Abstract

We explore theoretically and quantitatively, the role of competition and markups in determining access and use of credit by households. We find that positive spreads (markups) in the credit market are an essential ingredient of a market with high credit penetration, high defaults and chargeoffs. We explore whether increased competition in the credit card industry can explain a rise in bankruptcies, penetration and chargeoff rates, and a simultaneous drop in interest rates. We find that an essential ingredient of the story is household heterogeneity and expansion of credit on the extensive margin.

1 Introduction

The US market for unsecured credit has undergone dramatic change in the last few decades. Unsecured debt has grown from 5% to 9% of disposable income between the early 1980s and the late 1990s while credit card balances have grown at annual rates exceeding 10% since the 1970s. During the same period the bankruptcy rate has tripled and the
chargeoff rate (the fraction of debt charged off due to delinquency) has more than doubled.\textsuperscript{2}

Understanding the driving force behind these changes can inform a variety of policy initiatives, including bankruptcy policy and legal restrictions to the provision of credit to name two areas that have attracted attention in the last decade. However, the magnitude of the developments in the market for unsecured cannot be explained by changes in interest rates, demographics, the volatility of income and expense shocks or legal changes, such as the repeal of usury laws (Livshits, MacGee, and Tertilt (2010b)). Technological progress in the provision of credit and, in particular, the widespread adoption of the use of automated credit scoring models in evaluation of the riskiness of potential borrowers is a widely cited reason behind these developments.\textsuperscript{3}

We build a model with endogenous access to credit which provides a good qualitative match for the observed developments and we quantify our findings. The credit market is segmented in islands that are populated by one household type, where type refers to a household’s debt levels and income prospects. To access an island, lenders have to pay a cost representing the cost of screening applicants. Within each island there is complete information and lenders compete imperfectly, modeled as competitive search. Loan sizes, interest rates and lenders’ profits rates on an island are all determined in equilibrium and depend on the extent of lender entry.

We solve analytically a two period version of our model, to illustrate the main driving forces behind our framework. We prove that an equilibrium exists and is unique, that some islands exhibit no lender entry and that households’ utility increases with lender entry. Therefore, the borrowers who are so unprofitable that lenders can never recoup their entry costs have no access to the credit market. Islands with high levels of entry (due to households’ greater profitability) lender competition is more intense and therefore households receive a greater share of the surplus created by each loan. Furthermore, the risk-adjusted profit rate on a loan is different from zero, exactly because of the different level of competition across islands. This endogenous profit margin has not been explored in the theoretical literature.

\footnote{See Livshits, MacGee, and Tertilt (2010b) for a very good summary of the evidence. Also see Moss and Johnson (1999) and Sullivan, Warren, and Westbrook (2000).}
\footnote{See Livshits, MacGee, and Tertilt (2014) and references therein.}
but it has been documented in recent empirical studies, e.g. Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015).

We model technological progress as a reduction in the cost that lenders pay to enter an island. A reduction in this cost has both intensive and extensive margin effects. On the intensive margin, islands that were already served experience greater entry of lenders, leading to higher levels of competition, lower interest rates and higher loan sizes. This margin leads to greater indebtedness but does not necessarily increase the bankruptcy rate. On the extensive margin, islands that were previously shut from credit markets start being served by lenders. Bankruptcy rates are higher in these islands which would to push the overall bankruptcy rate higher.

To evaluate the quantitative importance of our theoretical predictions, we build and parameterize an equilibrium model of default à la Eaton and Gersovitz (1981) with the crucial modification that credit markets are not perfectly competitive. Instead, we embed our search and matching friction in the credit market and evaluate the importance of endogenous markup determination on the predictions of the model. In preliminary results, we find that the imperfectly competitive model has vastly different predictions for debt and delinquency when compared to the perfectly competitive benchmark. In particular, the model with positive markups generates the levels of bankruptcy, chargeoffs, credit penetration and interest rates that are consistent with the US data. When we shut down the friction in our model to go back to the frictionless benchmark, the common issues identified in the literature emerge, with the model having problems generating high levels of debt and bankruptcy at the same time.

The paper is organized as follows. In Section 2 we present a two-period version of the model that includes all of our methodological innovations. Section 3 develops the full model and section 4 presents our preliminary calibration.
2 Two periods

This section develops a simple two-period of the model. The purpose is to provide a sharp characterization of equilibrium and present the main economic forces in the presence of market segmentation and imperfect competition.

2.1 The model

There are two periods. The household’s income in the two periods is denoted by $y_1$ and $y_2$. The first-period income is the household’s cash-in-hand, i.e. his income net of debt repayment and non-discretionary expenses (such as medical expenses). The second-period income is stochastic and drawn from distribution $F_i(y_2)$ where $i$ denotes the household’s income process. The household’s type is $x = (y_1, i)$.

The household values consumption according to a CRRA utility function:

$$ U(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \beta \frac{c_2^{1-\sigma}}{1-\sigma} $$

where $\beta$ is the discount factor.

We make the following assumptions that facilitate equilibrium characterization and will be relaxed in the full model. We have log utility ($\sigma = 1$). First-period income is low enough so that the household does not want to save when offered interest rate $r$. Second period income is drawn from a uniform distribution with support $(\underline{y}, \bar{y}_i)$, where $\bar{y}_i > \underline{y} > 0$. Specifically we assume:

$$ y_1 \leq \frac{\bar{y}_i - y}{\beta(1+r)(\log(\bar{y}_i) - \log(\underline{y}))} $$

$$ F(y_2) = \frac{y_2 - \underline{y}}{\bar{y}_i - \underline{y}}, \quad y_2 \in [\underline{y}, \bar{y}_i] $$

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2.1.1 The loan

In the first period the household has access to the loan market (as mentioned above, our assumptions guarantee that the household will not want to save). In the second period the household decides whether to repay the loan.

We first describe the loan structure. A loan has two elements \((z_1, z_2)\), where \(z_1\) is the amount that the household receives in the first period and \(z_2\) is the amount that the household promises to repay (but may renege on) in the second period. The interest rate faced by the household is equal to \(\frac{z_2}{z_1} - 1\). We assume contract incompleteness in that the second period repayment cannot depend on the realization of second period income. This incompleteness might be due to the lack of verifiability of income shocks and follows the literature (e.g. Eaton and Gersoviz 1981, Linvshits, MacGee and Tertilt 2007 and Chaterjee et al 2007).

If the household defaults on its repayment it bears a utility cost which is equivalent to losing share \(\phi\) of second period income. It follows that a household which has to repay \(z_2\) and has drawn second period income of \(y_2\) will repay if and only if \(y_2 - z_2 \geq y_2(1 - \phi)\), or:

\[ y_2 \geq \frac{z_2}{\phi} \]

The probability that a type-\(x\) household makes a repayment of \(z_2\) is given by:

\[ \rho(z_2, i) = 1 - F_i\left(\frac{z_2}{\phi}\right) \]

Notice that the household’s first period income does not affect the repayment probability.

We can now determine the profit to the lender and the value to the household of a particular loan. The profits for a lender of offering loan \((z_1, z_2)\) to a type-\(x\) household are:

\[ \pi_x(z_1, z_2) = -z_1 + \frac{z_2}{1 + r} \rho(z_2, i) \]

where \(r\) is the cost of funds for the lender.
The value of autarky to a type-x household is:

\[ v_x^A = \log(y_1) + \beta \int \log(y_2) dF_i(y_2) \]

The value of loan \((z_1, z_2)\) to a type-x household over and above autarky is:

\[
v_x(z_1, z_2) = \log(y_1 + z_1) + \beta \int \log \left( \max \left[ (1 - \phi) y_2, y_2 - z_2 \right] \right) dF_i(y_2) - v_x^A
\]

\[
= \log \left( \frac{y_1 + z_1}{y_1} \right) + \beta \int \log \left( \max \left[ 1 - \phi, \frac{y_2 - z_2}{y_2} \right] \right) dF_i(y_2)
\]

### 2.1.2 Market structure

The loan market is segmented by household type and characterized by frictions. All households of a given type \(x\) are located on the same island. \(H_x\) denotes the measure of type-x households. There is a large (unbounded) number of potential lenders who are risk-neutral, access funds at the risk free rate \(r\) and can offer loans to borrowers. A lender can pay cost \(k\) to enter a particular island and offer one loan. \(L_x\) denotes the number of lenders who enter the island with type-x households and \(\theta_x \equiv \frac{H_x}{L_x}\) denotes the household-lender ratio in a type-x island.

The interpretation of segmentation is that \(k\) represents the cost of identifying a particular household of type \(x\), to which a loan can be targeted. A lender who has not paid this cost cannot target the loan to an appropriate household and will be swamped by (unmodeled) bad borrowers (frauds).

Competition among lenders is imperfect and modeled as in competitive search. First, each lender posts a loan \((z_1, z_2)\) and \(L_x(z_1, z_2)\) denotes the measure of lenders offering that loan. Second, each household observes all posted loans and decides which loan to apply for, where \(H_x(z_1, z_2)\) denotes the measure of households applying for loan \((z_1, z_2)\)\(^4\). The resulting household-lender ratio at each loan is given by \(\theta_x(z_1, z_2) = \frac{H_x(z_1, z_2)}{L_x(z_1, z_2)}\).

The probability that a lender succeeds in making a loan when the household-lender ratio

\(^4\)Consistency requires that \(L_x(z_1, z_2) \leq L_x\) and \(H_x(z_1, z_2) \leq H_x\).
is \( \theta \) is given by \( \alpha_L(\theta) \); the probability that a household receives a loan at such a contract is given by \( \alpha_H(\theta) \). We assume that \( \alpha_L(\theta) \) is increasing and concave in \( \theta \) and \( \alpha_H(\theta) \) is decreasing and convex and \( \alpha_L(\theta) = \theta \alpha_H(\theta) \). We denote the elasticity of \( \alpha_L(\theta) \) with respect to \( \theta \) by \( \epsilon_L(\theta) \) and assume that \( \epsilon_L(\theta) < 0 \), \( \epsilon_L(0) = 1 \) and \( \lim_{\theta \to \infty} \epsilon_L(\theta) = 0 \).

The household-lender ratio \( \theta_x(z_1, z_2) \) at each loan is determined by an indifference conditions for households. The expected value of household from applying to any loan is equal to their “market utility”, which is an equilibrium object. This implies that a household has lower probability of receiving a more desirable loan. Specifically the following condition holds for any loan \((z_1, z_2)\) that gives positive value to households:

\[
\alpha_H(\theta_x(z_1, z_2)) v_x(z_1, z_2) = U_x
\] (1)

\( U_x \) denotes the market utility of a type-\( x \) household.

The lender chooses the loan to offer \((z_1, z_2)\) to maximize expected profits

\[
\max_{z_1, z_2} \Pi_x(z_1, z_2) = \alpha_L(\theta(z_1, z_2)) \pi_x(z_1, z_2)
\] (2)

subject to (1).

The free entry condition requires that the expected profits of a lender who enters the type-\( x \) household are equal to the entry cost:

\[
\max_{z_1, z_2} \Pi_x(z_1, z_2) = k
\] (3)

2.1.3 Equilibrium and results

We are ready to define the equilibrium.

**Definition 1.** An equilibrium in the market for type-\( x \) households is a contract \( \{z^*_1, z^*_2\} \) and household-lender ratio \( \{\theta^*_x\} \) that solve (2) and (3).

\(^5\)A household remains in autarky if it does not receive a loan. See below.

\(^6\)See Galenianos and Kircher (2012) for the game-theoretic foundations of market utility.
The variables above suffice to define other outcomes such as the gross interest rate \((GIR_x)\), the default rate \((DR_x)\) and the profit rate \((PR_x)\):

\[
GIR_x = \frac{z_2^*}{z_1^*}
\]
\[
DR_x = \frac{z_2^* - y}{\bar{y} - y}
\]
\[
PR_x = \frac{\pi_x(z_1^*, z_2^*)}{z_1^*}
= (1 - DR_x) \frac{GIR_x}{1 + r} - 1
\]

Notice that the profit rate is the product of the default risk with the spread between the interest rate faced the household and the risk-free rate. In a model with a competitive pricing of loans, profit rates are constant across households and the spread exactly compensates for default risk. This condition need not hold in our model.

The main result is stated in the following proposition and proved in the next sections.

**Proposition 1.** We have:

1. For given entry cost \(k\) and household type \(x\):
   - An equilibrium exists and it is unique.
   - If \(L_x > 0\), the island is open and households’ value is above autarky.
   - If \(L_x = 0\), the island is closed and households receive their autarky value.

2. The effects of reducing the entry cost are:
   - Islands previously open experience greater lender entry which leads to bigger loans, lower interest rates and lower profit margins.
   - Some islands previously closed experience lender entry and households gain access to credit.
2.2 Equilibrium characterization

We proceed to characterize the equilibrium in the following steps. We begin by showing that there is a unique bilaterally efficient loan \((z_1, z_2)\) that maximizes lender profits subject to offering a particular level of value to households. Then we solve for the profit maximizing loan for a given level of lender entry and endogenize the measure of lenders.

A contract is bilaterally efficient if it maximizes lender profits subject to offering a particular level of value to households, \(v \geq 0\). Specifically, a bilaterally efficient contract solves:

\[
\max_{z_1, z_2} \pi_x(z_1, z_2) \\
\text{s.t. } v_x(z_1, z_2) \geq v
\]  

For future reference, denote by \(\pi^M_x\) the solution to this problem when \(v = 0\), i.e. the household only receives the value of autarky. This is the maximum level of profits that a lender can receive.

Bilateral efficiency is a necessary condition for any equilibrium. Proposition 2 provides a first characterization.

**Proposition 2.** We have:

1. For any household value \(v \geq 0\) there is a unique bilaterally efficient contract \((z_1(v), z_2(v))\).

2. Increasing the household value \(v\) reduces lender profits, increases the loan size to the household \((z'_1(v) > 0)\), reduces repayments to the lender \((z'_2(v) < 0)\) and reduces the interest rate.

**Proof.** See the appendix.

Figure 1 illustrates the result. The value of a loan \(v_x(z_1, z_2)\) increases in \(z_1\) and falls in \(z_2\) leading to upward sloping indifference curves (IC) which increase in the northwestern
direction. Profits fall in \( z_1 \) and increase in \( z_2 \) leading to upward slopping iso-profit curves (IP) which increase in the southeastern direction. The first part of the proposition proves that there is a unique tangency between the IC and IP curves. The second part of the proposition proves that the set of the tangency points has negative slope.

Figure 1: Bilaterally efficient contracts

Notice that the lowest indifference curve for the household delivers the value of autarky \((v_x(z_1, z_2) = 0)\), along which the household is equally well-off with the event of not receiving a loan \((z_1 = z_2 = 0)\). The corresponding iso-profit curve determines the lender’s maximum profits, which is the maximum possible from a type-\( x \) household. At the other extreme, the lowest profits that a lender might receive are equal to zero \((\pi_x = 0)\) which yield the
maximum value to the household and correspond to the competitive loan (recall that in a competitive loan market, as in Eaton and Gersovitz 1981 there is no entry cost or frictions in the meeting process).

We complete the characterization of equilibrium.

**Proposition 3.** In the market for type-$x$ households:

1. Given a household-lender ratio $\theta_x$, the lenders’ problem (2) has a unique solution.
2. Lenders’ expected profits are strictly decreasing in $\theta_x$.
3. If $\pi^M_x > k$, then there is positive lender entry. If $\pi^M_x \leq k$ then the island is closed.

*Proof.* See the appendix.

Figure 2 illustrates the result. The measure of households in a particular market is fixed and therefore the household-lender ratio depends on the extent of lender entry. Greater entry of lenders reduces the ratio and leads to lower profits and higher household value in equilibrium which corresponds to a move towards the competitive loan; and vice versa. If there are gains from trade, then the exact measure of lenders is determined at the point where lenders’ expected profits exactly equal $k$.

### 2.3 Outcomes of interest and comparative statics

We examine the effect of a reduction in the entry cost $k$. This is the way that we model technological progress (especially IT) that has facilitated the credit market in the past few decades.

**Proposition 4.** A reduction in $k$ has the following effects:

1. For a household type that already had access to credit:
   - There is greater lender entry.
2. For a household type that marginally had no access to credit:

- **There is lender entry and the household gains access to credit.**

These results are intuitive and they are direct outcomes of the previous section.
3 The full model

We present the full model which we calibrate to the data. The full model is a recursive version of the two-period model presented earlier, with the additional options of saving and defaulting.

A household enters the period with default status \( d_{-1} \in \{0, 1\} \) and state \( x = (b, y_{-1}) \). Status \( d_{-1} = 1 \) indicates that the household ended the previous period in default and \( d_{-1} = 0 \) indicates that it did not. Furthermore, \( b \) is the household’s debt level and \( y_{-1} \) is the previous period’s income.\(^7\) Let \( V_d(b, y_{-1}) \) denote the value of a household with state \( (b, y_{-1}) \) and default status \( d_{-1} = d \) at the beginning of the period.

A period has two stages. First, nature determines exogenous transitions, current period income level \( y \), utility cost of default \( \delta \), and expense shock \( e \) from some joint distribution \( G \). A household that starts the period in default \( (d_{-1} = 1) \) transits out of default \( (i.e. ~ d = 0) \) with probability \( \lambda \). The expense shock is non-discretionary \( (e.g. \text{ medical expenses}) \) and is directly added on the household’s debt from the previous period so that interim debt is given by \( \hat{b} = b + e, (e \geq 0) \).\(^8\) The utility cost of default and expense shocks are i.i.d. over time and are therefore not included in the household’s state.

Second, households make their decisions. Households that are not in default choose between defaulting (choice \( D \)), saving (choice \( S \)) or borrowing (choice \( B \)). Households in default choose how much to save.

When a household defaults the debt is erased \( (b' = 0) \), it transits to the default state \( (d = 1) \) which precludes borrowing or saving during the period \( (i.e. \text{ consumption equals current income}) \) and it incurs a one-off utility cost \( \delta \).\(^9\) We denote the value of defaulting by

\[^7\] In general, a household’s income consists of a persistent and a transitory \( \text{(i.i.d.)} \) component. The persistent component is the relevant part of the state as it has predictive power regarding future income realizations.

\[^8\] If a household is in default, then the income is reduced by the level of the expense shock. This assumption can be amended. Also, this expense does not directly affect utility. We don’t have to specify the seniority of debt versus expense shock because the punishment for default does not depend on the amount defaulted on: the household would never choose to default on one and not the other.

\[^9\] Potentially, his current income is also reduced by proportion \( \phi \), either as payment to creditors or as a pure utility cost. This is not included in the present version but could easily be added.
\[ V_D(0, y, \delta) = u(y) + \beta V_1(0, y) - \delta \]  

A household that chooses to save has access to a competitive saving market at the risk-free rate \( r \) with the restriction \( b' \leq 0 \). This is the only option available to households that begin the period in default and do not exogenously transit out of default. We denote the value of saving by \( V_d^S(\hat{b}, y) \):

\[
V_d^S(\hat{b}, y) = \max_{c, b'} \left( u(c) + \beta V_d(b', y) \right) \tag{6}
\]

\[ \text{s.t.} \quad b' \leq 0 \]

\[ \text{and} \quad y - \hat{b} = c - \frac{b'}{1 + r} \]

Note that the default status of a household choosing to save affects the decision only through the continuation value.

We denote the value of borrowing by \( V_B(\hat{b}, y) \). This process is described in detail below.

Summarizing the above, the value of a household entering period \( t \) is given by:

\[ V_0(b, y_{-1}) = \int_{y, e, \delta} \max \left[ V_D(0, y, \delta), V_0^S(b + e, y), V_B(b + e, y) \right] dG(y, e, \delta) \tag{7} \]

\[ V_1(b, y_{-1}) = \lambda \int_{y, e, \delta} \max \left[ V_D(0, y, \delta), V_0^S(b + e, y), V_B(b + e, y) \right] dG(y, e, \delta) \]

\[ + (1 - \lambda) \int_{y, e, \delta} V_1^S(b + e, y) dG(y, e, \delta) \tag{8} \]

### 3.1 The loan market

The loan market is very similar to the two-period version.

Each type of household resides in a separate island and the household’s type is described by its cash-in-hand \( y - b - e \) (equivalent to \( y_1 \) in the two-period model) and the expectations about next period’s income, summarized in \( y \) (equivalent to \( i \) in the two-period model).
Lenders pay cost $k$ to enter the island and compete for customers by posting one-period loans. A loan specifies the amount that the household receives in the current period $qb'$ (equivalent to $z_1$ in the two-period model) and the amount that the household repays in the following period $b'$ ($z_2$ in the two-period model). Households choose which loan to apply for and the probability of a match depends on the matching functions and the household-lender ratio at the chosen loan.

A difference with the two-period model is what happens when a household fails to match with a lender. In that case, a monopoly lender makes a take-it-or-leave-it loan offer to the household. The household’s outside option is the value of default or saving, whichever is highest (note that since the household preferred to borrow than save in the first place, becoming a saver means repaying debts and having $b' = 0$). Therefore, if the household’s search is unsuccessful, it receives the value of financial autarky (default or debt repayment) but without necessarily reducing debt levels to zero.

When searching for a loan, the household solves:

\[
V^B(\hat{b}, y) = \max_{(q,b')} \left( \alpha_B(\check{\theta}_x(q,b')) \left( u(y - \hat{b} + qb') + \beta V_0(b', y) \right) \right.
\]
\[
\left. + (1 - \alpha_B(\check{\theta}_x(q,b'))) \max \left[ V^S_0(\hat{b}, y), V^D(0, y, \delta) \right] \right)
\]

In equilibrium, all loans offer the same value. Therefore, the following indifference condition determines the household-lender ratio at any loan $(q,b')$ that attracts some households:

\[
V^B(\hat{b}, y) - \max \left[ V^S_0(\hat{b}, y), V^D(0, y, \delta) \right] = \alpha_B(\check{\theta}_x(q,b')) v_x(q,b') \tag{9}
\]

where the left-hand side is the household’s market utility and $v_x(q,b')$ is the household’s net value from receiving loan $(q,b')$:

\[
v_x(q,b') = u(y - \hat{b} + qb') + \beta V_0(b', y) - \max \left[ V^S_0(\hat{b}, y), V^D(0, y, \delta) \right]
\]
The lender’s problem is to solve:

$$\max \Pi_x(q, b') = \alpha_L(\theta_x(q, b')) \left( -qb' + \frac{b'\rho(b', y)}{1+r} \right)$$

subject to (9).

Free entry implies that

$$\max_{q,b'} \Pi_x(q, b') = k$$

The loan offered to a type-\(x\) household by the monopoly lender solves:

$$\max_{q,b'} \pi^M_x(q, b') = -qb' + \frac{\rho(b', y)b'}{1+r}$$ (10)

s.t. \(u(y - \hat{b} + qb') + \beta V_0(y, b') \geq \max[V^S_0(\hat{b}, y), V^D(0, y, \delta)]\]

Notice that if monopoly profits are below \(k\) then there is no lender entry in that market and the household can only choose between default and saving.

4 Parameterization and Results

4.1 Parameter Choices

Here, we detail the choices of our functional forms and parameter values. The utility function is CRRA, with relative risk aversion coefficient of \(\sigma = 2\):

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$ 

The matching function \(m(b, l)\) is

$$m(H, L) = LH(L^\gamma + H^\gamma)^{-1/\gamma}$$
which gives the matching probabilities as functions of the market tightness $\theta \equiv H/L$:

$$
\alpha_H = m(H, L)/H = (1 + \theta^\gamma)^{-1/\gamma}, \quad \alpha_L = m(H, L)/L = (\theta^{-\gamma} + 1)^{-1/\gamma}.
$$

In the benchmark parameterization, we set $\gamma = 0.4$.

The period in the model is one year, and accordingly, we set the discount factor $\beta = 0.94$, and the death hazard rate to 2% (so that households live for 50 years, i.e. enter the economy at 21 and live up to 71 in expectation). We set the real risk free rate to 1.44% which is the real yield on 3-month t-bills over the period 1989-2004. In the setup of the model, the household is excluded from borrowing and saving in the period of default and then from borrowing in the following period. After that, they regain access to credit markets. This is in consistency with evidence in Jagtiani and Li (2013) and Albanesi and Nosal (2015), who find that bankrupt households regain access to credit extremely fast. Finally, we set an upper bound for interest rates to be 100% annually.

We take the income process estimated for high-school graduates in Guvenen (2009), combine it with divorce and unexpected pregnancy shocks, as estimate in Livshits, MacGee, and Tertilt (2010b), and map it into a 4 state Markov chain using the method in Rouwenhorst (1995). Specifically, the process for effective income we are approximating is:

$$\ln(Y_t) = \ln(z_t) + \eta_t + \zeta_t$$

where the persistent part is

$$\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t$$

where the variance of the normal disturbance $\varepsilon$ is 0.011, and the transitory part $\eta$ is normal, i.i.d. with variance 0.052. The shock $\zeta$ is a Poisson shock with arrival rate of 1.74% annually, and the shock has one size equal to 26.4% of annual income (i.e. mean income excluding the $\zeta$ shock)(for details of estimation of these numbers, see Livshits, MacGee, and Tertilt (2010b)).

The remaining two parameters we need to determine is the utility cost of default, $\delta$ and
the screening cost $\kappa$. $\kappa$ is chosen so that the acquisition cost (cost of screening $\kappa$ divided by the probability of matching with a borrower) is equal to $50. Finally, we set $\delta$ to match the aggregate debt/income ratio of 8.2%, which is the debt to income ratio of the lowest income quintile in the 2004 SCF. This number corresponds to data estimates for the recent part of the sample. The resulting values of $\kappa$ and $\delta$ are 0.00025 and 1.03, respectively.

In table 1, we report aggregate statistics from the benchmark parameterization. The model generates high volume of risky debt, as shown by high default rate and charge off rate in the model. The bankruptcy statistic exceeds that in the data, as should be the case given that we use the income process of high school graduates in the model. The model comes close to matching the chargeoff rate, fraction of revolvers and the level of interest rates. In terms of the bankrupt sub-population, the model matches well the level of indebtedness of the average bankrupt, with debt/income ratio of 1.5.

Table 1: Benchmark parameterization and data.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Data</th>
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</thead>
<tbody>
<tr>
<td>Bankruptcy filing rate</td>
<td>6.8%</td>
<td>5.75%</td>
</tr>
<tr>
<td>Charge-off rate</td>
<td>4.4%</td>
<td>4.9%</td>
</tr>
<tr>
<td>Fraction with revolving balance</td>
<td>28%</td>
<td>26%</td>
</tr>
<tr>
<td>Interest rates</td>
<td>13.7%</td>
<td>12-16%</td>
</tr>
<tr>
<td>Debt/Income of bankrupts</td>
<td>1.5</td>
<td>0.8-1.5</td>
</tr>
</tbody>
</table>

Bankruptcy filing rate is the 2004 Chapter 7 filings relative to working age population (20–74 years old). Fraction of households with revolving balance is taken from the 2004 SCF for the lowest income quintile as the fraction of population who have a revolving balance on their credit cards. Interest rates are the range observed between 200-2004 as reported by the Federal Reserve Board. Debt to income ratio of bankrupts are taken from studies summarized in Sullivan, Warren, and Westbrook (2006).

To explore the role of imperfect competition for the predictions of the model, we consider a frictionless parameterization of our model by setting $\kappa = 0$. We then re-parameterize $\delta$ to match the same 8.2% aggregate debt to income target. The results are presented in table 2. The zero markup model has an extremely hard time generating high bankruptcy and chargeoff statistics, showcasing problems in matching high debt and high bankruptcy without expense shocks, as pointed out by Livshits, MacGee, and Tertilt (2010b), among
Table 2: Benchmark model, data and the frictionless model.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Data</th>
<th>Frictionless: no markups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcy filing rate</td>
<td>6.8%</td>
<td>5.75%</td>
<td>0.35</td>
</tr>
<tr>
<td>Charge-off rate</td>
<td>4.4%</td>
<td>4.9%</td>
<td>0.21%</td>
</tr>
<tr>
<td>Fraction with revolving balance</td>
<td>28%</td>
<td>26%</td>
<td>32%</td>
</tr>
<tr>
<td>Interest rates</td>
<td>13.7%</td>
<td>12-16%</td>
<td>2.66</td>
</tr>
<tr>
<td>Debt/Income of bankrupts</td>
<td>1.5</td>
<td>0.8-1.5</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Notes at the bottom of table 1 apply.
5 Conclusion

TBC

References


Jagtiani, Julapa, and Wenli Li, 2013, Credit access and credit performance after consumer bankruptcy filing: new evidence.


APPENDIX

A Solution Algorithm

0. The agent goes into the period with debt/savings \( b \). Then, \( \delta, y, \lambda \) and \( e \) are realized, giving rise to the state being: net debt \( \hat{b} \), income \( y, \delta \), and default state \( \{0, 1\} \).

BRANCH 1: IF YOU ARE NOT IN DEFAULT STATE

**Step 1.** Start with a guess for the functions \( \rho(b', y, \delta) \) (no hat), \( V_1(\hat{b}, y) \), \( V_0(\hat{b}, y, \delta) \).

Compute \( V^D \) from

\[
V^D(y, \delta) = u(y) + \beta E_y V_1(0, y') - \delta
\]

and \( V^{NC} \) from the maximization problem in the previous section. Denote by \( \Delta(\hat{b}, y, b') \) the difference:

\[
\Delta(\hat{b}, y, b') \equiv \max \{V^D(y, \delta), V^{NC}(\hat{b}, y)\} - \beta E_{y', e', \delta'} V_0(b' + e', y', \delta')
\]

Compute the value to the household of getting the Monopoly contract from

\[
\max_{q, b'} \left( -qb' + \frac{\rho(b', y, \delta)b'}{1 + r} \right)
\]

s.t. \( u(y - \hat{b} + qb') + \beta E_{y', e', \delta'} V_0(b' + e', y', \delta') \geq V^D(y, \delta) \)

and \( -qb' + \frac{\rho(b', y, \delta)b'}{1 + r} \geq k(x) \)

which is equivalent to

\[
\max_{q, b'} \left( -qb' + \frac{\rho(b', y, \delta)b'}{1 + r} \right)
\]

s.t. \( u(y - \hat{b} + qb') \geq \Delta(\hat{b}, y, b') \)

and \( -qb' + \frac{\rho(b', y, \delta)b'}{1 + r} \geq k(x) \)

We can further characterize the monopoly contract by using the participation constraint. The RHS does not depend on \( q \), and hence the optimal \( q(b', \hat{b}, y) \) is given by

\[
q(b', \hat{b}, y) = \arg \min \{u(y - \hat{b} + qb') \text{ s.t. } u(y - \hat{b} + qb') \geq \Delta(\hat{b}, y, b')\}
\]

Given that, the monopoly problem is choosing \( b' \) to solve for each \( \hat{b}, y \) (the reason it is not
very convenient to just use cash in hand is that \( y \) is potentially persistent

\[
\max_{q,b'} \left( -q(b',\hat{b},y)b' + \frac{\rho(b', y, \delta)b'}{1 + r} \right) \tag{13}
\]

\[
\text{s.t. } -q(b',\hat{b},y)b' + \frac{\rho(b', y, \delta)b'}{1 + r} \geq k(x)
\]

The solution to the problem (if it exists) gives

\[
V^M(\hat{b},y,\delta) = u(y - \hat{b} + qb') + \beta E_{y',e',\delta'} V_0(b' + e', y', \delta') - V^M(\hat{b},y).
\]

If the solution to the Monopoly problem doesn’t exist, then it means that default or positive savings is the optimal strategy for that state. The markets are not open, and from the construction of the \( \Delta \) above, we know if the household chooses default. Then, we update the \( P_D(b + e, y, \delta) \), and \( V_0 \) at that point. Otherwise, proceed to step 2.

**Step 2.** Here, the markets are open and we need to compute the value of search for the household. The first step is to compute profits from offering contract \( C = (b', q) \) to a household in state \( x \). That is, for all admissible \( b', q \) pairs, compute

\[
\pi(b', q|x) = -qb' + \frac{\rho(b', y, \delta)b'}{1 + r}.
\]

Then, focus only on the efficient contracts, to obtain profits as a function of delivered continuation value (if matched)

\[
\Pi(v|x) := \max_{b', q} \pi(b', q|x)
\]

\[
\text{s.t. } v = u(y - \hat{b} + qb') + \beta E_{y',e',\delta'} V_0(b' + e', y', \delta') - V^M(\hat{b},y).
\]

This step gives all the efficient pairs \((\pi, v)\) that deliver value \( v \) to the household in the most efficient way, generating profit \( \pi \).

**Step 3.** Solve the entry game. The point is to find a \( \theta_x \) and \( V^S \) such that at the solution \((v^*)\) to

\[
\max_v = \alpha_L(\theta(v))\Pi(v|x) \tag{14}
\]

\[
\text{s.t. } \alpha_B(\theta(v))v = V^S(\hat{b},y) - V^M(\hat{b},y,\delta)
\]

the free entry condition holds exactly when evaluated at \( \theta(v^*) \)

\[
\alpha_L(\theta(v^*))\Pi(v^*|x) = k(x) \tag{15}
\]

The way to implement it is as follows. First, guess a value for \( V^S(\hat{b},y) \)– this is just a real number. Then use the constraint in (14) to get \( \theta(v) \) (\( \alpha \) is a known function). Give that, just maximize (14), which gives \( (\theta^*, \Pi(v^*)) \). Plug that into (15). If it does not hold, adjust \( V^S \) accordingly. The solution of this procedure for all states \( x \) is the function \( V^S(\hat{b},y) \) and
θ(x). Given that, we can update \( V_0 \) according to
\[
V_0(\hat{b}, y, \delta) = \max \{ V_S(\hat{b}, y), V_D(y, \delta) \}.
\] (16)
which gives the new \( P_D \).

Step 4. Finally, we update the expected probability of repayment
\[
\rho(b', y, \delta) = E_{e', y', \delta'}(1 - P_D(b' + e', y', \delta'))
\] (17)

Branch 2: You are in default state
If in default state, solve
\[
V_1(\hat{b}, y) = \max_{c, b'} \left( u(c) + \beta E_{y', \delta'} W(b', y') \right)
\] (18)
s.t. \( b' \leq 0 \) and \( y - \hat{b} = c - \frac{b'}{1 + r} \)

where
\[
W(b', y') = \lambda \int_{y', e', \delta'} V_0(b' + e', y', \delta') dG + (1 - \lambda) \int_{y', e', \delta'} V_1(b' + e', y') dG
\]

Summary
We started with a guess for 3 functions: \( \rho, V_1, V_0 \). Our procedure gave us the solution to the model, and a new guess for these 3 functions, given by equations (17), (18) and (16).

B Proofs

Proof of Proposition 2. We first calculate the value of loan \((z_1, z_2)\) to a type-x household for the case where default is possible:
\[
v_x(z_1, z_2) = \log\left(\frac{y_1 + z_1}{y_1}\right) + \frac{\beta}{y_i - y} \left( \int_y^{y_i} \log(1 - \phi) dy_2 + \int_{\phi y_i}^{y_2} \log \left( \frac{y_2 - z_2}{y_2} \right) dy_2 \right)
\]
\[
= \log\left(\frac{y_1 + z_1}{y_1}\right) + \frac{\beta}{y_i - y} \left[ \left( \frac{z_2}{\phi} - y \right) \log(1 - \phi) + (y_2 \log(1 - \frac{z_2}{y_2}) - z_2 \log(y_2 - z_2)) \right] \]
\[
= \log\left(\frac{y_1 + z_1}{y_1}\right) + \frac{\beta}{y_i - y} \left[ y_2 \log \left( \frac{y_2 - z_2}{y_2} \right) - z_2 L(z_2) \right] \log(1 - \phi)
\]
where
\[
L(z_2) = \log\left( \frac{y_i}{z_2} - 1 \right) - \log\left( \frac{1}{\phi} - 1 \right)
\]
Note that \( L(0) = +\infty \), \( L'(z_2) = -\frac{y_i}{z_2(y_i - z_2)} < 0 \), \( L(\phi y_i) = 0 \) for \( z_2 \in (0, \phi y_i) \).
The value of the contract depends on \( z_1 \) and \( z_2 \) according to:

\[
\frac{\partial v(x)}{\partial z_1} = \frac{1}{y_1 + z_1} > 0
\]

\[
\frac{\partial v(x)}{\partial z_2} = \frac{\beta}{y_i - y} \left( - \frac{\bar{y}_i}{y_i - z_2} - L(z_2) + z_2 \frac{\bar{y}_i}{z_2(y_i - z_2)} \right)
\]

\[
= -\frac{\beta L(z_2)}{y_i - y} < 0
\]

The value of a contract increases in the first period payment and decreases in the second period repayment, as is intuitive. Therefore the indifference curves are decreasing in the \( z_1 - z_2 \) plane.

Profits depend on payments according to:

\[
\frac{\partial \pi(x)}{\partial z_1} = -1 < 0
\]

\[
\frac{\partial \pi(x)}{\partial z_2} = \frac{1 - \frac{2z_2}{\phi y_i}}{1 + r}
\]

The iso-profit curve is negatively sloped in the \( z_1 - z_2 \) plane, so long as \( z_2 < \frac{\phi y_i}{2} \). This condition will turn out to hold in all bilaterally efficient contracts (see below).

The Lagrangian of (4) is:

\[
L(z_1, z_2, \mu) = -z_1 + \frac{z_2(1 - \frac{z_2}{\phi y_i})}{1 + r} + \mu \left[ \log(y_1 + z_1) + \frac{\beta}{\bar{y}_i - y} \left( \bar{y}_i \log(\bar{y}_i - z_2) - z_2 L(z_2) \right) \right]
\]

\[
-\beta \left( 1 + \frac{r \log(y_1(1 - \phi))}{\bar{y}_i - y} \right) - v^A - v \]

Differentiating:

\[
\frac{\partial L}{\partial z_1} = -1 + \frac{\mu}{y_1 + z_1} \tag{19}
\]

\[
\frac{\partial L}{\partial z_2} = \frac{1 - \frac{z_2}{\phi y_i} - z_2 \frac{1}{\phi y_i}}{1 + r} + \mu \frac{\beta}{\bar{y}_i - y} \left( - \frac{\bar{y}_i}{y_i - z_2} - L(z_2) + z_2 \frac{\bar{y}_i}{z_2(y_i - z_2)} \right)
\]

\[
= \frac{1 - \frac{2z_2}{\phi y_i}}{1 + r} + \frac{\mu \beta}{\bar{y}_i - y} \left( \log(\bar{y}_i - z_2) - 1 - \log(1 - \phi) \right) \tag{20}
\]

Noting that bilateral efficiency requires \( \mu > 0 \) and combining equations (19) and (20) leads to:

\[
\frac{1 - \frac{2z_2}{\phi y_i}}{1 + r} \frac{\beta(y_1 + z_1)}{\bar{y}_i - y} \left( \log(\bar{y}_i - z_2) - 1 - \log(1 - \phi) \right) = 0
\]

Therefore a bilaterally efficient contract satisfies:

\[
Q(z_1, z_2) = 1 - \frac{2z_2}{\phi y_i} \frac{(y_1 + z_1) \beta(1 + r)L(z_2)}{y_i - y} = 0
\]
which implies $\phi y_i > 2z_2$ in equilibrium.

Notice that

\[
Q(z_1, 0) = -\infty \\
Q(z_1, \frac{\phi y_i}{2}) = -\frac{(y_1 + z_1)\beta(1 + r)}{\bar{y}_i - y} < 0 \\
\frac{\partial Q(z_1, z_2)}{\partial z_2} = -\frac{2}{\phi y_i} + \frac{y_1 + z_1}{z_2(\bar{y}_i - z_2)} \frac{\beta(1 + r)\bar{y}_i}{\bar{y}_i - y} < 0, \quad z_2 \in (0, \frac{\phi y_i}{2})
\]

Therefore $Q(z_1, z_2)$ is strictly concave in $z_2$, is negative at the minimum and maximum values of $z_2$ (when $z_2 = 0$ and when $z_2 = \frac{\phi y_i}{2}$) and therefore crosses zero twice, if a solution exists. The solution that maximizes profits is the one with the higher $z_2$ and the derivative of $Q(z_1, z_2)$ with respect to $z_2$ is negative at that solution.

Furthermore:

\[
\frac{\partial Q(z_1, z_2)}{\partial z_1} = -\frac{\beta(1 + r)L(z_2)}{\bar{y}_i - y} < 0
\]

This means that an increase in $z_1$ leads to an increase in the low solution and a decrease in the high solution. Therefore, across efficient contracts there is a negative relation between $z_1$ and $z_2$.

Notice that there is $\bar{z}_1$ such that $Q(z_1, z_2) < 0$ when $z_1 > \bar{z}_1$ for any $z_2 \geq 0$. Therefore, bilaterally efficient contracts have the feature that $z_1 \in (0, \bar{z}_1)$. Essentially, very large loans are never repaid because the agent always prefers to default. Similarly, $z_2 \in (\bar{z}_2, \bar{z}_2)$.

A bilaterally efficient contract that offers value $v$ to the agent is defined by the following equations:

\[
T_1(z_1, z_2, v) = 1 - \frac{2z_2}{\phi y_i} - \frac{(y_1 + z_1)\beta(1 + r)}{\bar{y}_i - y}\left(\log(\frac{\bar{y}_i}{z_2}) - 1 - \log(\frac{1}{\phi}) - 1\right) = 0 \\
T_2(z_1, z_2, v) = \log(y_1 + z_1) + \frac{\beta}{\bar{y}_i - y}\left(\log(\frac{\bar{y}_i - z_2}{z_2L(z_2)}) - \log(1 + \frac{y\log(y(1 - \phi))}{\bar{y}_i - y})\right) - vA - v = 0
\]

Let $z(v) = (z_1(v), z_2(v))$ so that:

\[
T(z, v) = \begin{pmatrix} T_1(z, v) \\ T_2(z, v) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\]

Using the implicit function theorem:

\[
D_z T \cdot z'(v) + D_v T = 0
\]

(21)

where the subscript denotes the partial derivative. This leads to:

\[
z'(v) = -(D_z T)^{-1}(D_v T)
\]
The derivatives are given by:

\[
D_v T = \begin{pmatrix} D_v T_1 \\ D_v T_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}
\]

and

\[
D_z T = \begin{pmatrix} D_z T_1 & D_z T_1 \\ D_z T_2 & D_z T_2 \end{pmatrix}
\]

\[
\Rightarrow (D_z T)^{-1} = \frac{1}{(D_z T_1)(D_z T_2) - (D_z T_1)(D_z T_2)} \begin{pmatrix} D_z T_2 & -D_z T_1 \\ -D_z T_2 & D_z T_1 \end{pmatrix}
\]

Therefore:

\[
z'_1(v) = -\frac{D_z T_1}{\Delta}
\]

\[
z'_2(v) = \frac{D_z T_1}{\Delta}
\]

where

\[
\Delta = (D_z T_1)(D_z T_2) - (D_z T_1)(D_z T_2)
\]

Furthermore:

\[
D_z T_1 = -\beta(1 + r) \frac{y_i}{y_i - y} L(z_2) < 0
\]

\[
D_z T_2 = -\frac{2}{\phi y_i} + \beta(1 + r) \frac{y_i + z_1}{y_i - y} \frac{y_i}{z_2(y_i - z_2)}
\]

\[
= -\frac{2}{\phi y_i} + \beta(1 + r) \frac{y_i + z_1}{y_i - y} \frac{y_i + z_1}{z_2(y_i - z_2)}
\]

\[
D_z T_2 = \frac{1}{y_1 + z_1} > 0
\]

\[
D_z T_2 = -\beta \frac{y_i}{y_i - y} L(z_2) < 0
\]

We showed earlier that \(D_z T_1(z_1, z_2, v) < 0\) at a bilaterally efficient contract. This implies that:

\[
\Delta > 0
\]

\[
z'_1(v) > 0
\]

\[
z'_2(v) < 0
\]

We have uniquely defined \(z_1(v)\) and \(z_2(v)\) for \(v \geq 0\).
The profitability of a contract and its dependance on \( v \) is given by:

\[
\begin{align*}
\pi_x(v) &= -z_1(v) + \frac{z_2(v)(1 - \frac{z_2(v)}{\phi y_i})}{1 + r} \\
\pi'_x(v) &= -z'_1(v) + \frac{z'_2(v)(1 - \frac{z_2(v)}{\phi y_i}) + z_2(v)(- \frac{z'_2(v)}{\phi y_i})}{1 + r} \\
&= -z'_1(v) + \frac{z'_2(v)}{1 + r} < 0
\end{align*}
\]

Therefore, the contract’s value is \( v \in [0, \bar{v}] \) where \( \pi_x(\bar{v}) = 0 \).

Proof of Proposition 3: We first show that profits are a strictly concave function of the household’s value. This will prove that the lenders’ problem has a unique solution. We will then consider entry.

We have:

\[
\begin{align*}
\pi''_x(v) &= -z''_1(v) + \frac{2z_2(v)}{\phi y_i} - (z'_2(v))^2 \left( \frac{2}{1 + r} \phi y_i \right)
\end{align*}
\]

To find \( z''_1(v) \) and \( z''_2(v) \) we differentiate (21) with respect to \( v \):

\[
\begin{align*}
\left( D_z^2 \mathbf{T} z'(v) + D_zv \mathbf{T} \right) z'(v) + D_z \mathbf{T} z''(v) + D_zv \mathbf{T} z'(v) + D_v^2 \mathbf{T} &= 0 \quad (22)
\end{align*}
\]

where \( D_z^2 \mathbf{T} \) is a vector-valued bilinear form. Specifically,

\[
D_z^2 \mathbf{T} (z'(v), z'(v)) = \begin{pmatrix} \sum_{i=1}^{2} \sum_{j=1}^{2} D_{zi} D_{zj} T_1 z'_i(v) z'_j(v) \\ \sum_{i=1}^{2} \sum_{j=1}^{2} D_{zi} D_{zj} T_2 z'_i(v) z'_j(v) \end{pmatrix} \equiv \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}
\]

and further

\[
D_zv \mathbf{T} = \begin{pmatrix} D_{z1v} T_1 & D_{z2v} T_1 \\ D_{z1v} T_2 & D_{z2v} T_2 \end{pmatrix}
\]

\[
D_v^2 \mathbf{T} = \begin{pmatrix} D_v^2 T_1 \\ D_v^2 T_2 \end{pmatrix}
\]

Note that \( D_v^2 \mathbf{T} = 0 \) and \( D_zv \mathbf{T} = 0 \). Therefore:

\[
\begin{align*}
z''(v) &= -(D_z \mathbf{T})^{-1} (B_1, B_2)'
\end{align*}
\]

\[
\begin{align*}
&= -\frac{1}{\Delta} \begin{pmatrix} D_{z2} T_2 & -D_{z2} T_1 \\ -D_{z1} T_2 & D_{z1} T_1 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \\
&= \frac{1}{\Delta} \begin{pmatrix} (D_{z2} T_2) B_1 + (D_{z2} T_1) B_2 \\ (D_{z1} T_2) B_1 - (D_{z1} T_1) B_2 \end{pmatrix}
\end{align*}
\]
As a result:

\[ z''_1(v) = \frac{1}{\Delta} \left[ - (D_{z_2}T_2)B_1 + (D_{z_2}T_1)B_2 \right] \]

\[ z''_2(v) = \frac{1}{\Delta} \left[ (D_{z_1}T_2)B_1 - (D_{z_1}T_1)B_2 \right] \]

Therefore:

\[ \pi''_x(v) = \frac{1}{\Delta} \left[ (D_{z_2}T_2)B_1 - (D_{z_2}T_1)B_2 + ((D_{z_1}T_2)B_1 - (D_{z_1}T_1)B_2) \frac{1 - \frac{2z_2(v)}{\phi y_i}}{1 + r} \right] - \frac{2(z'_2(v))^2}{(1 + r)\phi y_i} \]

\[ = \frac{1}{\Delta} \left[ B_1 ((D_{z_2}T_2) + (D_{z_1}T_2) \frac{1 - \frac{2z_2(v)}{\phi y_i}}{1 + r}) - B_2 ((D_{z_2}T_1) + (D_{z_1}T_1) \frac{1 - \frac{2z_2(v)}{\phi y_i}}{1 + r}) \right] - \frac{2(z'_2(v))^2}{(1 + r)\phi y_i} \]

Notice that:

\[ (D_{z_2}T_2) + (D_{z_1}T_2) \frac{1 - \frac{2z_2(v)}{\phi y_i}}{1 + r} = - \frac{\beta L(z_2)}{\bar{y}_i - \bar{y}} + \frac{1}{y_1 + z_1} \frac{1 - \frac{2z_2(v)}{\phi y_i}}{1 + r} = 0 \]

when \( T_1(z_1, z_2) = 0 \).

As a result:

\[ \pi''_x(v) = \frac{1}{\Delta} \left[ - B_2 ((D_{z_2}T_1) + (D_{z_1}T_1) \frac{1 - \frac{2z_2(v)}{\phi y_i}}{1 + r}) \right] - \frac{2(z'_2(v))^2}{(1 + r)\phi y_i} \]

Furthermore:

\[ B_1 = (D_{z_1z_1}T_1)(z'_1(v))^2 + 2(D_{z_1z_2}T_1)(z'_1(v))(z'_2(v)) + (D_{z_2z_2}T_1)(z'_2(v))^2 \]

\[ = \frac{1}{\Delta^2} \left( 2(D_{z_1z_2}T_1)(-D_{z_2}T_1)(D_{z_1}T_1) + (D_{z_2z_2}T_1)(D_{z_1}T_1)^2 \right) \]

\[ B_2 = (D_{z_1z_1}T_2)(z'_1(v))^2 + 2(D_{z_1z_2}T_2)(z'_1(v))(z'_2(v)) + (D_{z_2z_2}T_2)(z'_2(v))^2 \]

\[ = \frac{1}{\Delta^2} \left( (D_{z_1z_1}T_2)(-D_{z_2}T_2)^2 + (D_{z_2z_2}T_2)(D_{z_1}T_1)^2 \right) \]

since \( D_{z_1z_1}T_1 = D_{z_1z_2}T_2 = 0 \). Recall that:

\[ z'_2(v) = \frac{D_{z_1}T_1}{\Delta} \]
We can rearrange:

\[
\pi''_x(v) = \frac{1}{\Delta^3} \left[ - \left( (D_{z_1 T_2})(-D_{z_2 T_1})^2 + (D_{z_2 z_2 T_2})(D_{z_1 T_1})^2 \right) \left( (D_{z_2 T_1}) + (D_{z_1 T_1}) \frac{1 - 2z_2(v)}{\phi y_i} \right) - \frac{2(D_{z_1 T_1})^2}{(1 + r)\phi y_i} \left( (D_{z_1 T_1})(D_{z_2 T_2}) - (D_{z_1 T_2})(D_{z_2 T_1}) \right) \right] \\
= \frac{1}{\Delta^3} \left[ - (D_{z_1 T_2})(-D_{z_2 T_1})^2 \left( (D_{z_2 T_1}) + (D_{z_1 T_1}) \frac{1 - 2z_2(v)}{\phi y_i} \right) - (D_{z_1 T_1})^2 \left( (D_{z_2 z_2 T_2}) + (D_{z_1 T_1}) \frac{1 - 2z_2(v)}{\phi y_i} + 2 \left( \frac{((D_{z_1 T_1})(D_{z_2 T_2}) - (D_{z_1 T_2})(D_{z_2 T_1}))}{(1 + r)\phi y_i} \right) \right) \right] \\
= \frac{1}{\Delta^3} \left[ - (D_{z_1 T_2})(-D_{z_2 T_1})^2 \left( (D_{z_1 T_1}) + (D_{z_1 T_1}) \frac{1 - 2z_2(v)}{\phi y_i} \right) - (D_{z_1 T_1})^2 \left( (D_{z_2 z_2 T_2}) + (D_{z_1 T_1}) \frac{1 - 2z_2(v)}{\phi y_i} + 2 \left( \frac{((D_{z_1 T_1})(D_{z_2 T_2}) - (D_{z_1 T_2})(D_{z_2 T_1}))}{(1 + r)\phi y_i} \right) \right) \right] \\
= \frac{1}{\Delta^3} \left[ - (D_{z_1 T_2})(-D_{z_2 T_1})^2 \left( (D_{z_1 T_1}) + (D_{z_1 T_1}) \frac{1 - 2z_2(v)}{\phi y_i} \right) - (D_{z_1 T_1})^2 \left( (D_{z_2 z_2 T_2}) + (D_{z_1 T_1}) \frac{1 - 2z_2(v)}{\phi y_i} + 2 \left( \frac{((D_{z_1 T_1})(D_{z_2 T_2}) - (D_{z_1 T_2})(D_{z_2 T_1}))}{(1 + r)\phi y_i} \right) \right) \right]
\]

Note that

\[D_{z_1 z_1 T_2} = - \frac{1}{(y_1 + z_1)^2} < 0\]

and recall that \(D_{z_2 T_1} < 0\) and \(D_{z_1 T_1} < 0\). Therefore to prove concavity of the profit function, it suffices to show that \(B_{11} < 0\) and \(B_{12} < 0\).

We have:

\[
B_{11} = (D_{z_2 z_2 T_2}) - \frac{2(D_{z_1 T_2})}{(1 + r)\phi y_i}
= \frac{\beta y_i}{z_2(y_i - z_2)(\bar{y}_i - y)} - \frac{2}{(1 + r)\phi y_i(y_1 + z_1)}
= \frac{1}{(1 + r)(y_1 + z_1)} \left( - \frac{2}{\phi y_i} + \frac{\beta(1 + r)\bar{y}_i}{\bar{y}_i - y} \frac{y_1 + z_1}{z_2(y_i - z_2)} \right)
= \frac{1}{(1 + r)(y_1 + z_1)} D_{z_2 T_1} < 0
\]
and:

\[ B_{12} = (D_{z_2}T_2)\left(1 - \frac{2z_2}{\phi y_i}\right) + \frac{2(D_{z_2}T_2)}{\phi y_i} \]

\[ = \frac{\beta \bar{y}_i}{z_2(y_i - z_2)(y_i - y)} \left(1 - \frac{2z_2}{\phi y_i}\right) - \frac{2\beta L(z_2)}{\phi y_i(y_i - y)} \]

\[ = \frac{\beta L(z_2)}{y_i - y} \left( - \frac{2}{\phi y_i} \right) + \frac{\bar{y}_i(1 - \frac{2z_2}{\phi y_i})}{L(z_2)z_2(y_i - z_2)} \]

\[ = \frac{\beta L(z_2)}{y_i - y} \left( - \frac{2}{\phi y_i} + \frac{\beta(1 + r)\bar{y}_i}{y_i - y} \right) \]

\[ = \frac{\beta L(z_2)}{y_i - y} D_{z_2}T_1 < 0 \]

where we used \( T_1(z_1, z_2) = 0 \).

**Proposition 5.** There is a unique measure of lenders that enter market \( x \). If that measure is positive, then all lenders offer the same contract to agents. The value of the contract is an increasing function of the measure of lenders that enter the market.

We reformulate the lenders' problem as one where they choose the value \( v \) to offer to households.

The expected profits of a lender offering value \( v \) to households are given by

\[ \Pi_x(v) = \alpha_L(\theta_x(v))\pi_x(v) \]

s.t. \( U_x = \alpha_B(\theta(v))v \)

where \( U_x \) is the borrower's market utility.

Combining the above, the lender solves:

\[ \max_\theta \Pi_x(\theta) = \alpha_L(\theta)\pi_x(\frac{\theta U_x}{\alpha_L(\theta)}) \]

The first order conditions are:

\[ \Pi_x'(\theta) = \alpha'_L(\theta)\pi_x(v) + \alpha_L(\theta)\pi'_x(v)U_x\frac{\alpha_L(\theta) - \theta\alpha'_L(\theta)}{(\alpha_L(\theta))^2} \]

\[ = \frac{\alpha'_L(\theta)\pi_x(v) + \pi'_x(v)\frac{1 - \theta\alpha'_L(\theta)}{\alpha_L(\theta)}}{\alpha_L(\theta)} \]

\[ = \frac{\alpha'_L(\theta)\pi_x(v) + \pi'_x(v)U_x(1 - \epsilon_L(\theta))}{\alpha_L(\theta)} \]

The second order conditions are:

\[ \Pi_x''(\theta) = \alpha''_L(\theta)\pi_x(v) + \alpha'_L(\theta)\pi'_x(v)\frac{1 - \epsilon_L(\theta)}{\alpha_L(\theta)} + \frac{\pi''_x(v)U_x(1 - \epsilon_L(\theta))^2}{\alpha_L(\theta)} < 0 \]

since \( \alpha'_L(\theta) > 0, \alpha''_L(\theta) < 0, \pi'_x(v) < 0 \) and \( \pi''_x(v) < 0 \). Therefore, the individual lender's problem has a unique solution, conditional on the action of every other lender which is summarized in the
borrowers’ market utility $U_x$. In equilibrium, all lenders offer the same contract and therefore $\Pi_x(\theta) = 0$ when $\theta = \theta_x$.

Borrowers’ market utility is defined so that this equilibrium condition holds. In other words, if $U_x$ is such that $\Pi_x'(\theta_x) \neq 0$, then $U_x$ needs to adjust. Noting that

$$\frac{\partial \Pi_x'(\theta)}{\partial U_x} = \frac{\alpha_L'(\theta) \pi_x'(v) \theta}{\alpha_L(\theta)} + \frac{\theta U_x (1 - \epsilon_L(\theta))}{\alpha_L(\theta)} + \frac{\pi_x'(v) (1 - \epsilon_L(\theta))}{U_x} < 0$$

proves that there is a unique $U_x$ that satisfies the equilibrium condition (also: when $U_x = 0$, we have $\Pi_x' > 0$ while $\Pi_x' < 0$ for $U_x$ large enough). Furthermore, if $\Pi_x'(\theta_x) > 0$, then $U_x$ must increase and vice versa.

If there are very few lenders on an island ($L_x \approx 0$) then they are almost certain to meet a household and can extract all the surplus. Therefore

$$\lim_{\theta_x \to \infty} \Pi_x = \pi_x^M$$

If the right-hand side is less than $k$, then there is no lender entry in that island. □