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Nicolas Coeurdacier

*INTERNATIONAL MACROECONOMICS*



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Nicolas Coeurdacier, London Business School (LBS) and CEPR

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Centre for Economic Policy Research  
90–98 Goswell Rd, London EC1V 7RR, UK  
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999  
Email: [cepr@cepr.org](mailto:cepr@cepr.org), Website: [www.cepr.org](http://www.cepr.org)

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## **ABSTRACT**

### **Do Trade Costs in Goods Market Lead to Home Bias in Equities?\***

Two of the main puzzles in international economics are the consumption and the portfolio home biases. We solve for international equity portfolios in a two-country/two-good stochastic equilibrium model with trade costs in goods markets. We show that introducing trade costs, as suggested by Obstfeld and Rogoff (2000), is not sufficient to explain these two puzzles simultaneously. On the contrary, we find that trade costs create a foreign bias in portfolios for reasonable parameter values. This result is robust to the addition of non-tradable goods for standard calibrations of the preferences.

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Nicolas Coeurdacier  
London Business School  
Economics Subject Area  
Regent's Park  
London NW1 4SA  
Email: [ncoeurdacier@london.edu](mailto:ncoeurdacier@london.edu)

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# 1 Introduction

This paper is mainly motivated by two broad stylized facts:

1. People mainly consume domestically produced goods: the home bias in consumption puzzle
2. People hold a disproportionate share of domestic assets : the home bias in portfolios puzzle

The first fact is well known: looking at consumption baskets, countries are not very open to trade. The openness to trade ratio in the US measured by the sum of exports and imports over GDP is only 25% in 2005. Given that the US account for about a third of world production, they should import about two thirds of their GDP in the absence of frictions in goods markets. Then, the openness ratio should be higher than 120%! Without being so ambitious about market integration, even the US and Canada are far from being perfectly integrated (Mc Callum [1995]<sup>1</sup>).

The second fact is also well known: since the seminal paper of French and Poterba [1991], the home bias in equities is one of the most pervasive empirical observations in international economics. Although home bias could be mainly due to capital market segmentation in the eighties, this explanation might be less valid nowadays. Indeed, developed countries opened up their stock market to foreign investors in the eighties, followed by many emerging economies in the early nineties, leading to a large increase in cross-border asset trade (Lane and Milesi-Feretti [2003]). However, the home bias in equities has not decreased sizeably. In 2005, US investors still hold 82% percent of domestic equities and the home bias in equities is observed in all developed countries (see Sercu and Vanpee [2007] for a recent survey).

Moreover there is evidence that the two puzzles are related: countries which are more open to trade are also more financially open. In other words, everything else equal, countries with higher import (or export) shares have larger stocks of foreign assets. Lane [2000], Aizenman and Noy [2005], Heathcote and Perri [2007] show this result using panel data for a cross-section of countries. Looking at bilateral data on trade in goods and asset holdings, Portes and Rey [2005], Aviat and Coeurdacier [2007] and Lane and Milesi-Feretti [2004] show that country portfolios are strongly biased towards trading partners.

There is now quite a consensus that international trade costs understood in a broad sense (*i.e* transport costs, tariffs, “border effect”...) can explain the first fact. As shown by Anderson and Van Wincoop [2004], international trade costs are very large, as large as production costs for some products. Obstfeld and Rogoff [2000] argue that home bias in equities might also be due to frictions in international goods markets rather than frictions in financial markets. If this is true, one can solve two important puzzles in international economics with only one simple friction, namely trade costs.

In this paper, we ask whether the Obstfeld and Rogoff [2000] argument is valid, *i.e* whether trade costs in goods markets alone can generate substantial home bias in portfolios. We do it in a standard static two-country/two-good model with symmetric endowment economies (Home and Foreign). Each country produces one good but agents consume both goods, facing some trade costs when importing foreign goods.

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<sup>1</sup>see also Anderson and Van Wincoop [2003] who correct Mc Callum estimates controlling for multilateral resistance.

Agents trade claims (equities) of both countries in a frictionless financial market. Equities are claims to the future endowment of the good (Lucas trees). In a set-up essentially similar to ours, Obstfeld and Rogoff [2000] restrict their attention to the very specific case where the efficient Arrow-Debreu consumption allocation can be perfectly replicated with equities only and do not solve for portfolios of tradable assets in the general case. We propose here to fill this gap. Contrary to their findings, we show that in general trade costs actually worsen the home bias in portfolios puzzle. Trade costs help to solve the Home bias in consumption puzzle but at the expense of Foreign bias in equities. The main intuition for our result goes as follows: when Home output is high, so is Home consumption due to the presence of trade costs. In these states of nature, Home investors want to make capital losses on their portfolios to share output risk efficiently with Foreigners. Consequently, they bias their portfolio towards Foreign equities as Foreign equities have a lower pay-off than Home equities when Home output is higher.

Our result is closely related to the literature that analyzes equity portfolios in presence of real exchange rate fluctuations in partial equilibrium models (see Adler and Dumas [1983], Cooper and Kaplanis [1994] and Warnock and van Wincoop [2007]). In these models, real exchange rate fluctuations might lead to portfolios that deviate from the world market portfolio. This literature shows that investors more risk averse than log-investors bias their portfolios towards assets that pay more when their aggregate price index is higher, *i.e.* when the real exchange rate appreciates, in order to stabilize their purchasing power (and inversely, agents who are less risk averse than logarithmic investors will prefer assets that pay more when prices are low). Then, the key point for portfolio allocation is whether the Home equity returns (relative to the Foreign ones) and the (Home) real exchange rate covary positively or negatively: a positive covariance meaning a Home bias in equities when agents are more risk averse than log-investors. The exact same mechanism is at work here but the model goes one step further since the general equilibrium approach allows to analyze whether Home equity returns should be (or not) positively correlated with the (Home) real exchange rate in the presence of trade costs. We show that the sign of this correlation is affected by the size of trade costs. However, for trade costs consistent with international consumption patterns, the model predicts a negative correlation. When Home output is low, Home equity returns shrink relative to Foreign ones and the relative price of Home goods increases as they are scarcer: the (Home) real exchange rate appreciates. Low returns in the Home country are associated with higher Home prices. Thus, consumers are better insured against real exchange rate fluctuations by holding a larger share of foreign equities.

We investigate the robustness of our result in presence of non-tradable goods. This is a potential candidate to reconcile facts and theory as variations of the real exchange rate reflects changes in the price of tradable **and** non-tradable goods. Consequently, the hedging of real exchange rate fluctuations becomes more complex. We follow Obstfeld [2007] and extend our benchmark model by adding a non-tradable sector: each country is producing a tradable and a non-tradable good that are imperfect substitutes. We depart from Obstfeld [2007] and from existing literature by assuming that agents cannot trade separately

claims on the tradable and non-tradable sector.<sup>2</sup> We rather assume that agents trade Home and Foreign equities that are claims over the aggregate output produced in the economy. Home bias in equities can emerge although for reasonable calibrations it remains fairly small and not very robust across specifications. Such equity home bias is driven by the presence of non-tradable goods together with a low elasticity of substitution between tradable and non-tradable goods. Like in the model with tradable goods only, a key feature of the model with non-tradable goods is that there is equity home bias if and only if the Home real exchange rate and Home (relative) equity returns co-vary positively. This condition is rejected by the data, at least for the US (see van Wincoop and Warnock [2007]). Finally and most importantly, holdings of local stocks are decreasing with trade costs. In other words, in the presence of non-tradable goods, trade costs still lead to Foreign equity bias and this result is robust across the calibrations considered. We also discuss the main results of Obstfeld [2007] and Collard, Dellas, Diba and Stockmann [2007] when investors can buy separate claims on tradable and non-tradable output in each country. In these models, while there might be some home bias in equities in the non-tradable sector, trade costs still generate equity foreign bias in the tradable sector for reasonable calibrations of the preference parameters. Overall, we believe that trade costs cannot account for the low level of international diversification.

Our paper is related to the literature that solves for international equity portfolios in two-country general equilibrium models. Part of this literature has focused on the role of labor income as source of portfolio biases, generating Home or Foreign biases depending on the set-up (see Baxter and Jermann [1997], Engel and Matsumoto [2006], Heathcote and Perri [2007]). Abstracting from labor incomes, we investigate a different source of heterogeneity among investors that could lead to portfolio biases, namely the presence of trade costs in goods markets. The closest paper to ours is Uppal [1993]: he develops a dynamic equilibrium of two endowment economies with complete markets and trade costs. However, he restricts his attention to the case of perfect substitutability between Home and Foreign goods. We will show that this last assumption plays a crucial role and relaxing it leads to a richer and more complex portfolio allocation. In independent work, Kollmann [2006a] solves for international portfolios in a two-country/two-good dynamic equilibrium with home bias in preferences but he focuses on the case of a low elasticity of substitution between goods (smaller than 1) where trade costs are inconsistent with consumption and portfolio biases. Finally, recent work by Devereux and Sutherland [2006a,b] and Tille and Van Wincoop [2007] provides new methods to solve for international equity portfolios in a large variety of context. We will borrow their technique and apply it to our set-up.

In section 2, we present our benchmark model with tradable goods only and derive the exact conditions under which trade costs lead to home bias in equities. We show that these conditions are violated under standard preference parameters. In section 3, we investigate the robustness of our result in the presence of non-tradable goods. Section 4 discusses the results and concludes.

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<sup>2</sup>see Dellas and Stockmann [1989], Baxter, Jermann and King [1998], Serrat [2001], Pesenti and Van Wincoop [2002], Obstfeld [2007], Matsumoto [2007], Collard, Dellas, Diba and Stockmann [2007]. Besides the financial asset structure, the model is identical to Obstfeld [2007].

## 2 The benchmark model

### 2.1 Set-up of the model

The model is a two period ( $t = 0, 1$ ) endowment economy with two symmetric countries, Home and Foreign. Home variables are denoted with  $H$ , Foreign variables with  $F$ . Each country produces one tradable good and the representative agent in each country consumes both goods; goods can be shipped from one country to the other with some trade costs. In period  $t = 0$ , no output is produced and no consumption takes place, but agents trade stocks (equities) which are claims on the future endowment of the country.<sup>3</sup> In period  $t = 1$ , uncertainty is realized and country  $i$  receives an exogenous stochastic endowment  $y_i$  of good  $i$ . We assume  $E_0(y_i) = y^*$  for both countries, where  $E_0$  is the conditional expectation operator, given date  $t = 0$  information.

#### 2.1.1 Preferences

The country  $i$  household has the standard constant relative risk aversion preferences, where  $\gamma$  denotes coefficient of relative risk aversion and  $C_i$  is the aggregate consumption index in country  $i$ :

$$U_i = E_0 \left[ \frac{C_i^{1-\gamma}}{1-\gamma} \right]$$

In both countries, the representative household consumes a basket of Home and Foreign goods.

The aggregate consumption index of an agent in country  $i$  is:

$$C_i = \left[ c_{ii}^{(\phi-1)/\phi} + c_{ij}^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)}$$

where  $c_{ij}$  is the total consumption of goods from country  $j$  by a representative agent in country  $i$ . The parameter  $\phi$  is the elasticity of substitution between Home and Foreign goods. We will assume that this elasticity is strictly larger than one:<sup>4</sup>  $\phi > 1$ .

#### 2.1.2 Trade Costs, Terms-of-Trade and the Real Exchange Rate

Exports from country  $j$  to country  $i$  are subject to some exogenous trade costs  $\tau$  of iceberg-type: for each good shipped,  $1/(1 + \tau)$  goods arrive at destination. If  $p_i$  denotes the price of one unit of output in country  $i$  in terms of the numeraire, the price faced by consumers in country  $i$  over goods from country  $j$  is for  $i \neq j$ :

$$p_j^i = (1 + \tau)p_j$$

This features frictions in international goods markets such as transport costs or other barriers to international trade (trade policies, “border effect”...). Trade costs will be the only source of heterogeneity among investors and consequently the only reason why they might hold different equity portfolios.

The Home terms of trade  $q$  denotes the relative price of the Home tradable good in terms of the Foreign

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<sup>3</sup>A risk-free bond could be added but due to the symmetry of countries, no bonds would be held in equilibrium.

<sup>4</sup>Except in section 2.4.4 where we consider elasticities smaller than one.



tradable good:  $q = \frac{p_H}{p_F}$ . The consumption price index (CPI) of agent  $i = H, F$  is equal to (with  $j \neq i$ ):

$$P_i = \left[ (p_i)^{1-\phi} + [(1+\tau)p_j]^{1-\phi} \right]^{1/(1-\phi)}$$

The (Home) real exchange rate ( $REER$ ) is the ratio of Home over Foreign CPI:

$$REER = \frac{P_H}{P_F} = \left\{ \frac{q^{1-\phi} + (1+\tau)^{1-\phi}}{1 + [(1+\tau)q]^{1-\phi}} \right\}^{1/(1-\phi)} \quad (1)$$

An increase in  $REER$  is an appreciation of the Home real exchange rate. The Home real exchange rate appreciates when the Home terms-of-trade  $q$  increase. Without trade costs, the real exchange rate is constant (purchasing-power-parity holds and agents have identical preferences over the two goods).

### 2.1.3 Financial markets

There is trade in equities in period 0. In each country there is one stock (Lucas tree). Each stock represents a claim to the future endowment of the tree. The supply of each share is normalized at unity. Each household fully owns the local stock, at birth, and has zero initial foreign assets. The country  $i$  household thus faces the following budget constraint, at  $t = 0$ :

$$p_S \mu_{ii} + p_S \mu_{ij} = p_S, \quad \text{with } j \neq i \quad (2)$$

where  $\mu_{ij}$  is the number of shares of stock  $j$  held by country  $i$  at the end of period 0.  $p_S$  is the share price of stock that is identical for both stocks due to symmetry.

## 2.2 Goods and Asset markets equilibrium conditions

### 2.2.1 Goods market equilibrium conditions

In period 1 (after the realization of the output shocks), a representative consumer in country  $i$  maximizes  $\frac{C_i^{1-\gamma}}{1-\gamma}$  subject to a budget constraint (for  $j \neq i$ ):

$$p_i c_{ii} + (1+\tau)p_j c_{ij} \leq \mu_{ii} p_i y_i + \mu_{ij} p_j y_j \quad (\lambda_{i,1}) \quad (3)$$

$$P_i C_i \leq \mu_{ii} p_i y_i + \mu_{ij} p_j y_j \quad (\lambda_{i,1}) \quad (4)$$

where  $\lambda_{i,1}$  is the Lagrange-Multiplier associated to the budget constraint in period 1 for household  $i$ . The budget constraint equalizes consumption expenditures to aggregate (financial) incomes of the representative agent in country  $i$ , where financial incomes depend on the portfolio  $\{\mu_{ii}; \mu_{ij}\}$ . At this point, we take portfolios chosen in period 0 as given. The first-order conditions are:

For consumption:

$$1 = \lambda_{i,1} P_i C_i^\gamma \quad (5)$$

Intratemporal allocation across goods (for  $j \neq i$ ):

$$c_{ii} = \left( \frac{p_i}{P_i} \right)^{-\phi} C_i \quad c_{ij} = \left( \frac{(1+\tau)p_j}{P_i} \right)^{-\phi} C_i \quad (6)$$

Resource constraints for both goods are given by:

$$c_{HH} + (1 + \tau)c_{FH} = y_H \quad (7)$$

$$c_{FF} + (1 + \tau)c_{HF} = y_F. \quad (8)$$

Using equations (6) for both countries and market-clearing conditions for tradable goods (7) and (8) gives the relative demand over Home and Foreign goods:

$$q^{-\phi}\Omega \left[ \left( \frac{P_F}{P_H} \right)^\phi \frac{C_F}{C_H} \right] = \frac{y_H}{y_F} \quad (9)$$

where  $\Omega(x)$  is a continuous function of  $x$  such that:  $\Omega(x) = \frac{1+x(1+\tau)^{1-\phi}}{x+(1+\tau)^{1-\phi}}$ .

## 2.2.2 Asset market equilibrium conditions

Introducing  $(\lambda_{i,0})$  the Lagrange-multiplier of the period 0 budget constraint (2) in country  $i$ , we get the following Euler equations that equalizes the marginal cost of buying an additional unit of stock  $i$  in period 0 to the expected marginal gain in period 1. The Euler equations determine the demand for stocks  $H$  and  $F$  in country  $i$ :

$$\lambda_{i,0}p_s = E_0[\lambda_{i,1}p_H y_H] \quad (10)$$

$$\lambda_{i,0}p_s = E_0[\lambda_{i,1}p_F y_F] \quad (11)$$

We can rewrite the Euler equations in relative terms using (5) as follows:

$$E_0 \left[ \frac{C_i^{-\gamma}}{P_i} \left( \frac{p_H y_H - p_F y_F}{p_s} \right) \right] = E_0 \left[ \frac{C_i^{-\gamma}}{P_i} (R_H - R_F) \right] = 0 \quad (12)$$

where  $R_i = \frac{p_i y_i}{p_s}$  denotes the return on stock  $i$ . Taking the difference of equation (12) between the Home and Foreign household, we get:

$$E_0 \left[ \left( \frac{C_H^{-\gamma}}{P_H} - \frac{C_F^{-\gamma}}{P_F} \right) (R_H - R_F) \right] = 0 \quad (13)$$

Market clearing in asset markets for stocks requires:

$$\mu_{HH} + \mu_{FH} = \mu_{FF} + \mu_{HF} = 1 \quad (14)$$

Countries' symmetry implies that equilibrium portfolios are symmetric:  $\mu_{HH} = \mu_{FF}$ . In what follows, country's holdings of local stock are denoted by  $\mu$ .  $(1 - \mu)$  will be the internationally diversified part of the equity portfolio.  $\mu > \frac{1}{2}$  means that there is equity home bias.

## 2.3 Log-linearization around the symmetric equilibrium

We use the world price index as a numeraire to preserve symmetry:  $P = P_H^{\frac{1}{2}} P_F^{\frac{1}{2}} = 1$ . We consider an approximation around the symmetric equilibrium where both countries have the same endowment  $y^*$  (and the same goods prices:  $p_H^* = p_F^* = 1$ ). We denote with  $(\hat{\cdot})$  the deviations of the variables from the symmetric equilibrium (in percents):  $\hat{x} \equiv \log\left(\frac{x}{x^*}\right)$  where  $x^*$  is the value at the symmetric equilibrium.

To solve for portfolios, we follow Devereux and Sutherland [2006a] and Tille and Van Wincoop [2007]. They show that a first-order approximation of the non-portfolio equations (equations (4), (6) to (9)) and a second-order approximation of the Euler equations are needed to express the zero-order component of equilibrium portfolios.

### 2.3.1 First-order approximation of non-portfolio equations

Due to symmetry, it is convenient to express all variables in relative terms (Home over Foreign). Log-linearizing (9) around the symmetric equilibrium gives:

$$\frac{\widehat{y}_H}{\widehat{y}_F} = -\phi\widehat{q} + \theta \left[ (\phi - 1) \frac{\widehat{P}_H}{\widehat{P}_F} + \widehat{PC} \right] \quad (15)$$

where  $\widehat{PC} = \widehat{P}_H\widehat{C}_H - \widehat{P}_F\widehat{C}_F$  denotes relative consumption expenditures and  $\theta(\tau) = \left( \frac{1-(1+\tau)^{1-\phi}}{1+(1+\tau)^{1-\phi}} \right) \in [0; 1]$ .  $\theta$  is a monotonic transformation of trade costs, increasing in  $\tau$ . When  $\theta$  is close to zero, trade costs are very low whereas  $\theta$  converges to one when trade costs are infinite. One can easily show that in equilibrium,  $(1 - \theta)$  is equal to the openness to trade ratio defined as the sum of exports plus imports over GDP. From now on,  $\theta$  will be the relevant parameter to look at the impact of trade costs.

The log-linearization of price indices gives in country  $i = \{H, F\}$  (for  $j \neq i$ ):

$$\widehat{P}_i = \frac{1}{1 + (1 + \tau)^{1-\phi}} \widehat{p}_i + \frac{(1 + \tau)^{1-\phi}}{1 + (1 + \tau)^{1-\phi}} \widehat{p}_j \quad (16)$$

We deduce the following expression for the Home real exchange rate around the symmetric equilibrium:

$$\widehat{RER} = \widehat{P}_H - \widehat{P}_F = \theta\widehat{q} \quad (17)$$

Due to trade costs, an increase in domestic prices appreciates the real exchange rate.  $\theta$  measures the size of trade barriers but it is also the elasticity of the real exchange rate with respect to the terms-of trade. In presence of trade costs, an increase in the Home terms-of-trade is equivalent to a Home real exchange rate appreciation. This is consistent with a positive correlation between the terms-of-trade and the real exchange rate in industrialized countries (see Obstfeld and Rogoff [2000b]). Using (15) and (17), we get:

$$\widehat{y}_H - \widehat{y}_F = [-\phi(1 - \theta^2) - \theta^2]\widehat{q} + \theta\widehat{PC} \quad (18)$$

Then, the relative demand over Home and Foreign goods is decreasing with respect to the Home terms-of-trade and increasing in Home (over Foreign) aggregate consumption expenditures: when aggregate expenditures are higher at Home, so is the demand for Home goods due to the presence of trade costs ( $\theta > 0$ ). Holding constant (relative) aggregate consumption expenditures  $\widehat{PC}$ , Home terms-of-trade always decrease with an increase the relative supply of Home goods. Note that the response of the Home terms-of-trade to a (relative) output shock will be stronger when trade costs are high (higher  $\theta$ ) since the (relative) demand for Home goods is more inelastic (for  $\phi > 1$ ).

Then, Home excess returns (excess returns of the Home stock over the Foreign stock)  $\widehat{R} = \widehat{R}_H - \widehat{R}_F$  can

be derived from (18):

$$\widehat{R} = p_H \widehat{y}_H - p_F \widehat{y}_F = (1 - \phi)(1 - \theta^2)\widehat{q} + \theta \widehat{PC} \quad (19)$$

The last non-portfolio equation that must be log-linearized is the period 1 budget constraint (4). We express the period 1 budget constraint in relative terms which gives (using portfolio symmetry):

$$\widehat{PC} = (2\mu - 1)(p_H \widehat{y}_H - p_F \widehat{y}_F) = (2\mu - 1)\widehat{R} \quad (20)$$

Without bias in the equity portfolio ( $\mu = \frac{1}{2}$ ), relative consumption expenditures are equalized since household have identical financial incomes. Any portfolio bias drives a wedge between Home and Foreign consumption expenditures.

### 2.3.2 Second-order approximation of (relative) Euler equation

The second-order approximation of the portfolio equation (13) is:

$$cov(\widehat{PC}, \widehat{R}) = (1 - 1/\gamma)cov(\widehat{RER}, \widehat{R}) \quad (21)$$

where  $var(\widehat{R})$  denotes the variance of excess returns of the Home stock over the Foreign stock and  $cov(\widehat{x}, \widehat{R})$  denotes the covariance of variable  $x$  with Home excess returns. (21) gives using (20):

$$(2\mu - 1)var(\widehat{R}) = (1 - 1/\gamma)cov(\widehat{RER}, \widehat{R}) \quad (22)$$

With a bit of rewriting, we get:<sup>5</sup>

$$\mu = \frac{1}{2} \left[ 1 + (1 - \frac{1}{\gamma}) \frac{cov(\widehat{RER}, \widehat{R})}{var(\widehat{R})} \right] \quad (23)$$

$$= \frac{1}{2} \left[ 1 + (1 - \frac{1}{\gamma}) \theta \frac{cov(\widehat{q}, \widehat{R})}{var(\widehat{R})} \right] \quad (24)$$

The expression is similar to the one obtained by Van Wincoop and Warnock [2007]. Holdings of domestic equity  $\mu$  depend on two terms:

- the market portfolio (which is  $\frac{1}{2}$ ) due to diversification motive (as in Lucas [1982])
- the hedging component due to real exchange rate fluctuations, which is

$$\frac{1}{2} \left( 1 - \frac{1}{\gamma} \right) \theta \frac{cov(\widehat{q}, \widehat{R})}{var(\widehat{R})}$$

We get a standard result: a logarithmic investor ( $\gamma = 1$ ) is not affected by fluctuations of the real exchange rate and the hedging term disappears in this case. Of course, in absence of trade costs ( $\theta = 0$ ), the real exchange rate is constant and the hedging term also cancels out. If  $\gamma > 1$ , this term is positive when Home equity excess returns have a positive covariance with the Home real exchange rate. Home investors prefer the stock that yields higher returns when the Home price index is higher.

<sup>5</sup>Here we assume that  $var(\widehat{R}) = var(\widehat{R}_H - \widehat{R}_F)$  is non-zero. This might happen for a peculiar calibration of the preferences and trade costs (see below). In that case the portfolio is undetermined since Home and Foreign stocks are perfect substitutes.

## 2.4 Equilibrium Portfolios

### 2.4.1 Analytical expression and Portfolio Biases

To compute the equilibrium portfolio from the partial equilibrium expression (24), we need to compute the equilibrium variance-covariance ratio  $\frac{cov(\hat{q}, \hat{R})}{var(\hat{R})}$ . Using (19) and (20), we get the following expression:

$$\frac{cov(\hat{q}, \hat{R})}{var(\hat{R})} = \frac{1 - \theta(2\mu - 1)}{(1 - \phi)(1 - \theta^2)} \quad (25)$$

This variance-covariance ratio depends in turn on the equilibrium portfolio  $\mu$ . As shown in Tille and Van Wincoop (2007), we deduce the zero-order component of equity portfolio as solution of a fixed-point problem using (24) and (25). Rearranging terms, we get the equilibrium equity portfolio:

$$\mu = \frac{1}{2} \left[ 1 - \left( 1 - \frac{1}{\gamma} \right) \frac{\theta}{(\phi - 1)(1 - \theta^2) - \theta^2 \left( 1 - \frac{1}{\gamma} \right)} \right] \quad (26)$$

Real exchange rate fluctuations generate a portfolio bias for investors with non-logarithmic preferences. Assuming a coefficient of relative risk aversion larger than one, the direction the bias depends on the sign of the denominator  $(\phi - 1)(1 - \theta^2) - \theta^2(1 - \frac{1}{\gamma})$ . We have Foreign (resp. Home) bias in equities if this denominator is positive (resp. negative). We denote  $\theta^*$  the unique  $\theta \in (0; 1)$  such that this denominator is equal to zero:

$$\theta^* = \left( \frac{\phi - 1}{\phi - 1/\gamma} \right)^{\frac{1}{2}} \quad (27)$$

In this case, equity portfolios are undetermined as Home and Foreign equities have perfectly correlated returns. We have Home bias in equities if and only if  $\theta > \theta^*$ , i.e trade costs are above a certain threshold. Below that threshold, the portfolio is biased towards Foreign equities.

### 2.4.2 Intuition for the result

The key point to understand portfolio biases is the equilibrium covariance between Home excess returns and the Home real exchange rate. When this covariance is positive, Home investors prefer Home equities as they provide higher returns when the relative price of Home goods is higher. Equity Foreign bias emerges when this covariance is negative. In standard cases, one should expect this covariance to be negative: indeed, when output is high at Home, the Home terms-of-trade and the Home real exchange rate depreciate due to the scarcity of Foreign goods. But in the mean time, Home equity returns are higher since Home production is higher. Home excess returns and the Home real exchange rate move in opposite directions. Investors have a Foreign bias in equities. We will see in the calibration of the model (see below) that this case is the most likely.

However, while this reasoning holds for low level of trade costs ( $\theta < \theta^*$ ), this is not always true. For (very) high level of trade costs ( $\theta > \theta^*$ ), a lower price for Home goods (depreciated Home real exchange rate) is associated with lower equity returns in the Home country. When goods markets are very segmented, an increase in Home output leads to a larger fall in the Home terms-of-trade since it is

harder to sell the additional output in the Foreign market. Potentially, the fall in Home terms-of-trade is so large that Home equity returns are lower than Foreign returns despite higher Home production. This is a case of immiserizing growth where a rise in Home output essentially benefits to Foreign stocks through the fall in the Home terms-of trade. The covariance between Home excess returns and the Home real exchange rate is positive and optimal portfolios are Home biased.

For a level of trade costs such that  $\theta = \theta^*$ , an increase in Home output is exactly offset by the response of the Home terms-of-trade (an extension of Cole and Obstfeld [1991]): Home and Foreign equity returns are perfectly correlated and the equity portfolio is undetermined.

### 2.4.3 Calibration

Preferences	
Relative risk-aversion	$\gamma = 2$
Elasticity of substitution	$\phi = 5$
Trade frictions	
Trade costs	$\tau \in [0; 300\%]$

Table 1: Parameter values of the benchmark model

Calibration of the parameters is presented in table 1. The coefficient of relative risk aversion is set at  $\gamma = 2$  (like in Backus, Kehoe and Kydland [1994]). The elasticity of substitution between Home and Foreign goods is set to 5. Estimates of this elasticity vary a lot across studies. Estimates from micro (sectoral) trade data usually find much higher elasticities, ranging from 4 to 15 (see Harrigan [1993], Hummels [2001] among others). Baier and Bergstrand [2001] reports an estimate of 6.4 using aggregate trade flows between OECD countries. However, estimates from time-series macro data usually give much lower elasticities, ranging from 1 to 3 (Backus, Kehoe and Kydland [1994]). Imbs and Mejean [2008] reconcile these two literatures by pointing out an aggregation bias and stands for elasticities of roughly 5 when they control for heterogeneity. In line with Obstfeld and Rogoff [2000] and Imbs and Mejean [2008], we choose the lower bound of estimates from the trade literature. However for values of this elasticity larger than 1, qualitative results remain unchanged. For the US, the openness to trade, *i.e* the ratio of (exports+imports) over GDP is 25% in 2005, which corresponds to a steady-state import share of 12.5%.<sup>6</sup> The model matches the observed steady-state import share in the US with an average trade cost of 63%. Anderson and Van Wincoop [2004] estimate international trade costs in the range of 40% to 70% so the calibration used in the paper seems fairly reasonable.

This gives the equilibrium share of domestic equities in the portfolio  $\mu$  as a function of  $\tau$  shown in figure 1. Portfolios exhibit a foreign bias for reasonable trade costs (trade costs smaller than 142%): at

<sup>6</sup>In 2005, imports were higher than exports but since the model is approximated around the symmetric equilibrium where the trade balance is zero, we use  $\frac{\text{Exp}+\text{Imp}}{2\text{GDP}}$  for the steady-state import share.

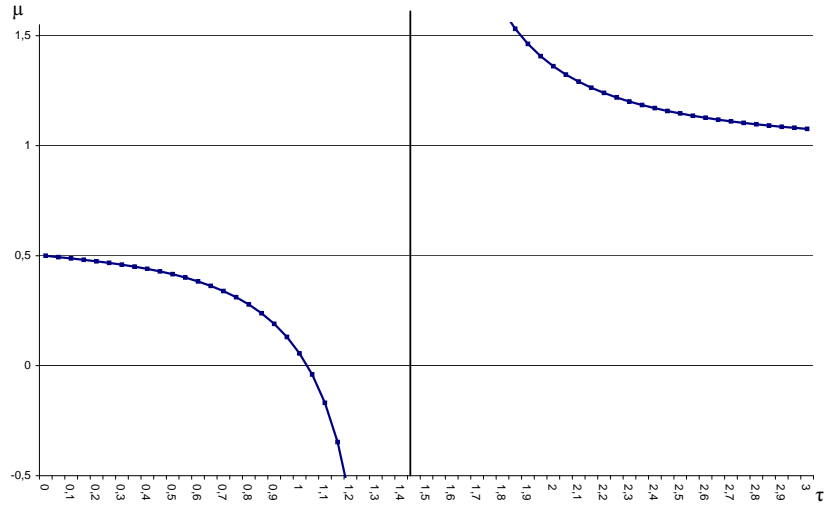


Figure 1: Holdings of local stocks  $\mu$  as a function of trade costs  $\tau$ . Benchmark Calibration:  $\gamma = 2$ ,  $\phi = 5$ .

the margin, an increase in trade costs  $\tau$  reduces  $\mu$  and increases the Foreign bias in portfolio. This is in sharp contradiction with Obstfeld and Rogoff [2000]. Moreover, the effect is rather large: increasing trade costs from 50% to 100% (or equivalently decreasing the import share by 10%) decreases the share of local equities from 41.5% to 5.5%!

For very high value of trade costs (trade costs higher than 142%), the equity portfolio is biased towards Home stocks. In this case, though, the share of Home equities is above unity and the model would actually deliver too much Home bias. The reason is that for a level of trade costs such that  $\theta$  is slightly above  $\theta^*$ , Home and Foreign stocks are almost perfect substitutes and the portfolio home bias is tremendously amplified: the investor would short Foreign assets.

#### 2.4.4 Robustness checks

- **How realistic is the case  $\theta > \theta^*$  ?**

For standard preferences ( $\gamma > 1$  and  $\phi > 1$ ),<sup>7</sup> trade costs in goods markets will lead to home bias in equities if and only if  $\theta > \theta^*$  ? (*i.e* trade costs are sufficiently high). This makes sense since in the neighborhood of trade autarky, portfolios should be fully biased towards local assets. We checked whether this case can be empirically realistic. Since  $\theta^*$  depends on the elasticity of substitution between Home and Foreign goods and on the coefficient of risk-aversion, we calculated  $\theta^*$  for different values of these parameters, the level of trade costs above which investors would short foreign assets and the corresponding import share (see table (4) in appendix 5.1). As shown in table (4), unless assuming a very high relative risk aversion, a low elasticity of substitution between Home and Foreign goods (and consequently incredibly large trade costs), goods markets are not closed enough to generate some Home bias in equities in this model. And the bottom-line is that the presence of trade costs in goods markets is certainly unable to deliver realistic Home bias in equities.

<sup>7</sup>Higher relative risk-aversion  $\gamma$  than one seems fairly uncontroversial. We will discuss the case of  $\phi < 1$ .

- **The role of the elasticity of substitution between home and foreign goods**

In the benchmark case, we consider an elasticity of substitution between Home and Foreign goods  $\phi$  larger than unity. Heathcote and Perri [2002] provides short-run estimates of this elasticity that are slightly smaller than one.<sup>8</sup> One could think that such a hypothesis would help us to generate Home bias in portfolios in this model. When Home output increases, the fall in Home terms-of-trade is stronger since consumers cannot substitute easily Home and Foreign goods and Home equity returns shrink. In this case, Home terms-of-trade and Home equity returns (relative to Foreign) are positively correlated. Again, this is a case of immiserizing growth: a good output shock is mainly beneficial to the foreign country. In particular, one can show that in the specific case where  $\phi = \frac{1}{\gamma} < 1$ , the share of foreign assets is exactly equal to the import share (case emphasized by Kollmann [2006a]). However, even in this case, at the margin, an increase in trade costs  $\tau$  reduces the equity home bias **and** the consumption home bias. Higher trade costs raise imports in value since the elasticity of demand with respect to imports is very low and people will hold more foreign stocks to stabilize their purchasing power on imports. Home consumers should tilt their consumption spending towards Foreign goods **and** their portfolio towards Foreign equities in presence of trade costs!

Then, given that many empirical works in international trade usually agree on larger elasticities of substitution across goods and put forward trade costs as one of the main explanation of the consumption home bias, from now on, we stick on the more standard case where  $\phi$  is above unity.

## 2.5 Related Literature

In Obstfeld and Rogoff [2000], portfolios are computed only when trade in equities are able to reproduce the complete markets allocation. This happens to be the case only if<sup>9</sup>  $\gamma = 1/\phi$ . They assume  $\phi > 1$  in their calibration, which implies  $\gamma < 1$ . In this case, calculus simplifies tremendously and we get:

$$\mu = \frac{1}{2}(1 + \theta)$$

When  $\gamma < 1$ , the “substitution effect” dominates and investors prefer equities that give higher returns when the price of their consumption bundle is lower. The hedging demand due to real exchange rate fluctuations leads to Home bias in equities and this bias is indeed increasing with trade costs:

$$\frac{\partial \mu}{\partial \tau} = \frac{1}{2} \frac{\partial \theta}{\partial \tau} > 0$$

What we have shown is that the Home bias they replicate under this specific calibration is far from being general (especially, under more standard calibrations, one would expect  $\gamma > 1$ ). For other parameter values, their approximate portfolios are actually far from the optimal ones computed in this paper.

<sup>8</sup>Corsetti, Dedola and Leduc [2004] and Kollmann [2006a] also assume an elasticity smaller than 1. See also Tille [2001] for some implications of this assumption.

<sup>9</sup>For other parameter values, two stocks are not enough to complete the market although, *up to the first-order*, period 1 Lagrange multipliers are equalized across countries.



Uppal [1994] describes equity portfolios in presence of trade costs in a one-good model. This corresponds to the case of  $\phi$  close to infinity (Home and Foreign goods are perfect substitutes). Since the limit of  $\theta^*$  when  $\phi$  approaches infinity is 1, Home excess returns and the Home real exchange rate are negatively related for any value of trade costs. Consequently, investors more risk averse than log- have a foreign bias in equities. More broadly, raising the elasticity of substitution increases the range of trade costs for which the investor exhibits a foreign portfolio bias. Indeed for  $\phi > 1$  and  $\gamma > 1$ , we have:

$$\frac{\partial \theta^*}{\partial \phi} > 0$$

As goods become closer substitutes, a small decrease in Home prices increases a lot the demand for Home goods, which dampens the response of Home terms-of-trade following an increase in Home output. This increases the range of trade costs for which Home terms-of-trade and Home excess returns move in opposite direction.

The benchmark model predicts a negative covariance between Home equity excess returns and the Home real exchange rate when goods markets are not “too closed”. This, in turn, leads to Foreign bias in equities. Van Wincoop and Warnock [2007] argue that this covariance is very close to zero in the data for the US. With zero covariance, the hedging term due to real exchange rate fluctuations in the portfolio disappears and investors should neither exhibit Foreign bias nor Home bias in equities. However, in both cases (zero or negative covariance), the main message of this section remains: one cannot explain the home bias in equities with trade costs alone.

In the next section, we look how our results are robust to the addition of non-tradable goods following Obstfeld [2007]. Indeed, the addition of non-tradable goods can potentially change the response of the real exchange rate to output shocks as the real exchange rate depends both on the terms-of-trade and the relative price of non-tradable goods.

### 3 Adding non-tradable goods to the benchmark model

While a large literature focuses on the role of non-tradable goods to generate equity home bias (see Dellas and Stockmann [1989], Baxter Jermann and King [1998], Serrat [2001],<sup>10</sup> Obstfeld [2007], Matsumoto [2007], Collard, Dellas, Diba and Stockmann [2007]), most of this literature does not consider the interaction between trade costs in the tradable sector and the presence of non-tradable goods (Obstfeld [2007] and Collard, Dellas, Diba and Stockmann [2007] are notable exceptions). Here, we want to investigate the robustness of our results to the addition of non-tradable goods. The framework is borrowed from Obstfeld [2007] but we depart from his analysis (and from existing literature) by assuming a different financial asset structure. We assume that agents trade shares in Home and Foreign mutual funds. Shares in Home (Foreign) mutual funds entitle investors to a share of aggregate Home (Foreign) output, but agents cannot trade separate claims on tradable and non tradable output in each country. This assump-

<sup>10</sup>see also Kollmann [2006b] for a correction of Serrat’s results

tion can be justified by the difficulty investors face when trying to distinguish between the exposure of equity to traded and non-traded goods sectors. This difficulty is all the more relevant given that, when allowing agents to trade separate claims on traded and non-traded output, optimal equity positions are very different for the traded and the non-traded sector.<sup>11</sup> This different structure of portfolios across traded and non-traded sectors seems “inconsistent with casual empiricism” as argued by Lewis [1999] (see also Baxter, Jermann and King [1998] for a similar criticism).

### 3.1 Set-up of the model with non-tradable goods

In a similar two-period ( $t = 0, 1$ ) model with two symmetric economies, each country now produces two goods, a tradable ( $T$ ) and a non-tradable good ( $NT$ ). At date 0, agents trade Home and Foreign equities, which are claims to aggregate Home and Foreign output. Equity returns of country  $i$  are a weighted share of returns in both sectors. At  $t = 1$ , country  $i$  receives an exogenous endowment  $y_i^T$  of the tradable good  $i$  and an exogenous endowment  $y_i^{NT}$  of the non-tradable good  $i$ . We assume:  $E_0(y_i^T) = y_i^{T*}$  and  $E_0(y_i^{NT}) = y_i^{NT*}$  for  $i = \{H, F\}$ . We will assume that the stochastic properties of the endowment shocks are symmetric across countries.

The country  $i$  household has the same CRRA preferences over the aggregate consumption index, but the aggregate consumption index is now a bundle of tradable and non-tradable goods:

$$C_i = \left[ \eta^{1/\epsilon} (c_i^T)^{(\epsilon-1)/\epsilon} + (1-\eta)^{1/\epsilon} (c_i^{NT})^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)}$$

where  $c_i^T$  is the consumption of a composite tradable good using Home and Foreign tradable goods and  $c_i^{NT}$  is the consumption of non-tradable goods.  $\epsilon$  is the elasticity of substitution between tradable and non-tradable goods.

The tradable consumption index for country  $i = \{H, F\}$  is still:

$$c_i^T = \left[ (c_{ii}^T)^{(\phi-1)/\phi} + (c_{ij}^T)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)}$$

where  $c_{ij}^T$  is country  $i$ 's consumption of the tradable good from country  $j$ .

The consumer price index that corresponds to these preferences is (for  $i = \{H, F\}$ ):

$$P_i = \left[ \eta (P_i^T)^{1-\epsilon} + (1-\eta) (P_i^{NT})^{1-\epsilon} \right]^{1/(1-\epsilon)}$$

where  $P_i^T$  is the price index over tradable goods in country  $i$  and  $P_i^{NT}$  is the price of non-tradable goods.

The tradable goods price index is defined by (for  $i = \{H, F\}$  and  $j \neq i$ ):

$$P_i^T = \left[ (p_i^T)^{1-\phi} + ((1+\tau)p_j^T)^{1-\phi} \right]^{1/(1-\phi)}$$

where  $p_i^T$  is the price of the tradable goods in country  $i$ . Home terms-of-trade are denoted by  $q$  :  $q = p_H^T/p_F^T$ .

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<sup>11</sup>See e.g. Obstfeld (2007) and Collard, Dellas, Diba and Stockmann [2007]. See section 3.5.2 for a summary of their results.

Resource constraints for tradable and non-tradable goods in country  $i = \{H; F\}$  are given by:

$$c_{ii}^T + (1 + \tau)c_{ji}^T = y_i^T \quad (28)$$

$$c_i^{NT} = y_i^{NT} \quad (29)$$

There is trade in stocks in period 0. In each country, a stock gives investors a claim on the aggregate output of the country at market value (where aggregate output at market value of country  $i$  is the sum of sales in the traded and non-traded sector,  $p_i^T y_i^T + P_i^{NT} y_i^{NT}$ ). The country  $i$  household fully owns the local stock and faces the following budget constraint, at  $t = 0$ :

$$p_s \mu_{ii} + p_s \mu_{ij} = p_s, \quad \text{with } j \neq i$$

where  $\mu_{ij}$  is the number of shares of stock of country  $j$  held by country  $i$  at the end of period 0.  $p_s$  is the share price of the stock, identical across countries due to symmetry. Market clearing in asset markets for the two stocks requires (supply of stocks normalized to unity) :

$$\mu_{HH} + \mu_{FH} = \mu_{FF} + \mu_{HF} = 1$$

Symmetry of preferences and shock distributions implies that equilibrium portfolios are symmetric:  $\mu_{HH} = \mu_{FF}$ . As in the benchmark model, we denote a country's holdings of local stock by  $\mu$  ( $\mu > \frac{1}{2}$  means that there is equity home bias).

### 3.2 Equilibrium conditions

In period 1, a representative consumer in country  $i$  maximizes  $\frac{C_i^{1-\gamma}}{1-\gamma}$  subject to a budget constraint (for  $j \neq i$ ):

$$P_i C_i = p_i^T c_{ii}^T + (1 + \tau)p_j^T c_{ij}^T + P_i^{NT} c_i^{NT} \leq \mu_{ii}(p_i^T y_i^T + P_i^{NT} y_i^{NT}) + \mu_{ij}(p_j^T y_j^T + P_j^{NT} y_j^{NT}) \quad (\lambda_{i,1}) \quad (30)$$

with  $\lambda_{i,1}$  the Lagrange-Multiplier of the period 1 budget constraint. The first-order conditions are:

For aggregate consumption:

$$1 = \lambda_{i,1} P_i C_i^\gamma \quad (31)$$

Intratemporal allocation across goods:

$$c_{ii}^T = \left( \frac{p_i^T}{P_i^T} \right)^{-\phi} c_i^T \quad c_{ij}^T = \left( \frac{(1 + \tau)p_j^T}{P_i^T} \right)^{-\phi} c_i^T \quad (32)$$

$$c_i^T = \eta \left( \frac{P_i^T}{P_i} \right)^{-\epsilon} C_i \quad c_i^{NT} = (1 - \eta) \left( \frac{P_i^{NT}}{P_i} \right)^{-\epsilon} C_i \quad (33)$$

Using budget constraint (30) and the symmetry of portfolios, we get:

$$P_H C_H - P_F C_F = (2\mu - 1) (p_H^T y_H^T + P_H^{NT} y_H^{NT} - p_F^T y_F^T - P_F^{NT} y_F^{NT}) \quad (34)$$

which implies that the difference between countries' consumption spending equals the difference between their financial incomes.

As in the benchmark model, we can express the Euler equations in relative terms (Home relative to Foreign) as follows:

$$E_0\left[\left(\frac{C_H^{-\gamma}}{P_H} - \frac{C_F^{-\gamma}}{P_F}\right)(R_H - R_F)\right] = 0 \quad (35)$$

where  $R_i = \frac{p_i^T y_i^T + P_i^{NT} y_i^{NT}}{p_s}$  denotes the return on stock  $i = \{H; F\}$ . Note that since investors cannot buy separate claims on the tradable and the non-tradable firms, equity returns in country  $i$  depend on the aggregate 'dividend' of the country  $[p_i^T y_i^T + P_i^{NT} y_i^{NT}]$ .

### 3.3 Solution method and equilibrium portfolios

As in the benchmark model, following Tille and Van Wincoop [2007] and Devereux and Sutherland [2007], one needs to solve for equilibrium equity portfolios  $\mu$  using the first-order approximation of non-portfolio equations and second-order approximation of the Euler equations. We do not detail the resolution and the approximation of the model (steps are similar to the benchmark model) and just present some key equations (see Obstfeld [2007] for similar derivations).

#### 3.3.1 First-order approximation of the non-portfolio equations

The log-linearization of the Home country's real exchange rate  $RER \equiv \frac{P_H}{P_F}$  gives:

$$\widehat{RER} = \frac{\widehat{P}_H}{\widehat{P}_F} = \eta\theta\widehat{q} + (1 - \eta)\widehat{P}^{NT}. \quad (36)$$

where  $P^{NT} = P_H^{NT}/P_F^{NT}$  is the relative price of Home non-tradable goods over Foreign non-tradable goods and  $\eta$  is the steady-state share of spending devoted to tradable goods.<sup>12</sup> The relative price of the tradable composite in both countries satisfies:  $\widehat{P}_H^T/\widehat{P}_F^T = \theta\widehat{q}$ . Importantly, the real exchange rate now depends positively on the Home terms-of-trade  $q$  as well as on the relative price of Home non-tradable goods  $P^{NT} = P_H^{NT}/P_F^{NT}$ .

Intratemporal allocation across goods (32) and (33) together with market-clearing conditions for tradable and non-tradable goods (28) and (29) imply the following equilibrium conditions in both sectors (see Obstfeld [2007]):

$$\widehat{y}^T = [(1 - \phi)(1 - \theta^2) + \theta^2(1 - \eta)(1 - \epsilon) - 1]\widehat{q} - \theta(1 - \eta)(1 - \epsilon)\widehat{P}^{NT} + \theta\widehat{PC} \quad (37)$$

$$\widehat{y}^{NT} = (\eta(1 - \epsilon) - 1)\widehat{P}^{NT} - \theta\eta(1 - \epsilon)\widehat{q} + \widehat{PC} \quad (38)$$

where  $\widehat{PC} = \widehat{P}_H\widehat{C}_H - \widehat{P}_F\widehat{C}_F$  denotes relative consumption expenditures and  $\widehat{y}^k = \widehat{y}_H^k - \widehat{y}_F^k$  denotes relative output in sector  $k = \{T, NT\}$ .

We finally log-linearize equation (34) and obtain:

$$\widehat{PC} = \widehat{P}_H\widehat{C}_H - \widehat{P}_F\widehat{C}_F = (2\mu - 1)\widehat{R} \quad (39)$$

<sup>12</sup>To simplify notations, we assume here that the share of spending devoted to tradable goods is the same as the weight of tradable goods in the consumption index. This is true only if in the steady state tradable and non tradable goods have the same price:  $p^{T*} = p^{NT*}$ . This assumption is however irrelevant for equity portfolios (see Obstfeld [2007]).

where  $\widehat{R} = \widehat{R}_H - \widehat{R}_F$  denotes Home aggregate excess returns and is a weighted sum of relative returns in the tradable and non-tradable sector:  $\widehat{R} = \eta(\widehat{q} + \widehat{y}^T) + (1 - \eta)(\widehat{P}^{NT} + \widehat{y}^{NT})$ .

It is convenient to rewrite the non-portfolio equations (37) and (38) in matrix form as follows:

$$\widehat{\mathbf{y}} = \mathbf{M}\widehat{\mathbf{p}} + \begin{pmatrix} \theta \\ 1 \end{pmatrix} \widehat{PC} \quad (40)$$

where  $\widehat{\mathbf{y}} = \begin{pmatrix} \widehat{y}^T \\ \widehat{y}^{NT} \end{pmatrix}$ ,  $\widehat{\mathbf{p}} = \begin{pmatrix} \widehat{q} \\ \widehat{P}^{NT} \end{pmatrix}$  and  $\mathbf{M} = \begin{pmatrix} (1 - \phi)(1 - \theta^2) + \theta^2(1 - \eta)(1 - \epsilon) - 1 & \theta(1 - \eta)(\epsilon - 1) \\ \theta\eta(\epsilon - 1) & \eta(1 - \epsilon) - 1 \end{pmatrix}$ .

### 3.3.2 Second-order approximation of the Euler equations

As in the benchmark case, the second order approximation of (35) gives:

$$\text{cov}(\widehat{PC}, \widehat{R}) = (1 - 1/\gamma)\text{cov}(\widehat{RER}, \widehat{R}) \quad (41)$$

A bit of rewriting and using (39) yields:

$$(2\mu - 1)\text{var}(\widehat{R}) = (1 - 1/\gamma)\text{cov}(\widehat{RER}, \widehat{R}) \quad (42)$$

As in the benchmark model, the Home investor will demand a larger quantity of stocks that covary positively with the Home real exchange rate. The equity home bias then depends on the covariance-variance ratio  $\text{cov}(\widehat{RER}, \widehat{R})/\text{var}(\widehat{R})$  (see also van Wincoop and Warnock [2007]).

### 3.3.3 Analytical expressions for equity portfolios

One needs to solve for equilibrium equity portfolio  $\mu$  using the first-order approximation of non portfolio equations (equations (37),(38),(39)) and the second-order approximation of the Euler equation (equation (41)). We have four endogenous variables (the two relative prices  $\widehat{q}$  and  $\widehat{P}^{NT}$ , relative consumption expenditures  $\widehat{PC}$  and the portfolio position  $\mu$ ) and four equations (3 non-portfolio equations and 1 portfolio equation).<sup>13</sup> As in Devereux and Sutherland [2007], it is convenient to rewrite Home excess returns and the Home real exchange rate in matrix form.

Using (39) and (40), we express aggregate Home excess returns  $\widehat{R}$  as a function of the vector of fundamental shocks  $\widehat{\mathbf{y}}$ :

$$\begin{aligned} \widehat{R} &= \begin{pmatrix} \eta & 1 - \eta \end{pmatrix} (\widehat{\mathbf{p}} + \widehat{\mathbf{y}}) \\ \widehat{R} &= \mathbf{R}_1\widehat{\mathbf{y}} + \mathbf{R}_2(2\mu - 1)\widehat{R} \\ \widehat{R} &= [1 - \mathbf{R}_2(2\mu - 1)]^{-1} \mathbf{R}_1\widehat{\mathbf{y}} \end{aligned} \quad (43)$$

with  $\mathbf{R}_1 = \begin{pmatrix} \eta & 1 - \eta \end{pmatrix} \begin{pmatrix} \mathbf{I} + \mathbf{M}^{-1} \end{pmatrix}$  and  $\mathbf{R}_2 = - \begin{pmatrix} \eta & 1 - \eta \end{pmatrix} \mathbf{M}^{-1} \begin{pmatrix} \theta \\ 1 \end{pmatrix}$  ( $\mathbf{I}$  denotes the  $2 \times 2$  identity matrix; note that  $\mathbf{R}_1$  is a  $1 \times 2$  vector while  $\mathbf{R}_2$  is a scalar).<sup>14</sup>

<sup>13</sup>Note that equilibrium (relative) equity returns are also unknown but solving for relative prices solve simultaneously for (relative) equity returns since endowments are exogenous.

<sup>14</sup>Note that for the values of  $\epsilon$  considered in the calibrations ( $\epsilon < 1$ ),  $\det(\mathbf{M}) > 0$  so the matrix  $\mathbf{M}$  is invertible.

Similarly, one can rewrite the real exchange rate using (36), (39), (40) and (43):

$$\begin{aligned}\widehat{REER} &= \begin{pmatrix} \eta\theta & 1-\eta \end{pmatrix} \widehat{\mathbf{P}} \\ \widehat{REER} &= \mathbf{P}_1 \widehat{\mathbf{y}} + \mathbf{P}_2 (2\mu - 1) \widehat{R}\end{aligned}$$

$$\widehat{REER} = [1 - \mathbf{R}_2(2\mu - 1)]^{-1} [\mathbf{P}_1 + (2\mu - 1)(\mathbf{P}_2 \mathbf{R}_1 - \mathbf{P}_1 \mathbf{R}_2)] \widehat{\mathbf{y}} \quad (44)$$

with  $\mathbf{P}_1 = \begin{pmatrix} \eta\theta & 1-\eta \end{pmatrix} \mathbf{M}^{-1}$  and  $\mathbf{P}_2 = -\begin{pmatrix} \eta\theta & 1-\eta \end{pmatrix} \mathbf{M}^{-1} \begin{pmatrix} \theta \\ 1 \end{pmatrix}$

From the expressions of Home excess returns (equation (43)) and the Home real exchange rate (equation (44)), one can deduce the following portfolio equation by substituting  $var(\widehat{R})$  and  $cov(\widehat{REER}, \widehat{R})$  in the second order approximation of the Euler equation (equation (41)):

$$(2\mu - 1) var(\mathbf{R}_1 \widehat{\mathbf{y}}) = (1 - 1/\gamma) cov((\mathbf{P}_1 + (2\mu - 1)(\mathbf{P}_2 \mathbf{R}_1 - \mathbf{P}_1 \mathbf{R}_2)) \widehat{\mathbf{y}}, \mathbf{R}_1 \widehat{\mathbf{y}})$$

The (zero-order) equilibrium portfolio is a fixed-point of the previous equation (Tille and van Wincoop [2007]). As both sides of the previous equation are linear in  $\mu$ , we get the following analytical expression for the local equity holdings as a function of exogenous parameters:

$$\mu = \frac{1}{2} \left[ 1 + \frac{(1 - 1/\gamma) cov(\mathbf{P}_1 \widehat{\mathbf{y}}, \mathbf{R}_1 \widehat{\mathbf{y}})}{var(\mathbf{R}_1 \widehat{\mathbf{y}}) (1 - (1 - 1/\gamma) \mathbf{P}_2) + (1 - 1/\gamma) \mathbf{R}_2 cov(\mathbf{P}_1 \widehat{\mathbf{y}}, \mathbf{R}_1 \widehat{\mathbf{y}})} \right] \quad (45)$$

As in the benchmark model, the equilibrium portfolio is the sum of the market portfolio (1/2) held by the log-investor and a bias due to fluctuations in the real exchange rate. Note that unlike Obstfeld [2007], the optimal portfolio is not independent of the stochastic properties of the endowment shocks. In fact, investors cannot span perfectly the uncertainty (even up to a first-order approximation) as the dimension of the uncertainty is strictly higher than the number of assets available (two endowment shocks in each country and only one stock). The key difference with our benchmark model is that the sign of the covariance between the real exchange rate and Home equity excess returns depends on the response of the terms-of-trade **and** the relative price of non-tradable goods to endowment shocks in both sectors and as shown below this covariance can be either positive or negative, depending on parameter values.

### 3.4 Description of equilibrium equity portfolios

#### 3.4.1 Calibration

Values for the parameters are shown in table 2. We keep the same elasticity of substitution  $\phi$  and the same risk aversion  $\gamma$  as in the benchmark model. According to the Bureau of Economic Analysis (BEA), the average share of services in total consumption expenditures is 55% over the period 1995-2005. Assuming that most services are non-tradable, we set the share of tradable goods in consumption<sup>15</sup> to  $\eta = 45\%$ . A key parameter is the elasticity of substitution between tradable and non tradable goods  $\epsilon$ . The existing literature focuses on low values for  $\epsilon$ , ranging from 0 to 1 for industrialized countries (see

<sup>15</sup>Stockman and Tesar [1995] provide slightly higher values for the share of tradable consumption (from 45% to 55%). Note however that our results do not depend qualitatively on  $\eta$  for a wide range of values.

Preferences	
Relative risk-aversion	$\gamma = 2$
Elasticity of substitution between tradables	$\phi = 5$
Elasticity of substitution between tradables and non-tradables	$\epsilon = \{0.25; 0.5; 0.75\}$
Share of tradable goods in consumption expenditures	$\eta = 0.45$
Trade frictions	
Trade costs	$\tau \in [0; 300\%]$
Volatility and correlation of shocks	
Volatility ratio	$\sigma_{NT}/\sigma_T = \{0.5; 1\}$
Corr( $\widehat{y}^T, \widehat{y}^{NT}$ )	$\rho = 0.3$

Table 2: Parameter values in the model with non-tradable goods

Van Wincoop [1999] and Matsumoto [2007] for a detailed discussion).<sup>16</sup> Consequently, we provide some sensitivity analysis of our results using values for  $\epsilon$  between 0 and 1.

While the stochastic properties of the shocks played no role for equilibrium portfolios in our benchmark model, they do matter in this case. Using US quarterly data from 1985 to 2005 from the BEA, we find that the volatility of (annualized) growth rates in the service sector (our proxy for the non-tradable goods sector) is slightly more than half of the volatility of growth rates of final goods sales (our proxy for the tradable sector).<sup>17</sup> Stockman and Tesar [1995] provide estimates for G7 countries of the volatility of output in both sectors: the ratio between the volatility of non-traded output and traded output is between 0.5 and 1 for all countries and the average is 0.6 (see also Van Wincoop [1999] and Serrat [2001] for similar estimates). The key parameter for portfolios is the ratio between the volatility of **relative** non-traded output  $\widehat{y}^{NT} = \widehat{y}_H^{NT} - \widehat{y}_F^{NT}$  and the volatility of **relative** traded output  $\widehat{y}^T = \widehat{y}_H^T - \widehat{y}_F^T$ . We denote this ratio by  $\sigma_{NT}/\sigma_T$ . Assuming that cross-country correlations of traded and non-traded output are similar in both sectors, we use 0.5 as a benchmark value for  $\sigma_{NT}/\sigma_T$  and provide some robustness checks when the volatility is the same in both sectors:  $\sigma_{NT}/\sigma_T = 1$ .<sup>18</sup> The correlation of output shocks within a country (correlation between output shocks in the tradable and non-tradable sector) also affects the equilibrium portfolio. Using the same BEA data, we find this correlation to be positive but quite low (equal to 0.3). We thus set the correlation of shocks across sectors (within a country) equal to 0.3.<sup>19</sup>

<sup>16</sup>Some representative estimates for  $\epsilon$  used in the existing literature are: 0.19 (Pesenti and Van Wincoop [2002]), 0.44 (Stockman and Tesar [1995]), 0.74 (Mendoza [1995] and Corsetti, Dedola and Leduc (2004), from 0.6 to 0.8 (Serrat [2001]). Ostry and Reinhart [1992] provides estimates for developing countries in the range of 0.6 to 1.4.

<sup>17</sup>Values for the two volatilities are 1.2% and 2.3%, respectively.

<sup>18</sup>This ratio depends on the volatility of shocks in each country but also on their cross-country correlation in each sector. Stockman and Tesar [1995] provide estimates of the cross-country correlation of traded and non-traded output that are similar in both sectors. If the cross-country correlation is the same in both sectors, then  $\sigma_{NT}/\sigma_T$  is simply the ratio of the volatility of output shocks in a given country and empirical evidence shows that this ratio is between 0.5 and 1.

<sup>19</sup>Stockman and Tesar [1995] estimates the correlation between innovations in the non-tradable sector and innovations in the tradable sector to be equal to 0.45. Our results are robust to a higher correlation.

### 3.4.2 Equity portfolios

Equilibrium equity positions  $\mu$  as a function of trade costs  $\tau$  are shown in figure (2) for values of  $\sigma_{NT}/\sigma_T = 0.5$  and in figure (3) for  $\sigma_{NT}/\sigma_T = 1$ . In both figures, we show the portfolios for the different values of  $\epsilon$  considered.

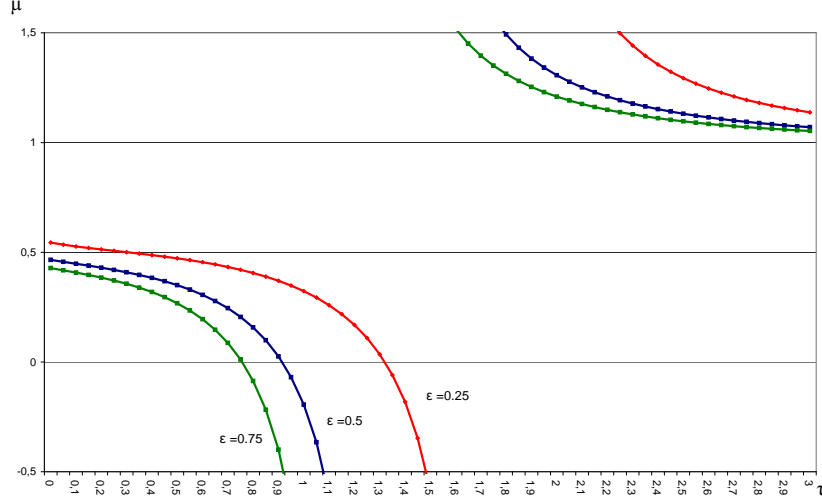


Figure 2: Holdings of local stocks  $\mu$  as a function of trade costs  $\tau$  for  $\sigma_{NT}/\sigma_T = 0.5$ . The upper curve (red) is for  $\epsilon = 0.25$ , the middle one (blue) for  $\epsilon = 0.5$  and the lower one (green) for  $\epsilon = 0.75$ . Other parameters are set to their benchmark value (see table (2)).

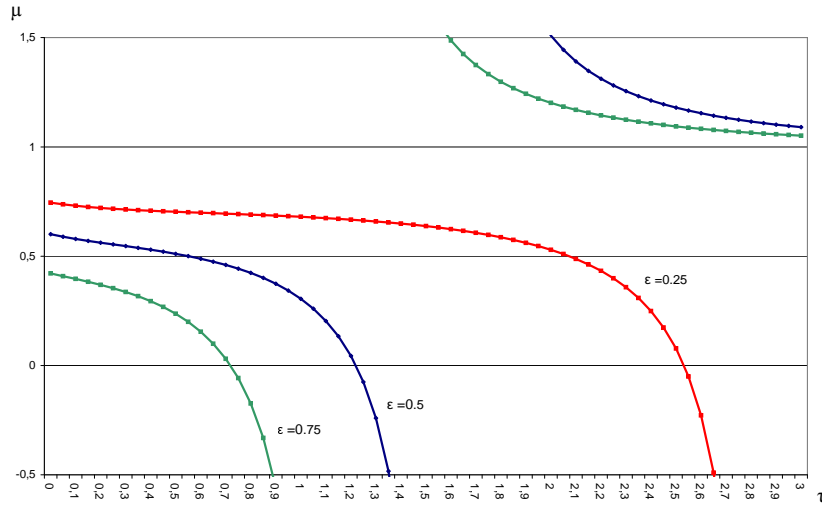


Figure 3: Holdings of local stocks  $\mu$  as a function of trade costs  $\tau$  for  $\sigma_{NT}/\sigma_T = 1$ . The upper curve (red) is for  $\epsilon = 0.25$ , the middle one (blue) for  $\epsilon = 0.5$  and the lower one (green) for  $\epsilon = 0.75$ . Other parameters are set to their benchmark value (see table (2)).

We will restrict our comments for trade costs below 100% as this is the most relevant case empirically but note that, as in the benchmark model, for trade costs above a certain threshold (higher than 100%), the model predicts too much Home equity bias and a short position in Foreign equity (when trade costs are infinite, we converge to the financial autarky portfolio  $\mu = 1$ ). For trade costs below that threshold, the portfolio  $\mu$  as a function of  $\tau$  is qualitatively very similar to our benchmark model: increasing trade



costs reduces Home equity holdings and potentially leads to some Foreign equity bias. Note however that the model can predict some Home bias in equities for certain parameter configurations (for low values of  $\epsilon$  and  $\tau$  and higher values of  $\sigma_{NT}/\sigma_T$ ; see figures (2) and (3)). Everything else constant, the lower is the elasticity of substitution between traded and non-traded goods  $\epsilon$  (resp. the higher is the ratio  $\sigma_{NT}/\sigma_T$ ), the higher are local equity holdings  $\mu$ .

To sum up, two main results emerge: 1) due to the presence of non-tradable goods, the model is able to generate some Home bias in equities for some calibrations but the direction of the bias is very sensitive to the values of the parameters; 2) increasing trade costs lowers Home equity holdings and leads to some Foreign bias, as in our benchmark case.

As equity positions depend crucially on the covariance between the real exchange rate and Home equity excess returns, we need to understand how these two variables respond to endowment shocks. Let us first understand why the model can generate some Home bias in equities when trade costs are low. To do so, it is useful to consider the case of zero trade costs. Then, the real exchange rate depends only on the relative price of non-traded goods  $\widehat{P}^{NT}$ . Home bias emerges if and only if Home equity returns are increasing when the price of Home non-tradable goods is increasing. Consider a fall in Home non-tradable output. Then the price of Home non-tradable goods increases. The response of Home equity returns is ambiguous: as the price Home non-tradable goods increases, dividends of Home non-tradable firms increase despite the fall in output ( $\epsilon$  is low enough). But since shocks to Home non-tradable and Home tradable output are positively correlated, there is a fall in Home tradable output and dividends. Depending on the strength of these two effects, Home (aggregate) equity returns can increase or decrease when the price of Home non-tradable goods increases. Thus, the sign of the covariance between the real exchange rate and Home equity excess returns is ambiguous and it is consequently also ambiguous whether the share of domestic equity held is lower or higher than 1/2. For  $\tau = 0$ , one can express the term  $cov(\mathbf{P}_1\widehat{\mathbf{y}}, \mathbf{R}_1\widehat{\mathbf{y}})$  which is key to understand the direction of the bias (see equation (45)) as follows:<sup>20</sup>

$$cov(\mathbf{P}_1\widehat{\mathbf{y}}, \mathbf{R}_1\widehat{\mathbf{y}}) = \eta\sigma_{NT}^2 \left[ \left( \frac{1-\eta}{1-\eta(1-\epsilon)} \right)^2 (1-\epsilon) - \rho \frac{\sigma_T}{\sigma_{NT}} \frac{1-\eta}{1-\eta(1-\epsilon)} \frac{\phi-1}{\phi} \right]$$

Hence, the sign of the equity bias is ambiguous depending on the sign of  $\left[ \left( \frac{1-\eta}{1-\eta(1-\epsilon)} \right)^2 (1-\epsilon) - \rho \frac{\sigma_T}{\sigma_{NT}} \frac{1-\eta}{1-\eta(1-\epsilon)} \frac{\phi-1}{\phi} \right]$ . This illustrates the two effects explained above. The first term is positive for  $\epsilon < 1$  (and decreasing with  $\epsilon$ ): following a fall in Home non-tradable output, Home price of non-tradable goods  $\widehat{P}^{NT}$  increases strongly ( $\epsilon$  is low), the Home real exchange rate appreciates and Home equity returns tend to rise because dividends in the non-tradable sector increase; this affects positively the covariance term. The second term is negative (for  $\phi > 1$ ): a fall in Home non-tradable output is associated with a fall in Home tradable output (for a correlation of shocks  $\rho > 0$ ) and Home dividends in the tradable sector decrease. Thus, the covariance between the Home real exchange rate and Home equity excess returns can have either sign. Note that the first effect tends to dominate for low values of  $\epsilon$ , low values of  $\rho$  and high values of

<sup>20</sup>Note that when  $\tau = 0$ ,  $\theta = 0$  and the matrix  $\mathbf{M}$  is diagonal, which simplifies the expression for the portfolio.

$\sigma_{NT}/\sigma_T$ . These are the cases where the presence of non-traded goods help to generate some equity home bias. Note however that in these cases, the home bias predicted by the model remains fairly small for the calibrations considered: if we take the mid-point value of 0.5 for  $\epsilon$  (as in Stockman and Tesar [1995]), the equity portfolio exhibits a small home bias of 5% for  $\tau = 30\%$  and  $\sigma_{NT}/\sigma_T = 1$ .<sup>21</sup> Moreover, as discussed below (see section 3.5.1), any calibration that would lead to a high degree of Home bias would contradict the very weak covariance between Home excess returns and Home real exchange rate observed in the data (Van Wincoop and Warnock [2007]).

Most importantly, the model predicts that local equity holdings are decreasing with trade costs. When trade costs are non-zero, the investor wants to hedge fluctuations in the Home terms-of-trade  $\hat{q}$  as well as fluctuations in the relative price of non-tradable goods  $\widehat{P}^{NT}$ . As in the benchmark model, an increase in output in the tradable sector leads to a fall of the Home terms-of-trade together with an increase in Home excess returns.<sup>22</sup> This negative co-movement between Home terms-of-trade and Home excess returns affects negatively the covariance between the Home real exchange rate and Home equity excess returns. Thus, we find that the result that trade costs do not help to generate equity home bias is robust to the case, where we allow for the presence of non tradable goods.

## 3.5 Related literature

### 3.5.1 A closer look at Van Wincoop and Warnock [2007]

Van Wincoop and Warnock [2007] computes the following statistic for the US:

$$\text{Covariance-variance ratio} = \frac{\text{cov}(\widehat{REER}, \widehat{R})}{\text{var}(\widehat{R})}$$

where  $\widehat{R}$  denotes US aggregate excess stock market return over Foreign stock markets and  $\widehat{REER}$  is the US real exchange rate. They find values between 0 and 0.32 depending on the method used.<sup>23</sup> In our benchmark model as well as in the extension with non-traded goods, any portfolio home bias must be consistent with this 'covariance-variance ratio' (remember that  $\mu - 1/2 = 1/2(1 - 1/\gamma)\text{cov}(\widehat{REER}, \widehat{R})/\text{var}(\widehat{R})$ ). This ratio is close to zero in the data, providing evidence against the view that equity home bias can be explained by the hedging of the real exchange rate, as Van Wincoop and Warnock(2007) already convincingly argued.

The benchmark model predicts a negative covariance between the Home real exchange rate and Home excess returns and the model is completely at odds with their 'covariance-variance ratio' (in the benchmark model with  $\tau = 60\%$ , the ratio is equal to  $= -0.72$ ; see table (3)). But with non-tradable goods, the model can arguably predict a 'covariance-variance ratio' that is closer to the one found in the data. While positive output shocks in both sectors depreciate the real exchange rate, these shocks can

<sup>21</sup>These are very conservative estimates: if we keep  $\tau = 30\%$  but use  $\sigma_{NT}/\sigma_T = 0.5$ , there is a foreign bias of 9%. If we increase  $\tau$  further, the equity foreign bias is larger.

<sup>22</sup>Note that while dividends in the tradable sector are increasing, dividends in the non-tradable sector might decrease depending on the response of the relative price of non-tradable goods but the latter effect is always dominated for the calibrations considered and Home stock returns increase with an increase in Home tradable output

<sup>23</sup>When they condition on nominal exchange rate movements or look at higher frequency, they find values closer to zero.

have ambiguous effects on Home excess returns. In particular, as seen above, a fall in the Home price of non-tradable goods due to an increase in Home non-tradable output is associated with a decrease (resp. an increase) in dividends in the non-tradable (resp. tradable) sector (for the low values of  $\epsilon$  considered). As a consequence, the covariance between Home excess returns and the Home real exchange rate can take any value depending on the relative size of the sectors, the preference parameters and the stochastic properties of the shocks.

		Tradable (Benchmark)	Tradable and non-tradable		
			$\epsilon = 0.25$	$\epsilon = 0.5$	$\epsilon = 0.75$
$\sigma_{NT}/\sigma_T = 0.5$	$\tau = 0\%$	0	<b>0.18</b>	-0.13	-0.29
	$\tau = 30\%$	-0.23	<b>0.01</b>	-0.36	-0.58
	$\tau = 60\%$	-0.72	<b>-0.18</b>	-0.77	-1.22
$\sigma_{NT}/\sigma_T = 1$	$\tau = 0\%$	0	0.98	0.40	-0.31
	$\tau = 30\%$	-0.23	0.85	<b>0.18</b>	-0.65
	$\tau = 60\%$	-0.72	0.79	<b>-0.04</b>	-1.38

Table 3: Covariance-variance ratio:  $cov(\widehat{RER}, \widehat{R})/var(\widehat{R})$ , where  $\widehat{R}$  denotes Home aggregate excess stock market return and  $\widehat{RER}$  is the Home real exchange rate.

All preference parameters are set to their benchmark value (see table (2)). The first column corresponds to the benchmark model without non-tradable goods.

We compute the 'covariance-variance ratio' for various levels of trade costs and different values for  $\epsilon$  and  $\sigma_{NT}/\sigma_T$  keeping other parameters at their benchmark value (see table (3)). The model now predicts ratios fairly close to zero (or slightly positive) for standard values of  $\epsilon$  (between 0.25 and 0.5; see bold figures in table (3)). Thus, the model with non-traded does better than our benchmark model in matching the 'covariance-variance ratio' but, as in the partial equilibrium approach of Van Wincoop and Warnock [2007], this confirms that trade frictions do not really help to generate equity home bias here, as a zero 'covariance-variance ratio' implies a fully diversified portfolio.

### 3.5.2 Comparison with Obstfeld [2007] and Collard, Dellas, Diba and Stockmann [2007]

Obstfeld [2007] extends the benchmark model with non-tradable goods in the same two-period set-up but assumes that agents can hold separate claims on tradable and non-tradable firms (see Collard, Dellas, Diba and Stockmann [2007] for a dynamic version): in their framework, there are two different stocks in each country, stocks of the tradable sector and stocks of the non-tradable sector, where each stock is a Lucas tree that represents a share in one of the future endowments (tradable or non-tradable).

Following Obstfeld [2007], if we normalize the supply of shares to unity in both sectors and assume symmetric countries, the vector  $(\mu^T; \mu^{NT})$  describes international equity portfolios where  $\mu^T$  (resp.  $\mu^{NT}$ )

denotes a country's holdings of local stock of tradables (resp. non-tradables).  $\mu^k > \frac{1}{2}$  means that there is equity home bias in sector  $k = \{T; NT\}$ .

With such a financial asset structure, Obstfeld [2007] derives the following equilibrium equity portfolios as a function of the parameters (see technical appendix 5.2.1):<sup>24</sup>

$$\mu^T = \frac{1}{2} \left[ 1 - \left(1 - \frac{1}{\gamma}\right) \frac{\theta(1-\epsilon)}{\Delta} \right] \quad (46)$$

$$\mu^{NT} = \frac{1}{2} \left[ 1 + \left(1 - \frac{1}{\gamma}\right) \frac{(\phi-1)(1-\theta^2) + \theta^2(\epsilon-1)}{\Delta} \right] \quad (47)$$

where  $\Delta = [(\phi-1)(1-\theta^2) - \theta^2((1-\eta)(1-\epsilon) + \eta(1-1/\gamma))][\eta(1-\epsilon) + (1-\eta)(1-1/\gamma)] + \eta(1-\eta)\theta^2(\epsilon-1/\gamma)^2$ . For the values considered in our calibrations and reasonable trade costs,  $\Delta$  is likely to be positive.<sup>25</sup> Note first that the equity portfolios in the tradable and non-tradable sector are again the sum of two terms: the fully diversified portfolio and a hedging component due to real exchange rate fluctuations. The hedging term cancels out for the logarithmic investor ( $\gamma = 1$ ). Secondly, the equity portfolios are now independent of the stochastic properties of the shocks: the reason is that with two stocks in each in each country, investors can perfectly span the four endowment shocks up to a first-order approximation (see Collard et al. [2007] for a similar result).

Keeping the same values of parameters as in table (2), equilibrium equity positions in the tradable and non-tradable sector as a function of trade costs  $\tau$  are shown in figure (4) for different values of  $\epsilon$  ( $\epsilon < 1$ ). The typical portfolio is Home biased in the non-tradable sector<sup>26</sup> and Foreign biased in the tradable sector if trade costs are below a threshold (the level of trade costs for which  $\Delta = 0$ , roughly 130%; see figure (4)). Hence, trade costs are still generating some equity foreign bias, at least in the tradable sector.

As emphasized by Obstfeld [2007] in his Ohlin lecture, when  $\epsilon$  is slightly above one,<sup>27</sup> the equity portfolio in the tradable sector exhibits some home bias that is increasing with trade costs. This is the only case where trade costs help to generate equity home bias. But as discussed above, the empirical evidence indicates that  $\epsilon$  is below unity. Moreover, for the benchmark calibrations used in the paper (setting  $\epsilon = 1.25$  and other values to their benchmark of table (2)), the 'covariance-variance ratio' predicted by the model of Obstfeld [2007] is far below the one measured by van Wincoop and Warnock [2007]: the ratio is equal to -0.38 for  $\tau = 30\%$  and -0.84 for  $\tau = 60\%$  if  $\sigma_{NT}/\sigma_T = 0.5$  (resp. -0.40 for  $\tau = 30\%$  and -0.49 for  $\tau = 60\%$  if  $\sigma_{NT}/\sigma_T = 1$ ).<sup>28</sup> Hence, while the case emphasized by Obstfeld [2007] is theoretically relevant, it hinges on non-standard values for  $\epsilon$  and predicts a 'covariance-variance ratio'

<sup>24</sup>Without non-tradables ( $\eta = 1$ ), one can easily verify that the equity portfolio of tradables is the same as in the benchmark model

<sup>25</sup>In particular  $\epsilon$  must be below a threshold slightly above unity. Note that we assume  $\Delta$  to be non-zero (otherwise portfolios are undetermined). Like in the previous section,  $\Delta$  switches sign for very high levels of trade costs.

<sup>26</sup>For the calibrations considered, depending on whether tradable and non-tradable consumption are substitutes ( $\epsilon < 0.5$ ) or complements ( $\epsilon > 0.5$ ), the equity portfolio invested in the non-tradable sector  $\mu_{NT}$  is above or below unity.

<sup>27</sup>He focuses on the case where  $\epsilon < 1 + \frac{1-\eta}{\eta}(1-1/\gamma)$ . With higher  $\epsilon$ ,  $\Delta$  switches sign and we have equity portfolios that are very similar to the benchmark model having equity foreign bias in both sectors.

<sup>28</sup>Note also that with a low elasticity of substitution between tradables and non-tradables ( $\epsilon$  around 0.5), the 'covariance-variance' ratio seem to be more in line with its empirical counterpart, i.e close to zero for reasonable trade costs.

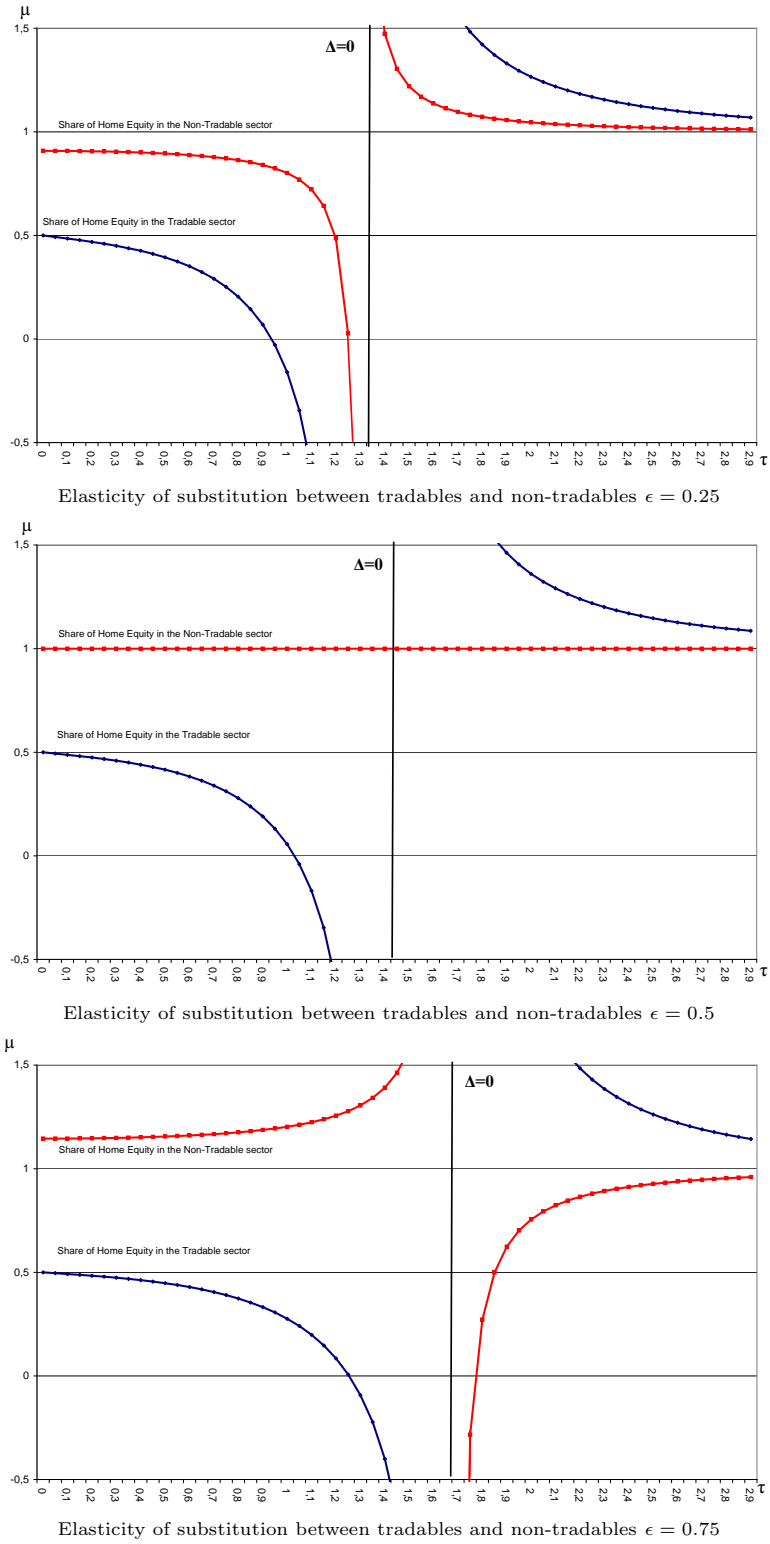


Figure 4: Holdings of local stocks in the tradable sector  $\mu_T$  (blue curve) and non-tradable sector  $\mu_{NT}$  (red curve) as a function of trade costs  $\tau$  with different elasticities of substitution between tradables and non-tradables  $\epsilon$ . Other parameters set to their benchmark value (see table (2)).

that is not consistent with the data. Overall, our main result also survives allowing trading separate claims on the two sectors: trade costs do not lead to Home bias in equities.

## 4 Conclusion

In this paper, we have shown that trade costs in goods markets alone cannot generate any home bias in equities under standard preferences in a two-good/two-country general equilibrium model. On the contrary, investors should bias their portfolios towards foreign equities. This is a very important result which goes against some conventional wisdom in international economics that has recently put forward trade costs as the relevant friction to solve the portfolio home bias puzzle. We investigated the robustness of this theoretical result by allowing trade costs to interact with the presence of non-tradable goods. Compared to existing literature, we focus on the case where investors cannot buy separate claims on tradable and non-tradable firms but hold claims over the aggregate stock market of a country. In this case, we find that for a reasonable calibration of the preference parameters, the main message of our paper remains: goods market frictions do not help to solve the equity home bias puzzle and if anything the puzzle is worsened. When investors can separate claims over traded and non-traded industries as in Obstfeld [2007], the presence of non-tradable goods cannot solve the home bias in equities in the tradable sector for a reasonable calibration of the preference parameters.

We used a static set-up. Whether the results are still valid in a dynamic two-country model is a meaningful question. In a dynamic setting, the terms-of-trade adjustment following output shocks work exactly in the same way (Pavlova and Rigobon [2007]). Moreover, as shown by Devereux and Sutherland [2006a, 2006b] and Tille and Van Wincoop [2007], up to a first-order approximation for the goods market equilibrium (the non-portfolio equations), portfolios are exactly the static ones, *i.e.* the one calculated in this paper. However, in a dynamic model, one can analyze second-order terms which make equilibrium portfolios time-varying. Analyzing portfolio re-balancing in this set-up is left for further research.

Finally, the model focuses on output shocks as source of uncertainty and extensions of the model with a larger array of shocks remain to be done. The results are certainly robust to any shock that increases the supply of goods. However, shocks to the quality of goods produced or demand shocks should affect the link between asset prices and the real exchange rate and consequently international portfolios.

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## 5 Appendix

### 5.1 Robustness check: How realistic is the case $\theta > \theta^*$ ?

$\gamma$	$\phi$	$\theta^*$	$\tau^*$	$(\frac{Imp}{GDP})^*$
2	2	0.82	889%	9%
2	5	0.94	142%	3%
2	8	0.96	78%	2%
4	2	0.77	619%	11.5%
4	5	0.92	119%	4%
4	8	0.95	69%	2.5%
6	2	0.74	564%	13%
6	5	0.91	114%	4.5%
6	8	0.94	66%	3%

Table 4: Is  $\theta(\tau) > \theta^*$  ? Trade costs ( $\tau^*$ ) and Import Shares such that  $\theta(\tau^*) = \theta^* = \left(\frac{\phi-1}{\phi-1/\gamma}\right)^{\frac{1}{2}}$  for various degrees of risk-aversion  $\gamma$  and elasticity of substitution across tradable goods  $\phi$

### 5.2 Technical appendix: not intended for publication

#### 5.2.1 Derivation of the portfolios of Obstfeld [2007] in the model with non-tradable goods

Here are the main derivations of the model with non-tradable goods following Obstfeld [2007]. We derive the portfolios when agents can trade separately stocks on tradable and non tradable firms as in Obstfeld [2007]. We also present more detailed derivations of the first-order approximation of non-portfolio equations than in the main text (as they are identical except the budget constraint).

#### First-order approximation of non portfolio equations

The representative consumer in country  $i$  maximizes  $\frac{C_i^{1-\gamma}}{1-\gamma}$  subject to a budget constraint (for  $j \neq i$ ):

$$P_i C_i = p_i^T c_{ii}^T + (1 + \tau) p_j^T c_{ij}^T + P_i^{NT} c_i^{NT} \leq \mu_{ii}^T p_i^T y_i^T + \mu_{ij}^T p_j^T y_j^T + \mu_{ii}^{NT} P_i^{NT} y_i^{NT} + \mu_{ij}^{NT} P_j^{NT} y_j^{NT} \quad (\lambda_{i,1})$$

with  $\lambda_{i,1}$  the Lagrange-Multiplier of the period 1 budget constraint. The first-order conditions are: For aggregate consumption:

$$1 = \lambda_{i,1} P_i C_i^\gamma$$

Intratemporal allocation across goods:

$$\begin{aligned} c_{ii}^T &= \left(\frac{p_i^T}{P_i^T}\right)^{-\phi} c_i^T & c_{ij}^T &= \left(\frac{(1+\tau)p_j^T}{P_i^T}\right)^{-\phi} c_i^T \\ c_i^T &= \eta \left(\frac{P_i^T}{P_i}\right)^{-\epsilon} C_i & c_i^{NT} &= (1-\eta) \left(\frac{P_i^{NT}}{P_i}\right)^{-\epsilon} C_i \end{aligned}$$

Intratemporal allocation across goods for both countries and market-clearing conditions for tradable goods and non-tradable goods give the following equilibrium conditions:

$$\frac{y_H^{NT}}{y_F^{NT}} = \left(\frac{P_H^{NT}}{P_F^{NT}}\right)^{-\epsilon} \frac{P_H^\epsilon C_H}{P_F^\epsilon C_F} \quad (48)$$

$$\frac{y_H^T}{y_F^T} = q^{-\phi} \Omega \left[ \left(\frac{P_F^T}{P_H^T}\right)^{\phi-\epsilon} \frac{P_F^\epsilon C_F}{P_H^\epsilon C_H} \right] \quad (49)$$

where  $\Omega(x) = \frac{1+x(1+\tau)^{1-\phi}}{x+(1+\tau)^{1-\phi}}$ .

The log-linearization of the Home country's real exchange rate  $REER \equiv \frac{P_H}{P_F}$  gives:

$$\widehat{REER} = \frac{\widehat{P}_H}{\widehat{P}_F} = \eta\theta\widehat{q} + (1-\eta)\widehat{P}^{NT}.$$

Log-linearizing equations (48) and using the expression for the real exchange rate implies:

$$\widehat{y}_H^{NT} - \widehat{y}_F^{NT} = -\epsilon\widehat{P}^{NT} + (\epsilon-1)\widehat{REER} + \widehat{PC} \quad (50)$$

$$\widehat{R}^{NT} = p_H^{NT}\widehat{y}_H^{NT} - p_F^{NT}\widehat{y}_F^{NT} = \eta(1-\epsilon)(\widehat{P}^{NT} - \theta\widehat{q}) + \widehat{PC} \quad (51)$$

where  $\widehat{PC} = P_H\widehat{C}_H - P_F\widehat{C}_F$  denotes relative consumption expenditures and  $\widehat{R}^{NT}$  denotes Home excess returns in the non-tradable sector:  $\widehat{R}^{NT} = \widehat{R}_H^{NT} - \widehat{R}_F^{NT}$ . We denote Home excess returns in the tradable sector by  $\widehat{R}^T$ . Similarly, log-linearizing (49) implies:

$$\widehat{y}_H^T - \widehat{y}_F^T = [-\phi(1-\theta^2) + \theta^2(\phi-\epsilon)]\widehat{q} + \theta(\epsilon-1)\widehat{REER} + \theta\widehat{PC} \quad (52)$$

$$\widehat{R}^T = p_H^T\widehat{y}_H^T - p_F^T\widehat{y}_F^T = [(1-\phi)(1-\theta^2) + \theta^2(1-\eta)(1-\epsilon)]\widehat{q} + \theta(1-\eta)(\epsilon-1)\widehat{P}^{NT} + \theta\widehat{PC} \quad (53)$$

The relative budget constraint is due to portfolio symmetry:

$$\widehat{PC} = P_H\widehat{C}_H - P_F\widehat{C}_F = \eta(2\mu^T - 1)\widehat{R}^T + (1-\eta)(2\mu^{NT} - 1)\widehat{R}^{NT} \quad (54)$$

### Second-order approximation of Euler (portfolio) equations

Like in the benchmark model, we can express the Euler equations in relative terms (Home relative to Foreign) in each sector  $k = \{T; NT\}$  as follows:

$$E_0\left[\left(\frac{C_H^{-\gamma}}{P_H} - \frac{C_F^{-\gamma}}{P_F}\right)(R_H^k - R_F^k)\right] = 0 \quad (55)$$

where  $R_i^k = \frac{p_i^k y_i^k}{p_s^k}$  denotes the return on stock  $i = \{H; F\}$  in sector  $k = \{T; NT\}$ . The second order approximation of (55) gives in each sector:

$$\text{cov}(\widehat{PC}, \widehat{R}^T) = (1-1/\gamma)\text{cov}(\widehat{REER}, \widehat{R}^T) \quad (56)$$

$$\text{cov}(\widehat{PC}, \widehat{R}^{NT}) = (1-1/\gamma)\text{cov}(\widehat{REER}, \widehat{R}^{NT}) \quad (57)$$

### Solving for equity portfolios with tradable and non-tradable goods

To solve for portfolios, note that if there is a portfolio such that, up to the first-order, (i)  $\widehat{PC} = (1-1/\gamma)\widehat{REER}$  and (ii) (51),(53),(54) holds, then this portfolio is the equilibrium zero-order portfolio since it satisfies the second order approximation of the Euler equations (56) and (57). Using this ‘‘guess and verify approach’’ simplifies the resolution. Rewriting Home excess returns in both sectors assuming

that  $\widehat{PC} = (1 - 1/\gamma)\widehat{RER}$  holds gives the following equilibrium equity excess returns as a function of relative prices:

$$\widehat{R}^{NT} = \eta\theta(\epsilon - 1/\gamma)\widehat{q} + [\eta(1 - \epsilon) + (1 - \eta)(1 - 1/\gamma)]\widehat{P}^{NT} \quad (58)$$

$$\widehat{R}^T = [(1 - \phi)(1 - \theta^2) + \theta^2(1 - \eta)(1 - \epsilon) + \theta^2\eta(1 - 1/\gamma)]\widehat{q} + \theta(1 - \eta)(\epsilon - 1/\gamma)\widehat{P}^{NT} \quad (59)$$

Using the relative budget constraint assuming that  $\widehat{PC} = (1 - 1/\gamma)\widehat{RER}$  holds, we get:

$$\widehat{PC} = (1 - 1/\gamma)(\theta\eta\widehat{q} + (1 - \eta)\widehat{P}^{NT}) = \eta(2\mu^T - 1)\widehat{R}^T + (1 - \eta)(2\mu^{NT} - 1)\widehat{R}^{NT} \quad (60)$$

Equations (58), (59) and (60) implies in matrix form (this expression is obtained through a projection on  $\widehat{q}$  and  $\widehat{P}^{NT}$ ):

$$\left(1 - \frac{1}{\gamma}\right) \begin{pmatrix} \theta\eta \\ 1 - \eta \end{pmatrix} = \begin{pmatrix} (1 - \phi)(1 - \theta^2) + \theta^2[(1 - \eta)(1 - \epsilon) + \eta(1 - 1/\gamma)] & \theta\eta(\epsilon - 1/\gamma) \\ \theta(1 - \eta)(\epsilon - 1/\gamma) & \eta(1 - \epsilon) + (1 - \eta)(1 - 1/\gamma) \end{pmatrix} \begin{pmatrix} \eta(2\mu^T - 1) \\ (1 - \eta)(2\mu^{NT} - 1) \end{pmatrix}$$

We deduce the following equity portfolio (assuming the matrix invertible i.e  $\Delta \neq 0$ ):

$$\begin{pmatrix} 2\mu^T - 1 \\ 2\mu^{NT} - 1 \end{pmatrix} = \left(1 - \frac{1}{\gamma}\right) \begin{pmatrix} (1 - \phi)(1 - \theta^2) + \theta^2[(1 - \eta)(1 - \epsilon) + \eta(1 - 1/\gamma)] & \theta(1 - \eta)(\epsilon - 1/\gamma) \\ \theta(1 - \eta)(\epsilon - 1/\gamma) & \eta(1 - \epsilon) + \frac{(1 - \eta)^2(1 - 1/\gamma)}{\eta} \end{pmatrix}^{-1} \begin{pmatrix} \theta \\ \frac{1 - \eta}{\eta} \end{pmatrix}$$

Inverting the matrix and rearranging terms give the expressions for equity portfolios of tradable and non-tradable (equations (46) and (47)) in the paper.