When Bonds Matter: Home Bias in Goods and Assets

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Abstract

This paper presents a model of international portfolios with real exchange rate and non financial risks that accounts for observed levels of equity home bias. Bonds matter: in equilibrium, investors structure their bond portfolio to hedge real exchange rate risks. Equity home bias arises when non-financial income risk is negatively correlated with equity returns, *after controlling for bond returns*. Our framework allows us to derive equilibrium bond and equity portfolios in terms of directly measurable hedge ratios. An empirical application to G-7 countries finds strong empirical support for the theory. We are able to account for a significant share of the equity home bias and obtain an aggregate currency exposure of bond portfolios comparable to the data.

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1. Introduction

Despite an unprecedented increase in cross-border financial transactions over the last thirty years, international portfolios remain heavily tilted toward domestic assets. This is the well-known equity home bias (see French and Poterba (1991) and Coeurdacier and Rey (2011) for a recent survey). As of 2008, the share of U.S. stocks in U.S. investors' equity portfolios was 77.2%, despite the fact that U.S. equity markets account for only 32% of world

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market capitalization.[2]

Two important strands of literature aim to account for the observed bias. In both approaches, investors depart from the perfectly diversified portfolio of frictionless general equilibrium models à la Lucas (1982), in order to insulate their consumption stream from asymmetric sources of risk. Generically, consider a risk-factor X that impacts *negatively* domestic wealth relatively more than foreign wealth. In equilibrium, the difference between domestic and foreign own-equity holdings (the degree of equity home bias) will be proportional to the following hedge ratio:

$$\frac{\mathrm{cov}\,(X,R)}{\mathrm{var}\,(R)}, \tag{1}$$

where R denotes the difference between domestic and foreign equity returns. Home equity bias arises when this relative equity return is *positively correlated* with X, that is, when domestic equities offer better protection to domestic investors against risk factor X.

The two strands of literature differ in the risk factor they consider. One approach, following Obstfeld and Rogoff (2000), explores the link between consumption expenditures and international portfolios in stochastic general equilibrium models where investors have different consumption baskets.[3] In their model, investors face real exchange rate risk: $X = (1 - 1/\sigma)\Delta \ln Q$ where $\Delta \ln Q$ is the rate of change of the real exchange rate (with the convention that an increase in Q denotes an appreciation), and σ is the coefficient of relative risk aversion. The hedge ratio takes the form $(1 - 1/\sigma)\mathrm{cov}(\Delta \ln Q, R)/\mathrm{var}(R)$. With a coefficient of relative risk aversion σ above unity, home equity bias arises when relative equity returns are *positively* correlated with the real exchange rate. The reason is simple: with $\sigma > 1$, efficient risk sharing requires that domestic consumption expenditures increase

[2] The equity home bias is a general phenomenon. The share of home equities in other G-7 countries portfolios in 2008 are as follows: 80.2% in Canada, 73.5% in Japan, 66% in France, 53% in Germany and 52% in Italy. All these countries account for less than 10% of world market capitalization.

[3] A non-exhaustive list of contributions –some of which precedes Obstfeld and Rogoff (2000)– includes Dellas and Stockman (1989), Uppal (1993), Baxter et al. (1998), Serrat (2001), Kollmann (2006), Obstfeld (2007), Heathcote and Perri (2013), Coeurdacier et al. (2009), Collard et al. (2007), Coeurdacier (2009) and Benigno and Nistico (2011). See also Kouri and Macedo (1978), Krugman (1981) and the references in Adler and Dumas (1983) for an early derivation in partial equilibrium.

with the real exchange rate.[4] If domestic equity returns are high precisely at that time, home equity bias follows. As shown by van Wincoop and Warnock (2010), this line of research faces a serious challenge: for many countries, the empirical correlation between excess equity returns and the real exchange rate is close to zero.

The second strand of literature focuses on the hedging properties of domestic stocks against fluctuations in domestic non-financial income (e.g. labor income).[5]. The risk factor is $X = -R^n$, where R^n is the return to domestic non-financial income, relative to the rest of the world. The hedge ratio takes the form $-\text{cov}(R^n, R)/\text{var}(R)$: if returns on domestic equities are high precisely when returns on non-financial wealth are low, then domestic investors will favor domestic equities. This line of research also faces an important empirical challenge: Baxter and Jermann (1997) find that financial and non-financial returns are positively correlated. Optimal portfolios should then be biased towards *foreign* equity.[6]

The first contribution of this paper is to merge and improve upon these two strands of literature by showing that many of the earlier results are not robust to the introduction of domestic and foreign bonds, whether nominal or real. We establish this point in a generic setting, characterizing jointly the optimal equity and bond portfolios in environments with multiple sources of risk and different degrees of completeness of financial markets. Our approach allows us to characterize the optimal equity and bond portfolios in terms of *sufficient statistics* that can easily be estimated, in the spirit of **?**. These sufficient statistics take precisely the form of the hedge ratios of Eq. (1), extended to the case of multiple assets.

The key economic insight of our paper is that in most models of interest, as well as in the data, nominal or real relative bond returns are strongly positively correlated with real exchange rate fluctuations. As a result, it is optimal for investors to use bond holdings to hedge real exchange rate risks. In that sense, bonds matter. All that is left for equities is to hedge the impact of any *additional* source of risk on investors' wealth. The precise structure of these additional risk factors matters for optimal portfolio holdings, but the

[4]Under efficient risk sharing, relative consumption expenditures satisfy $P_H C_H / P_F C_F = Q^{1-1/\sigma}$.

[5]A non-exhaustive list of contributions includes Bottazzi et al. (1996), Baxter and Jermann (1997), Julliard (2003), Heathcote and Perri (2013), Engel and Matsumoto (2009), **?** and **?**.

[6]Other empirical papers found more mixed results. See Bottazzi et al. (1996) and Julliard (2003).

general portfolio structure can be estimated independently of the specificities of the model. Generically, equity home bias arises if non-financial income risk is negatively correlated with equity returns, *after controlling for bond returns*. This conditioning is important: to the extent that *unconditional* and *conditional* hedge ratios for non-financial income risk are different in the data, bonds also matter for the insurance properties of equities against fluctuations in non-financial wealth.

The mapping from hedge ratios to structural parameters depends on the details of the model. We illustrate such a mapping in a two-country two-good model with stochastic endowments and redistributive shocks between capital and labor. This particular example serves to illustrate starkly how the failure to allow for trade in bonds can lead to incorrect inference on the structure of optimal equity portfolios. The same model without bond trading predicts that investors should short domestic equities, as in Baxter and Jermann (1997). By contrast, the model with equity and bond predicts full home equity bias.

The second important contribution of this paper is to confront the theory to the empirical evidence. The paper shows how to estimate the hedge ratios —and hence predicted portfolios— from observable data on bond returns, real exchange rates and estimated returns to financial and non-financial wealth. This provides an important link between recent theoretical work on international portfolios and data on asset returns. Our empirical exercise uses quarterly data on market returns, non-financial and financial income for the G-7 countries since 1970 to ask wether asset returns are theoretically consistent with observed portfolios. Since returns on non-financial and financial wealth are not directly observed, the paper considers a number of different approaches, such as Campbell and Shiller (1988) (our benchmark estimation), Baxter and Jermann (1997) or Lustig and Nieuwerburgh (2008).

For all G-7 countries, and across all specifications, allowing bond trading is key to obtaining more reasonable equity positions. Without bonds, 'the international diversification puzzle is worse than you think' as Baxter and Jermann (1997) argued. However, with bond trading, our estimates predict significant levels of equity home bias for all G-7 countries, in line with the data. Finally, our empirical estimates also predict short but fairly small domestic currency positions for a reasonable degree of relative risk aversion.

Section 2 presents our basic framework and characterizes optimal equity and bond portfolios in terms of hedge ratios. Section 3 presents a fully-specified version of the model with endowment and redistributive shocks and characterizes the equity-only and the full portfolios. Section 4 presents our empirical results. Section 5 concludes. An online Appendix provides many derivations and robustness checks.

2. A Benchmark Model

This section presents the benchmark model and derives equilibrium portfolios.

2.1. Set-up

Preferences. We consider a two-period model ($t = 0, 1$) with two symmetric countries, Home (H) and Foreign (F), each with a representative household. Country i's representative household has standard Constant Relative Risk Aversion (CRRA) preferences, with a coefficient of relative risk aversion $\sigma \geq 1$ defined over a consumption index C_i, and a discount factor $0 < \xi \leq 1$:

$$U_i = \frac{C_{i,0}^{1-\sigma}}{1-\sigma} + \xi E_0 \left[\frac{C_{i,1}^{1-\sigma}}{1-\sigma} \right], \tag{2}$$

where E_0 denotes expectations conditional on date $t = 0$ information. The ideal consumer price index in country $i = H, F$ is denoted $P_{i,t}$, in terms of an arbitrary numeraire.

Financial markets and budget constraints. Trade in financial assets occurs in period 0. In each country there is a 'Lucas tree' whose supply is normalized to unity. In both periods, a cash-flow $d_{i,t}^f$ is distributed to owners of this financial asset (stockholders) as dividend. Another cash-flow $d_{i,t}^n$ is distributed to households of country i as non-financial income. At the simplest level, one can think of $d_{i,t}^n$ as representing 'labor income.' More generally, it describes all of country i's income sources that cannot be capitalized into financial claims.

Agents can also trade Home and Foreign one-period bonds. Both bonds are in zero net supply. Buying one unit of the bond of country i in period $t-1$ yields a cash-flow $d_{i,t}^b$ at date t. These bonds are risk-free but pay in different units. If the bonds are risk-free in real terms, a unit of country i's bond purchased at date $t-1$ yields $d_{i,t}^b = P_{i,t}$ at date t, i.e. enough resources to purchase one unit of country i's consumption index.

The representative household from country i enters period $t = 0$ with an initial portfolio of stocks $\{S_{ij,0}\}$ and bonds $\{B_{ij,0}\}$ from country $j \in \{H, F\}$ and faces the following budget constraint:

$$P_{i,0}C_{i,0} + \sum_j \left(p_S^j S_{ij,1} + p_B^j B_{ij,1}\right) = d_{i,0}^n + \sum_j \left(S_{ij,0}\left(p_S^j + d_{j,0}^f\right) + B_{ij,0}d_{j,0}^b\right), \qquad (3)$$

where p_S^j (resp. p_B^j) denotes the price of a stock (resp. of the bond) from country j at date 0. The right hand side of Eq. (3) measures sources of funds, non-financial and financial. The left hand side captures uses of funds: consumption and portfolio investment.

At date $t = 1$, all income is spent:

$$P_{i,1}C_{i,1} = \sum_j \left(S_{ij,1}d_{j,1}^f + B_{ij,1}d_{j,1}^b\right) + d_{i,1}^n. \qquad (4)$$

Lastly, markets for stocks and bonds of country $i \in \{H, F\}$ clear:

$$\sum_j S_{ji,t} = 1; \ \sum_j B_{ji,t} = 0. \qquad (5)$$

2.2. Equilibrium portfolios

Portfolios decisions. The optimal portfolio allocation results from maximizing Eq. (2) subject to Eqs. (3) and (4). The optimality conditions for stocks and bonds holdings in country i are given by the usual Euler equations:

$$E_0\left(\mathcal{M}_i R_j^f\right) = E_0\left(\mathcal{M}_i R_j^b\right) = 1 \quad ; \quad i, j \in \{H, F\}, \qquad (6)$$

where $R_j^f = d_{j,1}^f / p_S^j$ and $R_j^b = d_{j,1}^b / p_B^j$ denote the gross return on stocks and bonds respectively in country j and $\mathcal{M}_i = \xi\left(P_{i,0}/P_{i,1}\right)\left(C_{i,1}/C_{i,0}\right)^{-\sigma}$ is the stochastic discount factor in country i.

Log-linearization of the budget constraint. We can characterize approximate optimal consumption and portfolio decisions around the symmetric equilibrium where both countries have the same distribution of financial, non-financial and bond cash flows, households hold similar initial portfolios and have no initial net foreign asset positions, using standard log-linearization techniques as in Devereux and Sutherland (2006) and Tille and van Wincoop

(2010).[7] Before doing so, let's introduce a bit of notation. First, Jonesian hats denote the log-deviation of a variable $x_{i,t}$ from its steady state value $\bar{x}_i$: $\hat{x}_{i,t} = log(x_{i,t}/\bar{x}_i))$. Second, variables without country indices denote *differences* across countries: $\hat{x}_t = \hat{x}_{H,t} - \hat{x}_{F,t}$. Finally, the operator Δ denotes first differences: $\Delta\hat{x} = \hat{x}_1 - \hat{x}_0$.

Define the Home country real exchange rate as the foreign price of the domestic good, $Q \equiv P_H/P_F$, so that an increase in the real exchange rate represents a real appreciation. Log-linearizing yields: $\hat{Q}_t = \hat{P}_{H,t} - \hat{P}_{F,t}$.

Define aggregate nominal expenditures $X_{i,t} = P_{i,t}C_{i,t}$, and denote $1 - \delta = \bar{d}_t^n/\bar{X}_t$ the steady state share of non-financial income in total expenditures, assumed common in both periods. Taking the difference between Home and Foreign budget constraints in both periods from Eqs. (3) and (4), log-linearizing around the symmetric equilibrium, and using the market clearing conditions (5) yields:

$$\hat{X}_t = (1-\delta)\,\hat{d}_t^n + (2S-1)\,\delta\hat{d}_t^f + 2b\hat{d}_t^b, \tag{7}$$

where $S = S_{ii,0} = S_{ii,1}$ and $B = B_{ii,0} = B_{ii,1}$ denote the (symmetric) optimal holdings of a country's own equities and real bonds and $b = B/\bar{X}_0$ denotes the steady state ratio of bond holdings to aggregate expenditures.

Define the (log-linearized) relative return on equities R^f, non-financial wealth R^n and bonds R^b as: $\hat{R}^f = \Delta\hat{d}^f - E_0\Delta\hat{d}^f$, $\hat{R}^n = \Delta\hat{d}^n - E_0\Delta\hat{d}^n$, and $\hat{R}^b = \Delta\hat{d}^b - E_0\Delta\hat{d}^b$. Taking first differences of Eq. (7), using the definition of the stochastic discount factor $\mathcal{M}_i = \xi\,(P_{i,0}/P_{i,1})\,(C_{i,1}/C_{i,0})^{-\sigma}$ and the fact that $\Delta\hat{X} = \Delta\hat{Q} + \Delta\hat{C}$ yields:

$$\Delta\hat{X} - E_0\Delta\hat{X} = \left(1 - \frac{1}{\sigma}\right)\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) - \frac{1}{\sigma}\left(\hat{\mathcal{M}} - E_0\hat{\mathcal{M}}\right) = (1-\delta)\hat{R}^n + (2S-1)\,\delta\hat{R}^f + 2b\hat{R}^b. \tag{8}$$

Eq. (8) is a key equation for our analysis. The first equality determines relative consumption expenditure growth as a function of the rate of change of the real exchange rate and the

[7] Formally, we log-linearize around $d_{i,0}^l = \bar{d}_0^l$ and $E_0 d_{i,1}^l = \bar{d}_1^l$, and assume that $S_{ii,0} = S$ and $B_{ii,0} = B$ for $i \in \{H, F\}$ and $l \in \{n, f, b\}$. Appendix A.5 derives the more general case where countries are asymmetric ex-ante. In that case, the optimal portfolio contains an additional intertemporal component.

relative stochastic discount factor. The second equality expresses relative income growth for a given portfolio choice (S, b), as a function of the relative return on non-financial wealth, financial wealth and bonds.

Hedge Ratios. If relative bond and equity returns are not perfectly correlated, it is always possible to 'project' the rate of change of the real exchange rate and the return on non-financial income on stock and bond returns.[8] This projection takes the form:

$$\begin{cases} \Delta\hat{Q} - E_0\Delta\hat{Q} & \equiv & \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f + u_Q \\ \hat{R}^n & \equiv & \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f + u_n \end{cases}, \tag{9}$$

where the residual terms u_i are orthogonal to asset returns $\hat{R}^j$, i.e. $E_0[u_i\hat{R}^j] = 0$ for $i \in \{Q, n\}$ and $j \in \{f, b\}$. The coefficients $\beta_{i,j}$ capture the loading of asset return $j = \{b, f\}$ on risk factor $i = \{Q, n\}$. These loading factors, also called *hedge ratios*, have the usual interpretation in terms of covariance-variance ratios:

$$\beta_{n,j} = \frac{\mathrm{cov}_{\hat{R}^l}\left(\hat{R}^n, \hat{R}^j\right)}{\mathrm{var}_{\hat{R}^l}\left(\hat{R}^j\right)} \; ; \; \beta_{Q,j} = \frac{\mathrm{cov}_{\hat{R}^l}\left(\Delta\hat{Q} - E_0\Delta\hat{Q}, \hat{R}^j\right)}{\mathrm{var}_{\hat{R}^l}\left(\hat{R}^j\right)},$$

where $j \neq l \in \{f, b\}$ and $\mathrm{cov}_z(x, y)$ (resp. $\mathrm{var}_z(x)$) denotes the covariance between x and y (resp. the variance of x), *conditional on* z. While these factor loadings are equilibrium objects and model-dependent, their empirical counterpart can be obtained simply from the reduced form Eq. (9) as regression coefficients, independently from the specifics of the model.

Equilibrium portfolios. From the Euler equations (6) of the investor problem, observe that the relative stochastic discount factor $\hat{\mathcal{M}}$ satisfies:

$$E\left[\hat{\mathcal{M}}\ \hat{R}^i\right] = 0 \text{ for } i \in \{b, f\}. \tag{10}$$

Using Eq. (10) to project the budget constraint (8) on relative asset returns $\hat{R}^f$ and $\hat{R}^b$, we obtain the following key property:[9]

[8]Appendix A.1 shows formally that a *rank condition* needs to be satisfied. This will generically be the case if the dimension of the underlying shocks is larger or equal to 2.

[9]The proof is relegated to Appendix A.1. It relies on observing that the relative stochastic discount factor is orthogonal to asset returns, using Eq.(10).

Property 1 (Optimal Portfolios in terms of Hedge Ratios). *Under the rank condition of Appendix A.1, the optimal portfolio is unique and can be expressed in terms of the hedge ratios $\beta_{i,j}$ as follows:*

$$b^* = \frac{1}{2}\left[\left(1-\frac{1}{\sigma}\right)\beta_{Q,b} - (1-\delta)\,\beta_{n,b}\right] \tag{11a}$$

$$S^* = \frac{1}{2}\left[1 + \frac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f} - \frac{1-\delta}{\delta}\beta_{n,f}\right]. \tag{11b}$$

Property 1 has a key implication for our analysis: the hedge ratios $\beta_{i,j}$ provide *sufficient statistics* for the optimal portfolios. The structural details of a general equilibrium model will generically provide a mapping of the loading factors into the primitive characteristics of the model. Yet the portfolio predictions remain identical across models, conditional on a set of loading factors $\beta_{i,j}$.

Let's now discuss the structure of equilibrium portfolios implied by Property 1. Consider first the bond portfolio b^* in Eq. (11a). It contains two terms. The first term $(1-1/\sigma)\,\beta_{Q,b}/2$ reflects the role of bonds in hedging real exchange rate risk. When $\sigma > 1$, the household's relative consumption expenditures $\hat{X}$ increase with the real exchange rate. If, after controlling for equity returns, domestic bonds deliver a high relative return when the currency appreciates (i.e. $\beta_{Q,b} > 0$), domestic bonds constitute a good hedge against real exchange rate risk. The second term $-(1-\delta)\,\beta_{n,b}/2$ captures the role of bonds in hedging non-financial income risk. When domestic bonds and the return to non-financial wealth are positively conditionally correlated ($\beta_{n,b} > 0$), investors want to short the domestic bond to hedge the implicit exposure to non-financial risks. Eq. (11a) indicates that investors will go long or short in their domestic bond holdings depending on the relative strength of these two effects.

Consider now the equilibrium equity position S^* in Eq. (11b). The first term inside the brackets represents the symmetric risk-sharing equilibrium of Lucas (1982): $S^* = 1/2$. This is the optimal equity portfolio if equities are not useful to hedge real exchange rate or non-financial risk ($\beta_{Q,f} = \beta_{n,f} = 0$).

The second term, $(1-1/\sigma)\beta_{Q,f}/\delta$, is similar to the term that has been emphasized in

Coeurdacier (2009) or Obstfeld (2007), with one important difference. It represents the demand for domestic equity that arises from hedging real exchange rate risk, corresponding to the hedge portfolio in Eq. (1). If $\beta_{Q,f}$ is positive, domestic stock returns are relatively high when the currency appreciates, contributing to home bias. The important difference is that this hedge ratio is *conditional on bond returns*. As we will see, conditional and unconditional hedge ratios can differ greatly, with important implications for optimal portfolios.

The last term, $-(1-\delta)\beta_{n,f}/\delta$, determines how equity portfolios are structured to hedge non-financial risk. Investors optimally want to undo the endowed equity exposure implicit to non-financial risks, measured by $\beta_{n,f}$. To fix ideas, consider the case where bonds are risk-free in real terms so that $d^b_{i,t} = P_{i,t}$. In that case, it is immediate, using Eq. (9), that $\beta_{Q,b} = 1$ and $\beta_{Q,f} = 0$ since $\hat{R}^b = \hat{Q} - E_0\hat{Q}$. In the absence of non-financial income (i.e. when $\delta \to 1$), the optimal portfolios are the same as in Adler and Dumas (1983). Since bonds hedge perfectly real exchange risk, risky asset holdings are fully diversified: $S^* = 1/2$. Eq. (11b) extends Adler and Dumas (1983) to the case with non-financial income ($\delta < 1$):

$$S^* = \frac{1}{2}\left(1 - \frac{1-\delta}{\delta}\beta_{n,f}\right). \tag{12}$$

This result is reminiscent of Baxter and Jermann (1997) who find that financial and non-financial returns are (*unconditionally*) positively correlated and conclude that the optimal portfolio should therefore be tilted towards foreign equities ($S^* < 0.5$). However, unlike Baxter and Jermann (1997), Eq. (12) indicates that the relevant hedge ratio is *conditional on bond returns*. Our model predicts that home equity bias arises if $\beta_{n,f} < 0$. To our knowledge, this condition has not been empirically investigated in the literature.[10]

Finally, observe that our approach is valid as long as equities and bonds are not redundant assets (the rank condition is satisfied), regardless of the degree of completeness of financial markets. Appendix A.1 shows that if an additional spanning condition is satisfied, markets are *locally* complete, in the sense that the efficient risk sharing condition of Backus and

[10] Engel and Matsumoto (2009) also note that this is the relevant condition in presence of bond holdings, or forward exchange contracts. See also Coeurdacier et al. (2009) and Coeurdacier et al. (2010).

Smith (1993) holds locally:

$$\hat{\mathcal{M}} = -\sigma \Delta \hat{C} - \Delta \hat{Q} = 0. \tag{13}$$

When this condition holds bonds and equities are sufficient to span the relevant sources of risk in the economy and the decomposition in Eq. (9) is exact: $u_i = 0$.

3. Closing the Model: The Case of Redistributive Shocks

Proposition 1 established that the hedge ratios $\beta_{i,j}$ provide sufficient statistics for a full characterization of optimal portfolios. By fleshing out the remaining details of the model, these hedge ratios can be linked to the structural parameters of the model. While providing a full fledged theory of the factor loadings $\beta_{i,j}$ is beyond the scope of this paper, this section presents such a mapping in an illustrative and simple model with endowment and redistributive shocks.[11] Importantly, the main result of our paper is both more modest and more general: Property 1 shows how to map the -empirically observable– hedge ratios into equilibrium portfolios, independently from the underlying model. Thus, readers only interested in the empirical implications of Section 2 can go directly to Section 4.

3.1. A model with endowment and redistributive shocks.

The model borrows all the elements introduced in Section 2. In addition, we assume the following endowment and demand structure.

Endowments and shocks. Each country receives an endowment of a country-specific tradable good each period. The endowment in country i at date t is denoted $y_{i,t}$. $y_{i,0}$ is known while $y_{i,1}$ is stochastic and symmetrically distributed with mean $\bar{y}_1$ common to both countries. Denote $p_{i,t}$ the price at date t of country i's good in terms of the numeraire. At each date, the financial cash-flow represents a share $\delta_{i,t}$ of output at market value $p_{i,t}y_{i,t}$: $d^f_{i,t} = \delta_{i,t}p_{i,t}y_{i,t}$. $\delta_{i,0} = \delta$ is known while $\delta_{i,1}$ is stochastic and symmetrically distributed with

[11] The working paper version **?** provides additional examples with, e.g., fiscal shocks, nominal shocks or non-traded goods. The case of redistributive shocks is similar to Coeurdacier et al. (2009) and Engel and Matsumoto (2009) with preset prices. The model in this section considers real bonds, which are by definition very effective at hedging real exchange rate risk. Nominal bonds can be less effective than real bonds to the extent that nominal shocks don't coincide with real shocks. This is discussed in more details in the working paper version.

mean δ. Shocks to $\delta_{i,t}$ represent redistributive shocks, i.e shocks to the share of total output distributed as financial income in country i.[12]

In each country the representative consumer enters period $t = 0$ with a given financial portfolio of financial assets, receive financial and non-financial income as described in Section 2, consume and trade financial claims. In period $t = 1$, stochastic endowments and stochastic shocks to δ are realized, households consume using the revenues from their financial portfolio and their non-financial endowment.

Preferences. Each representative household consumes both goods with a preference towards the domestic good. For $i, j \in \{H, F\}$ and $t = 0, 1$, the consumption index $C_{i,t}$ is a constant-elasticity aggregator: $C_{i,t} = \left[a^{1/\phi} c_{ii,t}^{(\phi-1)/\phi} + (1-a)^{1/\phi} c_{ij,t}^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)}$, where $c_{ij,t}$ denotes country i's consumption of the good from country j at date t. ϕ is the elasticity of substitution between the two goods and $1 \geq a \geq 1/2$ captures preference for the home good (mirror-symmetric preferences). With these preferences, the Fisher-ideal price index for consumption is:

$$P_{i,t} = \left[a p_{i,t}^{1-\phi} + (1-a) p_{j,t}^{1-\phi}\right]^{1/(1-\phi)}. \tag{14}$$

Financial and non-financial cash-flows. We assume that each country's bonds are risk-free in terms of that country's consumption index, that is $d_{i,t}^b = P_{i,t}$. With the notations of Section 2, financial and non financial cash-flows at date t are given by:

$$d_{i,t}^f = \delta_{i,t} p_{i,t} y_{i,t} \quad ; \quad d_{i,t}^n = (1 - \delta_{i,t})\, p_{i,t} y_{i,t} \quad ; \quad d_{i,t}^b = P_{i,t} \quad \text{for} \quad i \in \{H, F\}.$$

With these definitions of cash-flows, budget constraints can be written as in Eqs. (3) and (4) and portfolio equations as in Eq. (6).

Goods markets equilibrium. In each period, optimal intratemporal allocation of con-

[12] Our model considers exogenous redistributive shocks as in Ríos-Rull and Santaeulàlia-Llopis (2010). Fluctuations in income share can occur endogenously with a non-unitary elasticity of substitution between capital and labor, in presence of capital and labor augmenting productivity shocks or biased technical change (Young (2004)).

sumption requires:

$$c_{ii,t} = a\left(\frac{p_{i,t}}{P_{i,t}}\right)^{-\phi} C_{i,t} \quad ; \quad c_{ij,t} = (1-a)\left(\frac{p_{j,t}}{P_{i,t}}\right)^{-\phi} C_{i,t} \quad \text{for} \quad i \neq j. \tag{15}$$

Resource constraints are given by:

$$c_{ii,t} + c_{ji,t} = y_{i,t} \quad \text{for } \; i \in \{H, F\}, j \neq i. \tag{16}$$

Define q_t as Home's terms of trade, i.e. the relative price of the Home tradable good in terms of the Foreign tradable good: $q_t \equiv p_{H,t}/p_{F,t}$. An increase in q represents an improvement in Home's terms of trade. Using (15) together with the resource constraints (16) yields the following expression for relative output:

$$\frac{y_{H,t}}{y_{F,t}} = q_t^{-\phi}\Omega_a\left[(\frac{P_{F,t}}{P_{H,t}})^{\phi}\frac{C_{F,t}}{C_{H,t}}\right], \tag{17}$$

where $\Omega_a(x) \equiv \left[1 + x(\frac{1-a}{a})\right] / \left[x + (\frac{1-a}{a})\right]$. Without home bias in preferences ($a = 1/2$), Eq. (17) simplifies to $y_{H,t}/y_{F,t} = q_t^{-\phi}$: the price elasticity of relative output is ϕ, independently of the distribution of relative expenditures. As emphasized by Obstfeld (2007), the term $\Omega_a(.)$ captures the Keynesian *transfer effects* due to consumption home-bias: with $a > 0.5$, a reallocation of wealth towards the home country improves the domestic terms of trade since it shifts relative demand towards the domestic good.

Log-linearization of returns. Appendix A.1 shows that the model with independent endowment and redistributive shocks satisfies the rank and spanning conditions so that markets are locally complete. Formally, it implies that the Backus and Smith (1993) efficient risk sharing condition, Eq. (13), holds locally: $\Delta\hat{C} = -1/\sigma\Delta\hat{Q}$.

Log-linearizing the goods' market equilibrium condition Eq. (17) substituting the above expression, and using the fact that $\Delta\hat{Q} = (2a-1)\Delta\hat{q}$ from Eq. (14), yields a relationship between relative output and the terms of trade (or the real exchange rate):

$$\Delta\hat{y} = -\lambda\Delta\hat{q} = -\lambda(2a-1)^{-1}\Delta\hat{Q}. \tag{18}$$

In this expression, $\lambda \equiv \phi\left(1-(2a-1)^2\right) + (2a-1)^2/\sigma > 0$ represents the equilibrium terms of trade elasticity of relative output. Without home bias in preferences ($a = 1/2$), $\lambda = \phi$, the elasticity of substitution between Home and Foreign goods. When $a > 1/2$, the additional term $(2a-1)^2(1/\sigma - \phi)$ captures the required change in the terms of trade needed to accommodate transfer effects.

Define aggregate nominal income $x_{i,t} = p_{i,t} y_{i,t}$. We can write the (log-linearized) relative return on equities $\hat{R}^f$, bonds $\hat{R}^b$ and non-financial income $\hat{R}^n$ as:

$$\begin{aligned} \hat{R}^f &= \Delta\hat{\delta} - E_0\Delta\hat{\delta} + \Delta\hat{x} - E_0\Delta\hat{x} \qquad (19a) \\ \hat{R}^n &= -\frac{\delta}{1-\delta}\left(\Delta\hat{\delta} - E_0\Delta\hat{\delta}\right) + \Delta\hat{x} - E_0\Delta\hat{x} \qquad (19b) \\ \hat{R}^b &= \Delta\hat{Q} - E_0\Delta\hat{Q} \qquad (19c) \end{aligned}$$

Projection of risk factors on asset returns. Using Eq. (18), yields immediately that $\Delta\hat{x} = (1-\lambda)(2a-1)^{-1}\Delta\hat{Q}$. Substituting into asset returns, following hedge ratios obtain:

$$\beta_{Q,b} = 1 \quad ; \quad \beta_{Q,f} = 0 \quad ; \quad \beta_{n,b} = \frac{1-\lambda}{(1-\delta)(2a-1)} \quad ; \quad \beta_{n,f} = -\frac{\delta}{1-\delta} \qquad (20)$$

In this equation, two elements are essential: first, since investors can trade real risk-free bonds, relative bond returns and the real exchange rate are perfectly correlated ($\beta_{Q,b} = 1$ and $\beta_{Q,f} = 0$). Second and more importantly, despite positive co-movements between financial and non-financial returns driven by innovations to nominal income growth ($\Delta\hat{x} - E_0\Delta\hat{x}$), the loading of non-financial income risk on financial asset returns $\beta_{n,f}$ is always strictly negative because of the redistributive shocks.

Equilibrium portfolios. Substituting the equilibrium loadings Eq. (20) into Eq. (11), the optimal portfolio satisfies:

$$S^* = 1 \quad ; \quad b^* = \frac{1}{2}\left[1 - \frac{1}{\sigma} + \frac{\lambda - 1}{2a-1}\right]. \qquad (21)$$

Since purely redistributive shocks only affect the distribution of total output, but not its size, the optimal hedge is for the representative domestic household to hold all the domestic equity. Consequently, the model implies full equity portfolio home bias.

Observe that this result does not depend upon the size of the redistributive shock. If

$\upsilon^2 = \sigma_\delta^2/\sigma_y^2$ denotes the relative variance of redistributive and endowment shocks, then the model predicts that $S^*(\upsilon) = 1$ as long as $\upsilon > 0$.[13]

The optimal bond position is the outcome of two forces: first, investors hedge real exchange risk when $\sigma \neq 1$. This is the term $(1 - 1/\sigma)/2$. Second, investors are fully exposed to domestic endowment shocks given their equity holdings. The bond portfolio makes sure that endowment risk is equally shared between home and foreign investors. This is the term $(2a - 1)^{-1}(\lambda - 1)/2$. The overall bond position can be long or short depending on whether $\lambda < 1 - (1 - 1/\sigma)(2a - 1)$ or not, i.e depending on whether relative income growth co-move positively or negatively with relative bonds returns, or equivalently the real exchange rate.

3.2. The pitfalls of equity-only models

To illustrate the pitfalls of using equity-only models, consider what happens in the previous model if households can only trade equities. Following the same steps as in Section 2, one can derive the equilibrium equity-only optimal portfolio:

$$S^u(\upsilon) = \frac{1}{2}\left[1 - \frac{1-\delta}{\delta}\beta^u_{n,f} + \frac{1-\frac{1}{\sigma}}{\delta}\beta^u_{Q,f}\right].$$

This equation is similar to Eq. (11b) except that the loadings $\beta^u_{i,f} = \text{cov}(\hat{R}^i, \hat{R}^f)/\text{var}(\hat{R}^f)$ are *unconditional* loadings. Consider now the limit of $S^u(\upsilon)$ as $\upsilon \to 0$, i.e. as redistributive shocks become vanishingly small. Intuitively, in that case financial and non-financial returns become positively correlated (see Eq. (19)) so that $\beta^u_{n,f} > 0$.[14] In the limit of $\upsilon = 0$, markets are locally complete again, and following the same steps as before, one can establish that:

$$\beta^u_{n,f} = 1 \quad ; \quad \beta^u_{Q,f} = (2a-1)/(1-\lambda). \tag{22}$$

It follows that the optimal equity portfolio of the equities-only model satisfies:

$$S^u(0) = \frac{1}{2}\left[1 - \frac{1-\delta}{\delta} + \frac{1-\frac{1}{\sigma}}{\delta}\frac{(2a-1)}{1-\lambda}\right]. \tag{23}$$

[13] When $\upsilon = 0$, portfolios are indeterminate: equity and bond returns are perfectly correlated and the model fails the rank condition of App. A.1.

[14] Appendix A.2 shows how to solve for the unconditional hedge ratios $\beta^u_{i,f}$ using the approach of Devereux and Sutherland (2006).

As before, this portfolio is the sum of three terms: a Lucas pooled portfolio (1/2), a term due to hedging of non-financial income risk ($-(1-\delta)/(2\delta)$) and a term hedging real exchange rate risk $(1-1/\sigma)(a-1/2)/(\delta(1-\lambda))$). In the absence of bond trading, the hedging term for non-financial income risk always imparts a large foreign equity bias since $\beta^u_{n,f}=1$. While in principle the last term can be positive or negative depending on parameters values, van Wincoop and Warnock (2010) shows that the unconditional loading factor $\beta^u_{Q,f}$ is positive but small in the data, so that the portfolio $S^u(0)$ should typically exhibit foreign bias.[15] Thus, as in Baxter and Jermann (1997), the equity-only model cannot account for the home-equity bias for υ sufficiently small. With bond trading, the same model predicts full equity home bias, independently from model parameters. This striking example shows the crucial role of bond trading for the composition of equity portfolios.[16]

From an empirical perspective, our portfolio results are driven by the stark difference between *unconditional* and *conditional* hedge ratios: in the above example, $\beta^u_{n,f}>0$ for small values of υ while $\beta_{n,f}=-\delta/(1-\delta)<0$. If data were generated by such a model, measuring the *unconditional* hedge ratio would lead to the conclusion that the international diversification puzzle is worse than we think as in Baxter and Jermann (1997), while measuring the *conditional* hedge ratio would lead to the opposite conclusion.

More broadly, the message is that optimal equity portfolios depend on the menu of assets available to investors allowing them to diversify the risks they face. Allowing for bonds is essential since they provide a very natural hedge against real exchange rate risk—a point also noted by Adler and Dumas (1983). Other tradable assets may be relevant besides risk-free bonds if they have attractive hedging properties: long term bonds, housing, derivatives... The empirical approach developed in the next section aims to maintain a parsimonious framework. Our results indicate that we can provide a reasonable account of observed equity

[15] van Wincoop and Warnock (2010) estimate $\beta^u_{Q,f}=0.32$.

[16] Appendix A.4 relaxes the assumptions that (a) markets are complete and (b) bond returns correlate perfectly with the real exchange rate by introducing a relative preference shock. Equilibrium portfolios now depend on the relative size of shocks. The results described above – that equity portfolio exhibits a significant home bias in presence of bonds and foreign bias without bond trading– still holds for parameters values in a range consistent with the empirical evidence on factor loadings.

portfolios simply by allowing for trade in short term bonds. This does not preclude more sophisticated models from achieving an even better fit with the data.

4. Estimating Optimal Portfolios

We now show how to use our theoretical results to construct optimal equity and bond portfolios. Doing so requires estimating the reduced-form loading factors $\beta_{Q,i}$ and $\beta_{n,i}$ for $i = f, b$ for the G-7 countries. According to Property 1, this is all we need to characterize equilibrium portfolios.

4.1. From theory to data

Two issues arise when mapping the theory into the data, one theoretical, the other empirical. On the theoretical side, one might wonder if our results, derived in a two-period environment survive in a dynamic setting. On the empirical side, our two period model does not allow for time-varying expected returns, an important feature of the data. Appendix A.6 shows that our results are robust to a dynamic environment with complete markets and i.i.d returns, in a continuous-time model à la Merton (see **?** and Adler and Dumas (1983)). Optimal portfolios satisfy Property 1. The property holds with factor loadings computed on *total returns*, and thus including any time-varying expected return component — a finding that will matter when computing the empirical counterpart of returns on non-financial wealth.[17] In summary, our results hold in a static context with (locally) complete or incomplete markets, and also in a dynamic context, but under complete markets. Ideally, one would like to derive the equivalent of Property 1 for optimal portfolios in a dynamic model with incomplete markets, multiple agents and time-varying expected returns. This is a challenging task that is beyond the scope of this paper.[18]

[17] In the dynamic model of Appendix A.6, returns are iid log-normal. Expected returns can be time-varying as long as returns of a given asset are driven by a *unidimensional* Brownian motion to preserve market completeness. Otherwise, the derived portfolio is only valid in the log-case.

[18] See for instance the discussion in **?**.

4.2. The data.

We collect quarterly data for all G-7 countries over the period 1970:1-2008:3, stopping short of the global financial crisis.[19] We consider each member of the G-7 as the Home country in turn, aggregating the remaining countries into a 'Foreign country'.

4.2.1. The easy part: bond returns, real exchange rates, financial and non-financial income.

We measure gross real bond returns, R_i^b, as the ex-post gross return on 3-month domestic Treasury-bill converted in constant U.S. dollars. The (log) of the real exchange rate Q_i for country i is defined as the difference between the (log) of the consumer price index in country i, P_i, and the (log) of the consumer price index for the rest of the world, defined as a GDP-weighted average of the price indices of the remaining countries, where all price indices are converted into U.S. dollars:[20] $\ln Q_i = \ln P_i - \sum_{j \neq i} \alpha_{ji} \ln P_j$, where α_{ji} represents the share of country $j's$ output in the rest of the world outside country i. With this definition, an increase in Q_i represents a real appreciation of the currency of country i. Figure 1 reports the real exchange rate for the G-7 countries, normalized to 100 in 2001Q1.

Next, we decompose each country's gross domestic product into a financial and a non-financial components using National Income Account data.[21] All variables are converted in U.S. dollars using nominal exchange rates.

Using these measures yields estimates of the share of financial income δ. Table 1 summarizes our estimates for the G-7 countries. These estimates range from 13.1 percent for Germany to 25.4 percent for Italy, with an unweighted average of 16.7 percent. For comparison, the table also reports the 'naïve' estimate of δ, defined as one minus the share of compensation of employees in output measured at factor prices. It is much higher, with an average of 41.3 percent.

In what follows, financial and non-financial income are normalized by population, and express them in constant U.S. dollars. Figure 1 reports non-financial income per capita for

[19] See appendix B.1 for a detailed description of data sources.

[20] Short-term government bond yields and dollar nominal exchange rates are obtained from the *Global Financial Database.*, Consumer Price Indices from the *OECD Main Economic Indicators*.

[21] Data is obtained from the OECD quarterly national income and from U.N. national account statistics. Details are in appendix B.1.

each country relative to the non-financial income of the remaining G-7 countries. Relative non-financial income exhibits marked fluctuations over the period. For instance, for the U.S., it fluctuates between 0.9 and 2.4. It is also strikingly correlated with the real exchange rate, also reported on the same figure.[22]

4.2.2. The harder part: returns to financial and non-financial wealth

We now construct empirical counterparts to the return on financial and non-financial wealth since neither returns are directly observable. Consider first the return to financial wealth, R^f, where the country subscript i is omitted to ease notation. In general, that return is not equal to the return on aggregate equity R^e. In the model, the two are equal because financial wealth is entirely capitalized in the equity market. In practice, firms are levered, financed with a mix of equity and corporate debt, among other instruments.[23] What is needed is an estimate of the financial return to the firm. Our benchmark method looks at the liability side of the firms' balance sheet, using observable equity and corporate bond market data. Specifically, the gross return to financial wealth, R^f is constructed as a weighted average of the country's equity (R^e) and corporate debt (R^d) gross constant dollar returns, where the weight μ_t reflects the share of corporate debt in the total value of the firm. These weights are estimated for each country using balance sheet data for non-financial firms from Compustat.[24] Our measure of returns to financial wealth for each country is then:

$$r^f_{t+1} \equiv \log(R^f_{t+1}) = \log\left[(1-\mu_t)R^e_{t+1} + \mu_t R^d_{t+1}\right]. \tag{24}$$

Section 4.6 presents alternative estimates of R^f as robustness checks.

Consider next the return to non-financial wealth, R^n. In a dynamic context, that return differs from the growth rate of real non-financial income per capita $\Delta \ln W$: the latter rep-

[22]The correlation ranges between 0.68 for Italy and 0.96 for Japan with an average of 0.85

[23]Appendix A.3 shows that if firms' financing decisions are irrelevant for the value of the firm then the presence of corporate debt has no impact on equity portfolio decisions.

[24]See appendix B.1 for details. The average share of debt in total liabilities is 67.1 percent (Canada), 75.2 percent (France), 75.3 percent (Germany), 76.2 percent (Italy), 70.7 percent (Japan), 59.2 percent (U.K.), 71.8 percent (U.S.). The country equity and corporate debt returns are obtained from the Global Financial Database. For Italy and Japan, corporate bond markets developed only in the late 1980s and the corporate debt return is proxied by the holding return on long-term government debt.

resents only the dividend component and not the total return on the corresponding asset.[25] Measuring the total return on non-financial wealth is a difficult issue. We tackle it from a variety of perspectives. Our benchmark approach follows the present-value method of Campbell and Shiller (1988). Under the assumption that the dividend-price ratio on non-financial wealth is stationary, and using lower case variable for logs, one can derive the following present-value relationship:

$$r^n_{t+1} - E_t r^n_{t+1} = (E_{t+1} - E_t) \sum_{s=0}^{\infty} \rho^s \Delta w_{t+1+s} - (E_{t+1} - E_t) \sum_{s=1}^{\infty} \rho^s r^n_{t+1+s}. \tag{25}$$

In this expression, r^n_{t+1} denotes the (log) gross return on non-financial wealth and ρ is a scalar slightly smaller than 1.[26]

This expression makes clear that the innovation to the return on non-financial wealth depends positively upon revisions to the path of future expected real non-financial income growth—the *cash-flow* component represented by the first summation on the right hand side—and negatively upon revisions to the path of future expected real returns—the *discount* component represented by the second summation on the right hand side.

Our approach consists in constructing the empirical counterpart of Eq. (25) for each country using a Vector-Auto-Regression (VAR) in first differences of the following form:[27] $\mathbf{Z}_{t+1} = \mathbf{A}\ \mathbf{Z}_t + \epsilon_{t+1}$, where $\mathbf{Z}_t = (\tilde{r}_t, \Delta w_t, \Delta k_t, \Delta \ln Q_t, \mathbf{x}'_t)'$. In this expression, $\tilde{r}_{t+s}$ represents a possible proxy for the *expected* return on non-financial wealth at time $t+s$, in the sense that $E_t r^n_{t+s} = E_t \tilde{r}_{t+s}$. This proxy is necessary to construct the second summation on the right hand side of (25). Our benchmark approach sets $\tilde{r} = r^f$, that is, it assumes that expected financial and non-financial future returns are equal. $\Delta w_t, \Delta k_t$ and $\Delta \ln Q_t$ denote respectively the rate of change of non-financial income, financial income and the real exchange rate. Finally, $\mathbf{x}_t$ denotes a vector of additional controls used to forecast factor income growth

[25] See Baxter and Jermann (1997, p. 175)

[26] One can show that $\rho = 1/(1+\phi)$ where ϕ is the steady state dividend price ratio for non-financial wealth. We set $\rho = 0.98$ in line with standard estimates in the literature. Our results are robust to changes in the value of ρ.

[27] Standard Akaike and Schwarz lag-selection criteria indicate that a VAR of order 1 is the preferred specification for all countries.

and returns.[28]

Our VAR specification first-differences financial and non-financial income. Appendix B.3 discusses in details why this is the appropriate empirical specification. In short, it shows that while we cannot reject the null hypothesis that w and k are integrated processes, there isn't any statistical evidence of a co-integration relationship between the two variables. This is a point of departure from Baxter and Jermann (1997) who estimate a Vector Error Correction Mechanism (VECM) on financial and non-financial income, imposing the cointegration relationship that $k - w$ is stationary. This assumption is appealing on theoretical grounds since the share of financial income is bounded between 0 and 1. The null of co-integration is, however, strongly rejected in the data, indicating a very persistent process for income shares, with no apparent error-correction term. Therefore, a stationary VAR in first-differences is appropriate.[29]

With estimates of $\mathbf{A}$ and ϵ_{t+1} in hand, the empirical counterpart to $r^n_{t+1} - E_t r^n_{t+1}$ can be obtained from (25) as:

$$r^n_{t+1} - E_t r^n_{t+1} = \left(\mathbf{e}'_{\Delta w} - \rho \mathbf{e}'_{\tilde{r}} \mathbf{A}\right) \left(\mathbf{I} - \rho \mathbf{A}\right)^{-1} \epsilon_{t+1}, \tag{26}$$

where $\mathbf{e}'_y$ is a row-vector that 'selects' variable y in $\mathbf{Z}$, i.e. such that $\mathbf{e}'_y \mathbf{Z} = y$. Figure 2 reports the return to non-financial wealth $r^n_{t+1} - E_t r^n_{t+1}$ for the U.S., together with the growth rate of non-financial income Δw. The correlation between the two series is high (0.66), but the striking fact is that the return innovation exhibits much more volatility.[30]

The last step consists in measuring bond, financial and non-financial returns relative to

[28] Based on our reading of the literature on financial return predictability, a comprehensive set of potential controls for future asset returns is considered: consumption growth; the relative T-bill rate (the difference between the yield on 3-month T-bill rate and a 4-quarter moving average); the term premium (the spread between 10 year and 3 months government yields); the yield spread (the spread between the yield on long-term corporate bonds and that on 10-year government bonds); *cay*, the fluctuations in U.S. aggregate consumption-wealth ratio as measured by Lettau and Ludvigson (2001); and *nxa*, the Gourinchas and Rey (2007) measure of U.S. external imbalances, extended to 2008. These controls and their selection are described in Appendix B.3.

[29] As discussed in appendix B.3, the assumption that $k - w$ is stationary is also rejected. Section 4.6 considers a VECM alternative, similar to Baxter and Jermann (1997) — results are largely unchanged.

[30] The standard deviation of the return innovations is 3.09% vs. 1.01% for non-financial income growth.

the rest of the world. To this effect, define the relative returns $\hat{r}_i^l$ of country i as follows: $\hat{r}_{i,t+1}^l = \left(r_{i,t+1}^l - E_t r_{i,t+1}^l\right) - \sum_{j\neq i} \alpha_{ji}\left(r_{j,t+1}^l - E_t r_{j,t+1}^l\right)$, for $l \in \{b, f, n\}$, where α_{ji} is the output weight of country j in the rest of the world outside of country i.

4.3. Estimating the loadings on the real exchange rate

We are now in a position to estimate the key loading parameters in Eq. (9). The loadings on the real exchange rate, $\beta_{Q,j}$ for $j = f, b$, are estimated by the following simple regression for each country i:

$$\Delta \ln Q_{i,t} - E_{t-1}\Delta \ln Q_{i,t} \equiv \mathbf{e}'_{\mathbf{\Delta q}}\epsilon_{\mathbf{i,t}} = \beta_{Q,0}^i + \beta_{Q,b}^i \hat{r}_{i,t}^b + \beta_{Q,f}^i \hat{r}_{i,t}^f + u_{i,t}. \tag{27}$$

where $u_{i,t}$ captures the fluctuations in the real exchange rate that are not spanned by relative bond and financial returns.[31]

Results of regression (27) for each countries are displayed in Table 2. Our empirical results confirm the results of van Wincoop and Warnock (2010) for all the countries in our sample: relative bond returns capture most of the variations of the real exchange rate. The coefficient on the relative bond returns in panel A, $\beta_{Q,b}$ is often not statistically different from one, between 0.82 for the U.K and 1.01 for Japan. The R^2 of the regression is also very strong, between 0.86 for UK and 0.95 for France and Japan. Moreover, *conditional on bond returns*, the hedge ratio of financial returns for real exchange rate risk, $\beta_{Q,f}$ is almost never statistically different from zero.[32]

Panel B of the table reports the unconditional loading on the real exchange rate $\beta_{Q,f}^u$ obtained from a regression *only* on the relative financial return $\hat{r}^f$. The coefficients are significantly positive for all countries, between 0.38 (U.K.) and 0.73 (U.S.). This re-emphasizes the importance of properly conditioning on relative bond returns. Finally, the last column of the table reports the results from a pooled regression with country fixed effects. This can

[31] It is important to note how Eq. (27) differs from a standard test of uncovered interest rate parity (Fama (1984)). Denote $\check{r}_{t-1}^b$ the *ex-ante* real interest rate differential between $t-1$ and t, expressed in *local units*. Then $\hat{r}_t^b = \check{r}_{t-1}^b + \Delta \ln Q_t$ and the coefficient $\beta_{Q,b}$ will be close to 1 if most of the variation in ex-post real interest rate differential $\hat{r}_t^b$ comes from movements in the real exchange rate, regardless of whether uncovered interest rate parity holds. However, under uncovered interest rate parity, $\check{r}_{t-1}^b = -E_{t-1}\left[\Delta \ln Q_t\right]$ so that $\hat{r}_t^b = \Delta \ln Q_t - E_{t-1}\Delta \ln Q_t$ measures the innovation to the rate of depreciation and $\beta_{Q,b} = 1$.

[32] The exception is the U.K. but even in this case $\beta_{Q,f}$ remains economically very small, less than 7 percent.

be interpreted as an average loading for all G-7 countries. The estimates, $\beta_{Q,b} = 0.94$ and $\beta_{Q,f} = 0.01$, confirm the strong correlation between relative bond returns and real exchange rates.

4.4. Estimating the loadings on the return to non-financial wealth

We now use the returns to non-financial wealth estimated for each country i to estimate the loadings of (relative) bond returns and (relative) returns to financial wealth by estimating the following equation:

$$\hat{r}^n_{i,t} = \beta^i_{n,0} + \beta^i_{n,b}\hat{r}^b_{i,t} + \beta^i_{n,f}\hat{r}^f_{i,t} + v_{i,t}, \tag{28}$$

where $v_{i,t}$ is attributed both to measurement error in the construction of the return on non-financial wealth, and to fluctuations in relative non-financial income risk not spanned by relative bond returns and relative returns to financial wealth.

Results of the regression (28) for each countries are shown in Table 3. Panel B reports the estimate of the unconditional loading factor $\beta^{i,u}_{n,f} = cov(\hat{r}^n_i, \hat{r}^f_i)/var(\hat{r}^f_i)$. This coefficient is positive and significant for all countries except Italy, with a pooled estimate of 0.41. This indicates that returns to non financial wealth are positively correlated with returns to financial wealth as in Baxter and Jermann (1997) and the international diversification puzzle is 'worse than you think' when using equities only.

However, the loading factor conditional on bond returns $\beta^i_{n,f}$ reported in panel A is negative and strongly significant for all countries, except Germany. It varies between -0.05 (Germany) and -0.55 (Italy) with a pooled estimate of -0.23. As the previous analysis emphasized, this negative conditional hedge ratio indicates that in all these countries domestic equities constitute a good hedge against shocks to non financial wealth.

Moreover, the positive loadings of (relative) bond returns $\beta_{n,b} > 0$ implies that shorting the local currency bond, and going long in the foreign currency bond, constitutes a good hedge against fluctuations in returns to non-financial wealth (see Eq. (11)). This is not surprising: in our model, a potentially large part of relative non-financial income co-moves with the real exchange rate (see Figure 1), and it is well-known that relative bond returns track almost perfectly the real exchange rate.

4.5. Implied equity and bond portfolios

The previous estimates allow us to back out the implied equity and bond positions using equations (11) when all countries are symmetric. Allowing for different country sizes, Eq. (11) must be rewritten as follows (see Appendix (A.5)):

$$\begin{cases} b^* &= (1-\omega_i)\left(1-\frac{1}{\sigma}\right)\beta_{Q,b} - (1-\omega_i)(1-\delta)\beta_{n,b} \\ S^* &= \omega_i + (1-\omega_i)\left(\frac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f} - \frac{1-\delta}{\delta}\beta_{n,f}\right) \end{cases} \tag{29}$$

where ω_i is the relative size of country i in world market capitalization.

The implied equity bias and bond portfolios are summarized in table 4 using the loading coefficients from our baseline estimates. As Eq. (29) indicates, the optimal bond position requires an estimate of the degree of risk aversion σ. We consider the plausible value of $\sigma = 2$ in our benchmark calibration and use the average share of financial income across G-7 countries in the more recent period (2000-2008) for the share of financial income, yielding $\delta = 0.191$.

The model is very successful at predicting a significant degree of equity home bias for all countries when bond trading is allowed. Consider first panel B, which excludes bonds, as commonly done in the literature. The baseline refers to the first term in Eq. (29), that is, a predicted portfolio share equal to the share in world market capitalization ω_i. The second term (Bias due to Q) reflects the contribution of the real exchange rate hedging component: $(1-\omega_i)(1-1/\sigma)\beta^{i,u}_{Q,f}/\delta$. Given the positive unconditional correlation between financial returns and exchange rates ($\beta^{i,u}_{Q,f} > 0$ in table 2), this term is positive, indicating a potential source of home bias. The second term (Bias due to r^n) reflects the contribution of the non-financial income hedging component: $-(1-\omega_i)(1-\delta)\beta^{i,u}_{n,f}/\delta$. Since $\beta^{i,u}_{n,f}$ is strongly positive (see table 3), this term contributes negatively to the optimal equity portfolio and dominates the real exchange rate hedge. The result, as in Baxter and Jermann (1997) is a strong predicted foreign bias, $S^i - \omega_i$ ranging from -8.6 percent for France to -91.5 percent for Germany, in total contrast to the data.[33,34]

By contrast, Panel A shows that the estimated model accounts for a large share of observed equity home bias once bond trading is allowed. The hedge portfolio is now dominated by the non-financial income component. This term is strongly positive since $\beta^i_{n,f} < 0$ in table 3. The predicted equity portfolio (S) is 29% for Germany, between 59% and 101% for Canada, Japan, U.K. and the U.S. and quite above 100% for France and Italy.[35] Available empirical evidence indicates a home equity position between 55% (Germany) and 85.6% (Canada).[36] Except for Germany, the equity bias predicted by the model is comparable to the amount of bias in the data. Using $\beta_{Q,f}$ and $\beta_{n,f}$ estimated on pooled data for all countries, equity portfolios are close to 90% for all countries, fairly close to the data.

The last line (ΔS) reports the change in the predicted equity position between the equity only and the full model. In all cases, the predicted equity position increases substantially, moving the model closer to the data. For instance, in the case of the U.S., in the model with equity only, investors should have a strong foreign bias ($S = 12\%$) while the full model predicts 101% domestic equity holdings, much closer to the empirical estimate (83.2%).

Panel A also reports the model predictions for bond holdings. As for equities, it decomposes the predicted bond position into a real exchange rate hedge component ($(1-\omega_i)(1-1/\sigma)\beta_{Q,b}$) and a non financial income component ($-(1-\omega_i)(1-\delta)\beta_{n,b}$).

We find a strong positive demand for local currency bonds arising from real exchange rate hedging, given the positive loading factor $\beta_{Q,b}$, but an even stronger incentive to borrow

[33] The exception is Italy, where the unconditional loading $\beta^{\mathrm{u}}_{n,f}$ is insignificant and therefore the model predicts more home bias than observed.

[34] Our baseline estimates ignores time-variation of the equity home bias over the period considered. In our framework, a change in the equity home bias should come from changes in the hedge ratios. This hypothesis can be tested by splitting the sample in two, from 1970Q1 to 1989Q1 and from 1989Q2 to 2008Q3 — and estimating hedge ratios over the two sub-samples. Our results suggest a very slight fall in the equity home bias over the period for the average country, although not statistically significant. We conclude that while the model can account for the cross section of equity home bias, it does not account for the decline in equity home bias in the time-series. Focusing on shorter time periods reduces variations in the data, which makes statistical inference harder and renders difficult any comparison across time-periods.

[35] The results for Italy are perhaps to be taken with some caution since the return on corporate bonds is proxied by the return on Italian T-bills.

[36] Data are from Coeurdacier and Rey (2011).

in local currency bonds to hedge non-financial income risk, given the positive $\beta_{n,b}$. While each of these component can be large relative to output, they offset each other and imply net currency exposure of bond portfolios of reasonable magnitude. The model predicts that countries' net currency exposure ranges between -19.6 percent (U.S.) and -54.5 percent (Italy) of domestic output, with a (size-weighted) average of -29% of GDP. Data regarding the net currency exposure in portfolio debt positions from Lane and Shambaugh (2010) indicates that G-7 countries are on average short in domestic currency, as predicted by the model, although the positions are both smaller than predicted by the model and more heterogenous. The (size-weighted) average net currency bond exposure is only $b = -7.9\%$ of GDP over 2000-2004, ranging from -16.40 percent (U.K) to 9.90 percent (France).[37] Overall, the fit of the benchmark model in terms of bond portfolios seems less impressive. In particular, the model is not able to match the heterogeneity in observed bond positions across countries with some countries long and some countries short in domestic currency exposure.[38]

4.6. Using Different Measures of Returns to Financial and Non-Financial Wealth

A key element of our analysis is the construction of returns to financial and non financial wealth r^f and r^n. This section investigates the robustness of our results to various alternative measures of financial and non financial returns.

A first point of departure would be to construct returns to financial wealth using the same approach as for non financial returns, with national income data as in Baxter and Jermann (1997). This approach yields the following expression for the return to financial

[37] In the data, countries often have leveraged external debt position. The counterpart of b in the model is $(b_{HH} - b_{HF})/2$ where b_{HH} denotes the net domestic currency debt exposure, that is, the difference between domestic currency denominated debt assets and domestic currency denominated debt liabilities– and b_{HF} denotes the net foreign currency debt exposure, that is, the difference between foreign currency debt assets and foreign currency debt liabilities. This counterpart generates the same wealth transfer towards a country whose currency depreciates by 1% with respect to all other currencies as in our model.

[38] The model assumes that there is no sovereign risk. While this is a reasonable assumption for G-7 countries, we note that sovereign risk is likely to reduce equilibrium portfolio bond holdings, forcing countries to rely more on equity holdings to hedge real exchange rate and non-financial income risk. While this would reduce observed bond holdings, it would also reduce home bias in equities. A full analysis of the model with sovereign risk is beyond the scope of this paper.

wealth:

$$r_{t+1}^{f} - E_t r_{t+1}^{f} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta k_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+s}^{f}. \tag{30}$$

Using the same VAR specification as in section (4.2.2), the empirical estimates of the returns to financial wealth becomes:[39]

$$r_{t+1}^{f} - E_t r_{t+1}^{f} = \left(\mathbf{e}_{\Delta k}' - \rho \mathbf{e}_{\tilde{r}}' \mathbf{A}\right) \left(\mathbf{I} - \rho \mathbf{A}\right)^{-1} \epsilon_{t+1}. \tag{31}$$

The returns on the firm thus obtained may be noisy and imperfectly estimated. Our second approach instruments the returns in Eq. (31) with the country's equity and corporate debt returns, forcing the weights to sum to one. This is approximately equivalent to choosing different weights $\hat{\mu}$ in Eq. (24), measuring the leverage implied by national accounts data according to a first stage regression:

$$r_t^f = (1 - \hat{\mu}) r_t^e + \hat{\mu} r_t^d + \nu_t. \tag{32}$$

The predicted component $\left((1 - \hat{\mu}) r_t^e + \hat{\mu} r_t^d\right)$ of (32) becomes our proxy for returns to financial wealth. This method identifies the variations in financial wealth estimated from national accounts that are reflected in market returns and is therefore potentially more robust to measurement error.

A third approach approach simply sets $\mu = 0$, equating the return to financial wealth with observed equity returns: $r_t^f = r_t^e$. This approach has the merit of simplicity, but as argued earlier, there are good theoretical reasons why equity returns may differ from the returns to the firm.

Lastly, we also consider three different approaches to constructing returns to non financial wealth. The first one assumes that non-financial wealth is discounted using the holding return on long term government bonds, denoted r^{lb}. It follows the exact same methodology as in our benchmark estimates but sets $\tilde{r} = r^{lb}$ to construct estimates of returns to non-financial

[39]The implementation still requires the use of observed market returns to form expectations of future returns. In practice, we use the returns on the firm constructed in the previous section as a proxy.

wealth.

The second approach borrows from Lustig and Nieuwerburgh (2008). The basic idea is to recover the unobserved innovation to non financial wealth from the joint behavior of consumption and market returns, under the assumption that aggregate consumption satisfies the first-order condition of an optimizing representative household. The last approach imposes, as in Baxter and Jermann (1997), that financial and non-financial incomes are co-integrated and estimates a VECM. These last two approaches are detailed in appendix B.3.

Results from regressions (27) and (28) are displayed in table 5 and table 6 for the different specifications and the different countries. Our empirical results confirm the previous results across all specifications: relative bond returns capture most of the variations of the real exchange rate and claims on financial income are not used to hedge real exchange rate changes (see table 5). Moreover, *conditional on bond returns*, the loadings of non-financial wealth on financial wealth are negative across all specifications and significantly so for most of the countries, implying home bias in our model (see table 6). This confirms the important role of bond holdings as an hedging instrument. Hence, qualitatively, results using these alternative measures of returns are very similar to our benchmark case.

Quantitatively, the magnitude of the loadings $\beta^i_{n,f}$ in table 6 are similar to our benchmark case when using the projection of financial returns estimated from national accounts on market returns (panel B, the pooled estimate of $\beta^i_{n,f}$ is equal to -0.177), when using long term government bond returns to discount non-financial wealth (panel D, pooled estimate of $\beta^i_{n,f}$ equal to -0.245), when using the method of Lustig and Nieuwerburgh (2008) (panel E, pooled estimate of $\beta^i_{n,f}$ equal to -0.191) or when using a cointegration approach (panel F, pooled estimate of $\beta_{n,f} = -0.245$). As reported in table 7, under all of these specifications, the amount of equity home bias generated by our estimates are in line with or even larger then the home bias data for most countries.

The results are marginally weaker when using national accounts data (panel A) or equity returns (panel C). More generally, one could also argue that these are noisier measure of financial returns causing attenuation bias on our estimates of the loadings. When using equity returns, the pooled estimates of $\beta^i_{n,f}$ is equal to -0.08 (panel C), roughly 40% of the value of our benchmark. Hence in this specification, the model can still explain a significant share of equity home bias (around 40%; see table 7). The estimation using national account data to estimate returns to financial wealth performs qualitatively similarly as our benchmark, except for Italy.[40] When looking at the U.S. more specifically, table 7 indicates that the equity portfolio implied by the model are respectively 82% of domestic equity when using national account data and 70% when using equity returns, only slightly below the measured ones.[41]

As a final check, we consider the relative importance of the cash flow and discount components in Eq. (25). Unlike our benchmark result, Benigno and Nistico (2011) find that, for the U.S., returns to non-financial wealth are largely uncorrelated with financial returns, even after controlling for bond returns. Their approach ignores the contribution of revisions to the path of future expected real returns to the return on non-financial wealth –the second term in Eq. (25). Conceptually, it is not clear why one would wish to assume that the expected return to non-financial wealth remains constant given the large body of evidence on time-varying asset returns. Further, as the robustness checks presented above illustrates, our results are robust to many plausible alternative assumptions regarding expected future non-financial returns (equal to expected financial return, expected government bond return, or determined by consumption innovations). Lastly, our results are qualitatively robust to the restriction that expected non-financial returns are constant. Setting the second summation in (25) equal to zero, the conditional loading of non-financial returns on financial ones remains negative and significant for most countries, although not the U.S. or Germany, accounting perhaps for the findings in Benigno and Nistico (2011).[42]

[40]In panel A, Italy is an obvious outlier with $\beta_{n,f} = 0.51$. However, recall for that country, we do not have a good measure of corporate returns which affects the way non-financial wealth is discounted. When dropping Italy from our pooled estimation, $\beta^i_{n,f}$ is equal to -0.1 and is highly significant.

[41]Like in our benchmark regression, the unconditional loadings (non-reported) for these two specifications are positive and highly significant ($\beta^{i,u}_{n,f} > 0$) implying a very large foreign bias in the model without bonds.

[42]The estimates vary between 0.004 for Germany and -0.20 for France. Results available upon request from the authors.

5. Conclusion

What drives portfolio equity home bias? This paper merges and improves upon two strands of literature. The first strand focused on risks to non-financial wealth. It concluded that home equity positions should be even more tilted towards foreign equity since non financial and financial returns appear to be positively correlated. The second strand looked at frictions in goods markets and emphasized real exchange rate risks. In this class of models, efficient risk sharing requires holding securities delivering high returns when the domestic currency appreciates. However, the correlation between stock returns and exchange rates is too low to generate significant portfolio biases. This class of models has thus been challenged for its lack of empirical support.

This paper shows that both strands of the literature are related, but incomplete. It starts from the observation that relative bond returns (nominal or real) are strongly correlated with real exchange rates. It follows that, in a world where investors can trade both equities and bonds, they will hedge real exchange rate risk with the latter. And once this is achieved, the equilibrium equity position will be a function of the residual risks that investors face, namely the risk to their non-financial wealth, *conditional* on bond returns. Equity home bias will arise if non financial risk is negatively correlated with equity returns, after controlling for bond returns. The paper derives this prediction in a fairly general model and characterizes equilibrium portfolios as a simple function of hedge ratios that can easily be estimated from data on real exchange rates and returns on bonds, financial and non-financial wealth. This paper implements this empirical strategy for the countries of the G-7 and shows that under many reasonable specifications, the conditional correlation between financial and non-financial returns is such that it can empirically account for a significant share of the observed equity home bias. For most countries, the conditional correlation between financial and non financial returns is negative and economically significant. In other words, the international diversification puzzle is not so puzzling anymore! The model also makes predictions about equilibrium bond positions. Here, although the overall currency exposure of bond portfolios is broadly in line with the empirical evidence, the model fails to capture the heterogeneity in currency exposure across countries.

It is possible to interpret our results in a broader perspective. Nominal exchange rates present a deep source of puzzles in international finance. They are too volatile and largely uncorrelated with their fundamental determinants — the exchange rate disconnect puzzle. To the extent that nominal exchange rate movements drive real exchange rate fluctuations, real exchange rates too, do not behave as predicted in our models —the Mussa (1986) puzzle. For instance, relative real consumption is not correlated with real exchange rate movements as models of risk sharing predict—the Backus and Smith (1993) puzzle. In the context of international portfolios, this implies that real exchange rates fluctuations are both uncorrelated with relative financial returns, and that relative financial and non-financial returns are positively correlated, since a given change in the nominal exchange rate affects both returns in the same direction. Our paper shows that, once currency fluctuations are controlled for through the use of nominal or real bonds, the structure of international equity portfolios conforms to the predictions of standard portfolio models. This provides a qualified success for the theory, since an empirically successful theory of exchange rate fluctuations remains elusive.

We left open an obvious step for future research. One would want to go back and enrich/discriminate among existing models to fully account for the hedge ratios we obtain from the data. Such a model would be consistent both with observed portfolios (quantities) and with their corresponding loadings, i.e the covariance structure of exchange rates and asset returns (prices). Going from the reduced form estimates to the structural parameters of the model requires taking a stand on the 'correct' model of the economy. A full-fledged structural estimation lies beyond what we attempt in this paper.

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	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Average
δ	16.4	14.1	13.1	25.4	16.1	18.5	13.3	16.7
naïve-δ	39.9	39.9	40.5	50.9	43.5	36.7	37.8	41.3

Table 1: Estimates of the share of financial income in output δ (in percent), defined as the share of financial income $(\Pi + D + (1-\lambda)\,M - I)$ in output at product prices net of investment $(Y - T - I)$. The naïve share is estimated as one minus the share of compensation of employees $(COMP)$ in output at factor prices $(Y - T)$. Source: OECD Quarterly National Income and U.N. National Account Statistics. Authors' calculations.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
	Panel A: Conditional Loadings							
$\beta_{Q,f}$	-0.036	-0.011	0.007	-0.017	-0.030	0.064***	-0.013	0.006
(s.e.)	(0.025)	(0.024)	(0.026)	(0.018)	(0.028)	(0.022)	(0.038)	(0.009)
$\beta_{Q,b}$	1.003***	0.944***	0.946***	0.969***	1.012***	0.821***	0.944***	0.942***
(s.e.)	(0.033)	(0.028)	(0.031)	(0.026)	(0.034)	(0.039)	(0.040)	(0.012)
R^2	0.941	0.947	0.940	0.944	0.947	0.863	0.918	0.929
	Panel B: Unconditional Loadings							
$\beta^u_{Q,f}$	0.579***	0.591***	0.616***	0.447***	0.658***	0.376***	0.733***	0.554***
(s.e.)	(0.040)	(0.043)	(0.043)	(0.042)	(0.040)	(0.034)	(0.048)	(0.016)
R^2	0.579	0.557	0.573	0.424	0.641	0.453	0.611	0.535
Obs.	153	153	153	153	153	153	153	1071

Table 2: Loadings on **real exchange rate changes**: $\Delta \ln Q_{i,t} - E_0 \Delta \ln Q_{i,t} = \beta^i_{Q,b} \hat{r}^b_{i,t} + \beta^i_{Q,f} \hat{r}^f_{i,t} + u_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Unconditional loadings impose $\beta_{Q,b} = 0$. Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
				Panel A: Conditional Loadings				
$\beta_{n,f}$	-0.186***	-0.327***	-0.053	-0.551***	-0.171***	-0.081**	-0.252***	-0.227***
(s.e.)	(0.072)	(0.057)	(0.062)	(0.098)	(0.053)	(0.036)	(0.099)	(0.026)
$\beta_{n,b}$	1.262***	1.122***	1.073***	1.295***	0.970***	0.967***	1.073***	1.096***
(s.e)	(0.094)	(0.069)	(0.075)	(0.140)	(0.065)	(0.062)	(0.103)	(0.034)
R^2	0.709	0.719	0.759	0.366	0.769	0.706	0.595	0.600
				Panel B: Unconditional Loadings				
$\beta^u_{n,f}$	0.588***	0.389***	0.637***	0.068	0.489***	0.286***	0.595***	0.411***
(s.e.)	(0.064)	(0.060)	(0.060)	(0.089)	(0.046)	(0.043)	(0.074)	(0.024)
R^2	0.362	0.219	0.429	0.004	0.428	0.223	0.300	0.213
Obs.	153	153	153	153	153	153	153	1071

Table 3: Loadings on **non-financial returns**: $\hat{r}^n_{i,t} = \beta^i_{n,b}\hat{r}^b_{i,t} + \beta^i_{n,f}\hat{r}^f_{i,t} + v_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Unconditional loadings $\beta^u_{n,f}$ impose $\beta_{n,b} = 0$. Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.
	Panel A: With Bonds and Equity						
Equity							
Baseline (Market Cap Weights)	5.13	7.30	5.67	3.30	15.71	12.35	50.53
Bias due to:							
Q	-8.96	-2.72	1.77	-4.25	-6.64	14.59	-1.64
r^n	74.56	128.36	21.27	225.88	61.01	30.25	52.76
Total (S)	70.73	132.95	28.71	224.93	70.08	57.18	101.66
Data for (S) (2000-2008)	85.60	71.40	55.40	59.50	84.30	65.20	83.20
Bond							
Bias due to:							
Q	47.58	43.74	44.59	46.85	42.65	35.99	23.35
r^n	-96.85	-84.11	-81.85	-101.33	-66.14	-68.55	-42.92
Total (b)	-49.27	-40.37	-37.26	-54.48	-23.49	-32.56	-19.57
Data for (b) (2000-2004)	9.30	9.90	8.90	-2.70	-12.70	-16.40	-10.90
	Panel B: Equities Only						
Baseline (Market Cap Weights)	5.13	7.30	5.67	3.30	15.71	12.35	50.53
Bias due to:							
Q	145.25	142.53	143.62	108.09	144.20	90.82	104.86
r^n	-238.74	-151.72	-240.46	-26.71	-173.30	-111.90	-137.79
Total (S)	-89.35	-1.29	-85.91	88.97	-12.77	-13.33	12.44
ΔS	160.08	134.24	114.62	135.96	82.84	70.52	89.22

Table 4: Implied Portfolio Equity (S) and bond (b) position for G-7 countries. Calculations are done under the assumption that $\delta = 0.19$ and $\sigma = 2$. (S) refers to the percentage of domestic stocks held by domestic residents (data for (S) are averaged over the period 2000-2008). ΔS refers to the difference between the implied S in a model with bonds and equity and the implied S with equities only. (b) refers to the net domestic currency exposure of bond portfolios (as a % of GDP). Data for (b) are computed from Lane and Shambaugh (2010) and refers to the average between net debt assets in domestic currency and net debt liabilities in foreign currency as a % of GDP (averaged over 2000-2004): $b = \frac{b_{HH} - b_{HF}}{2}$.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
	Panel A: Financial returns estimated using national accounts							
$\beta_{Q,f}$	0.011	0.017	0.037	0.051	0.032	0.047***	0.000	0.027
(s.e.)	(0.010)	(0.010)	(0.014)	(0.012)	(0.013)	(0.019)	(0.019)	(0.005)
$\beta_{Q,b}$	0.954***	0.921***	0.911***	0.923***	0.953***	0.846***	0.934***	0.924***
(s.e.)	(0.022)	(0.019)	(0.025)	(0.019)	(0.022)	(0.035)	(0.027)	(0.009)
R^2	0.941	0.948	0.942	0.950	0.949	0.862	0.918	0.931
	Panel B: Projection of returns from panel A on market returns							
$\beta_{Q,f}$	-0.050	-0.021	-0.001	-0.049***	-0.043	0.040	-0.010	-0.020
(s.e.)	(0.028)	(0.028)	(0.026)	(0.019)	(0.033)	(0.033)	(0.043)	(0.011)
$\beta_{Q,b}$	1.018***	0.953***	0.953***	1.008***	1.026***	0.843***	0.943***	0.968***
(s.e.)	(0.035)	(0.032)	(0.033)	(0.028)	(0.039)	(0.051)	(0.046)	(0.014)
R^2	0.941	0.947	0.939	0.946	0.948	0.857	0.918	0.929
	Panel C: Financial returns based on equity returns							
$\beta_{Q,f}$	-0.003	0.003	0.008	0.011	-0.015	0.040***	-0.007	0.006
(s.e.)	(0.012)	(0.009)	(0.012)	(0.008)	(0.011)	(0.014)	(0.018)	(0.004)
$\beta_{Q,b}$	0.971***	0.935***	0.939***	0.946***	0.996***	0.878***	0.943***	0.946***
(s.e.)	(0.022)	(0.019)	(0.022)	(0.020)	(0.023)	(0.030)	(0.026)	(0.009)
R^2	0.945	0.953	0.935	0.941	0.944	0.889	0.924	0.932
	Panel D: Non-financial returns using bond return discounting: $\tilde{r} = r^b$							
$\beta_{Q,f}$	-0.030	0.001	0.007	-0.018	-0.033	0.053**	-0.030	0.001
(s.e.)	(0.024)	(0.024)	(0.028)	(0.018)	(0.030)	(0.026)	(0.037)	(0.010)
$\beta_{Q,b}$	1.002***	0.929***	0.939***	0.971***	1.006***	0.769***	0.964***	0.937***
(s.e.)	(0.031)	(0.029)	(0.034)	(0.026)	(0.037)	(0.044)	(0.039)	(0.013)
R^2	0.947	0.944	0.931	0.945	0.939	0.807	0.924	0.921
	Panel E: Non-financial returns estimated using Lustig and Nieuwerburgh (2008)							
$\beta_{Q,f}$	-0.027	-0.017	0.017	-0.008	-0.030	0.072***	-0.013	0.011
(s.e.)	(0.028)	(0.024)	(0.028)	(0.018)	(0.030)	(0.023)	(0.038)	(0.010)
$\beta_{Q,b}$	0.980***	0.947***	0.925***	0.958***	1.005***	0.802***	0.945***	0.931***
(s.e.)	(0.037)	(0.029)	(0.034)	(0.026)	(0.036)	(0.040)	(0.040)	(0.013)
R^2	0.927	0.944	0.927	0.945	0.940	0.855	0.918	0.923
Obs.	153	153	153	153	153	153	153	1071
	Panel F: Vector Error-Correction Mechanism							
$\beta_{Q,f}$	-0.037	-0.022	0.007	-0.013	-0.039	0.065***	-0.016	0.004
(s.e.)	(0.028)	(0.028)	(0.026)	(0.018)	(0.028)	(0.023)	(0.037)	(0.010)
$\beta_{Q,b}$	0.990***	0.934***	0.946***	0.967***	1.019***	0.819***	0.948***	0.939***
(s.e.)	(0.037)	(0.034)	(0.031)	(0.026)	(0.034)	(0.039)	(0.039)	(0.013)
R^2	0.926	0.922	0.939	0.946	0.947	0.861	0.922	0.925
Obs.	153	153	153	153	153	153	153	1071

Table 5: Loadings on **real exchange rate changes** for alternative measures of returns: $\Delta \ln Q_{i,t} - E_0 \Delta \ln Q_{i,t} = \beta^i_{Q,b} \hat{r}^b_{i,t} + \beta^i_{Q,f} \hat{r}^f_{i,t} + u_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
	Panel A: Financial returns estimated using national accounts							
$\beta_{n,f}$	-0.093***	-0.041	-0.074**	0.506***	-0.118***	-0.027	-0.149***	-0.004
(s.e.)	(0.027)	(0.026)	(0.034)	(0.062)	(0.025)	(0.031)	(0.047)	(0.015)
$\beta_{n,b}$	1.156***	0.847***	1.104***	0.459***	0.901***	0.900***	0.981***	0.877***
(s.e.)	(0.061)	(0.052)	(0.059)	(0.100)	(0.041)	(0.056)	(0.070)	(0.027)
R^2	0.718	0.662	0.765	0.470	0.786	0.698	0.603	0.572
	Panel B: Projection of financial returns from panel A on market returns							
$\beta_{n,f}$	-0.149	-0.331***	-0.065	-0.256**	-0.219***	-0.094	-0.179	-0.177***
(s.e.)	(0.080)	(0.071)	(0.062)	(0.111)	(0.063)	(0.052)	(0.113)	(0.031)
$\beta_{n,b}$	1.224***	1.119***	1.089***	1.043***	1.019***	0.992***	1.021***	1.056***
(s.e.)	(0.102)	(0.079)	(0.166)	(0.074)	(0.081)	(0.119)	(0.069)	(0.031)
R^2	0.703	0.700	0.759	0.258	0.772	0.702	0.584	0.584
	Panel C: Financial returns based on equity returns							
$\beta_{n,f}$	-0.109***	-0.053***	0.014	-0.125***	-0.076***	-0.028	-0.099**	-0.080***
(s.e.)	(0.043)	(0.020)	(0.033)	(0.023)	(0.026)	(0.026)	(0.049)	(0.012)
$\beta_{n,b}$	1.287***	1.032***	1.168***	1.240***	0.375***	0.995***	0.926***	0.952***
(s.e.)	(0.079)	(0.044)	(0.058)	(0.058)	(0.053)	(0.055)	(0.071)	(0.025)
R^2	0.678	0.805	0.762	0.751	0.256	0.726	0.571	0.600
	Panel D: non-financial returns using bond returns discounting: $\tilde{r} = r^b$							
$\beta_{n,f}$	-0.148***	-0.289***	-0.100	-0.590***	-0.259***	-0.079	-0.298***	-0.245***
(s.e.)	(0.074)	(0.083)	(0.071)	(0.120)	(0.060)	(0.061)	(0.095)	(0.032)
$\beta_{n,b}$	1.076***	0.951***	0.917***	1.076***	1.073***	0.981***	1.046***	1.012***
(s.e.)	(0.096)	(0.100)	(0.085)	(0.171)	(0.074)	(0.106)	(0.099)	(0.041)
R^2	0.634	0.459	0.612	0.212	0.732	0.457	0.577	0.451
	Panel E: non-financial returns estimated using Lustig and Nieuwerburgh (2008)							
$\beta_{n,f}$	-0.172***	-0.218***	-0.122***	-0.204***	-0.216***	-0.199***	-0.179***	-0.191***
(s.e.)	(0.042)	(0.037)	(0.052)	(0.037)	(0.037)	(0.028)	(0.040)	(0.014)
$\beta_{n,b}$	1.084***	1.141***	0.985***	1.163***	1.165***	1.113***	1.124***	1.116***
(s.e.)	(0.055)	(0.044)	(0.063)	(0.053)	(0.046)	(0.048)	(0.041)	(0.018)
R^2	0.836	0.884	0.765	0.814	0.906	0.815	0.920	0.853
Obs.	153	153	153	153	153	153	153	1071
	Panel F: Vector Error-Correction Mechanism							
$\beta_{n,f}$	-0.284***	-0.463***	-0.004	-0.531***	-0.150***	-0.127***	-0.228**	-0.245***
(s.e.)	(0.094)	(0.094)	(0.078)	(0.085)	(0.055)	(0.041)	(0.11)	(0.029)
$\beta_{n,b}$	1.369***	1.229***	0.994***	1.265***	0.946***	0.994***	1.012***	1.095***
(s.e.)	(0.123)	(0.114)	(0.094)	(0.122)	(0.068)	(0.072)	(0.033)	(0.038)
R^2	0.597	0.492	0.651	0.422	0.752	0.632	0.517	0.545
Obs.	153	153	153	153	153	153	153	1071

Table 6: Loadings on **non-financial returns** for alternative measure of returns: $\hat{r}^n_{i,t} = \beta^i_{n,b}\hat{r}^b_{i,t} + \beta^i_{n,f}\hat{r}^f_{i,t} + v_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.
	Implied Equity (S) under alternative estimation methods						
Baseline (Market Cap Weights)	5.13	7.30	5.67	3.30	15.71	12.35	50.53
Benchmark estimates	70.73	132.95	28.71	224.93	70.08	57.18	101.66
National Accounts	45.54	27.57	44.34	-191.01	64.94	33.08	81.77
Projection of financial returns	51.19	137.32	32.70	79.24	86.95	58.09	91.05
Equity returns	48.26	28.93	1.99	57.33	39.62	31.96	70.30
Bond returns discounting	57.43	121.10	47.52	240.61	100.76	53.90	109.01
Method of Lustig et al (2008)	67.46	88.67	58.63	84.81	86.32	102.79	86.39
Vector Error-Correction	109.92	183.60	9.07	217.43	60.70	74.23	96.12
Data for S (2000-2008)	85.60	71.40	55.40	59.50	84.30	65.20	83.20
	Implied Bond (b) under alternative estimation methods						
Benchmark estimates	-49.27	-40.37	-37.26	-54.48	-23.49	-32.56	-19.57
National Accounts	-18.43	-29.79	-17.49	-50.01	-26.99	-20.56	-13.11
Projection of financial returns	-45.32	-40.56	-38.34	-29.40	-26.85	-33.84	-18.31
Equity returns	-52.67	-34.02	-44.87	-51.26	16.39	-32.07	-13.71
Bond returns discounting	-35.04	-28.27	-25.68	-37.27	-30.77	-35.86	-17.99
Method of Lustig et al (2008)	-36.71	-41.62	-31.52	-44.69	-37.10	-43.78	-21.61
Vector Error-Correction Model	-58.14	-48.91	-31.23	-52.18	-21.55	-34.57	-17.06
Data for b (2000-2004)	9.30	9.90	8.90	-2.70	-12.70	-16.40	-10.90

Table 7: Implied Portfolio Equity (S) and bond (b) position for G-7 countries under alternative methods to compute financial and non-financial returns. Calculations are done under the assumption that $\delta = 0.19$ and $\sigma = 2$. (S) refers to the percentage of domestic stocks held by domestic residents (data for (S) are averaged over the period 2000-2008). (b) refers to the net domestic currency exposure of bond portfolios (as a % of GDP). Data for (b) are computed from Lane and Shambaugh (2010) and refers to the average between net debt assets in domestic currency and net debt liabilities in foreign currency as a % of GDP (averaged over 2000-2004): $b = \frac{b_{HH} - b_{HF}}{2}$.

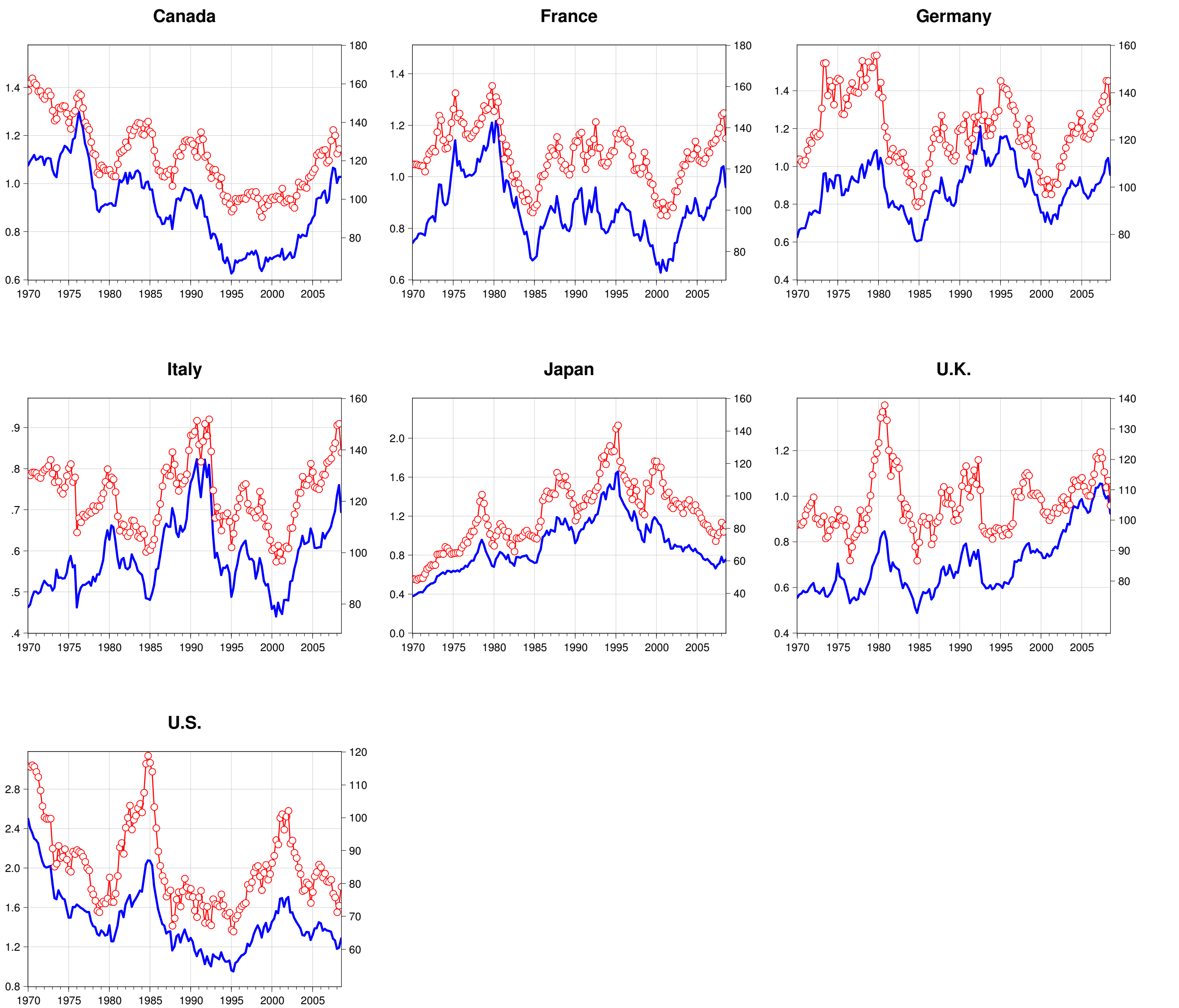

Figure 1: Relative non-financial income (left) [–] and real exchange rate [-o-] (100 in 2001Q1, right), G-7 countries, 1970:1-2008:3. Data Sources: Global Financial Database, OECD Quarterly National Accounts and UN National Account Statistics. Authors' calculations.

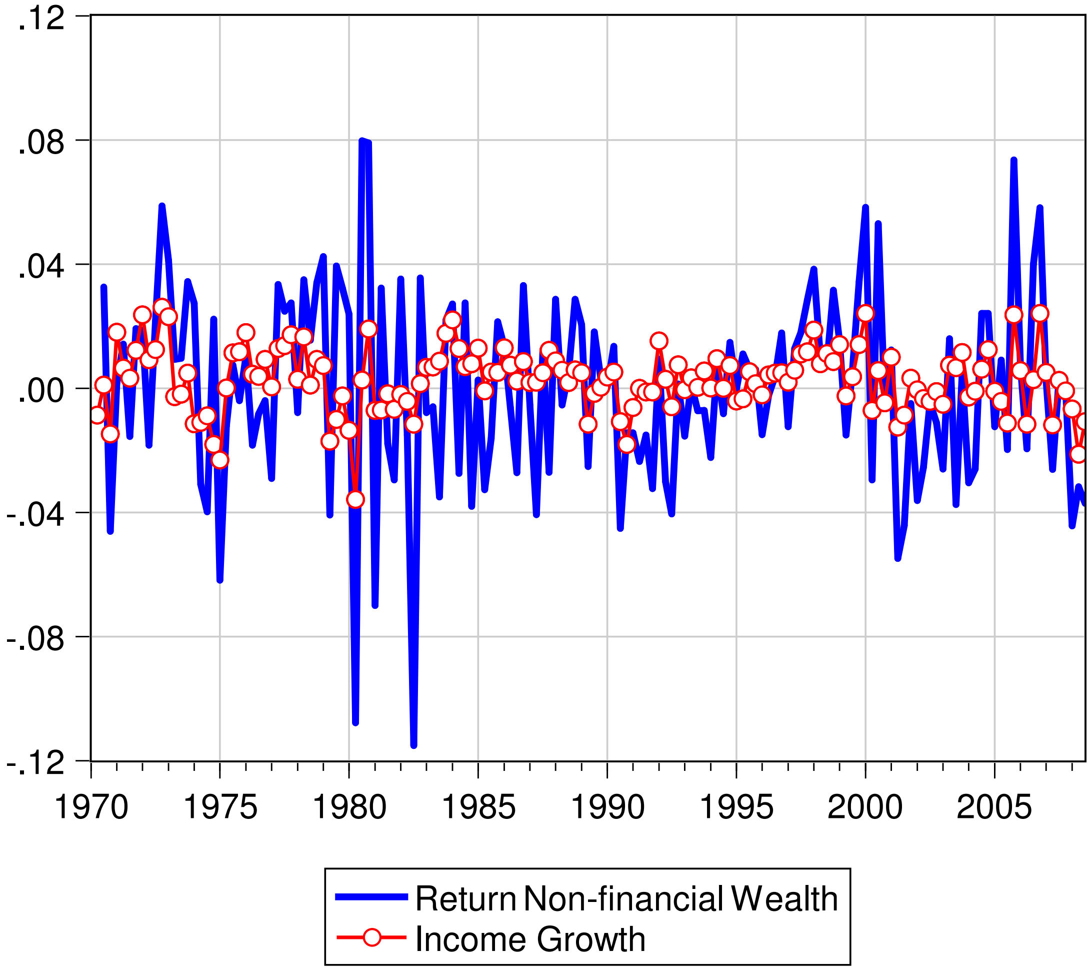

Figure 2: Innovations to returns on non-financial wealth $r^n_{t+1} - E_t r^n_{t+1}$, and non-financial income growth Δw, United States, 1970:1-2008:3.

Appendices [Not for Publication]

A. Theoretical Derivations

A.1. Optimal portfolios in the benchmark model

We use Jonesian hats ($\hat{x} \equiv log(x/\bar{x})$) to denote the log-deviation of a variable x from its mean value $\bar{x}$. Variables x without country indices denotes difference across countries: $\hat{x} = \hat{x}_H - \hat{x}_F$. We denote Δx for first-differences : $\Delta\hat{x} = \hat{x}_1 - \hat{x}_0$.

We assume that countries are symmetric ex-ante, that is $E_{-1}y_{i,t} = \bar{y}$. However, as of time 0, countries can have different expected growth rates. That is, we allow $E_0 y_{i,1}$ to differ across countries. Appendix (A.5) considers the case of ex-ante asymmetries in sizes.

We apply a similar method to Devereux and Sutherland (2006) (see also Tille and van Wincoop (2010)) to characterize equilibrium portfolios. This method relies on deriving:

1. First-order approximation for non-portfolio equations,
2. Second-order approximation of the Euler equations.

Non-portfolio equation. In our generic model of Section 2, there is only one non-portfolio equation, the relative budget constraint. Taking the difference between Home and Foreign budget constraints (4) and using the asset market clearing conditions implies:

$$\begin{aligned} X_0 =& d_0^n + \sum_j \left(2S_{Hj,0} - 1\right) d_{j,0}^f + 2\sum_j \left(S_{Hj,0} - S_{Hj,1}\right) p_S^j \\ & - 2\sum_j p_B^j B_{Hj,1} + 2\sum_j d_{j,0}^b B_{Hj,0} \end{aligned} \tag{A.1a}$$

$$X_1 = d_1^n + \sum_j \left(2S_{Hj,1} - 1\right) d_{j,1}^f + 2\sum_j B_{Hj,1} d_{j,1}^b \tag{A.1b}$$

Denote (S, B) the optimal holdings of stocks and bonds in the perfectly symmetric equilibrium, i.e. the case considered in the main text where $S = S_{ii,t}$ and $B = B_{ii,t}$. Because we allow for differences in output growth (as of time 0), we can write the optimal stock and bond holdings as $S_{ii,t} = S$ and $B_{ii,0} = B$, $B_{ii,1} = B(1 + \hat{B}_i)$. $\hat{B}_i$ denotes the portfolio rebalancing component of the bond portfolio due to *intertemporal* smoothing between period 0 and 1. Note that with fully symmetric country, as in Section 2, countries have no incentives for reblancing their portfolio, i.e $\hat{B}_i = 0$.[43]

Log-linearizing (A.1a) and (A.1b) and neglecting second-order terms, yields (using the steady-state share of non financial income $1 - \delta = \bar{d}_t^n / \bar{X}_t$):

$$\hat{X}_0 = (1-\delta)\,\hat{d}_0^n + (2S-1)\,\delta\hat{d}_0^f + 2\left(\hat{b}_H - \hat{b}_F\right) b p_B + 2b\left(\hat{d}_0^b - p_B\hat{p}_B\right)$$

$$\hat{X}_1 = (1-\delta)\,\hat{d}_1^n + (2S-1)\,\delta\hat{d}_1^f + 2b\hat{d}_1^b + \underbrace{2\left(\hat{b}_H - \hat{b}_F\right)\hat{d}_1^b}_{\text{2nd order}\approx 0}$$

[43] This *intertemporal* smoothing component could also be implemented through equity holdings. Contrary to the zero-order portfolio, at this order of approximation the rebalancing portfolio $\hat{B}$ is not unique. At this order of approximation, only the change in the net foreign asset position (and not its composition) is pinned down.

where p_S and p_B denotes stocks and bond prices in the symmetric ex-ante equilibrium and $b = B/X$ denotes the ratio of bond holdings over aggregate expenditure.

Taking the difference between period $t = 1$ and $t = 0$ and introducing the relative stochastic discount factor $\mathcal{M} = \mathcal{M}_H/\mathcal{M}_F$, with $\mathcal{M}_i = \xi\left(P_{i,0}/P_{i,1}\right)\left(C_{i,1}/C_{i,0}\right)^{-\sigma}$, we obtain:

$$\left(1 - \frac{1}{\sigma}\right)\Delta\hat{Q} - \frac{1}{\sigma}\hat{\mathcal{M}} = (1-\delta)\Delta\hat{d}^n + \delta\left(2S-1\right)\Delta\hat{d}^f + 2b\left(\Delta\hat{d}^b - p_B\hat{p}_B\right) - 2\hat{b}bp_B \tag{A.3}$$

where $\hat{b} = \left(\hat{b}_H - \hat{b}_F\right)$ is the intertemporal smoothing term between $t = 0$ and $t = 1$.

Portfolio equations. Let us now turn to the second-order approximation of the portfolio equations. The Euler equations for asset returns in country $i \in \{H, F\}$ and asset type $k \in \{f, b\}$ from country $j \in \{H, F\}$ is:

$$E_0\left[\mathcal{M}_i R_j^k\right] = 1$$

Taking differences across country for each asset type $k \in \{f, b\}$, we obtain:

$$E_0\left[\left(\mathcal{M}_H - \mathcal{M}_F\right)\left(R_H^f - R_F^f\right)\right] = E_0\left[\left(\mathcal{M}_H - \mathcal{M}_F\right)\left(R_H^b - R_F^b\right)\right] = 0$$

The second-order approximation of this expression yields:

$$E\left[\hat{\mathcal{M}}\ \hat{R}^i\right] = 0 \text{ for } i \in \{f, b\} \tag{A.4}$$

The optimal portfolio is a vector $\left(S, b, \hat{b}\right)$ such that the first order of the non-portfolio condition (A.3) and the second-order portfolio conditions (A.4) are satisfied.

Locally complete markets. Note that, if markets are (locally) complete, we are looking for a portfolio $\left(S, b, \hat{b}\right)$ such that the following risk-sharing condition holds:

$$\hat{\mathcal{M}} = -\sigma\Delta\hat{C} - \Delta\hat{Q} = 0. \tag{A.5}$$

Such a portfolio trivially satisfies the two (second-order) Euler equation approximations (A.4).

Let us assume that it is possible to find a portfolio such that markets are locally complete. Note that this also implies $\Delta\hat{X} = (1 - 1/\sigma)\Delta\hat{Q}$.

Using Eq. (A.3) and Eq. (A.5), the zero-order portfolio (S, b) must be such that the following equation holds for any (first-order approximation) of the returns innovations $\hat{R}^j$ for $j = \{n, f, b\}$:

$$\left(1 - 1/\sigma\right)\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) = (1-\delta)\hat{R}^n + \delta\left(2S-1\right)\hat{R}^f + 2b\hat{R}^b \tag{A.6}$$

The rebalancing portfolio $\hat{b}$ makes sure that Eq. (A.3) holds in expectations together with $E_0\hat{\mathcal{M}} = 0$:

$$\left(1 - \frac{1}{\sigma}\right)E_0\Delta\hat{Q} = (1-\delta)E_0\Delta\hat{d}^n + \delta\left(2S-1\right)E_0\Delta\hat{d}^f + 2bE_0\Delta\hat{d}^b - p_B\left(\widehat{p_B} + \hat{b}\right) \tag{A.7}$$

The rebalancing portfolio $\hat{b}$ only affects the expected component: in other words, deviations form the zero-order bond portfolio will be used for intertemporal smoothing and will be such that:

$$2\left(\hat{p_B}+\hat{b}\right)bp_B=\left[(1-\delta)E_0\Delta\hat{d}^n+\delta\left(2S-1\right)E_0\Delta\hat{d}^f+2bE_0\Delta\hat{d}^b-\left(1-\frac{1}{\sigma}\right)E_0\Delta\hat{Q}\right] \quad \text{(A.8)}$$

The zero-order portfolio (S,b) is the one used for risk-sharing *across states of nature*. The key question is wether one can verify (A.6) in all states of nature. To answer this question, write relative returns $\hat{R}^j$ and the innovation to the real exchange rate $\Delta\hat{Q}-E_0\Delta\hat{Q}$ as a (log-linearized) function of a vector of structural shocks $\hat{\epsilon}$ of dimension $k\times 1$:[44]

$$\hat{R}^j=\mathbf{v^{j\prime}}\hat{\epsilon}, \quad j=f,b,n; \text{ and } \quad \Delta\hat{Q}-E_0\Delta\hat{Q}=\mathbf{v^{q\prime}}\hat{\epsilon}, \quad \text{(A.9)}$$

where the vectors $\mathbf{v}$ are also of dimension $k\times 1$.

The portfolio restrictions encoded in (A.6) can be rewritten as:

$$\mathbf{V}\left(\begin{array}{c}\delta\left(2S-1\right)\\ 2b\end{array}\right)=\left(1-1/\sigma\right)\mathbf{v^q}-\left(1-\delta\right)\mathbf{v^n}, \quad \text{(A.10)}$$

where $\mathbf{V}=\left(\mathbf{v^f},\mathbf{v^b}\right)$ is the matrix of loadings for equities and bonds. Because we have two instruments (S and b), markets are locally complete if we have at most two sources of risk: $k\leq 2$. This is our Spanning Condition.

Spanning Condition: The spanning condition holds when $k\leq 2$.

The second condition for the portfolio to be unique and determined is that $k=2$ and that the matrix $\mathbf{V}$ is invertible.

Rank Condition: The rank condition for the portfolio to be uniquely determined is $k=2$ and $\det\mathbf{V}\neq 0$.

The Rank Condition is equivalent to assuming that equity and bond excess returns, $\hat{R}^f$ and $\hat{R}^b$, are not perfectly correlated (as well cross-country returns within an asset class). In that case, the unique equilibrium portfolio $(S^*;b^*)$ is determined as follows:

$$\left(\begin{array}{c}\delta\left(2S^*-1\right)\\ 2b^*\end{array}\right)=\mathbf{V}^{-1}\left[\left(1-1/\sigma\right)\mathbf{v^q}-\left(1-\delta\right)\mathbf{v^n}\right]$$

Note that, by identifying appropriately the coefficients of vectors $\mathbf{V}^{-1}\mathbf{v^q}$ and $\mathbf{V}^{-1}\mathbf{v^n}$, one can easily show that this last expression is equivalent to:

$$\left(\begin{array}{c}\delta\left(2S^*-1\right)\\ 2b^*\end{array}\right)=\left(\begin{array}{c}\left(1-1/\sigma\right)\beta_{Q,f}-\left(1-\delta\right)\beta_{n,f}\\ \left(1-1/\sigma\right)\beta_{Q,b}-\left(1-\delta\right)\beta_{n,b}\end{array}\right)$$

Where the hedge ratios $\beta_{i,j}$ are uniquely defined when the *rank* and *spanning* conditions are satisfied, and such that:

[44] Without lack of generality, we assume that the structural shocks $\hat{\epsilon}$ are not perfectly correlated.

$$\begin{cases} \Delta\hat{Q} - E_0\Delta\hat{Q} & \equiv & \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f \\ \hat{R}^n & \equiv & \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f \end{cases}$$

This ends the proof of Property 1 when markets are locally complete.

Incomplete markets. First note that the rebalancing portfolio $\hat{b}$ is the same as in Eq. (A.8), such that $E_0\hat{\mathcal{M}} = 0$. As a consequence, the zero-order portfolio (S, b) is such that the non-expected component of Eq. (A.3) holds:

$$(1-1/\sigma)\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) - \frac{1}{\sigma}\left(\hat{\mathcal{M}} - E_0\hat{\mathcal{M}}\right) = (1-\delta)\hat{R}^n + \delta\,(2S-1)\,\hat{R}^f + 2b\hat{R}^b \qquad \text{(A.11)}$$

Markets are not complete (even locally) when $k > 2$ (i.e. there are more shocks than assets). This implies that the risk-sharing condition (A.5) cannot be verified in all states. However, the equilibrium portfolio still has the same expression as in Property 1, as long as a similar Rank Condition is satisfied.

Rank Condition: the equilibrium portfolio under (locally) incomplete markets ($k > 2$) is uniquely determined as long the following rank condition is satisfied: $\text{rank}(\mathbf{V}) = 2$, where $\mathbf{V} = (\mathbf{v^f}, \mathbf{v^b})$ and $\hat{R}^j = \mathbf{v^{j}}'\hat{\epsilon}$ for $j \in \{f, b\}$.

This rank condition ensures that $\hat{R}^f$ and $\hat{R}^b$ are not perfectly correlated. In that case, one can always span the vector of structural shocks $\hat{\epsilon}$ on the following basis $(\hat{R}^f, \hat{R}^b, \check{\epsilon}_1 ..., \check{\epsilon}_{k-2})$ with $E_0(\check{\epsilon}'_s\ \hat{R}^i) = 0$ for $s = \{1; ...; k-2\}$ and $i = \{f, b\}$.

Let us rewrite the risk factors in this transformed basis of innovations (with $\check{\epsilon}$ the $(k-2)\times 1$ vector of innovations $\check{\epsilon}_s$ for $s = \{1; ...; k-2\}$):

$$\begin{cases} \Delta\hat{Q} - E_0\Delta\hat{Q} & \equiv & \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f + \check{\mathbf{v}}^{\mathbf{q}\prime}\check{\epsilon} \\ \hat{R}^n & \equiv & \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f + \check{\mathbf{v}}^{\mathbf{n}\prime}\check{\epsilon} \end{cases}$$

Note that this expression is equivalent to Eq. (9) with $u_Q = \check{\mathbf{v}}^{\mathbf{q}\prime}\check{\epsilon}$ and $u_n = \check{\mathbf{v}}^{\mathbf{n}\prime}\check{\epsilon}$.

The projection of Eq. (A.11) on returns innovations $\hat{R}^i$ for $i = \{f, b\}$, using the portfolio-equation (Eq. (A.4)) gives:

$$\begin{aligned} (1-1/\sigma)\beta_{Q,b} &= (1-\delta)\beta_{n,b} + 2b \\ (1-1/\sigma)\beta_{Q,f} &= (1-\delta)\beta_{n,f} + \delta\,(2S-1) \end{aligned}$$

Such a portfolio implies, by construction, $\frac{1}{\sigma}\left(\hat{\mathcal{M}} - E_0\hat{\mathcal{M}}\right) = (1-1/\sigma)\check{\mathbf{v}}^{\mathbf{q}\prime}\check{\epsilon} - (1-\delta)\check{\mathbf{v}}^{\mathbf{n}\prime}\check{\epsilon}$. This insures that the portfolio-equation (A.4) holds and that (A.11) holds in all states.[45]

Thus, this last expression gives the *unique* equilibrium portfolio as long as the *Rank condition* is verified. This ends the proof of Property 1 when markets are incomplete.

[45]Note that, by construction, $E_0(\check{\epsilon}'_s\ \hat{R}^i) = 0$ for $s = \{1; ...; k-2\}$ and $i = \{f, b\}$—or equivalently, the stochastic discount factor is only correlated with uninsurable risks.

A.2. Closing the model: optimal portfolios with endowment and redistributive shocks

Bonds and equity portfolios. We use the Devereux and Sutherland (2006) approach to characterize the optimal equity and bond positions. To do so, we use the first-order approximations of the non-portfolio equations (see reduced-form of the model below) and the second order approximation of the Euler equations. This pins down a unique equilibrium portfolio. In this appendix, we implement the solution method in a slightly more general model than the one developed in Section 3: asset returns can be driven by structural shocks of dimension higher or equal to two. We do so to show that our derivations do not rely on the locally complete markets assumption of Section 3.

Non portfolio equations. In the general equilibrium of that section, there is an additional non-portfolio equation, which is derived from the optimal intratemporal condition for the allocation of consumption across goods and the market clearing condition in goods markets (Eq. (17)). The log-linear first-order approximation yields:

$$\hat{y}_t = \left[-\phi + (2a-1)^2(\phi-1)\right]\hat{q}_t + (2a-1)\hat{X}_t \tag{A.12}$$

Taking first differences, we obtain a system of two non portfolio equations:

- Intratemporal allocation across goods:

$$\Delta\hat{y} = -\phi\Delta\hat{q} + (2a-1)[\Delta\hat{X} - (1-\phi)\Delta\hat{Q}] \tag{A.13}$$

- Budget constraint:

$$\Delta\hat{X} - E_0\Delta\hat{X} = (1-\delta)\hat{R}^n + \delta\,(2S-1)\,\hat{R}^f + 2b\hat{R}^b$$

Using a more general expressions of returns than in Section 3, we can express the returns on equities, bonds and non-financial wealth as follows:

$$\begin{cases} \hat{R}^f &=& \Delta\hat{x} - E_0\Delta\hat{x} & + & \gamma_f'\hat{\varepsilon} \\ \hat{R}^b &=& (2a-1)\,(\Delta\hat{q} - E_0\Delta\hat{q}) & + & \gamma_b'\hat{\varepsilon} \\ \hat{R}^n &=& \Delta\hat{x} - E_0\Delta\hat{x} & + & \gamma_n'\hat{\varepsilon} \end{cases}, \tag{A.14}$$

where $\hat{\varepsilon}$ is a N-dimensional vector of shocks and γ_i for $i = \{b, f, n\}$ is a $N \times 1$ vector that controls the impact of $\hat{\varepsilon}$ on assets returns and non-financial wealth. $\Delta\hat{x} - E_0\Delta\hat{x}$ denotes innovations on (relative) income growth. In Section 3, $\hat{\varepsilon}$ is unidimensional and equal to $\hat{\delta} - E_0\hat{\delta}$ so that the loadings γ_i satisfy: $\{\gamma_f;\gamma_b;\gamma_n\} = \left\{1; 0; -\frac{\delta}{1-\delta}\right\}$.

Portfolio equations. Due to symmetry, we can write the Euler equations in relative terms as follows for asset $i = \{f, b\}$:

$$E_0(\mathcal{M}\ R^i) = 0 \text{ for } i = \{f, b\} \tag{A.15}$$

where M is the difference between stochastic discount factor across countries. The second-order approximation of Euler equations is thus:

$$E\left[\hat{\mathcal{M}}\ \hat{R}^i\right] = 0 \text{ for } i = f, b \tag{A.16}$$

Solution method. Using the budget constraint, the intratemporal condition can be rewritten as follows, where $\hat{\xi} = \delta\,(2S-1)\,\hat{R}^f + 2b\hat{R}^b = \begin{pmatrix} \delta\,(2S-1)\,, & 2b \end{pmatrix}\binom{\hat{R}^f}{\hat{R}^b}$ denotes portfolio excess returns:

$$\Delta\hat{q} - E_0\Delta\hat{q} = q_y\,(\Delta\hat{y} - E_0\Delta\hat{y}) + \mathbf{q}_\varepsilon'\hat{\varepsilon} + q_\xi\hat{\xi}$$

where q_y, $\mathbf{q}_\varepsilon$ and q_ξ are derived by substituting portfolio excess returns and the budget constraints into the equilibrium goods market condition.[46] If we rewrite the reduced form model using Devereux and Sutherland (2006)'s notations, we get the following expression for the vector excess returns:

$$\begin{pmatrix} \hat{R}^f \\ \hat{R}^b \end{pmatrix} = \mathbb{R}_1 \hat{\xi} + \mathbb{R}_2 \begin{pmatrix} \Delta\hat{y} - E_0 \Delta\hat{y} \\ \hat{\varepsilon} \end{pmatrix}$$

where $\mathbb{R}_2 = \begin{pmatrix} 1+q_y & \mathbf{q}_\varepsilon' + \gamma_f' \\ (2a-1)q_y & (2a-1)\mathbf{q}_\varepsilon' + \gamma_b' \end{pmatrix}$ and $\mathbb{R}_1 = \begin{pmatrix} q_\xi \\ (2a-1)q_\xi \end{pmatrix}$.

The first-order approximation of the difference between stochastic discount factor across countries gives:

$$-\frac{\hat{\mathcal{M}}}{\sigma} = \Delta\hat{X} - E_0\Delta\hat{X} + (1/\sigma - 1)(2a-1)\left(\Delta\hat{q} - E_0\Delta\hat{q}\right) = D_1\hat{\xi} + \mathbf{D}_2 \begin{pmatrix} \Delta\hat{y} - E_0\Delta\hat{y} \\ \hat{\varepsilon} \end{pmatrix}$$

where $D_1 = 1 + [(1-\delta) + (2a-1)(1/\sigma - 1)]q_\xi$ is a scalar and and $\mathbf{D}_2 = \begin{pmatrix} (1-\delta)(1+q_y) + (2a-1)(1/\sigma-1)q_y & (2a-1)(1/\sigma-1)\mathbf{q}_\varepsilon' + (1-\delta)\gamma_n' \end{pmatrix}$ is a $1 \times N{+}1$ vector.

Following Devereux and Sutherland (2006), we define $\tilde{\mathbb{R}}_2 = \mathbb{R}_1\tilde{\mathbf{H}} + \mathbb{R}_2$ and $\tilde{\mathbf{D}}_2 = D_1\tilde{\mathbf{H}} + \mathbf{D}_2$ with $\tilde{\mathbf{H}} = (\mathbf{1} - \begin{pmatrix} \delta(2S-1) & 2b \end{pmatrix}\mathbb{R}_1)^{-1} \begin{pmatrix} \delta(2S-1) & 2b \end{pmatrix}\mathbb{R}_2$.

Then using the second-order approximation of the Euler equation, we get the following quadratic equation:

$$\tilde{\mathbb{R}}_2 \Sigma \tilde{\mathbf{D}}_2' = 0$$

where Σ is the $(N+1)\times(N+1)$ variance-covariance matrix of the vector of innovations $(\Delta\hat{y} - E_0\Delta\hat{y}, \hat{\varepsilon})'$. Rearranging terms, this equation simplifies into the following expression for portfolios:

$$\begin{pmatrix} \delta(2S-1) \\ 2b \end{pmatrix} = \left(\mathbb{R}_2\Sigma\mathbf{D}_2'\mathbb{R}_1' - D_1\mathbb{R}_2\Sigma\mathbb{R}_2'\right)^{-1}\mathbb{R}_2\Sigma\mathbf{D}_2'$$

where we assume that the 2×2 matrix $[\mathbb{R}_2\Sigma\mathbf{D}_2'\mathbb{R}_1' - D_1\mathbb{R}_2\Sigma\mathbb{R}_2']$ is invertible (**Rank condition**). When this rank condition is satisfied, the equilibrium portfolio is unique and bond and equity excess returns are not collinear. There also exists a unique decomposition such that:

$$\begin{aligned} \Delta\hat{Q} - E_0\Delta\hat{Q} &\equiv \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f + u_Q \\ \hat{R}^n &\equiv \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f + u_n \end{aligned}$$

where u_i for $i = \{Q, n\}$ is orthogonal to $\hat{R}^j$ for $j = \{b, f\}$: $E_0\left[u_i\hat{R}^j\right] = 0$. This decomposition allows to rewrite the portfolio as in Section 2, using Property 1.

Equity only portfolios. Using a similar solution technique, one can be the equilibrium portfolio in a model with equities only.

Non portfolio equations.

[46] $q_y = \frac{[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}[(2a-1)(1-\delta)-1]}{1-[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}$, $\mathbf{q}_\varepsilon' = \frac{[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}{1-[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}\gamma_n'$ and $q_\xi = \frac{[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}{1-[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}$.

- Intratemporal allocation across goods:

$$\Delta\hat{y} = -\phi\Delta\hat{q} + (2a-1)[\Delta\hat{X} - (1-\phi)\Delta\hat{Q}]$$

- Budget constraint:

$$\Delta\hat{X} - E_0\Delta\hat{X} = (1-\delta)\hat{R}^n + \delta\,(2S-1)\,\hat{R}^f$$

Portfolio equations. We can write Euler equations in relative terms as follows for asset f:

$$E_0(\mathcal{M}R^f) = 0 \tag{A.17}$$

We use similar expressions as Eq. (A.14) to express returns on financial and non-financial wealth. In the example developed in the core of the paper, $\hat{\varepsilon}$ is unidimensional and equal to $\hat{\delta} - E_0\hat{\delta}$ and $\{\gamma_f;\gamma_n\} = \left\{1; -\frac{\delta}{1-\delta}\right\}$.

Solution method: Using the budget constraint, the intratemporal condition can be rewritten as follows, where we introduce portfolio excess returns $\hat{\xi} = \delta\,(2S-1)\,\hat{R}^f$:

$$\Delta\hat{q} - E_0\Delta\hat{q} = q_y^u\,(\Delta\hat{y} - E_0\Delta\hat{y}) + \mathbf{q}_\varepsilon^{u\prime}\hat{\varepsilon} + q_\xi^u\hat{\xi}$$

where q_y^u, $\mathbf{q}_\varepsilon^{u\prime}$ and q_ξ^u are simply derived from the non-portfolio equations where $\delta\,(2S-1)\,\hat{R}^f$ has been substituted by $\hat{\xi}$. If we rewrite the reduced form model (Eq. (A.14)) using Devereux and Sutherland (2006)'s notations, we get the following expression for the vector excess returns:

$$\hat{R}^f = \mathbb{R}_1^u\hat{\xi} + \mathbb{R}_2^u\begin{pmatrix}\Delta\hat{y} - E_0\Delta\hat{y}\\ \hat{\varepsilon}\end{pmatrix}$$

where $\mathbb{R}_2^u = \begin{pmatrix} 1+q_y^u & \mathbf{q}_\varepsilon^{u\prime} + \gamma_f^\prime \end{pmatrix}$ and $\mathbb{R}_1 = q_\xi^u$.

The first-order approximation of the difference between stochastic discount factor across countries gives:

$$-\frac{\hat{\mathcal{M}}}{\sigma} = D_1^u\hat{\xi} + \mathbf{D}_2^u\begin{pmatrix}\Delta\hat{y} - E_0\Delta\hat{y}\\ \hat{\varepsilon}\end{pmatrix}$$

where $D_1^u = 1 + [(1-\delta) + (2a-1)(1/\sigma - 1)]q_\xi^u$ is a scalar and and $\mathbf{D}_2^u = \begin{pmatrix} (1-\delta)\left(1+q_y^u\right) + (2a-1)(1/\sigma-1)q_y^u & (2a-1)(1/\sigma-1)\mathbf{q}_\varepsilon^{u\prime} + (1-\delta)\gamma_n^\prime \end{pmatrix}$ is a $1\times N+1$ vector.

Following Devereux and Sutherland (2006), we define $\tilde{\mathbb{R}}_2^u = \mathbb{R}_1^u\tilde{\mathbf{H}}^u + \mathbb{R}_2^u$ and $\tilde{\mathbf{D}}_2^u = D_1^u\tilde{\mathbf{H}}^u + \mathbf{D}_2^u$ with $\tilde{\mathbf{H}}^u = (\mathbf{1} - (\delta\,(2S^u-1))\,\mathbb{R}_1^u)^{-1}\,(\delta\,(2S^u-1))\,\mathbb{R}_2^u$.

Then using the second-order approximation of the Euler equation, we get the following quadratic equation:

$$\tilde{\mathbb{R}}_2^u\Sigma\tilde{\mathbf{D}}_2^{u\prime} = 0$$

where Σ is the (N+1) x (N+1) variance-covariance matrix of the vector of innovations $\begin{pmatrix}\Delta\hat{y} - E_0\Delta\hat{y}\\ \hat{\varepsilon}\end{pmatrix}$. Rearranging terms, this equation simplifies into the following expression for portfolios:

$$\delta\,(2S^u-1) = \left(\mathbb{R}_2^u\Sigma\mathbf{D}_2^{u\prime}\mathbb{R}_1^{u\prime} - D_1^u\mathbb{R}_2^u\Sigma\mathbb{R}_2^{u\prime}\right)^{-1}\mathbb{R}_2^u\Sigma\mathbf{D}_2^{u\prime}$$

where we assume that the 2×2 matrix $[\mathbb{R}_2^u\Sigma\mathbf{D}_2^{u\prime}\mathbb{R}_1^{u\prime} - D_1^u\mathbb{R}_2^u\Sigma\mathbb{R}_2^{u\prime}]$ is invertible (**Rank condition** — which states that equity returns across countries are not collinear). When this rank condition is satisfied, the equilibrium equity portfolio is unique. There also exists a unique decomposition such that:

$$\Delta\hat{Q} - E_0\Delta\hat{Q} \equiv \beta^u_{Q,f}\hat{R}^f + u^u_Q$$
$$\hat{R}^n \equiv \beta^u_{n,f}\hat{R}^f + u^u_n$$

where u_i for $i = \{Q, n\}$ is orthogonal to $\hat{R}^j$ for $j = \{b, f\}$: $E_0\left[u_i\hat{R}^j\right] = 0$. This decomposition allows to rewrite the portfolio as in Section 2.

In the example of the main text (with $\hat{\varepsilon}$ unidimensional and equal to $\hat{\delta} - E_0\hat{\delta}$ and $\{\gamma_f; \gamma_n\} = \left\{1; -\frac{\delta}{1-\delta}\right\}$), we have:

$$\begin{aligned}
q^u_y &= -\frac{1-(2a-1)(1-\delta)}{\phi+(1-\phi)(2a-1)^2-(2a-1)(1-\delta)} \\
\mathbf{q}^u_\varepsilon &= \frac{(2a-1)\delta}{\phi+(1-\phi)(2a-1)^2-(2a-1)(1-\delta)} \\
q^u_\xi &= \frac{(2a-1)}{\phi+(1-\phi)(2a-1)^2-(2a-1)(1-\delta)}
\end{aligned}$$

together with: $\mathbb{R}^u_2 = \begin{pmatrix} 1+q^u_y & 1+\mathbf{q}^u_\varepsilon \end{pmatrix}$ and $\mathbb{R}_1 = q^u_\xi$; $D^u_1 = 1 + [(1-\delta)+(2a-1)(1/\sigma-1)]q^u_\xi$ and $\mathbf{D}^u_2 = \begin{pmatrix} (1-\delta)\left(1+q^u_y\right)+(2a-1)(1/\sigma-1)q^u_y & (2a-1)(1/\sigma-1)\mathbf{q}^{u\prime}_\varepsilon - \delta \end{pmatrix}$.

Equity only portfolio when $\upsilon^2 \to 0$. One can verify that when $\upsilon^2 \to 0$, only the terms in y remain as $\Sigma \to \begin{pmatrix} \sigma^2_y & 0 \\ 0 & 0 \end{pmatrix}$. Keeping only the y terms, to compute $(\mathbb{R}^u_2\Sigma\mathbf{D}^{u\prime}_2\mathbb{R}^{u\prime}_1 - D^u_1\mathbb{R}^u_2\Sigma\mathbb{R}^{u\prime}_2)^{-1}$ and $\mathbb{R}^u_2\Sigma\mathbf{D}^{u\prime}_2$, we make use of:

$$\begin{aligned}
1+q^u_y &= \frac{(\phi-1)\left(1-(2a-1)^2\right)}{\phi+(1-\phi)(2a-1)^2-(2a-1)(1-\delta)} \\
(2a-1)(1/\sigma-1)q^u_y &= -\frac{(2a-1)(1/\sigma-1)\left(1-(2a-1)(1-\delta)\right)}{\phi+(1-\phi)(2a-1)^2-(2a-1)(1-\delta)} \\
(2a-1)(1/\sigma-1)q^u_\xi &= \frac{(2a-1)^2(1/\sigma-1)}{\phi+(1-\phi)(2a-1)^2-(2a-1)(1-\delta)}
\end{aligned}$$

Thus, $\delta\left(2S-1\right) = (\mathbb{R}^u_2\Sigma\mathbf{D}^{u\prime}_2\mathbb{R}^{u\prime}_1 - D^u_1\mathbb{R}^u_2\Sigma\mathbb{R}^{u\prime}_2)^{-1}\,\mathbb{R}^u_2\Sigma\mathbf{D}^{u\prime}_2$ can be rewritten as—noticing that denominator $(\mathbb{R}^u_2\Sigma\mathbf{D}^{u\prime}_2\mathbb{R}^{u\prime}_1 - D^u_1\mathbb{R}^u_2\Sigma\mathbb{R}^{u\prime}_2)^{-1}$ and numerator $\mathbb{R}^u_2\Sigma\mathbf{D}^{u\prime}_2$ are both multiplied by the term: $\frac{\left(1+q^u_y\right)\sigma^2_y}{\phi+(1-\phi)(2a-1)^2-(2a-1)(1-\delta)}$:

$$\begin{aligned}
\delta\left(2S^u(0)-1\right) &= -\frac{(1-\delta)\left(\phi-1\right)\left(1-(2a-1)^2\right)-(2a-1)(1/\sigma-1)\left(1-(2a-1)(1-\delta)\right)}{\left(\phi-1\right)\left(1-(2a-1)^2\right)+(2a-1)^2(1/\sigma-1)} \\
&= -\frac{(1-\delta)\left(\lambda-1\right)-(2a-1)(1/\sigma-1)}{\lambda-1} \\
&= -\left(1-\delta\right)+\left(1-1/\sigma\right)\frac{2a-1}{1-\lambda}.
\end{aligned}$$

This is the expression shown in the main text. One can easily verify by identification with the portfolio formula that $\beta^u_{n,f} = 1$ and $\beta^u_{Q,f} = \frac{2a-1}{1-\lambda}$.

A.3. Optimal portfolios with equity and corporate debt

Consider the benchmark model of Section 2 under locally complete markets.[47]

Assume that firms in country i issue a given amount of corporate debt. We call $D_{i,t}$ the debt payments that have to be paid in period t in country i (we preserve symmetry across countries, i.e $D_{i,0} = D$ and $E_0(D_{i,1}) = D$ but results regarding equity home bias do not depend on this assumption).[48]

We call S_D the (zero-order) share of corporate debt in country i held by country i. Market clearing in the corporate debt market implies that country j holds a share $(1 - S_D)$.

In this environment, Modigliani-Miller theorem holds. This means that equilibrium firms values are independent of the amount of debt issued. In particular, the log-linearized expressions for returns are unchanged and so are the loadings β_{ij} under locally complete markets:

$$\left\{ \begin{array}{lcl} \Delta\hat{Q} - E_0\Delta\hat{Q} & \equiv & \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f \\ \hat{R}^n & \equiv & \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f \end{array} \right.$$

Note that $\hat{R}^f$ is *not* relative equity returns anymore (if $D_{i,1}$ non-zero in some states) but relative financial returns, i.e cross country difference in the sum of the returns on equity and returns on corporate debt.

Using similar notations, we introduce $d^f_{i,t}$ the financial income in country i (sum of equity dividends $\left(d^f_{i,t} - D_{i,t}\right)$ and corporate debt payments $D_{i,t}$).

Ignoring portfolio rebalancing terms (assuming perfect symmetry ex-ante and that countries start with the optimal steady state portfolio as in our benchmark case), the budget constraint at date t in country i can be written as:

$$X_{i,t} = d^n_{i,t} + S\left(d^f_{i,t} - D_{i,t}\right) + (1-S)\left(d^f_{j,t} - D_{j,t}\right) + S_D D_{i,t} + (1-S_D)D_{j,t} + B(d^b_{i,t} - d^b_{j,t}) \tag{A.18}$$

$$= d^n_{i,t} + Sd^f_{i,t} + (1-S)d^f_{j,t} + (S_D - S)\,D_{i,t} + (S - S_D)\,D_{j,t} + Bd^b_t \tag{A.19}$$

Taking the difference across countries and the first-difference across time gives in log-linearized terms a similar equation to (A.6) once we project on innovations (under locally-complete markets):

$$(1-\frac{1}{\sigma}\ )\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) = \delta\,(2S-1)\ \ \hat{R}^f\ + (1-\delta)\hat{R}^n\ + (S_D - S)\,\Delta\hat{D}\ + 2b\hat{R}^b$$

If we find a portfolio (S, S_D, b) such that the previous equation holds for arbitrary realizations of the shock innovations (or equivalent asset returns under the Rank condition), markets are locally-complete and such a portfolio is the equilibrium one.

The portfolio $(S, S_D, b) = (S^*, S^*, b^*)$ obviously satisfies this condition and is unique, where (S^*, b^*) are the ones derived in Section (2) (see equations (11a) and (11b)). The intuition for the result is quite straightforward: the presence of corporate debt only redistributes income from shareholder to debt holders in some states (without any impact on total financial returns). By

[47] Note that our results also hold in the general equilibrium model of Section 3.

[48] We assume that debt issued is bounded above such that at period t equity payments are strictly positive: $\left(d^f_{i,t} - D_{i,t}\right) > 0$ for all states.

holding corporate debt and equity in the same proportion, investors insulate their consumption expenditures from this redistribution.

We can conclude that the presence of corporate leaves the degree of home bias unchanged as well as the expression in terms of factor loadings once we compute the aggregate financial returns $\hat{R}_f$ (equity returns plus corporate debt returns). We also obtain that the (equilibrium) home bias in corporate debt is equal to the one in equity. One assumption is key for this result: returns on financial incomes $hatR_f$ are independent on the capital structure (i.e Modigliani-Miller holds) which makes the projection on the risk factors (and hence the portfolio) unchanged.

A.4. Closing the model: optimal portfolios with endowment, redistributive and preference shocks

Consider now the benchmark set-up with endowment and redistributive shocks of section 3 and add preference shocks as in Coeurdacier et al. (2009). The aggregate consumption index C_i, for $i = H, F$, is given by:

$$C_{i,t} = \left[a^{1/\phi}\left(\Psi_{i,t}c_{ii,t}\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi}\left(\Psi_{j,t}c_{ij,t}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)}$$

where $\Psi_{i,t}$ is an exogenous worldwide shocks to the preference for the country i good, $i = H, F$. We assume $E_0(\Psi_{i,1}) = \Psi_0 = 1$.[49] We denote$\Delta\hat{\Psi}$ the (relative) change in preference shocks, where $\Psi_t = \frac{\Psi_{H,t}}{\Psi_{F,t}}$. The associated ideal price index in country i, for $i = H, F$ is defined as follows:

$$P_{i,t} = \left[a\left(p_{i,t}/\Psi_{i,t}\right)^{1-\phi} + (1-a)\left(p_{j,t}/\Psi_{j,t}\right)^{1-\phi}\right]^{1/(1-\phi)}. \tag{A.20}$$

Non-portfolio equations and returns. The intratemporal allocation across goods (equivalent to Eq. (17)) yields:

$$\Psi_t\frac{y_{H,t}}{y_{F,t}} = \left(\frac{q_t}{\Psi_t}\right)^{-\phi}\Omega_a\left[\left(\frac{P_{F,t}}{P_{H,t}}\right)^{\phi}\frac{C_{F,t}}{C_{H,t}}\right] \tag{A.21}$$

Using Eq.(A.20), the welfare-based real exchange rate is equal to:

$$\Delta\hat{Q} = (2a-1)\left(\Delta\hat{q} - \Delta\hat{\Psi}\right) \tag{A.22}$$

The budget constraints (Eq. (8)) are unchanged and can be written as follows using the notations of Devereux and Sutherland (2006):

$$\Delta\hat{X} - E_0\Delta\hat{X} = (1-\delta)\hat{R}^n + \hat{\xi} \tag{A.23}$$

where $\hat{\xi} = \delta\left(2S-1\right)\hat{R}^f + 2b\hat{R}^b = \left(\delta\left(2S-1\right),\ 2b\right)\binom{\hat{R}^f}{\hat{R}^b}$ denotes the portfolio excess returns. We assume that bonds in each country pays one unit of the good **not** adjusted for preference shocks. This assumption is crucial for the results since it generates a wedge between relative bond returns

[49]Note that the shock $\Psi_{i,1}$ can also have a more supply oriented interpretation, as a shock to the quality of good i.

$\hat{R}^b$ and real exchange rate changes $\Delta\hat{Q}$. Returns can be summarized as follows:

$$\left\{\begin{array}{lcl} \hat{R}^f & = & \Delta\hat{x} - E_0\Delta\hat{x} + \Delta\hat{\delta} - E_0\Delta\hat{\delta} \\ \hat{R}^b & = & (2a-1)\left(\Delta\hat{q} - E_0\Delta\hat{q}\right) \\ \hat{R}^n & = & \Delta\hat{x} - E_0\Delta\hat{x} - \frac{\delta}{1-\delta}\left(\Delta\hat{\delta} - E_0\Delta\hat{\delta}\right) \end{array}\right. ,$$

To obtain more compact expressions, we express variables as a function of portfolio excess returns $\hat{\xi}$ and the vector of shock innovations $\hat{\varepsilon}$, with $\hat{\varepsilon}' = \left(\Delta\hat{y} - E_0\Delta\hat{y} \;;\; \Delta\hat{\delta} - E_0\Delta\hat{\delta} \;;\; \Delta\hat{\Psi} - E_0\Delta\hat{\Psi} \right)$. Differentiating across time and log-linearizing Eq. (A.21), using Eqs. (A.22) and (A.23), gives the following equilibrium terms-of-trade:

$$\Delta\hat{q} - E_0\Delta\hat{q} = \frac{(2a-1)}{\Gamma}\hat{\xi} + \left(\begin{array}{ccc} q_y & -\frac{(2a-1)\delta}{\Gamma} & 1+q_y \end{array} \right)\hat{\varepsilon} \qquad \text{(A.24)}$$

with $\Gamma = \phi(1-(2a-1)^2) + (2a-1)^2 - (2a-1)(1-\delta)$ and $q_y = -\frac{1-(2a-1)(1-\delta)}{\Gamma} < 0$. Note that here, due to market incompleteness, one cannot solve for equilibrium terms-of-trade independently of the portfolio solution. Using Eq. (A.24), financial and bond returns satisfy:

$$\begin{pmatrix} \hat{R}^f \\ \hat{R}^b \end{pmatrix} = \mathbb{R}_1\hat{\xi} + \mathbb{R}_2\hat{\varepsilon}$$

where $\mathbb{R}_2 = \left(\begin{array}{ccc} 1+q_y & 1-\frac{(2a-1)\delta}{\Gamma} & 1+q_y \\ (2a-1)q_y & -\frac{(2a-1)^2\delta}{\Gamma} & (2a-1)\left(1+q_y\right) \end{array} \right)$ and $\mathbb{R}_1 = \frac{(2a-1)}{\Gamma}\left(\begin{array}{c} 1 \\ 2a-1 \end{array} \right)$.

Stochastic discount factor and equilibrium portfolios with bonds and equity. To solve for equilibrium portfolios, one needs to express the stochastic discount factor $\hat{\mathcal{M}}$ as a function of portfolio excess returns $\hat{\xi}$ and shock innovations $\hat{\varepsilon}$, where $\hat{\mathcal{M}}$ is defined as follows:.

$$-\frac{1}{\sigma}\hat{\mathcal{M}} = \Delta\hat{X} + (1/\sigma - 1)\Delta\hat{Q}$$

Using Eqs. (A.22), (A.23) and (A.24), this is equivalent to:

$$-\frac{1}{\sigma}\hat{\mathcal{M}} = D_1\hat{\xi} + \mathbf{D}_2\hat{\varepsilon}$$

with $D_1 = \left(1 + \frac{(2a-1)}{\Gamma}(1-\delta+(1/\sigma-1)(2a-1))\right)$ a scalar and $\mathbf{D}_2 = \left(\begin{array}{ccc} D_{21} & D_{22} & D_{23} \end{array} \right)$ a 1×3 matrix with the following coefficients:

$$\begin{array}{lcl} D_{21} & = & D_{23} = (1-\delta)\left(1+q_y\right) + (2a-1)(1/\sigma-1)q_y \\ D_{22} & = & -\delta\left[1 + \dfrac{(2a-1)}{\Gamma}(1-\delta+(1/\sigma-1)(2a-1))\right] \end{array}$$

As shown by Devereux and Sutherland (2006), the portfolio equations defined by Eq. (10) are verified for a unique portfolio if the matrix $(\mathbb{R}_2\Sigma\mathbf{D}_2'\mathbb{R}_1' - D_1\mathbb{R}_2\Sigma\mathbb{R}_2')^{-1}$ is invertible **(Rank Condition)**:

$$\left(\begin{array}{c} \delta\left(2S-1\right) \\ 2b \end{array} \right) = \left(\mathbb{R}_2\Sigma\mathbf{D}_2'\mathbb{R}_1' - D_1\mathbb{R}_2\Sigma\mathbb{R}_2'\right)^{-1}\mathbb{R}_2\Sigma\mathbf{D}_2'$$

with Σ the 3×3 variance-covariance matrix of shock innovations $\hat{\varepsilon}$.

Equity only portfolios. If only claims on financial incomes are traded, the equilibrium equity portfolio satisfies:

$$\delta\left(2S-1\right)=\left(\mathbb{R}_2^u\Sigma\mathbf{D}_2'\mathbb{R}_1^{u\prime}-D_1\mathbb{R}_2^u\Sigma\mathbb{R}_2^{u\prime}\right)^{-1}\mathbb{R}_2^u\Sigma\mathbf{D}_2'$$

with $\mathbb{R}_2^u$ a 1×3 matrix corresponding to the first line of $\mathbb{R}_2$ and $\mathbb{R}_1^u$ a scalar corresponding to the first (top) element of $\mathbb{R}_1$.

Numerical solutions. We use the following values for the home-bias in preferences, the capital share and the degree of risk aversion: $a = 0.80$; $\delta = 0.4$ and $\sigma = 2$. In our baseline simulation, we set ϕ to 1.5 as in **?** but, given the lack of consensus in the literature regarding the value of ϕ, we provide sensitivity analysis for a low value of 0.5 and a high value of 2.5. We express the volatility of shocks relative to the volatility σ of endowment shocks and assume a variance-covariance matrix of shock innovations Σ of the following form:

$$\Sigma=\sigma^2\begin{pmatrix}1 & 0 & 0\\ 0 & \theta_\delta^2 & 0\\ 0 & 0 & \theta_\Psi^2\end{pmatrix}$$

with $\theta_\Psi < \theta_\delta < 1$, such that the main source of fluctuations in returns is driven by endowment risk, while the preference shocks which drive a wedge between relative bond returns and the real exchange rate remain small. This is consistent with the empirical analysis in section 4 which shows that relative bond returns and real exchange rate changes are very highly correlated. Moreover, we keep the redistributive shocks of moderate magnitude as otherwise conditional and unconditional covariance between financial returns and non-financial returns would be *both* negative while in the data the unconditional covariance remains positive. In our baseline, all shocks are orthogonal for simplicity. We also investigated portfolios when shocks are correlated, as results were barely affected, we do not report these results which are available upon request.

We perform two types of comparative statics. In the first one, we set θ_Ψ to a low value of 0.1 and let θ_δ varies between 0.1 and 1. In the second one, θ_δ is set to the value of 0.5 and we let θ_Ψ vary between 0 and 0.5. The first numerical simulation is close to the case developed in the main text of the paper, with arbitrarily small preference shocks but letting the size of redistributive shocks vary. The second numerical simulation corresponds to a case where preference shocks becomes larger and thus bonds are less of an efficient hedging instrument. For each comparative simulation, we plot the portfolios when both bonds and equities are traded and when only equities are traded.

Results are shown for different values of ϕ in Figure 3. First and foremost, the equity portfolio is very different when bonds are traded (solid line) compared to the case without bond trading (dotted-line). While the latter exhibit a potentially large amount of foreign bias, largely driven by the hedging of non-financial returns, the former exhibit a significant degree of home bias for most parameter values.[50] In that sense, we confirm our result that bond matters. Further, the degree of home bias in the model with both assets remains high unless the volatility of $\Psi-$ shocks is of the same magnitude as $\delta-$shocks. In the extreme case where the $\delta-$shocks becomes negligible compared to $\Psi-$shocks ($\theta_\delta/\theta_\Psi \to 0$, not shown), bonds are not used at all since their hedging properties disappear and the model behaves like the equity-only model — the bond position goes to zero and the equity portfolio with bond trading converges to the one without bond trading.

[50] If the volatility of $\delta-$shocks is close enough to the one of endowment shocks (θ_δ close to one, see left-panels of Fig. 3), the equity-only model starts to exhibit some significant home bias.

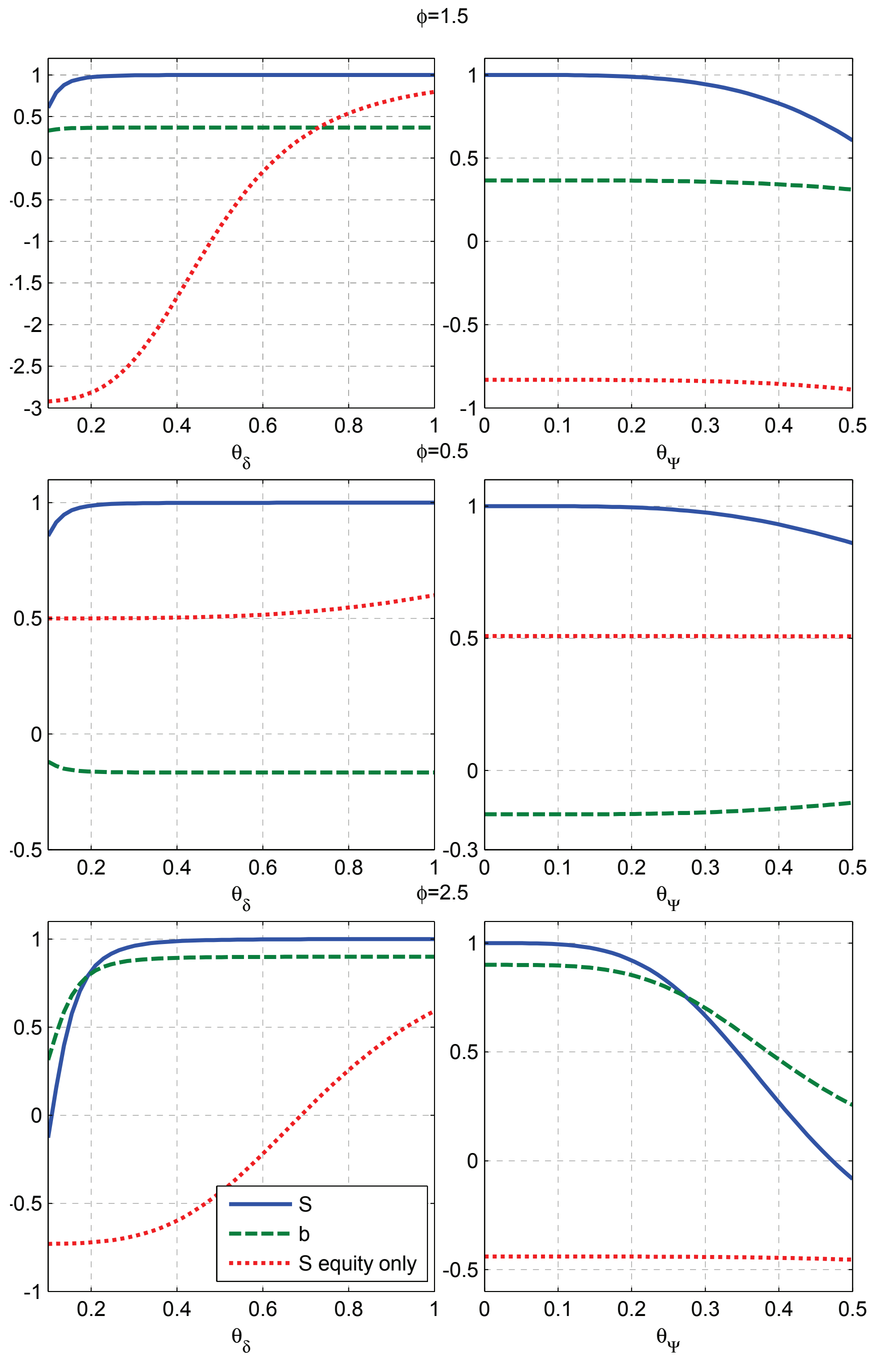

Figure 3: Equity and bond portfolios compared to equity portfolio in an equity-only model for different values of relative volatility of shocks (θ_Ψ and θ_δ) and different values of ϕ.

Notes: In the left-panels, θ_Ψ is set to a low value of 0.1 and θ_δ varies. In the right-panels, θ_δ is set to a value of 0.5 and θ_Ψ varies. ϕ is set to 1.5 (top-panel, baseline), a low value of 0.5 (medium-panel) or a high value of 2.5 (bottom-panel). Other parameters are set to their baseline value: $a = 0.80$, $\delta = 0.4$ and $\sigma = 2$.

A.5. Derivation of portfolios for countries of different sizes

We extend our benchmark model of Section 2 by log-linearizing around the case where countries are of different country sizes. We assume that unconditional means of financial and non-financial incomes, $E(d^j_{i,t})$, $j = f, n$ and $i = H, F$, are not equal across countries. Calling total income (financial and non financial) in country i, $x_{i,t} = d^n_{i,t} + d^f_{i,t}$, the size of countries is measured by the unconditional mean of $x_{i,t}$, $E(x_{H,t}) = \overline{x_H}$ and $E(x_{F,t}) = \overline{x_F}$. We solve the model when countries start in period zero from their unconditional mean of income $\overline{x_i}$, focusing on the non-expected part of the budget constraints. We denote by ω_i the relative size of country i: $\omega_i = \frac{\overline{x_i}}{\overline{x_i}+\overline{x_j}}$, with $\omega_H + \omega_F = 1$.

Keeping the same notations as in the case of our benchmark model with locally complete markets, projection of the log-linearized budget constraints on shocks innovations in country i gives (using market clearing conditions in the asset market) for $i \neq j$:

$$\Delta\hat{X}_i - E_0\Delta\hat{X}_i = (1-\delta)\hat{R}^n_i + \delta S_{ii}\hat{R}^f_i + \frac{\omega_j}{\omega_i}\delta\left(1 - S_{jj}\right)\hat{R}^f_j + \overline{b_{ii}}\hat{R}^b_i - \frac{\omega_j}{\omega_i}\overline{b_{jj}}\hat{R}^b_j$$

where $\hat{R^b_i}$ denotes the return on the bond of country i, $\overline{b_{ii}}$ denotes bonds i held by country i normalized by the unconditional expenditures $\overline{X_i}$ of country i (in the benchmark model $\overline{X_i} = \overline{X}$ and $\overline{b_{ii}} = b_{ii} = b$).[51]

Taking the difference across countries and using $\hat{\mathcal{M}} = 0$, we get:

$$\begin{aligned}(1-\frac{1}{\sigma})\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) &= (1-\delta)\hat{R}^n_i + \delta\hat{R}^f_i\left(S_{HH} - \frac{\omega_H}{\omega_F}(1 - S_{HH})\right) \\ &- \delta\hat{R}^f_j\left(S_{FF} - \frac{\omega_F}{\omega_H}(1 - S_{FF})\right) + \left(1 + \frac{\omega_H}{\omega_F}\right)\overline{b_{HH}}\hat{R}^b_H - \left(1 + \frac{\omega_F}{\omega_H}\right)\overline{b_{FF}}\hat{R}^b_F\end{aligned}$$

Rewrite the equilibrium portfolios as:

$$\begin{aligned}S_{ii} &= S = \omega_i + \Omega^f(1-\omega_i) \\ \overline{b_{ii}} &= b = \Omega^b(1-\omega_i)\end{aligned}$$

Ω^f and Ω^b are measures of the size of portfolio biases. Then, keeping the same notations:

$$(1-\frac{1}{\sigma})\left(\Delta\hat{Q} - E_0\Delta\hat{Q}\right) = (1-\delta)\hat{R}^n + \Omega^f\delta\hat{R}^f + \Omega^b\hat{R}^b$$

Assuming the following loadings on $\hat{R}^n$ and $\Delta\hat{Q}$ (see main text):

$$\begin{aligned}\Delta\hat{Q} - E_0\Delta\hat{Q} &= \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f \\ \hat{R}^n &= \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f\end{aligned}$$

[51] Note that the steady-state shares of financial and non-financial income are assumed to be constant across countries, only the unconditional mean of total income is different across countries.

Projection on $\hat{R}^n$ and $\Delta\hat{Q}$ gives Ω^f and Ω^b :

$$\Omega^f = -\frac{1-\delta}{\delta}\beta_{n,f} + \frac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f}$$
$$\Omega^b = \left(1-\frac{1}{\sigma}\right)\beta_{Q,b} - (1-\delta)\,\beta_{n,b}$$

Using $S = \omega_i + \Omega^f(1-\omega_i)$ and $b = \Omega^b(1-\omega_i)$ gives equilibrium portfolios for countries of different sizes:

$$\begin{cases} b^* &= (1-\omega_i)\left(1-\frac{1}{\sigma}\right)\beta_{Q,b} - (1-\omega_i)(1-\delta)\,\beta_{n,b} \\ S^* &= \omega_i + (1-\omega_i)\left(\frac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f} - \frac{1-\delta}{\delta}\beta_{n,f}\right) \end{cases}$$

A.6. A dynamic portfolio model with complete markets

Set-up. We use a dynamic continuous time portfolio model à la Merton (see **?** and Adler and Dumas (1983)). Investors have a CRRA utility flow $\frac{c_t^{1-\sigma}}{1-\sigma}$ at date t. We consider two countries $i = \{H, F\}$. Countries are ex-ante *symmetric* except in terms of their initial aggregate wealth W_{it}. Aggregate wealth of country i at date t is the sum of financial wealth W_{it}^f and non-financial wealth W_{it}^n:

$$W_{it} = W_{it}^f + W_{it}^n$$

Each country issues two assets, one bond and one claim on financial wealth. We denote by $k = \{f, b\}$ the asset class. Asset ik denotes the asset of country $i = \{H, F\}$ of class $k = \{f, b\}$. A riskless asset in zero-net supply is traded internationally with riskless rate r, paying in units of the numeraire good. All returns are expressed in the numeraire good. Each asset ik has iid log-normal returns (with common drift across countries):

$$R_{it}^k = \mu^k dt + \sigma_i^k dz_i^k$$

The return on non financial wealth R_{it}^n in country $i = \{H, F\}$ is iid log-normal with common drift across countries:

$$R_{it}^n = \mu^n dt + \sigma_i^n dz_i^n$$

The change in the price index of country $i = \{H, F\}$, denoted π_{it} is iid log-normal with common drift across countries:

$$\pi_{it} = \mu^\pi dt + \sigma_i^\pi dz_i^\pi$$

Complete markets assumption. *The two sources of local risk, returns on human wealth and inflation, are perfectly spanned by the set of risky securities:*

$$R_{it}^n = \lambda^n dt + \sum_{j=\{H,F\}}\sum_{k=\{f,b\}} \beta_{ijk}^n R_{jt}^k \tag{A.25}$$

$$\pi_{it} = \lambda^\pi dt + \sum_{j=\{H,F\}}\sum_{k=\{f,b\}} \beta_{ijk}^\pi R_{jt}^k \tag{A.26}$$

with, for $r = \{n, \pi\}$, $\lambda^r = \left(\mu^r - \sum_{j=\{H,F\}}\sum_{k=\{f,b\}} \beta_{ijk}^r \mu^k\right)$ *and* $\sigma_i^r dz_i^r = \sum_{j=\{H,F\}}\sum_{k=\{f,b\}} \beta_{ijk}^r \sigma_j^k dz_j^k$.

Note that, following to our notations, the change in the real exchange rate for country H is $\Delta \ln Q_t = \pi_{Ht} - \pi_{Ft}$.

Portfolio choice. The total share of aggregate wealth α_{ijk} invested by country $i = \{H, F\}$ in security j is:

$$\alpha_{ijk,t} = \frac{1}{\sigma}\theta_{jk} + \left(1 - \frac{1}{\sigma}\right)\beta^{\pi}_{ijk} - \beta^{n}_{ijk}\left(1 - \frac{W^f_{it}}{W_{it}}\right) \quad \text{(A.27)}$$

where θ_{jk} is the market price of risk of security jk defined as the corresponding line of $\Omega^{-1}[\mu - \mathbf{r}]$ with Ω the (4×4) matrix of variance-covariance of returns and $[\mu - \mathbf{r}]$ the vector of excess returns of assets jk.

The proof of this result is immediate using Adler and Dumas (1983) as the share of aggregate wealth (financial and non financial, $\alpha_{ijk} + \beta^{n}_{ijk}\left(1 - \frac{W^f_{it}}{W_{it}}\right)$) invested in asset jk has to be equal to $\frac{1}{\sigma}\theta_{jk} + \left(1 - \frac{1}{\sigma}\right)\beta^{\pi}_{ijk}$. The second term of Eq. (A.27) is the hedge portfolio of inflation risk and the last term is the hedge portfolio of non-financial wealth risk.

Rewriting Eq. (A.27) by taking the difference within an asset class k for a country i $(i \neq j)$ gives:

$$(\alpha_{iik,t} - \alpha_{ijk,t}) = \left(1 - \frac{1}{\sigma}\right)\left(\beta^{\pi}_{iik} - \beta^{\pi}_{ijk}\right) - \left(\beta^{n}_{iik} - \beta^{n}_{ijk}\right)\left(1 - \frac{W^f_{it}}{W_{it}}\right) \quad \text{(A.28)}$$

assuming that, due to symmetry, assets within an asset class $k = \{f, b\}$ have the same market price of risk $(\theta_{ik} - \theta_{jk} = 0)$.

Rewrite the projection of returns on non-financial wealth (Eq. (A.25)) as follows:

$$R^n_t = \sum_{j=\{H,F\}} \sum_{k=\{b,f\}} \left(\beta^n_{Hjk} - \beta^n_{Fjk}\right) R^k_{jt}$$

where $R^s_t = R^s_{Ht} - R^s_{Ft}$ for $s = \{n, f, b\}$ is the cross-country differential in returns. Symmetry across countries implies: $\beta^n_{HHk} - \beta^n_{FHk} = \beta^n_{FFk} - \beta^n_{HFk} = \beta_{n,k}$. Thus,

$$R^n_t = \sum_{k=\{b,f\}} \beta_{n,k} R^k_t \; .$$

Similarly, using Eq. (A.26):

$$\Delta \ln Q_t = \sum_{k=\{b,f\}} \beta_{Q,k} R^k_t$$

with $\beta^{\pi}_{HHk} - \beta^{\pi}_{FHk} = \beta^{\pi}_{FFk} - \beta^{\pi}_{HFk} = \beta_{Q,k}$. Eq. (A.28) can be rewritten as:

$$(\alpha_{iik,t} - \alpha_{ijk,t}) = \left(1 - \frac{1}{\sigma}\right)\beta_{Q,k} - \beta_{n,k}\left(1 - \frac{W^f_{it}}{W_{it}}\right) \quad \text{(A.29)}$$

Asset market clearing. The market clearing condition in the asset market is for $i \neq j$:

$$\alpha_{iik,t} W_{it} + \alpha_{jik,t} W_{jt} = M^k_{it}$$

with M^k_{it} the total market value of security ik at date t. Equivalently:

$$\alpha_{ijk,t} = \frac{M^k_{it}}{W_{jt}} - \alpha_{iik,t}\frac{W_{it}}{W_{jt}}$$

with $\alpha_{iik,t} = \alpha_{jjk,t}$ and $\alpha_{jik,t} = \alpha_{ijk,t}$ (symmetry).

Equilibrium portfolios. Using market clearing, Eq. (A.29) leads to (abstracting from time indices):

$$\left(\frac{\alpha_{iik}}{1-\omega_i} - \frac{\delta}{1-\omega_i}\left(\frac{S_i^k}{W_H^f + W_F^f}\right)\right) = \left(1-\frac{1}{\sigma}\right)\beta_{Q,k} - \beta_{n,k}\left(1-\delta\right)$$

where $\delta = \left(\frac{W_i^f}{W_i}\right) = \frac{W_j^f}{W_j}$ and $\omega_i = \frac{W_i}{W_H + W_F} = \omega_{i,0}$ (complete markets). Using $M_i^b = 0$ (bonds in net zero supply) and calling b_i the share of wealth invested in domestic bonds and $S_i = \frac{\alpha_{iik}}{\delta}$ the share of financial wealth invested in domestic claims on financial wealth.

$$\begin{aligned} b_i &= (1-\omega_i)\left[\left(1-\frac{1}{\sigma}\right)\beta_{Q,b} - \beta_{n,b}\left(1-\delta\right)\right] \\ S_i &= \left(\frac{S_i^k}{\sum\limits_{i=\{H,F\}} S_i^k}\right) + \left(\frac{1-\omega_i}{\delta}\right)\left[\left(1-\frac{1}{\sigma}\right)\beta_{Q,f} - \beta_{n,f}\left(1-\delta\right)\right] \end{aligned}$$

These expressions are identical to our static two-period model if the relative stock market size of country i is equal to the relative share of country i in aggregate wealth $\omega_i = \left(S_i^k / \sum\limits_{i=\{H,F\}} S_i^k\right)$.

B. Empirical Appendix

B.1. Data description.

All data are quarterly, between 1970-Q1 and 2008-Q3.

- Government bond returns: gross return on 3-month domestic Treasury-bill, from Global Financial Database.
- Nominal exchange rates: from Global Financial Database.
- Consumer Price Index (CPI): from OECD Main Economic Indicators
- Gross Domestic Product: OECD Quarterly National Accounts, Seasonally adjusted, except as noted. Notes: Germany: data for West Germany before 1991:Q1. Japan: data before 1979 from 1999 OECD Statistical compendium and seasonally adjusted with X-12 method; Italy: data before 1979 from the 1999 OECD Statistical Compendium (seasonally adjusted).
- Compensation of Employees: OECD Quarterly National Accounts, Seasonally adjusted.except as noted. Notes: Japan: Japan: data before 1998 from 1999 OECD Statistical compendium and seasonally adjusted with X-12 method. Italy: data before 1979 from the 1999 OECD Statistical Compendium (seasonally adjusted). France: data before 1977 from the 1999 OECD Statistical Compendium (seasonally adjusted). Germany: data for West Germany before 1991:Q1.
- Mixed Income:
    - US: fraction of net operating surplus plus mixed income from OECD Quarterly National Accounts. Fraction calculated as the share of mixed income in (mixed income + gross operating surplus - consumption of fixed capital) annual data from UN National Income System of National Accounts. Ratio after 2007 is the average for 2004-2006.

- UK: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1987-1994. Annual data from 2007 National Accounts Statistics Part III, pp836 for 1995-2005. Ratio after 2005 is the average for 2003-2005. Ratio before 1987 is the average for 1987-1989.
- Japan: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1980-2003. Ratio after 2003 is the average for 2001-2003. Ratio before 1980 is the average for 1980-1982.
- Italy: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1980-2003. Ratio after 2003 is the average for 2001-2003. Ratio before 1980 is the average for 1980-1982.
- Germany: fraction of net operating surplus + mixed income. Uses data from West Germany before 1991:Q1. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1991-2002. Ratio after 2002 is the average for 2000-2002. Ratio before 1991 is the average for 1991-1993.
- France: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1978-2003. Ratio after 2003 is the average for 2001-2003. Ratio before 1978 is the average for 1978-1980.
- Canada: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1970-2005. Ratio for 2005-2007 from StatCan. Data after 2007 is the average for 2005-2007.

• Net Operating Surplus and mixed income: from OECD Quarterly National Accounts, seasonally adjusted, except as noted below.

- France: before 1978 GDP minus compensation of employees, depreciation and indirect taxes.
- Italy: before 1980 GDP minus compensation of employees, depreciation and indirect taxes.
- Japan: Before 1998:Q3, net operating surplus + mixed income from OECD Statistical Compendium quarterly data, seasonally adjusted with X-12 routine. After 1998:Q3, defined as GDP minus compensation of employees, depreciation and indirect taxes.
- United Kingdom: before 1988, GDP minus Compensation of employees, Depreciation and Indirect Taxes.

• Depreciation: OECD Quarterly National Accounts, Consumption of Fixed Capital. Seasonally adjusted, except as noted below.

- France: Before 1978, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from 1999 OECD Annual National Accounts.
- Germany: Data for West Germany before 1991:Q1.
- Italy: Before 1980, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from 1999 OECD Annual National Accounts.
- Japan: Before 1998:Q2 from the 1999 OECD Statistical Compendium. After 1998, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from United Nations system of national accounts annual data.
- United Kingdom: Before 1988, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from 1999 OECD Annual National Accounts.

• Indirect Taxes: Taxes less Subsidies on Production and Imports from OECD Quarterly National Accounts, seasonally adjusted, except as noted below.

  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 uses data for West Germany.
  - Italy: before 1980, calculated as fraction of GDP, where the fraction is computed annually as the ratio of indirect taxes to GDP from 1999 OECD Annual National Accounts.
  - Japan: before 1998:Q2, from OECD Statistical Compendium quarterly data, seasonally adjusted with X-12 routine. After 1998:Q2 calculated as fraction of GDP, where the fraction is computed annually as the ratio of indirect taxes to GDP from United Nations system of national accounts annual data.

• Gross Fixed Capital Formation: from OECD Quarterly National Accounts, seasonally adjusted, except as noted below.

  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 uses data for West Germany
  - Italy: before 1980, from 1999 OECD Statistical Compendium.
  - Japan: before 1994 from ESRI National Accounts Office Total Fixed Investment.

• Residential Investment: OECD Quarterly National Accounts, seasonally adjusted, except as noted below.

  - Canada: before 1980, assumed to be 25% of total investment.
  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 data for West Germany
  - Italy: before 1980, constructed backwards from the growth rate of total construction, from 1999 OECD Statistical Compendium.
  - Japan: before 1994, from ESRI National Accounts Office Residential Investment.
  - United States: before 1990, from 1999 OECD Statistical Compendium.

- Consumption: OECD Quarterly National Accounts, seasonally adjusted, except as noted below.
    - France: before 1978, from 1999 OECD Statistical Compendium.
    - Germany: before 1991 data for West Germany.
    - Italy: before 1980, from 1999 OECD Statistical Compendium.
    - Japan: before 1980, from OECD Statistical Compendium quarterly data, seasonally adjusted with X-12 routine.
- Equity Returns: Global Financial Database Total Return index.
- Corporate Bond Returns: quarterly holding return on corporate bond, converted into U.S. dollar, assuming a 10 year maturity. except for Italy and Japan where we use the quarterly holding return on government bonds. Yields on corporate debt from Global Financial Database. Yields on government bonds from IFS (line 61).
- Compustat Weights: For each country and each available year, we construct the share of corporate debt as 1 minus the share of stockholder's equity in total assets for non-financial firms listed in Compustat North America (for the U.S. and Canada) and Compustat Global (for France, Germany, Italy, Japan and the UK). Data start in 1970 for Canada and the U.S., 1987 for Japan and the UK, 1988 for Germany and France and 1989 for Italy.

B.2. Constructing Financial and Non-Financial Income

The decomposition of output Y by income satisfies:

$$Y = COMP + M + \Pi + D + T, \tag{B.1}$$

where $COMP$ refers to the compensation of employees, M to mixed income, Π to the net operating surplus, D to the consumption of fixed capital, and T to taxes minus subsidies on production and imports. According to the 1993 United Nations's System of National Accounts, the net operating surplus Π represents the profits of incorporated entities.[52] By contrast, mixed income M denotes income from self-employment as well as proprietary income.[53] In the model, non-financial income denotes the component of aggregate income that cannot be capitalized into financial claims. We follow Gollin (2002) and construct an empirical counterpart W as the sum of the compensation of employees $COMP$, plus a fraction ν of mixed income M:[54] $W = COMP + \nu M$.

[52] It is defined as "the surplus or deficit accruing from production before taking account of any interest, rent or similar charges payable on financial or tangible non-produced assets borrowed or rented by the enterprise, or any interest, rent or similar receipts receivable on financial or tangible non-produced assets *owned by the enterprise*."

[53] It is defined as "the surplus or deficit accruing from production by *unincorporated enterprises owned by households*; it implicitly contains an element of remuneration for work done by the owner, or other members of the household, that cannot be separately identified from the return to the owner as entrepreneur but it excludes the operating surplus coming from owner-occupied dwellings."

[54] ν is assumed equal to $COMP/\left(COMP + \Pi\right)$. The results are very robust to alternative measures of ν, including the polar cases where all mixed incomes are treated as non-financial income ($\nu = 1$) and all mixed incomes are treated as financial income ($\nu = 0$).

Financial income K is then defined as gross operating profits $\Pi + D$ plus the remainder of mixed income $(1-\nu)\,M$, net of non-residential gross capital formation I:[55]

$$K = \Pi + D + (1-\nu)\,M - I.$$

Using these measures, we construct estimates of the share of financial income δ as $K/(Y-T-I)$. The 'naïve' estimate of δ is defined as one minus the share of compensation of employees in output measured at factor prices, that is $1 - COMP/(Y-T)$.

B.3. Empirical Issues

B.3.1. VAR diagnostic tests

We specify our Vector Auto Regression in first differences for $\ln w$ and $\ln k$ (see section 4.2.2). This is empirically valid given that:

- w and k are integrated of order 1;
- w and k are *not* co-integrated.

We verify that these conditions are satisfied as follows (detailed results available upon request):

- we conduct Augmented Dickey-Fuller tests of unit roots for both variables. We cannot reject the null of a unit root, except for $\ln w$ in Japan.
- We perform Johansen tests of co-integration for (w, k). We find no cointegration relationship, except for Germany.
- Since theory suggests that the only correct co-integration vector is $w - k$ (see Baxter and Jermann (1997)), we directly test for stationarity for this variable. Using Augmented Dickey-Fuller tests, we cannot reject the null of a unit root, except for Germany (with a p-value of 5.3%).

We conclude from these diagnostic tests that a VAR in first difference is appropriate. Although theory suggests that $w-k$ should be stationary, this variable is extremely persistent even over long periods of time, suggesting that the correcting mechanism does not play an important role at least over the period we consider.

B.3.2. Variable Selection

We consider the following additional variables in the VAR: consumption growth; the relative T-bill rate (the difference between the yield on 3-month T-bill rate and a 4-quarter moving average); the term premium (the spread between 10 year and 3 months government yields); the yield spread (the spread between the yield on long-term corporate bonds and that on 10-year government bonds); *cay*, the fluctuations in U.S. aggregate consumption-wealth ratio as measured by Lettau and Ludvigson (2001); and *nxa*, the Gourinchas and Rey (2007) measure of U.S. external imbalances, extended to 2008. In order to maintain a parsimonious and statistically significant representation, our selection of variables is as follows. First we exclude variables that appear integrated, based on Augmented-ADF tests, since this would violate our stationary VAR assumptions. Second, we select predictive variables based on the Least Angle Regression (LARS) approach of Efron et al. (2004)

[55] We substract gross capital formation to compute the part of income that flows to owners of financial claims on capital. We adjust gross capital formation for residential investment since the latter does not reflect investment decisions of corporations but of households.

applied to the financial return equation of the Vector Auto Regression. This selection algorithm efficiently selects a parsimonious subset of predictive variables. In our final specification, only one predictive variable remains: the term premium for the U.S and Italy.[56]

When set $r^f = \tilde{r} = r^e$, we re-run the LARS algorithm to select predictive variables. The following variables are added to the VAR: the relative T-bill rate for Japan, the yield spread and the term-premium for the U.S.

In specification with $\tilde{r} = r^{lb}$, the following variables are added to the VAR according to our LARS algorithm: term-premium (Italy and the U.S.), consumption growth (Japan, U.S.), relative T-bill (UK) and yield spread (U.S.).

B.3.3. The Lustig and Nieuwerburgh (2008) approach.

Lustig and Nieuwerburgh (2008) propose an alternative approach to measuring the returns to human wealth. The key identification assumption consists in assuming that consumption choices are consistent with the choices of a representative agent faced with financial and non-financial wealth. In other words, aggregate consumption satisfies the Euler equation of the representative household when using the total return to the agent's wealth. Since this return is a combination of the return to financial wealth (observable) and non-financial wealth (non-observable), one can then back out the innovation to the return on non-financial wealth.

The Lustig and Nieuwerburgh (2008) method starts with the two equations below:

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r^m_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \tag{B.2a}$$

$$E_t \Delta c_{t+1} = \mu_m + \sigma^{-1} E_t r^m_{t+1} \tag{B.2b}$$

where the first equation is a log-linearization of the intertemporal budget constraint following Campbell (1993), under the assumption that $c_t - v^m_t$ is stationary, where c_t is log-consumption, v^m_t is log-total wealth and $r^m_t = \ln(R^m_t)$ is the return on total wealth: $V^m_{t+1} = R^m_{t+1}(V^m_t - C_t)$. ρ is related to the steady state consumption wealth ratio as $\rho = 1 - \exp(c - v^m)$. Crucially, V^m_t includes non-financial wealth. The second equation is the log-linearized form of the Euler equation that characterizes the slope of the consumption profile. σ is the coefficient of relative risk aversion (inverse of the intertemporal elasticity of substitution) and μ_m captures all variance-covariance terms, assumed constant.

Substituting the Euler equation into the budget constraint, one obtains an expression for the innovation to consumption:

$$c_{t+1} - E_t c_{t+1} = r^m_{t+1} - E_t r^m_{t+1} + \left(1 - \sigma^{-1}\right)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^m_{t+1+j} \tag{B.3}$$

The next step consists in writing the (log) return on total wealth as :

$$r^m_{t+1} = (1 - \kappa_t) r^f_{t+1} + \kappa_t r^n_{t+1} \tag{B.4}$$

where r^f_{t+1} is the return on financial wealth and r^n_{t+1} the return on non-financial wealth and κ_t is the share of non-financial wealth in total wealth (possibly time-varying). Following the usual steps,

[56] At the suggestion of a referee, we also explored a specification including the first difference of control variables, when the latter appear integrated. The results, available upon request, are largely unchanged.

the innovation to the return on non-financial wealth satisfies:

$$r^n_{t+1} - E_t r^n_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta w_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^n_{t+1+j} \tag{B.5}$$

The central idea is to recover $(E_{t+1} - E_t)\, r^n_{t+1+j}$ from the consumption innovations in (B.3). Substituting (B.4) and (B.5) into (B.3), and assuming constant portfolio shares ($\kappa_t = \kappa$), we obtain:

$$\begin{aligned} c_{t+1} - E_t c_{t+1} = (1-\kappa)\left(r^f_{t+1} - E_t r^f_{t+1}\right) + \left(1 - \sigma^{-1}\right)(1-\kappa)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^f_{t+1+j} \\ + \kappa (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta w_{t+1+j} - \kappa\sigma^{-1} (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^n_{t+1+j}. \end{aligned} \tag{B.6}$$

Assuming that financial returns are observed (as in the benchmark case), we can invert this expression to obtain an expression for the innovation to non-financial returns that does not involve expected future non-financial returns:[57]

$$\begin{aligned} r^n_{t+1} - E_t r^n_{t+1} = (1-\sigma)(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta w_{t+1+j} - \sigma\left(\kappa^{-1} - 1\right)\left(r^f_{t+1} - E_t r^f_{t+1}\right) \\ - (\sigma - 1)\left(\kappa^{-1} - 1\right)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r^f_{t+1+j} + \sigma\kappa^{-1}(c_{t+1} - E_t c_{t+1}). \end{aligned} \tag{B.7}$$

One can estimate the innovation to non-financial wealth in (B.7) using a Vector Autoregression of the form $\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \epsilon_{t+1}$ with $\mathbf{z}'_t = \left(\Delta w_t, \Delta k_t, \Delta c_t, r^f_t, \Delta \ln Q_t, \mathbf{x}'_t\right)$ as:

$$\begin{aligned} r^n_{t+1} - E_t r^n_{t+1} = &\ (1-\sigma)\, \mathbf{e}'_{\Delta w} (\mathbf{I} - \rho\mathbf{A})^{-1} \epsilon_{t+1} \\ &- \left(\kappa^{-1} - 1\right) \mathbf{e}'_{r^f} \left[\sigma + (\sigma - 1)\rho\mathbf{A}(\mathbf{I} - \rho\mathbf{A})^{-1}\right] \epsilon_{t+1} \\ &+ \sigma\kappa^{-1} \mathbf{e}'_{\Delta c} \epsilon_{t+1} \end{aligned} \tag{B.8}$$

We implement this VAR estimation in section 4.6 under the assumption that $\sigma = 1$ and $\kappa = 1 - \delta$. In that case, the expression for the innovations simplifies substantially:

$$r^n_{t+1} - E_t r^n_{t+1} = -\left(\kappa^{-1} - 1\right) \mathbf{e}'_{r^f} \epsilon_{t+1} + \kappa^{-1} \mathbf{e}'_{\Delta c} \epsilon_{t+1}. \tag{B.9}$$

B.3.4. The cointegration approach.

Consider the following VECM(1) representation:

$$\mathbf{\Delta Z_{t+1}} = \mathbf{A \Delta Z_t} + \mathbf{b}\chi' \mathbf{Z_t} + \epsilon_{\mathbf{t+1}}$$

where $\mathbf{Z_{t+1}} = (\tilde{r}^c_t, k_t, w_t, \ln Q_t, \mathbf{x'_t})'$ and $\tilde{r}^c_t$ is the cumulation of the proxy return on non-financial wealth. χ captures the co-integration restriction that the ratio of labor and capital income needs

[57] A similar derivation can be obtained in the case where financial returns are not observed, using Eq. (30).

to be stationary: $\chi'\mathbf{Z_t} = k_t - w_t$. Given estimates of $\mathbf{A}$ and $\mathbf{b}$, one can construct estimates of $\mathbf{J_1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \mathbf{\Delta Z_{t+j+1}}$ and $\mathbf{J_2} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \mathbf{\Delta Z_{t+j+1}}$ as:

$$\begin{aligned} \mathbf{J_1} &= \left[\mathbf{I} - \rho\mathbf{A} - \frac{\rho}{1-\rho}\mathbf{B}\right]^{-1} \\ \mathbf{J_2} &= \mathbf{J_1} - \mathbf{I}. \end{aligned}$$