SET 5
Money and fluctuations
Cash-in-advance model

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A Cash-in-advance Model

Objective: we want to study money and business cycle fluctuations.


- Does money and the form of money supply rules affect the nature and amplitude of the business cycle?
- How does *anticipated inflation* affect the nature and amplitude of the business cycle?
Money does not affect the nature and the amplitude of the business cycles when money supply follows a constant growth rule with a positive interest rate.

On the contrary, shocks to money supply have an impact on anticipated inflation by agents and affect the response of labor and consumption through the demand of cash for transactions. No role for sticky prices mechanisms.
The model

Let’s consider an economy populated by a representative household seeking to maximize an intertemporal utility function

\[ E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t - Bh_t), \quad 0 < \beta < 1 \]

depending on consumption \( c_t \) and labor effort \( h_t \), and where \( E_0 \) is the expectation operator with respect to information available at date 0.
The household faces the budget constraint:

\[ c_t + x_t + m_t/p_t \leq w_t h_t + r_t k_t + (m_{t-1} + (g_t - 1) M_{t-1})/p_t \]

where \( x_t \) is the investment, defined by \( k_{t+1} = (1 - \delta) k_t + x_t \), \( 0 \leq \delta \leq 1 \), \( m_t \) the nominal money balances, \( p_t \) a price index, \( w_t \) the real wage, \( r_t \) the real interest rate and \( k_t \) the capital stock (\( k_0 \) given).

Per capita money supply, \( M_t \), evolves according to \( M_t = g_t M_{t-1} \), with \( g_{t+1} = g_t^\alpha \tilde{g}^{1-\alpha} \exp(\xi_{t+1}) \), \( \xi_t \ iid \) with variance \( \sigma_\xi^2 \).

The household faces another constraint on cash good:

\[ p_t c_t \leq m_{t-1} + (g_t - 1) M_{t-1} \]
Give a rationale for a cash-in-advance constraint. What is the basic difference with the money-in-utility-function paradigm?
The cash-in-advance paradigm

There are two main timing conventions

- Lucas: Households invest in the financial market after observing current period shocks but before purchasing cash. This means households can adjust their holdings of cash in the financial market to match their consumption needs.

- Svensson: Goods market opens before the asset market. Households have to purchase goods using cash they carried over from the previous period. Holdings of money are chosen before households observe the current period shock. There is a precautionary motive for holding cash.
**CIA vs MIU models**

The main difference

- **MIU**: households gain utility flows from their money holdings, no matter how they use cash.
- **CIA**: give a description of what people do with their money holdings.
- **Ex. Cooley Hansen (1989)**: if agents want to reduce their cash holdings in presence of a higher interest rate (opportunity cost to hold money \(↑\)), they have to reduce their consumption (because of the CIA constraint).
Question 2

Write the lagrangian for the household problem and derive the first order conditions for an optimal path. The Lagrangian of the problem is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t - Bh_t \\
+ \lambda_t \left( w_t h_t + r_t k_t + \frac{m_{t-1} + (g_t - 1) M_{t-1}}{p_t} \\
- c_t - k_{t+1} + (1 - \delta) k_t - \frac{m_t}{p_t} \right) \\
+ \mu_t \left( \frac{m_{t-1} - (g_t - 1) M_{t-1}}{p_t} - c_t \right) \right]$$
The first-order conditions with respect to $c_t$, $h_t$, $m_t$ and $k_{t+1}$ are then respectively (with the expectation operator implicitly appended):

$$(c_t) \quad \frac{1}{c_t} = \lambda_t + \mu_t$$

$$(h_t) \quad \frac{B}{w_t} = \lambda_t$$

$$(m_t) \quad E_t \left\{ \frac{p_t}{p_{t+1}} \beta (\lambda_{t+1} + \mu_{t+1}) \right\} = \lambda_t$$

$$(k_{t+1}) \quad E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} (r_{t+1} + 1 - \delta) \right\} = 1$$

1. cost of holding one more unit of money (loss in consumption) to its future benefits (larger future consumption and loosened cash constraint)
2. trade-off between labour and leisure
3. asset pricing equation
4. Euler equation
In which variables one can found nominal trend? Propose a method to make the problem stationary.

There is a nominal trend in the optimal level of cash holdings. The problem can be made stationary by defining:

\[ \hat{m}_t = \frac{m_t}{M_t} \quad \text{and} \quad \hat{p}_t = \frac{p_t}{M_t} \]

Recall that per capita money supply, \( M_t \), evolves according to \( M_t = g_t M_{t-1} \), with \( g_{t+1} = g_t^\alpha \bar{g}^{1-\alpha} \exp(\xi_{t+1}) \).
The firm in the economy produces output according to a Cobb-Douglas production function $y_t = \exp(z_t)k_t^\theta h_t^{1-\theta}$, $0 \leq \theta \leq 1$, with $z_{t+1} = \gamma z_t + \varepsilon_{t+1}$, $0 \leq \gamma \leq 1$ and $\varepsilon_t \ iid$ with mean 0 and variance $\sigma_{\varepsilon}^2$. Give the first order conditions for the firm’s problem.

Optimal level of production input are given by

$$w_t = (1 - \theta) \frac{y_t}{h_t}$$

and

$$r_t = \theta \frac{y_t}{k_t}$$
Question 5: compute the steady state

At the steady-state, we have by definition: \( g = \bar{g}, \bar{z} = 0, \bar{m} = 1, \bar{x} = \delta \bar{k} \). Then, from FOCs:

\[
\bar{r} = \frac{1}{\beta} - 1 + \delta
\]

\[
\bar{\nu} = (1 - \theta) \left( \frac{\theta}{\bar{r}} \right)^{\frac{\theta}{1-\theta}}
\]

\[
\bar{\lambda} = \frac{B}{\bar{\nu}}
\]

\[
\bar{\mu} = \frac{1}{\bar{c}} - \bar{\lambda}
\]

\[
\bar{c} = \frac{\beta \bar{g}}{\bar{\lambda}}
\]

\[
\bar{p} = \frac{1}{\bar{c}}
\]

\[
\bar{h} = \frac{\bar{c}}{\bar{\nu} + (\bar{r} - \delta) \left( \frac{\theta}{\bar{r}} \right)^{\frac{1}{1-\theta}}}, \bar{y} = \frac{\bar{\nu} \bar{h}}{1 - \theta}, \bar{k} = \theta \frac{\bar{y}}{\bar{r}}
\]
Simulations

We use the following calibration

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\theta$</th>
<th>$\delta$</th>
<th>$B$</th>
<th>$\gamma$</th>
<th>$\sigma_\epsilon$</th>
<th>$\bar{g}$</th>
<th>$\alpha$</th>
<th>$\sigma_\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.36</td>
<td>0.025</td>
<td>2.86</td>
<td>0.95</td>
<td>0.00721</td>
<td>1.015</td>
<td>0.48</td>
<td>0.009</td>
</tr>
</tbody>
</table>

Question 6
Simulate the model with 50 draws of 115 periods long. After logging and detrending, compute the standard deviation and correlation with output for the following variables: output, consumption, investment, capital stock, hours, productivity, and price level.
## Results

<table>
<thead>
<tr>
<th>Series</th>
<th>Std Dev.</th>
<th>Corr w. Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.79(0.19)</td>
<td>1.00(0.00)</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.66(0.07)</td>
<td>0.70(0.05)</td>
</tr>
<tr>
<td>Investment</td>
<td>5.88(0.58)</td>
<td>0.97(0.01)</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.50(0.09)</td>
<td>0.07(0.05)</td>
</tr>
<tr>
<td>Hours</td>
<td>1.36(0.14)</td>
<td>0.98(0.00)</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.53(0.07)</td>
<td>0.87(0.03)</td>
</tr>
<tr>
<td>Price level</td>
<td>1.94(0.27)</td>
<td>-0.26(0.17)</td>
</tr>
</tbody>
</table>

**Table:** Simulation 1, 50 draws, 115 periods
Redo the simulations with $\sigma_\xi = 0$. Compare with the results in the indivisible labor model of Hansen (1985):

<table>
<thead>
<tr>
<th>Series</th>
<th>$\sigma_\xi \neq 0$</th>
<th>$\sigma_\xi = 0$</th>
<th>Hansen(1985)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.79(0.19)</td>
<td>1.00(0.00)</td>
<td>1.76(0.24)</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.66(0.07)</td>
<td>0.70(0.05)</td>
<td>0.51(0.09)</td>
</tr>
<tr>
<td>Investment</td>
<td>5.88(0.58)</td>
<td>0.97(0.01)</td>
<td>5.72(0.81)</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.50(0.09)</td>
<td>0.07(0.05)</td>
<td>0.47(0.12)</td>
</tr>
<tr>
<td>Hours</td>
<td>1.36(0.14)</td>
<td>0.98(0.00)</td>
<td>1.34(0.17)</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.53(0.07)</td>
<td>0.87(0.03)</td>
<td>0.51(0.10)</td>
</tr>
<tr>
<td>Price level</td>
<td><strong>1.94</strong>*(0.27)</td>
<td>-0.26*(0.17)</td>
<td><strong>0.50</strong>*(0.08)</td>
</tr>
</tbody>
</table>

**Table:** Simulation 2, 50 draws, 115 periods, $\sigma_\xi = 0$
Money does affect the business cycle through the cash-in-advance constraint.

Consumption, investment and price level are more volatile when money supply is stochastic ($\sigma_\xi \neq 0$), but consumption is less correlated with output.

Hansen(1985): most of the fluctuations in worked hours occur at the extensive margin (change in the employment level more than per household worked hours).