Microeconometric Search-Matching Models and Matched Employer-Employee Data

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Outline of the talk

1. **WAGE DISPERSION**
   - Matched employer-employee data and the variance of wages.
   - The contribution of structural job search models.
     [Here, a lot of research has been done and we begin to know a few things.]

2. **WAGE DYNAMICS**
   - Worker panels and the persistence of income shocks.
   - The contribution of structural job search models.
     [Here, research is just taking off and we still know very little.]

3. **FOR THE FUTURE...**
   - Stuff about which we know next to nothing
     [and to which job search theory might be usefully applied.]
Part I: 
WAGE DISPERSION
Matched Employer-Employee Data

In the past ten years, after Abowd, Kramarz and Margolis’s (*Econometrica*, 1999) initial push, many matched employer-employee datasets have been constructed in the US, Denmark, Italy, Sweden, Austria, etc, to estimate wage equations.

They are Matched Panel Data (MPD) with the following basic structure:

- One panel of workers with individual index $i \in \{1, \ldots, I\}$ and time index $t \in \{1, \ldots, T\}$,
- One panel of firms with individual index $j \in \{1, \ldots, J\}$ and same time index $t \in \{1, \ldots, T\}$,
- A matching function $j(i, t) \in \{1, \ldots, J\}$ defining worker $i$’s employer at time $t$.

They usually are register data (employer payroll reports collected for tax purposes).

Matching wage register data with firm data (accounting data like EBIT, book value, etc.), feasible by availability of firm ID, is not always done. Matching with other individual social security or health insurance data is realised in some countries (Denmark, Sweden).
Basic model used by AKM is an error-component model with firm fixed-effects:

\[
\begin{align*}
    w_{it} &= x_{it}\beta + \psi_{j(i,t)} + \alpha_i + u_{it} \\
    &= x_{it}\beta + \sum_{j=1}^{J} \psi_j d_{it}^j + \alpha_i + u_{it},
\end{align*}
\]

where:

- \( w_{it} \) is individual log wage,
- \( x_{it} \) is a vector of time-varying individual characteristics (experience, tenure),
- \( d_{it}^j, j \in \{1, \ldots, J\} \), are indicator variables:
  \[
  d_{it}^j = \begin{cases} 
  1 & \text{if } j(i,t) = j \\
  0 & \text{otherwise}
  \end{cases}
  \]
- AKM estimate \( \beta \), person effects \( \alpha = (\alpha_1, \ldots, \alpha_N) \) and firm effects \( \psi = (\psi_1, \ldots, \psi_J) \) by OLS.
Consistency of OLS

- OLS estimator of $\psi$ consistent for fixed $J$, large $I$, if worker assignment to firms is strictly exogenous:

$$d_{it}^j \perp u_{it}, \forall i \in \{1, \ldots, N\}.$$  

[Acceptable if $u_{it}$ is transitory.]

- OLS estimator of $\alpha$:

$$\hat{\alpha}_i = w_i \cdot x_i \cdot \hat{\beta} - \sum_{j=1}^{J} \hat{\psi}_j d_{i\cdot}^j.$$  

consistent for large $T$.

**Conclusion:** If too few individuals, too few time periods and too little mobility:

- Imprecise fixed effect estimates;
- Spurious negative correlation between $\hat{\alpha}_i$ and $\hat{\psi}_{j(i,t)}$ (across individuals).
### Correlations of Components of Real Annual Wage Rates


#### France 1976-1996

<table>
<thead>
<tr>
<th></th>
<th>$StD.$</th>
<th>$\ln w$</th>
<th>$x \beta$</th>
<th>$x = z \gamma + \theta$</th>
<th>$z \gamma$</th>
<th>$\theta$</th>
<th>$\psi$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log real annual wage rate</td>
<td>0.9772</td>
<td>1.0000</td>
<td></td>
<td></td>
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<tr>
<td>Experience</td>
<td>0.4087</td>
<td>0.5377</td>
<td>1.0000</td>
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<td></td>
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</tr>
<tr>
<td>Person effect</td>
<td>0.5217</td>
<td>0.4569</td>
<td>0.0698</td>
<td>1.0000</td>
<td>0.2917</td>
<td>1.0000</td>
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</tr>
<tr>
<td>Schooling</td>
<td>0.1522</td>
<td>0.1510</td>
<td>-0.0469</td>
<td>0.2917</td>
<td>1.0000</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Unobservable</td>
<td>0.4990</td>
<td>0.4316</td>
<td>0.0872</td>
<td>0.9565</td>
<td>0.0000</td>
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<tr>
<td>Firm effect</td>
<td>0.4665</td>
<td>0.4287</td>
<td>0.1670</td>
<td>-0.2225</td>
<td>0.0293</td>
<td>-0.2415</td>
<td>1.0000</td>
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<tr>
<td>Residual</td>
<td>0.5545</td>
<td>0.5675</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.0000</td>
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</table>

#### US 1990-1999

<table>
<thead>
<tr>
<th></th>
<th>$StD.$</th>
<th>$\ln w$</th>
<th>$x \beta$</th>
<th>$x = z \gamma + \theta$</th>
<th>$z \gamma$</th>
<th>$\theta$</th>
<th>$\psi$</th>
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<tbody>
<tr>
<td>Log real annual wage rate</td>
<td>0.8941</td>
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<td>Experience</td>
<td>0.6965</td>
<td>0.2305</td>
<td>1.0000</td>
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<tr>
<td>Person effect</td>
<td>0.8434</td>
<td>0.4871</td>
<td>-0.6085</td>
<td>1.0000</td>
<td>0.2748</td>
<td>1.0000</td>
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<tr>
<td>Schooling</td>
<td>0.2317</td>
<td>0.1733</td>
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<td>0.2748</td>
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<td>Unobservable</td>
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<td>0.9615</td>
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<td>0.0635</td>
<td>0.0445</td>
<td>0.0824</td>
<td>0.0228</td>
<td>1.0000</td>
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<tr>
<td>Residual</td>
<td>0.3614</td>
<td>0.4042</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

- **Strong evidence of both firm- and worker-effects.** The Law of One Price does not hold in the labour market.
We use French wage register data matched with firm accounting data.

Regress log wages $w_{it}$ on employer’s mean log productivity (measured by mean log value-added per worker):

$$w_{it} = x_{it}\beta + \alpha_i + \gamma \bar{y}_j(i,t) + u_{it},$$

where $\bar{y}_j = \frac{1}{T} \sum_{t=1}^{T} y_{jt}$.

<table>
<thead>
<tr>
<th>France, 1990-2000</th>
<th>St.D.</th>
<th>ln $w_{it}$</th>
<th>$x_{it}\beta$</th>
<th>$\alpha_i$</th>
<th>$\gamma \bar{y}_j(i,t)$</th>
<th>$u_{it}$</th>
</tr>
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<tbody>
<tr>
<td>Log real annual labor cost ($w_{it}$)</td>
<td>0.477</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quartic in age ($x_{it}\beta$)</td>
<td>0.077</td>
<td>0.139</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Person effect ($\alpha_i$)</td>
<td>0.420</td>
<td>0.888</td>
<td>-0.029</td>
<td>1.000</td>
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<tr>
<td>Firm effect ($\gamma \bar{y}_j(i,t)$)</td>
<td>0.022</td>
<td>0.290</td>
<td>0.047</td>
<td>0.269</td>
<td>1.000</td>
<td></td>
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<tr>
<td>Residual ($u_{it}$)</td>
<td>0.204</td>
<td>0.428</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Sample: all private sector employees, 20-50 in initial year.
Equilibrium search models offer a natural framework in which to analyse this multiform wage dispersion.

These models rest upon two basic principles:

- Labor market competition between employers is the fundamental determinant of wages.
- Competition is limited by search frictions, meaning any information imperfection on job offers giving employers market power.

This framework encompasses two extreme cases:

- Competitive wages. When workers can freely force employers into competition, workers get paid their marginal productivity.
- Monopsony wages. When the cost of finding alternative employers is infinite, firms offer unemployed workers their reservation wages (Diamond, *JET*, 1971).
Basic Assumptions

- The labour market has a unit-mass of homogeneous, infinitely lived workers.
- Time is continuous.
- Unemployed workers sample job offers sequentially at some exogenous Poisson rate $\lambda_0 > 0$.
- There is on-the-job search. Job offers also accrue to employed workers at a rate $\lambda_1 > 0$.
- Firm-worker matches are dissolved at rate $\delta > 0$. Upon match dissolution, the worker becomes unemployed.
- Each job offer is a wage draw $w$ from a sampling distribution $F$. Any wage draw from $F$ is preferred to unemployment.
- Job offer of $w$ is preferred to job offer of $w'$ iff $w > w'$. 
The Steady-State Hypothesis

- **Unemployment rate**, $u$. Equal flows in and out of stock of unemployed workers:

  \[ \lambda_0 u = \delta (1 - u) \quad \iff \quad u = \frac{\delta}{\delta + \lambda_0}. \]

- **Cross-sectional wage distribution**, $G$. Equal flows in and out of steady-state stock of employees paid less than $w$, $(1 - u) \, G(w)$:

  \[ [\text{out}] \quad [\delta + \lambda_1 \, \overline{F}(w)] (1 - u) \, G(w) = \lambda_0 u F(w) \quad [\text{in}] \]

  \[ G(w) = \frac{F(w)}{1 + \kappa \, \overline{F}(w)} \quad \text{or} \quad F(w) = \frac{1 + \kappa \, G(w)}{1 + \kappa \, \overline{G}(w)}, \]

  where $\kappa = \frac{\lambda_1}{\delta}$ is the average number of job offers that a worker receives between two job destruction shocks (index of search frictions), $\overline{F} = 1 - F$ and $\overline{G} = 1 - G$. 
Unemployment Rates From Stocks and Flows
(French LFS and US CPS)

From stocks: Fraction of unemployed in March of year t
From flows: t/t+1 job destruction rate divided by t/t+1 job destruction rate plus t/t+1 re-employment rate
Rates are computed by comparing the state in March of year t+1 to the state in March of year t.
Notes:

- Uses European Community Household Panel.
- Estimates employees’ wage density $g = G'$ nonparametrically from a cross-section of employees’ wages.
- Estimates wage offer density $f = F'$ nonparametrically from the flow of recently employed workers.
- Estimates $\kappa$ from either spell duration data or from the steady-state $F$-$G$ relationship:

$$\kappa \equiv \frac{F(w) - G(w)}{G(w) F(w)}.$$

- Predicts $f$ from $g$ using the above formula (in differentiated version).
Wage Posting Models

Burdett and Mortensen (*IER*, 1998)

- Posting wages: firms decide ex ante what wage to offer, then commit to a given wage offer.
- Employers do not counter outside offers.
- Noncooperative Nash equilibrium.

**Burdett-Mortensen important result:**
The equilibrium wage offer distribution $F$ is a continuous, nondegenerate distribution even if all workers and firms are identical.

[Idea of a proof: If $w$ were a mass point, any firm offering $w$ would profit from offering slightly more because the inflow of extra workers attracted from other firms through on-the-job search would more than compensate the marginal loss in profit per worker.]
BM Model with Heterogeneous Productivity

- BM show that if all firms have equal productivity $p$ and all workers have the same reservation wage $\phi$, then the equilibrium wage offer density is increasing over $[\phi, p]$.
- **Hence**: productivity heterogeneity necessary to fit wage data.
- Let $\Gamma(p)$ denote the sampling distribution (cdf) of productivity (upon reception of a job offer).
- **Equilibrium** of the wage posting game is such that any firm with $p \in \text{Supp}(\Gamma)$ offers $w(p)$ so as to maximise steady-state profit flow:

$$w(p) = \arg \max_w (p - w) \ell(w),$$

where $\ell(w)$ is steady-state employment:

$$\ell(w) = \frac{(1 - u) g(w)}{f(w)} = \frac{(1 - u)(1 + \kappa)}{[1 + \kappa F(w)]^2}.$$

- Optimality condition yields a differential equation that is solved subject to the condition: $F(w) = \Gamma(p)$. 

Postel-Vinay, Robin  
- Bowlus, Kiefer, Neumann (*JAE*, 1995, *IER*, forth.) assume a discrete distribution of productivity (very cumbersome if many support points).


- BRVdB (*IER*, 1999) have both heterogeneous productivity and heterogeneous leisure costs (but same job offer arrival rate for employed and unemployed).

- Christensen, Lentz, Mortensen, Neumann, Werwatz (*JOLE*, 2005) estimate an extension of the BM model with endogenous search intensity (declines with wage). The estimation technique draws on BRVdB.
BM Model—Estimation Procedure
Bontemps, Robin, and Van den Berg (*IER*, 2000)

1. Estimate $G$ nonparametrically;

2. Maximize likelihood of workers’ turnover and wage mobility, treating $F$ as a nuisance parameter preestimated as

\[
\hat{F}(w; \kappa) = \frac{(1 + \kappa) \hat{G}(w)}{1 + \kappa \hat{G}(w)};
\]

3. Estimate $\Gamma$ as $\hat{\Gamma}(p) = \hat{F}[w(p); \hat{\kappa}]$ where $w(p)$ satisfies the maximal profit condition:

\[
\begin{cases}
(p - w) \ell'(w) - \ell(w) = 0 \\
\ell(w) = \frac{(1 - u)(1 + \kappa)}{[1 + \kappa \hat{F}(w)]^2}.
\end{cases}
\]
BM Model—Main Empirical Lesson

- BM models fits turnover and wage distributions well.

- But estimated distribution of firm productivity implausible: exceedingly long right tail.

**Why?**

- High-productivity firms have a lot of market power in the BM model.

- This tends to concentrate wages towards the upper part of the distribution.

- In order to generate the very long, thin tails of observed wage distributions, productivity distributions with much longer and thinner tails are thus necessary.

- Irreconcilable with actual productivity distributions.
Sequential Auctions
Postel-Vinay and Robin (*Econometrica*, 2002)

- Workers differ in ability $\varepsilon$ and are **perfect substitutes** (buys *no sorting* at equilibrium).
- Marginal productivity of a match $(\varepsilon, p)$ is $p\varepsilon$, where $p$ is firm-specific.
- Workers draw type $p$ firms according to the same sampling distribution $F$, whatever their type.
- Wage contracts are negotiated between employers and employees under complete information about each other’s type and can be renegotiated by mutual consent only.
- Employers have full market power.
  [Assumption relaxed in Cahuc, Postel-Vinay and Robin, 2005.]
Let $V(w; \varepsilon, p)$ denote the lifetime value of current wage $w$ at firm $p$.

When a worker paid $w$ in firm $p$ receives an offer from a firm $p'$:

- If $p < p'$, moves to $p'$ for wage $\phi(\varepsilon, p, p')$ (possibly lower than $w$).
- If $p > p'$ and $w < \phi(\varepsilon, p', p)$, no mobility but wage rises to $\phi(\varepsilon, p', p)$.
- If $\phi(\varepsilon, p', p) < w$ nothing happens.

Wage value $\phi(\varepsilon, p, p')$, $p < p'$, solves the equation:

$$V(\phi; \varepsilon, p') = V(\varepsilon; \varepsilon, p).$$

[Similar to a second-price auction.]

Solves as:

$$\phi(\varepsilon, p, p') = \varepsilon \cdot \left( p - \frac{\lambda_1}{\rho + \delta} \int_{p}^{p'} F(x) \, dx \right) \quad [p' > p].$$
Equilibrium wage distribution equals distribution of $\phi(\varepsilon, q, p)$, where:

- $\varepsilon$: person effect;
- $p$: firm effect (independent of $\varepsilon$, i.e. no sorting);
- $q$: persistent component (random process of outside offers causing wage increases).

Main results

- Use French wage register data (DADS, same as AKM).
- Person effect explains 40% of $\text{Var} (\ln w)$ for managers and quickly drops to 0 for unskilled categories.
- Contribution of $q \approx 50\%$. 
Sequential Auctions (cont’d)
Cahuc, Postel-Vinay and Robin (WP, 2005)

- Match administrative data on wages with accounting firm data to obtain independent productivity estimates.
  
  \[ p = \text{observed labour productivity constructed from estimated production function.} \]

- Sequential auctions with positive worker bargaining power \((\beta \in [0, 1])\). Assess the relative importance of between-firm competition and Nash-type bargaining in wage determination.

**Main results**

- \( \beta = 0 \) for low-skill workers.
- \( \beta \) between 0 and 0.3 for high-skill workers (depending on particular industry).
Microeconometric Search-Matching Models

Wage Dispersion  Wage Dynamics  For the Future...

Microeconometric Search-Matching Models

Wage Dispersion
Wage Dynamics For the Future...

Part II: WAGE DYNAMICS
Long tradition of fitting ARMA-type models to individual income trajectories in worker panel data (e.g. Abowd and Card, *Econometrica* 1989).

Archetypal ARMA decomposition:

\[
\begin{align*}
    w_{it} &= \alpha_i + s^P_{it} + s^T_{it}, \\
    s^P_{it} &= s^P_{i,t-1} + u_{it}, \quad \text{with } u_{it} \text{ i.i.d.} \\
    s^T_{it} &= \sum_{\ell=0}^{q} \theta_{\ell} \varepsilon_{i,t-\ell}, \quad \text{with } \varepsilon_{it} \text{ i.i.d. and typically } q = 0 \text{ or } 1.
\end{align*}
\]

Theoretically consistent in Walrasian markets with perfectly substitutable workers and individual productivity shocks.
P-VR model with no firm-specific match productivity component but with \textit{i.i.d.} productivity shocks \([\sim F]\).

- \textbf{Renegotiation by mutual consent} implies the following dynamic structure for wages:
  
  - \textbf{Wage cut} when sufficiently negative shock to make firm’s threat of dismissing the worker credible at current wage;
  
  - \textbf{Wage increase} when sufficiently good outside offer to make worker’s threat of quitting credible at current wage.
P-VT model: Results

- Model predicts following acceptance/rejection scheme of i.i.d. shocks $v'$:

$$w_{it} = \alpha_i + v_{it}, \quad \text{with} \quad v_{i,t+1} | v_{it} = \begin{cases} v_{it} & \text{with prob. } F(v_{it}) - \frac{\lambda_1}{1-\delta} F(v_{it})^2 \\ v' < v_{it} & \text{with density } f(v') \\ v' > v_{it} & \text{with density } \frac{2\lambda_1}{1-\delta} f(v') F(v'). \end{cases}$$

- Illustration: P-VT estimate the structural model from BHPS data.

<table>
<thead>
<tr>
<th>Cov ($\Delta w_{it}, \Delta w_{i,t+s}$):</th>
<th>$s = 0$</th>
<th>$s = 1$</th>
<th>$s = 2$</th>
<th>$s = 3$ ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data (BHPS)</td>
<td>0.062</td>
<td>-0.021</td>
<td>0.001</td>
<td>-0.003</td>
</tr>
<tr>
<td>Simulated</td>
<td>0.066</td>
<td>-0.023</td>
<td>-0.001</td>
<td>-0.004</td>
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<table>
<thead>
<tr>
<th>ARMA parameters: RW comp: Var ($u_{it}$)</th>
<th>MA(0) comp: Var ($\varepsilon_{it}$)</th>
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</thead>
<tbody>
<tr>
<td>Data (BHPS)</td>
<td>0.014 (0.002)</td>
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<td>Simulated</td>
<td>0.013 (0.002)</td>
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<td>0.018 (0.002)</td>
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<td></td>
<td>0.022 (0.001)</td>
</tr>
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</table>
Sequential Auctions (One Last Time)
Bagger, Fontaine, Postel-Vinay and Robin (ongoing research)

- Serious shortcoming of the theory developed so far: overlooks human capital.
- Assuming piece-rate wages in P-VR yields the following structural log-wage equation:

\[ w_{it} = \alpha_i + k_{it} + p_{j(i,t)} - \int_{q_{it}}^{p_{j(i,t)}} \left[ 1 + \frac{\lambda_1}{\rho + \delta} \bar{F}(x) \right] dx, \]

where:
- \( k_{it} \) is worker \( i \)'s human capital and follows any dynamic process we like;
- \( p_{j(i,t)} \) and \( q_{it} \) are firm productivity levels that evolve along with the occurrence of outside job offers following similar rules as in P-VR.

**Research question**: Role of human capital accumulation vs. market forces in explaining wage/tenure and wage/experience profiles?
Conclusion:
FOR THE FUTURE...
Where do we go from there?

- **Wage posting vs. sequential auctions?** Maybe, Mortensen’s monopoly wage posting model is a good description of the market for unskilled workers, while the sequential auction model is better at describing the market for skilled workers.

- Equilibrium search models so far carefully assume away any potential source of *sorting*. Well, what about it, really? (Lots of theoretical work, including Sattinger, Shimer and Smith, Teulings... Hardly any empirical work save for Abowd and Kramarz and Teulings and Gautier.)

- Possible to **close the model** by modelling labour demand to endogenise job offer arrival rates. See Mortensen (2000), “Equilibrium Unemployment with Wage Posting: BM meet Pissarides”. Crucial move to policy analysis.