Financial Frictions in Production Networks

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Overview

- Business Cycles Studies have placed Financial Frictions:
  - Households (Lorenzoni-Guerrieri, Midrigan-Phillipon)
  - Investment (Bernanke et al., Kiyotaki-Moore)
  - Capital and Labor Reallocation (Eisfeldt-Rampini, Moll)
  - Labor (Jermann-Quadrini, Bigio)
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- This paper:
  - Departs from literature: role of interfirm trade
  - Physical environment: production is organized through network
  - Financial Frictions: interrupt interfirm trade
Motivation

1. Aggregate Production Organized through Highly Complex Network

2. Network Empirics suggest cross-sectoral spill overs
   - Financial Constraints $\rightarrow$ affects supply and demand of firms
   - Foerster et al. (2013)

3. Common theme in Macro-Finance Models
   - Profession in Search of Multipliers
What we will show...

**Theory:**
- Financial Shocks on GDP:
  - Input Misalloc vs. Labor Wedge
- How location of matters:
  - Size vs. Location
- Sensitivity to Aggregate Shocks:
  - Upstream Firms are more sensitive

**Quantitative:**
- Key Parameters and Magnitudes
  - Multiplier of 2.5
- Most Important:
  - Health Care vs. Construction
- Most affected:
  - Durables and Commodities
What we will show...

Theory:
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• Quantitative: 2008-2012 crisis analysis
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What we don’t do

- CES not general, always equals 1
- No dynamics
  - ...equilibrium here is steady-state
- Frictions only on firms...
  - ...easy to generalize to Household Consumption or Investment
Today’s Agenda

- Place Financial Frictions on Production Networks

- Analytic Examples: distill effects

- General Network with Financial Frictions
  - Generalization of Acemoglu et al. (2012) and Jones (2011)
  - Characterization: Production Possibility Frontier and Labor Wedge

- Quantitative Analysis

- Great Recession Application
Model
Production Network Economy

- General input-output Network

- Representative household (Sector 0)

- $N$ production sectors
  - goods are differentiated across sectors
  - each sector consists of a continuum of identical competitive firms.
• Within each $i \in \{1, 2, ..., N\}$

• Production function

\[ y_i = z_i x_i^\kappa_i. \] (1)

• $y_i$ output and $x_i$ intermediate input composite

• Intermediate input composite

\[ x_i = \prod_{j=0}^N x_{ij}^{\alpha_{ij}} \]

• $\alpha_{ij}$: good $j$ share of total intermediate input by firm $i$

  ▶ CRS (wlog): $\sum_{j \in N_i} \alpha_{ij} = 1$
The Firm’s Problem

Problem

\[ \pi_i = \max p_i x_i^{\kappa_i} - \sum_{j=1}^{n} p_j x_{ij} \]

s.t. financial constraint:

\[ \sum_{j=0}^{N} p_j x_{ij} \leq \chi_i p_i y_i \]

intermediate composition

\[ x_i = \prod_{j=0}^{N} x_{ij}^{\alpha_{ij}} \]
The Firm’s Problem

Problem

\[ \pi_i = \max p_i x_i^k - \sum_{j=1}^{n} p_j x_{ij} \]

s.t. financial constraint:

\[ \sum_{j=0}^{N} p_j x_{ij} \leq \chi_i p_i y_i \]

intermediate composition

\[ x_i = \prod_{j=0}^{N} x_{ij}^{\alpha_{ij}} \]

- Where do constraints come from?
Representation of Financial Constraint

- **Financial Constraint:**
  \[\sum_{j=0}^{n} p_j x_{ij} \leq \chi_i p_i y_i\]
  
  - **Microfoundations:**
    - Limited Enforcement (Kiyotaki and Moore 08)
    - Limited Enforcement/Asymmetric Information (Bigio 15)
    - Moral Hazard (Holmstrom Tirole 07)
    - Working Capital + Spread (Christiano et al 14)

- **Cash-In-Advance:**
  \[\sum_{j=0}^{n} p_j x_{ij} \leq M_i\]
  
  - Delivers Multiplicity, \(\chi_i\) is one such solution

- Also: **markups** (Basu 94)
Household - I

- Sector 0 (special) supplies labor endowment
- Preferences:
  \[ U(x_0, y_0) \equiv \frac{x_0^{1-\gamma}}{1-\gamma} - \frac{y_0^{1+\nu}}{1+\nu}. \]
  - \( x_0 \) consumption basket
  - \( y_0 = L \) is labor supply
- \( \sigma = 0 \) no income effect GHH
- \( \nu \to \infty \) labor endowment case
Household -II

- Consumption Basket:
  \[ x_0 = \prod_{j=1}^{N} x_{0j}^{\alpha_{0j}} \]

- Budget constraint:
  \[ \sum_{j=1}^{N} p_j x_{0j} = p_0 y_0 + \sum_{i=1}^{N} \pi_i \]

- Note:
  - No frictions on household
  - Add (artificial) final goods sector
Market Clearing

- **Commodities**

\[ y_i = x_{0i} + \sum_{j=1}^{n} x_{ji} \]

- **Labor**

\[ \sum_{i=1}^{n} x_{i0} = y_o = L \]
Definition

A **competitive equilibrium** is
a collection of prices and quantities
\[
\{\{p_j, l_j, x_j\}_{j=1:N}, \{x_{i,j}\}_{i=1:N, j=1:N}, L, x_0\}
\] such that:

(i) firms maximizes profits **subject to liquidity constraint**

(ii) households maximize utility

(iii) markets clear
Three Examples
1. Organization of Production Matters for Diffusion of Shocks

2. Two Forces:
   - Input Misallocation
   - Labor Wedge

3. Predominance of Forces depends on Network Architecture
   - Propagation of shocks
Consider three economies

1. Pure Production Chain (c)
   - Single Input Path

2. Vertical Economy (v)
   - Single Path for Commodities - All Sectors use Labor

3. Horizontal Economy (h)
   - Firms operating in isolation

4. \( \{c, v, h\} \equiv E \)
Chain Economy

Figure: Vertical Economy - with use Labor
Chain Economy

- Three firms only.

- Downstream firms only use commodity

\[ x_{21} = y_1 = z_1 L^{\kappa_1} \]

\[ x_{32} = y_2 = z_2 x^{\kappa_2}_{21} \]

\[ x_{03} = y_3 = z_3 x^{\kappa_3}_{32} \]
Chain Economy

- Three firms only.

- Downstream firms only use commodity

\[ x_{21} = y_1 = z_1 L^{\kappa_1} \]
\[ x_{32} = y_2 = z_2 x_{21}^{\kappa_2} \]
\[ x_{03} = y_3 = z_3 x_{32}^{\kappa_3} \]

- Household Consumption:

\[ x_0 = x_{03} \]
Figure: Vertical Economy - with use Labor
Vertical Economy

Figure: Vertical Economy - with use Labor
Vertical Economy

- Labor used throughout production chain...
- Then endowment of upstream firm:

\[
x_{21} = y_1 = z_1 l_1^{\kappa_1}
\]

\[
x_{32} = y_2 = z_2 (l_2^{\alpha_20} x_{21}^{\alpha_21})^{\kappa_2}
\]

\[
x_{03} = y_3 = z_3 (l_3^{\alpha_30} x_{32}^{\alpha_32})^{\kappa_3}
\]
Vertical Economy

- Labor used throughout production chain...
- Then endowment of upstream firm:

\[ x_{21} = y_1 = z_1 l_1^{k_1} \]

\[ x_{32} = y_2 = z_2 (l_2^{\alpha_{20} x_{21}^{\alpha_{21}}})^{k_2} \]

\[ x_{03} = y_3 = z_3 (l_3^{\alpha_{30} x_{32}^{\alpha_{32}}})^{k_3} \]

- Labor resources:

\[ y_0 = L = \sum_{i=1}^{3} l_i \]

- Household Consumption:

\[ x_0 = x_{03} \]
Vertical with Labor Uses

Figure: Vertical Economy - with use Labor
Horizontal Economy

Figure: Horizontal Economy
Horizontal Economy

- Only labor input

\[ x_01 = y_1 = z_1 l_1^{K_1} \]

\[ x_02 = y_1 = z_2 l_2^{K_2} \]

\[ x_03 = y_1 = z_3 l_3^{K_3} \]
Horizontal Economy

- Only labor input

\[ x_{01} = y_1 = z_1 l_1^k \]

\[ x_{02} = y_1 = z_2 l_2^k \]

\[ x_{03} = y_1 = z_3 l_3^k \]

- Labor resources:

\[ y_0 = L = \sum_{i=1}^{3} l_i \]

- Household Consumption:

\[ x_0 = \prod_{j=1}^{3} x_{0j}^{a_{0j}} \]
Frictionless Benchmark

• Assume $\chi_i = 1$
Frictionless Benchmark

- Assume $\chi_i = 1$
- Application of Neoclassical Production:
Frictionless Benchmark

- Assume $\chi_i = 1$
- Application of Neoclassical Production:

Corollary 1.
- Parameter restrictions: same production set
- Same frictionless equilibrium $(x_0, L)$

Summary: Organization of Production irrelevant
Frictionless Benchmark

- Assume $\chi_i = 1$
- Application of Neoclassical Production:

**Proposition**

*All $e \in \mathcal{E}$ have *Technological-Production Possibility Frontier:*

\[
\text{Output} = x_0 = \bar{z}_e \tilde{\kappa}_e
\]

**Corollary**

1. *Parameter restrictions: same production set*
2. *Same frictionless equilibrium $(x_0, L)$*

- Summary: Organization of Production *irrelevant*
Equilibrium with Frictions

- Assume $\chi_i \leq 1$

- Firms face liquidity constraint
  
  expenditure on inputs $\leq \chi$ revenue

- Define distance to FOC:

  $$\phi_i = \min \left\{ 1, \frac{\chi_i}{\kappa_i} \right\}$$
Inefficient Outcome
Equilibrium with Frictions

- Adjustment to Neoclassical Production

**Proposition**

All $e \in \mathcal{E}$ have **Financially Feasible-Production Possibility Frontier**:

$$\text{Output} = x_0 = \bar{z}_e \zeta(\phi, e) L^\kappa_e$$

*input misalloc*

**Remark**

*Misallocation* $\zeta(\phi, e)$ depends on network

- Organization of Production **Does Matter**! How?
- Location of Shocks **Does Matter**! Where more?
Proposition

(i) Chain economy (c) PPF

\[ \zeta(\phi, c) = 1 \]

(ii) Vertical economy (v) PPF

\[ \zeta(\phi, v) = \frac{\prod_{i=1}^{3} \prod_{j=i}^{3} \phi_j \prod_{j=i}^{3} \alpha_{j+i+1} \alpha_{0i} \kappa_j \kappa_i}{(1 - \sum_{i=1}^{3} \alpha_{0i}(1 - \phi_i))^{\bar{\kappa}_v}} \]

(iii) Horizontal economy (h) PPF

\[ \zeta(\phi, h) = \frac{\prod_{i=1}^{3} \phi_i^{\kappa_i}}{(1 - \sum_{i=1}^{3} \alpha_{0i}(1 - \phi_i))^{\kappa_h}} \]
Financially-Feasible PPF

1. Chain economy, $\phi$ no effect on $\zeta_c(\phi)$.

2. Vertical economy, $\phi$ have different effects on $\zeta_v(\phi)$.
   ▶ Big effect if same shock!

3. Horizontal $\phi$ have symmetric effects on $\zeta_h(\phi)$
   ▶ No effect if same shock!
Aggregate Labor Wedge
Aggregate Labor Wedge

- So far: only changes in production given L
- What about labor supply?

Definition

The aggregate labor wedge is given by

$$(1 - \tau_e) \tilde{\kappa} x_o L = V'(L) U'(x_o)$$

This wedge is important in explaining recessions.

Chari, Kehoe, McGrattan (2007)

In frictionless economy, $\tau_e = 0$. 

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Aggregate Labor Wedge

• So far: only changes in production given L

• What about labor supply?

Definition
Aggregate labor wedge \((1 - \tau_e)\) satisfies

\[
(1 - \tau_e) \frac{\kappa x_o}{L} = \frac{V'(L)}{U'(x_o)}
\]

• Aggregate labor wedge is important in explaining recessions
  
  Chari, Kehoe, McGrattan (2007)

• In frictionless economy, \(\tau_e = 0\).
1. Chain economy: $\phi$ have non-zero and equal effects on wedge.
2. Vertical: $\phi_3$ has the greatest effect and $\phi_1$ has the smallest effect.
3. Horizontal: $\phi$ symmetric effects individually.
Equilibrium Allocation

- Overall effect on output depends on:

- Effect on FF-PPF vs. Labor Wedge
  - Overall effect depends on \((\gamma, \sigma)\)
Equilibrium Output

Proposition

Output depends on labor wedge and PPF movements:

\[ x_{0,e} = \left[ \bar{\kappa}_{e} \left( 1 - \tau_{e} \left( \phi \right) \right) \right]^{\frac{\bar{\kappa}}{1 - \bar{\kappa}(1 - \sigma) + \nu}} \left( \bar{z}_{e} \zeta_{e} \left( \phi \right) \right)^{\frac{1 + \nu}{1 - \bar{\kappa}(1 - \sigma) + \nu}} \]
Effect on Output

1. Chain Economy: only operates through labor wedge

2. Horizontal Economy: wedge + asymmetric shocks move feasible production set

3. Vertical Economy: compounds effects
Vertical Economy - Intuition

1. The more vertical transactions
   → greater effects of liquidity?
   → more liquidity needed for same allocation

2. In the vertical economy there are spill-over effects
   → works like an adverse demand shock for firms $j < i$
   → works like an adverse supply shock for firms $j > i$.

3. Compounded Effects!

4. Chain: no input distortion (only one route)

5. Horizontal: no amplification through chain, only relative prices
Summary of Analytic Examples
We have shown

- Network Structure Matters
  - Distortions on Real Wage
  - Distortions on Input Use
Back to General Network...
Recall Production

- Production:
  \[ x_i = z_i \left( \prod_{j=0}^{n} x_{ij}^{\alpha_{ij}} \right)^{\kappa_i}. \]

- \( \alpha_{ij} \): good \( j \) share of total intermediate input by firm \( i \).

- CRS assumption: \( \sum_{j \in N} \alpha_{ij} = 1 \)

- Firm solves:
  \[ \pi_i = \max p_i x_i^{\kappa_i} - \sum_{j=0}^{N} p_j x_{ij} \]
  subject to financial constraint
  \[ \sum_{j=0}^{N} p_j x_{ij} \leq \phi_i \kappa_i p_i x_i \]
Input-output matrix $\alpha_{ij}$:

\[
A = \begin{bmatrix}
\alpha_{01} & \alpha_{02} & \cdots & \alpha_{0N} \\
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1N} \\
\alpha_{21} & \alpha_{22} & \ddots & \\
\vdots & \vdots & \ddots & \alpha_{N1} \\
\alpha_{N1} & \cdots & \cdots & \alpha_{NN}
\end{bmatrix}
\]
Key Objects

- Vector of DRS ($\kappa_i$):

$$\kappa_I = \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ \vdots \\ \kappa_N \end{bmatrix}$$

- Vector of financial shocks ($\phi_i$)

$$\phi_I = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix}$$

- $e$ is unitvector
Problem (Optimal Inputs given Uses)

\[ \pi_i(U_i) = \max_{x_{ij}} p_i y_i - U_i \]
subject to:

\[ x_i = \prod_{j \in I_i} x_{ji}^{\alpha_{ij}} \]
\[ \sum_{j \in I} p_j x_{ij} \leq U_i \]

Lemma (Solution Optimal Input Use)

\[ p_j x_{ij} = \alpha_{ij} U_i \]
\[ y_i = z_i \left( \prod_{j \in I_i} \left( \frac{\alpha_{ij}}{p_j} \right)^{\alpha_{ij}} U_i \right)^{\kappa_i} \]
Separation of Input-Choice and Scale of Production

- Sector i’s uses \( u_i = \sum_{j=0}^{N} p_j x_{ij} \)
- Sector i’s revenues \( g_i = p_i y_i \)

Lemma

Total expenditure and total revenue:

\[ u_i = \phi_i \kappa_i g_i \]
Solving it

1. Obtain Gross Revenue as function of L
2. Obtain Prices from Gross Revenue and Firm FOC
3. Back-out GDP from Prices and Gross Revenues
4. Obtain equilibrium L

Modify to allow non-unit CES
Equilibrium - Revenues and Expenditures

- Revenues and Expenditures Connection:

\[ G_i = p_i y_i = p_i \sum_{j \in N} x_{ji} = \sum_{j \in N} \alpha_{ji} U_j \]

- In Vector form:

\[ G = (U'A)' = A'U. \] (2)

where:

\[ U_i = \phi_i \circ \kappa_i \circ G_i. \]

and \( \circ \) is Haddamard Product

- For Household:

\[ U_0 = L + \sum_{i \in I} G_i (1 - \phi_i \alpha_i) \]
Equilibrium sales:

**Proposition**

$L$ given, $G_I$ solves:

$$G^* = \left[ I - A' \circ e [\phi_I \circ \kappa_I]' - \alpha_0 (e - \phi_I \circ \kappa_I)' \right]^{-1} \alpha_0 L$$

**Financial influence vector:** $G^* = v(\phi)L$
Revenue Fixed Point

- Equilibrium sales:

**Proposition**

$L$ given, $G_I$ solves:

$$G^* = \left[ \mathbb{I} - A' \circ e [\phi_I \circ \kappa_I]' - \alpha_0 (e - \phi_I \circ \kappa_I)' \right]^{-1} \alpha_0 L$$

**Financial influence vector:** $G^* = v(\phi)L$

- Vector $v(\phi)$ determines influence on gross output
- Note this is independent of TFP!
Prices

- Definition

\[ G_i = p_i y_i = z_i \max_{x_{ij}} \left( \prod_{j \in I_i} \left( \frac{\alpha_{ij}}{p_j} \right)^{x_{ij}} \right)^{\kappa_i} \]

- In vector of logs:

\[ \tilde{G}_I = \tilde{p} + \tilde{z} + \kappa_I \circ \left( \text{diag} \left( A \tilde{A}' \right)_I - A_I \tilde{p} + \tilde{U}_I \right) \]

Proposition

Equilibrium log-prices are:

\[ \tilde{p} = -\left[ I_N - A \circ \kappa_I e' \right]^{-1} \left[ \tilde{z} - \tilde{G} + \kappa_I \circ (\phi + \kappa_I + \tilde{G}) + \kappa_I \circ \text{diag} \left( A \tilde{A}' \right) \right] / \tilde{v} \]

Physical influence vector is \( \tilde{v} \).
Solving for the General Financial Feasible PPF

- So far know: \((\tilde{G}, \tilde{p})\)

- **Sectoral Gross Outputs:** \(\tilde{y} = \tilde{G}_I - \tilde{p}\)

- **Value Added:**
  \[
  \tilde{x}_o = \tilde{U}_0 - \log \tilde{p}_o
  \]

- **Value Added:**
  \[
  U_0 = \left(1 + (e - \phi \circ \kappa_I)' v(\phi)\right) L
  \]
  
  Markups on Income
The FF-PPF

Proposition

The financially-feasible production possibility frontier is:

\[ \tilde{x}_0 = \tilde{v}'\tilde{z} \]

Effects of TFP Shocks

\[ + \log \left( 1 + (e - \phi \circ \kappa_I)' v^I (\phi) \right) + \tilde{v}' \left( \kappa_I \circ \tilde{\phi} - (e - \kappa) \circ \tilde{v}^I (\phi) \right) \]

Effects of Financial Shocks

\[ + \left[ 1 - \tilde{v}' (e - \kappa) \right] \tilde{L} \]

Effects on hours

Technology Constant
The FF-PPF

Remark

Note output is of the form:

\[ \tilde{x}_0 = A(\phi, z) + B\tilde{L} + C. \]

or in levels PPF:

\[ x_0 = \tilde{z} \zeta(\phi, e) L^{\bar{\kappa}} \]
Relationship with Networks literature

- Acemoglu-Carvalho et al. (ECMA, 12)
- Physical Influence Vectors determines Diffusion of Resources:
  \[ \tilde{v}' \hat{Z} \]
- Where does Physical Influence Vectors come from?
  \[ \tilde{v}' = \left[ \mathbb{I}_N - A \circ \kappa e' \right]^{-1} \]
Relationship with Networks literature

• Acemoglu-Carvalho et al. (ECMA, 12)

• Physical Influence Vectors determines Diffusion of Resources:
  \( \tilde{v}' \tilde{z} \)

• Where does Physical Influence Vectors come from?
  \( \tilde{v}' = \left[ I_N - A \circ \kappa_1 e' \right]^{-1} \)

• Diffusion of Financial Shocks:

\[
\log \left( 1 + (e - \phi \circ \kappa_I)' v^I (\phi) \right) + \tilde{v}' \begin{pmatrix}
\kappa_I \circ \tilde{\phi} & (e - \kappa_I) \circ \tilde{\phi}^I (\phi) \\
\text{Prod. Scale} & \text{Input Distortion}
\end{pmatrix}
\]

Household Income
Labor Decision

- So far, L exogenous
- Otherwise:

\[ \tilde{L} = -\frac{\sigma}{\nu} \tilde{U}_0 - \left( \frac{1-\sigma}{\nu} \right) \log \bar{p} \]

- Recall PPF:

\[ \tilde{x}_0 = A(\phi, z) + B\tilde{L} + C. \]

**Proposition (Output)**

\[ \tilde{x}_0 = \frac{(A(\phi, z) + C)(1+\nu) - [1 - \bar{v}'(e - \kappa_I)] A_1(\phi)}{\nu + \sigma + (1-\sigma) \bar{v}'(e - \kappa_I)} \]
Quantitative Analysis
Quantitative Motivation

- Why look at financial frictions on networks?

- Motivation
  - Different Network Structure $\rightarrow$ Different Liquidity Multiplier
  - Important Multiplier
Quantitative Challenges for the Literature

- Investment
  - $I/K$ is not large. Even if we shut down $I$ completely
    Lower bound on output loss: $(1 - \delta)^{1/3}$
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  - Flows in Eisfeldt-Rampini are small
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- **Intermediate Inputs**
  - **This paper**
  - Jones (2009 and 2011) for growth
Quantification of Network Effects

1. Calibrate using US Input-Output matrix
   - Spirit of Jones (2011) but applied to Great Recession
2. Ask how big effects are and why?
3. Ask what sectors are more vulnerable and cause vulnerabilities?
4. Recession Analysis
5. Are we measuring Financial Frictions
The Numbers

- Frisch elasticity of labor 2, CRRA 2.
- Data Source: Input-Output tables (BEA)
  - 2, 3 and 4 digits IO matrix,
  - 12, 65 and 512 industries
  - years: 1997-2013
  - Consumption Shares:

\[ \hat{\alpha}_{0j}^{Data} = \frac{p_j x_{0j}}{\sum_{n=1:N} p_n x_{0n}} = \frac{u_{0j,t}}{\sum_{n=1:N} u_{0n,t}} \]

- \( u_{on,t} \) use of commodity \( n \) for final goods
The Numbers

- Cannot identify \((\alpha_j, \phi_j)\) independently. But can obtain

  \[ \eta_{i,t} = 1 - \text{Profit Share}_i. \]

- Jones (2011) writes:
  
  “There is a fundamental identification problem: we see data on observed intermediate goods shares, and we do not know how to decompose that data into distortions and differences in technologies. This identification problem is not solved in anything I have done.”

- Our strategy:
  
  \( \rightarrow \) movements in \( \eta_{i,t} \) over Great Recession due to \( \phi_j \)

- Average to obtain labor share
Validating our Assumption?

• Are measures $\phi_j'$s associated with sectoral output drops?
• Variable $\phi_j$ is effectively markups
• (TEST) Are measured $\phi_j'$s related to:
  1. ...financial expenses?
  2. ...bonds spreads?
  3. ...firm-specific financial data?
  4. ...Rajan-Zingales measures
Measured Markups

- How measured $\phi_j$'s look?
• Are measures $\phi_j$'s associated with sectoral output drops?

![Graph showing the relationship between $\Delta \%$ in Industry Output vs. $\phi_t - \phi_{2006}$](image-url)
RJ - Regression

- Take Rajan-Zingales Measure of Financial Dependence...
- Correlated with markups during recession?
• Take Rajan-Zingales Measure of Financial Dependence...
• Correlated with markups during recession?
Suppose liquidity is drawn down by in 1% in every sector.

How does the response in US output compare to a horizontal economy?

<table>
<thead>
<tr>
<th>Model</th>
<th>Labor Market</th>
<th>Horizontal</th>
<th>3AR</th>
<th>5AR</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Wealth</td>
<td>ν = 2</td>
<td>-1.99%</td>
<td>-5.3%</td>
<td>-5.5%</td>
<td>-5.44%</td>
</tr>
<tr>
<td>Wealth</td>
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<td>-.50%</td>
<td>-1.1%</td>
<td>-1.3%</td>
<td>-1.11%</td>
</tr>
<tr>
<td>PPF</td>
<td></td>
<td>0%</td>
<td>-0.82%</td>
<td>-0.87%</td>
<td>-0.75%</td>
</tr>
</tbody>
</table>

Critical Parameter: Decreasing Ret Scale, Preferences
Question 2: Vulnerable Sectors

• Suppose we tighten credit constraints by 1% in all sectors. Which sector has the greatest decline in output?

• Similar to Vertical Economy

• Greatest decline: Upstream Manufacturing sectors
  ▶ Metals, chemicals, etc.

• Lowest decline: Retail, Hospitals
Question 2: Vulnerable Sectors

Sectoral Responses (Selected 3 Digit Level)

Highest Sensitivity Sectors

Lowest Sensitivity Sectors

% Sector Output

Prime Mets
Chemicals
Fab Mets
Paper
Plst & Rubb
Pet and Coal Pry
Oil Gaz Ext
Textiles
Wood
Motors

Sectors: Merchant St

Social Ass

Supermarkets

Auto Deals

FED GOV

Amb Health

Recreation

Mine Support

Oth Retail

Lawyers
Question 3: Influential Sectors

• Suppose we tighten each sector’s credit constraint individually. Which sector has the greatest effect on aggregate output?

• Answer: Gov and Hospitals (size), Construction (location), Wholesale (both)

• As theory predicts—most downstream sector
Question 3: Most Influential Sectors

Sectoral Responses (3 Digit Level)
Question 3: Influential Sectors

• Suppose we tighten each sector’s credit constraint proportionally to their sales?

• More bang for buck?

• Answer: motors industry
Question 3: Most Influential Sectors

Aggregate Effect of Sector Shock (5 Digit Level)

% Output Drop Relative to Sector Sales

- Cigarettes
- Distilleries
- Animals Prod.
- Oilseed
- Biological Prods
- FED Gov.
- Wild Animals
- Book Publishers
- Oil and Gas
- Gambling Industries
- Trailer
- Other Motor Parts
- Kitchen Cabinets
- Cars
- House Furniture
- Optical Instruments
- Car Seating and Interior
- Heavy Trucks
- Motor Homes
- Vehicle bodies

0 500 1000 1500
The Great Recession
Question 4: Liquidity Drop to Generate Great Recession

• How much would aggregate $\phi$ have to tighten to explain GR?

• Answer: Liquidity would have to drop by 1.3% of sales on average at trough
Question 4: Liquidity Drop to Generate Great Recession
Question 5: Key Sectors During Great Recession

Sectoral Responses (3 Digit Level)
Conclusion

- Effect of credit constraints amplified through network structure
- Can cause Big Recession but hard to even notice them
- Pending Work: identify increase in wedges with Financial Constraints
- Extension: Dynamics and Concentration of Liquidity