ON INCOME AND WEALTH TAXATION IN A LIFE-CYCLE MODEL WITH EXTENSIVE LABOUR SUPPLY*

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In a stationary life-cycle model with extensive labour supply, two forms of taxation are studied: non-linear income taxation and linear wealth taxation. In the life-cycle model, the social weights of the dynasties depend on their permanent incomes, not on the observed taxable current income. A tax on wealth then can complement income tax as a redistributive tool. The derivative of social welfare with respect to the wealth tax rate at the no-tax point is computed. It is positive whenever permanent income is positively correlated with aggregate life time savings. This result is illustrated on a numerical example.

One function of the modern state is to make more equal the lifetime welfare of individuals. Although private insurance may go a long way into smoothing out the hazards of existence, pre-existing differences between the members of the economy are not insurable and can only be reduced by government interventions, provided it has information and power to detect and correct these differences (Varian, 1980).

In second best environments, optimal taxation theory has developed a set of tools that allow to derive the properties of tax schemes given a specific information structure. A large part of the literature, however, is static and does not directly address the intertemporal side of the above problem. In the most popular intensive model, laid down by Mirrlees, the government observes income, the product of wage with hours worked, but has no direct information separately on the two components. Some dynamic features, for example, see Werning (2007), have been incorporated to the intensive model. Here a different path is followed, that of the extensive model, where the workers’ choice is discrete, to work or not to work.

This article considers a stationary model in continuous time with generations of consumer-workers reproducing identically. At any point in time, the typical agent has an instantaneous productivity, observable and taxable by the government if she works, and a privately known cost of working. She decides whether to work or not, comparing her after-tax income net of cost when working with the subsistence allowance paid to the unemployed. Given her intertemporal resources and perfect financial markets, she chooses a smooth consumption profile over her lifetime.

The article studies some of the tax arrangements that a utilitarian government might want to implement in this context. The attention is focused on simple schemes, close to what is observed in practice. The basic tool is supposed to be a time invariant non-linear income tax: this is a tax schedule that to any (current) before tax income associates a value of (current) disposable income. The government has no memory or does not use

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its knowledge of the income history of the tax payer. To design such a tax, the tax
authority only needs to know the joint distribution of agents’ types, productivities and
opportunity costs of work, which by assumption on the evolution of the economy is
identical in cross-sections or during the lifespan of a cohort. As a consequence, the
optimal tax schedule is derived from similar principles as in a static model. For a given
level of gross income, an important element in the computation is the social weight (or
the marginal utility of income) of the workers earning this income, as seen by the
government. This is a mathematical expectation that reflects the fact that the tax
authority does not observe the costs of working, and has to infer the truncated distri-
bution of these costs, knowing that the workers choose to participate in the labour
force. At the optimum, the workers pay (positive) taxes when their social weight is
smaller than average, while their work is subsidised (they face a negative participation
tax rate) if their social weight is larger than average. The extent of the tax or subsidy
depends on the value of the weight and on the elasticity of labour supply, the latter
being directly related to the distribution of the unobservable opportunity costs of work.
A major difference with the static model, however, is in the social weights of the agents,
which here depend on their lifetime permanent incomes, instead of the current
incomes. We describe a set of assumptions on the fundamentals of the economy that
lead to optimal tax schedules with similar shapes as those obtained in the static model
by a redistributive government. Nevertheless, compared with the static model, the
redistributive scope of the tax authority here is limited by the fact that its information,
current income, is a noisier signal of its ultimate objective, lifetime income. This
phenomenon also seems to be present in Gorry and Oberfield (2008) who study the
properties of the optimal tax schedule in a similar life-cycle model with an intensive
labour supply à la Mirrlees.

It is therefore natural to seek other information and tax tools that might comple-
ment income tax to facilitate redistribution. There is a large literature on the topic,
recently surveyed in Banks and Diamond (2010). It has been suggested and experi-
mented to base taxes on a smoothed income (the average of the past three years, say,
instead of only the current one). But this does not seem to work in practice (Vickrey,
1939). Recent works, Micheletto (2008) and Weinzierl (2008), have looked at the effect
of having taxes depend both on income and on age. Following a different track,
keeping in mind the textbook property of the life-cycle model that accumulated savings
serve to finance consumption during retirement, a tax on wealth is a natural instru-
ment. Note that the underlying motive is different from the mechanism put forward by
Saez (2002) and Spinnewijn (2009): these authors advocate a tax on wealth based on
the idea of heterogeneity in discount rates, the rich being more patient and therefore
saving more than the poor. Here, the consumption–income profiles of the life-cycle
model are the relevant underlying feature, not impatience. Indeed, as shown by Erosa
and Gervais (2002) in a non-stationary Ramsey framework, the celebrated Chamley–
Judd optimality of non-taxation of capital does not extend to the life-cycle model. In
the stationary framework studied here, one can easily compute the derivative of the
government social welfare function when financial markets are perfect: at the point of
zero tax on wealth, this derivative is equal to the opposite of the covariance between the
dynastic social weights and their total savings, summed up over their lifetime. If the
high productivity types have wage and consumption profiles who make them high

savers, wealth taxation is a valuable redistributive tool. An illustrative example shows that this may be the case in the presence of borrowing constraints, and that a channel for redistribution is the taxation of retirement accounts.

Capital taxation is a controversial and lively topic. The point made here is theoretical and it would be of interest, but difficult, to supplement it with a fully fledged calibration exercise, using some of the computation tools advocated by Judd and Su (2006). It would also be good to know whether the mechanism underlined here is at work in the empirical life-cycle models of İmrohoroğlu (1998) and Conesa et al. (2009).

Finally, a more ambitious theoretical undertaking would be to look for the full dynamic optimal tax schedule in the extensive labour supply model, using all the information potentially available to the government, replicating here the type of analysis that has been carried out for the intensive model à la Mirrlees, see, for example, Golosov et al. (2006) or Grochulski and Kocherlakota (2007). The government would keep track of the history of employment and incomes, and design an optimal tax scheme depending on age, current income and this whole history. This should be the subject of further research.

The article is organised as follows. The model and notations are set in the next section. Non-linear income taxation is studied in Section 3, whereas Section 4 examines whether, in addition, a linear tax on wealth may be desirable. Section 5 briefly discusses the scope for a consumption tax.

1. The Description of the Economy

Consider an economy in continuous time where all the agents have the same fixed length of life \( A \). Agents differ through a characteristic \( \alpha \) determined at birth. They have a concave increasing instantaneous utility function \( u(c, \alpha) \), where \( c \) is their non-negative consumption. At each date, labour supply \( \ell \) is either zero or one. At age \( a \), the typical agent has a productivity \( \omega_2(a) \) and a private cost of going to work \( \delta_2(a) \), measured in units of good. Neither labour nor the good is storable. There is no discounting and there are perfect financial markets with a zero interest rate.

The parameter \( \alpha \) is a way to identify the exogenous trajectory \( [\omega_2(a), \delta_2(a)], a \in [0, A] \). In the study of the non-linear income tax, this trajectory may be random, provided that \( \alpha \) identifies the life time distribution of \((\omega, \delta)\), under the assumption of perfect insurance markets. For simplicity, when turning to the wealth tax, the attention is restricted to the particular case where the trajectory \([\omega_2(a), \delta_2(a)], a \in [0, A] \) is deterministic and known from birth. In the absence of government intervention agent \( \alpha \) chooses the consumption and labour supply profile solution of the programme

\[
\begin{align*}
\max & \int_0^A u[c(a), \alpha] \, da \\
\text{s.t.} & \int_0^A \{c(a) - [\omega_2(a) - \delta_2(a)]\ell(a)\} \, da \leq 0. 
\end{align*}
\]

By concavity of the utility function, agents choose a constant consumption flow. They decide to work, \( \ell(a) = 1 \), whenever net production \( \omega_2(a) - \delta_2(a) \) is positive, not to work, \( \ell(a) = 0 \), whenever \( \omega_2(a) - \delta_2(a) \) is negative, so that at laissez faire
\[ c_x^* = \frac{1}{A} \int_{\omega_x(a) - \delta_x(a) \geq 0} [\omega_x(a) - \delta_x(a)] da. \]

The economy is stationary. At each instant a new cohort, with the same characteristics as all the previous cohorts, is born. There is an infinitely lived government with powers of taxation that wants to redistribute welfare between the various \( z \) members of a cohort. A fundamental information asymmetry limits the government actions. It does not know the type \( z \) of the agents and it never observes the cost of going to work \( \delta \). But it sees the incomes of the workers, equal to their productivities \( \omega \) under suitable incentive conditions. In this setup, this study focuses on two tax instruments. First, taxation of current gross income: the results of the static model, if of any relevance, should be useful in this stationary environment. The attention therefore here will be on the limits of myopic income taxation to redistribute lifetime welfare, linked to the correlation between current and permanent income: a correlation of 1 between current and permanent income would make equivalent the static and the dynamic model, whereas a 0 correlation would make myopic income tax useless for intertemporal redistribution. A natural extension then is to suppose that the government can observe and tax savings. Conditions under which a positive linear taxation of savings is optimal are given.

2. Income Tax

Under the stationarity assumption, the distribution of agents’ characteristics \((\omega, \delta)\) for a given type \( z \) in a cross-section is identical to that faced by a new born \( z \) over her life time. To study the optimal tax schedule, it is therefore simpler to describe the economy through the distributions of \((\omega, \delta)\) conditional on \( z \), together with the marginal distribution of \( z \), rather than with the functions \([\omega_x(a), \delta_x(a)]\), \( a \) in \([0, A]\) introduced above. Indeed, we can altogether forget about age for the time being.

Although the economy is fully described with a probability distribution on the space \((\omega, \delta, z)\), in the computations some sections of this distribution will only be needed. The letter \( F \) will be used for the cdf of \( \delta \), so that

\[ F(\delta | \omega, z) \]

is the fraction of her lifetime, an agent of type \( z \) has work opportunity cost at most equal to \( \delta \) when her productivity is equal to \( \omega \). Similarly, \( f(\delta | \omega, z) \) is the associated pdf. \( G \)

The letter \( G \) is associated with distributions of productivity \( \omega \). The quantity

\[ G(\omega | z) \]

is the fraction of her lifetime, an agent of type \( z \) has productivity smaller than or equal to \( \omega \). The marginal distribution of productivities will be denoted \( G(\omega) \). The letter \( g \) is used for the corresponding pdf’s. Similarly,

\[ H(z | \omega) \]

is the cdf of characteristic \( z \), given a productivity level \( \omega \): it reflects the information that the tax authority can infer on the value of \( z \) of someone of current income \( \omega \). The marginal distribution of \( z \) is \( H(z) \). The letter \( h \) is used to denote the associated pdf’s.
2.1. The Optimal Taxation Programme

The tax authority at any date is faced with a stationary distribution of productivities and work opportunity costs \((\omega, \delta)\). Its objective is to maximise the sum of the agents lifetime utilities.

Since it does not observe the \(\delta\)'s, and only knows the productivities of the workers, it announces a non-decreasing\(^1\) after-tax income schedule \(R(\cdot)\) and a subsistence income level \(s\) common to all those out of work.

Facing a time invariant tax subsidy schedule \([R(\cdot), s]\), the typical agent maximises her consumption flow which is given by her intertemporal budget constraint

\[
\epsilon = \int_\omega \int_0^{R(\omega) - s} [R(\omega) - \delta]f(\delta|\omega, z)g(\omega|z)d\delta d\omega + s \int_\omega \int_0^{1 - \ell(\omega, \delta)} [1 - \ell(\omega, \delta)]f(\delta|\omega, z)g(\omega|z)d\delta d\omega,
\]

with respect to her labour supply \(\ell(\omega, \delta)\). The optimal labour supply therefore is to work \((\ell = 1)\) whenever the financial incentive to work \(R(\omega) - s\) is larger than the work opportunity cost \(\delta\). The government intervention, setting a positive \(s\) and often announcing a value of \(R(\omega)\) smaller than \(laisses faire\ \omega\), reduces labour supply. The consumption of consumer \(z\), as a function of the tax schedule, is given by

\[
C(R, s; z) = \int_\omega \int_0^{R(\omega) - s} [R(\omega) - \delta]f(\delta|\omega, z)g(\omega|z)d\delta d\omega \\
+ s \int_\omega \int_0^{1 - \ell(\omega, \delta)} [1 - \ell(\omega, \delta)]f(\delta|\omega, z)g(\omega|z)d\delta d\omega. \tag{2}
\]

It is useful for further reference to compute the derivative of consumption both with respect to a change in \(R(\omega)\) in an interval \([\omega, \omega + d\omega]\) and to a change in \(s\):

\[
\frac{\partial C}{\partial R(\omega)} = F[R(\omega) - s|\omega, z]g(\omega|z) \\
\frac{\partial C}{\partial s} = \int_\omega 1 - F[R(\omega) - s|\omega, z]g(\omega|z)d\omega. \tag{3}
\]

The optimal taxation problem of the government is then to find the couple \([R(\cdot), s]\) that maximises

\[
\int_z u[C(R, s; z), z]dH(z),
\]

where lifetime consumption of type \(z\) is given by (2), subject to the budget constraint

\[
\int_z \int_\omega [\omega - R(\omega) + s]F[R(\omega) - s|\omega, z]g(\omega|z)d\omega dH(z) \geq s. \tag{4}
\]

A marginal increase of the consumption of a person of type \(z\) increases the social objective by \(u_z[C(R, s; z), z]\). This quantity, which we shall also note \(u_z(z)\) to alleviate notations, is the social weight of person \(z\).

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\(^1\) We suppose that once she has paid the fix cost of work, a worker of productivity \(\omega\) is indifferent between producing any before tax income less than \(\omega\), in the interval \([0, \omega]\). Incentive compatibility then obtains when the after-tax income schedule is a non-decreasing function of before tax income \(\omega\), see Choné and Laroque (2011) for a detailed argument.
2.2. First-Order Conditions

Let $\lambda$ be the Lagrange multiplier associated with the government budget constraint, which measures the marginal cost of public funds. The Lagrangian of the programme is

$$L(R, s) = \int_x u[C(R, s; x), x] \, dH(x)$$

$$+ \lambda \int_\omega \int_x [(\omega - R(\omega) + s)F[R(\omega) - s|\omega, x] - s]g(\omega|x) \, d\omega \, dH(x).$$  \hspace{1cm} (5)

A small change in income affecting equally everyone in the economy [d$R(\omega)$=ds for all $\omega$] must leave the Lagrangian unchanged at the margin, which using equation (2) yields the first-order condition

$$\int_x u_e(x) \, dH(x) = \lambda.$$  \hspace{1cm} (6)

The marginal cost of public funds is equal to the average of the social weights.

Denote $\partial L/\partial R(\omega)$ the change in the Lagrangian induced by a change $dR(\omega)$ in a small interval [$\omega, \omega + d\omega$], normalised by $dR(\omega)d\omega$. We have

$$\frac{\partial L}{\partial R(\omega)} \, d\omega = \int_x u_e \frac{\partial C}{\partial R(\omega)} h(x) d\omega dx$$

$$+ \lambda \int_x [(\omega - R(\omega) + s)F[R(\omega) - s|\omega, x] - F[R(\omega) - s|\omega, x]]g(\omega|x) \, d\omega h(x) \, dx.$$  

Using (3) and the conditional densities property $g(\omega|x)h(x) = h(x|\omega)g(\omega)$, this yields

$$\frac{\partial L}{\partial R(\omega)} = \int_x [u_e - \lambda F[R(\omega) - s|\omega, x]h(x|\omega)] \, dx g(\omega)$$

$$+ \lambda [(\omega - R(\omega) + s) \int_x f[R(\omega) - s|\omega, x]h(x|\omega) \, dx g(\omega)].$$  

It is useful to define the (average) social weight of the workers of productivity $\omega$, that is, the average of the marginal utility of income $u_e(x)$ on all the agents of productivity $\omega$ and opportunity costs of work smaller than $R(\omega) - s$. It is given by

$$p_e(R, s|\omega) = E[u_e(x)|\omega, \delta \leq R(\omega) - s] = \int_x u_e(x) h(x|\omega) \, dx \delta \leq R(\omega) - s, \omega$$

$$= \int_x u_e(x) F[R(\omega) - s|\omega, x]h(x|\omega) \, dx$$

$$\int_x F[R(\omega) - s|\omega, x]h(x|\omega) \, dx.$$  

Noting that $\int F(\delta|\omega, x)h(x|\omega) \, dx$ is simply the marginal $F(\delta|\omega)$ and similarly with the pdf $f$, the first-order condition can be rewritten as

$$\frac{1}{g(\omega)} \frac{\partial L}{\partial R(\omega)} = [p_e(R, s|\omega) - \lambda F[R(\omega) - s|\omega] + \lambda (\omega - R(\omega) + s) f[R(\omega) - s|\omega].$$  \hspace{1cm} (7)

At any point where $R(\omega)$ is strictly increasing, a small enough variation $dR$ is admissible without violating the monotonicity constraint bearing on the function $R$. At any such point, $\partial L/\partial R(\omega)$ is equal to zero along the optimal solution. This expression is identical to that of the static model (see Choné and Laroque, 2011). It is the sum of two terms. The first term is the direct effect associated with the income transfer which accrues to all the workers whose number is $F[R(\omega) - s(\omega)]$, has social value equal to $p_E$, but costs $k$ per unit of transfer. The second term is the distortionary effect. There are $f[R(\omega) - s(\omega)]$ new workers entering the labour market, on each of which the government earns $[\omega - R(\omega) + s]$, which is valued at the marginal cost of public fund $\lambda$. The crucial difference with the static model comes from the social weights, which are derived here from the underlying life-cycle model. The game is to see whether the assumptions made for the static model in Choné and Laroque (2011) on the social weights $p_E$ have natural counterparts in the life-cycle setup.

2.3. Optimal Participation Tax Rates

An individual of productivity $\omega$ has a net gain $R(\omega) - s$ from working, so that she faces a participation tax rate

$$\tau(\omega) = \frac{\omega - R(\omega) + s}{\omega}.$$ 

The elasticity of the labour supply $F(R-s(\omega))$ of the agents of productivity $\omega$ with respect to after-tax income $R$ is

$$\varepsilon_R = R \frac{f(R-s(\omega))}{F(R-s(\omega))}.$$ 

With these two definitions, (7) can be rewritten as in Choné and Laroque (2011)

$$\omega \tau(\omega) = \frac{R}{\varepsilon_R} \left[ 1 - \frac{p_E(R, s(\omega))}{\lambda} \right].$$ 

At a productivity level where the after-tax schedule is strictly increasing and there are both workers and non-workers $[0 < F(R - s(\omega) < 1]$, the financial incentive to work is larger (resp. smaller) than productivity and the participation tax rate is negative (resp. positive) whenever the social weight of the workers is larger (resp. smaller) than the marginal cost of public funds.

2.4. The Redistributive Case

The overall shape of the optimal tax schedule depends on the social weights of the employed agents $p_E(R, s(\omega))$. Without restrictions on these weights, (7) indicates that essentially anything can happen: the shape of the after-tax income is unrestricted.

However, a natural redistributive case, stemming from utilitarianism with concave utility indices, is one where the function $p_E$ is non-increasing both in $R$ and in $\omega$. Under

\footnote{In this article, the case of pooling where the function $R$ has flat portions are neglected for the sake of simplicity.}

these two assumptions, the after-tax schedule has a simple shape described in Chone and Laroque (2011), which is recalled below. First, a set of restrictive assumptions adapted to the overlapping generations configuration is presented under which the redistributive properties hold.

### 2.4.1. Deriving redistributive properties from the fundamentals

A first task is to pin down how consumption varies with $a$. A sufficient condition for consumption to be an increasing function of $a$ is that the distribution of $(\omega, -\delta)$ be first order stochastically increasing in the unidimensional parameter $x$. This is spelled out in Assumption 1 below. This is restrictive. When $a$ increases and people get richer, the whole distribution of their income over their lifetime stochastically increases. It is not difficult to imagine situations where education implies a low market productivity when young, to get high returns later: then the distributions of productivities of educated and non-educated persons would not be comparable according to the stochastic dominance criterion. Also the distribution of work opportunity costs stochastically decreases with $a$: the richer and more productive types are the ones who are the less work averse.

#### Assumption 1.

The distribution of $\delta$ conditional on $(\omega, x)$ is first order stochastically nonincreasing in $(\omega, x)$. The distribution of $\omega$ conditional on $a$ is first order stochastically increasing in $a$.

#### Lemma 1.

Under Assumption 1, for any non-decreasing after-tax income schedule $R(\cdot)$, consumption $C(R, s; x)$ is a non-decreasing function of $x$.

**Proof.** From (2),

$$C(R, s; x) = \int_\omega \int_\delta \max[R(\omega) - \delta, s] f(\delta|\omega, x) g(\omega|x) \, d\delta \, d\omega.$$  

Let

$$\tilde{A}(r, \omega, x) = \int_\delta \max[r - \delta, s] f(\delta|\omega, x) \, d\delta,$$

$$A(\omega, x) = \tilde{A}[R(\omega), \omega, x],$$

and

$$B(a, x) = \int_\omega A(\omega, a) g(\omega|x) \, d\omega.$$  

By definition $\tilde{A}$ is non-decreasing in $r$, and by stochastic dominance it is non-decreasing in $(\omega, x)$. Then, $A$ is non-decreasing in $(\omega, x)$ by the monotonicity of $R(\cdot)$. Finally, $B$ is non-decreasing in both its arguments, the first one by the monotonicity of $A$, the second by monotonicity of $A$ and first-order stochastic dominance. The result follows from the remark that $C(R, s; x) = B(x, x)$.

We now turn to the properties of the function \( p_E \), which describes the social weights of the workers as seen by the government. It relies on guessing the types \( z \) of the agents that work, that is, have an opportunity cost smaller than \( R-s \), at productivity \( \omega \). It is natural to favour redistribution if a larger income \( \omega \), given the same financial incentive to work \( R-s \), selects richer types, as assumed below.

**Assumption 2.** The distribution of \( z \) conditional on \([\omega, \delta \leq R-s]\) is first order stochastically increasing in \( \omega \).

**Lemma 2.** Under Assumption 2, the function \( p_E(R, s \mid \omega) \) is non-increasing in \( \omega \) provided that consumption is non-decreasing with \( z \).

**Proof.** This is a direct consequence of the formula
\[
p_E(R, s \mid \omega) = \int_x u_c(z) dH[x \mid \delta \leq R(\omega) - s, \omega],
\]
since the marginal utility of consumption \( u_c(z) \) decreases with \( z \).

Increasing the current income of the employed increases their permanent income, and therefore, lowers their social weights. But it also brings into the labour force some previously unemployed persons. The next assumption, which is similar to Assumption in Chone and Laroque (2011), warrants that the arrival of the newcomers does not prevent the function \( p_E \) to be decreasing in \( R \).

**Assumption 3.** The distribution of \( z \) conditional on \((\omega, \delta = R-s)\) first order stochastically dominates the distribution of \( z \) conditional on \((\omega, \delta \leq R-s)\).

**Lemma 3.** Under Assumption 3, the function \( p_E(R, s \mid \omega) \) is non-increasing in \( R \) provided that consumption is non-decreasing with \( z \).

**Proof.** We use the expression
\[
p_E(R, s \mid \omega) = \frac{\int_x u_c(z) F(R-s \mid \omega, z) h(z \mid \omega) dz}{\int_x F(R-s \mid \omega, z) h(z \mid \omega) dz}.
\]
Differentiating with respect to \( R \) gives
\[
\frac{d p_E}{dR} = \frac{\int_x u_c(z) (dC/dR) F(R-s \mid \omega, z) h(z \mid \omega) dz + \int_x u_c(z) f(R-s \mid \omega, z) h(z \mid \omega) dz}{\int_x F(R-s \mid \omega, z) h(z \mid \omega) dz} - \frac{\int_x u_c(z) F(R-s \mid \omega, z) h(z \mid \omega) dz}{\int_x F(R-s \mid \omega, z) h(z \mid \omega) dz}.
\]
The first term is negative by concavity of the utility function. Multiplying through the second line of the formula by \( R(a)F(R/C0) \) shows that it has the same sign as \( Z_a u_c(a)h(a) \).

The desired result follows from Assumption 3 and the fact that \( u_c \) is decreasing in \( a \).

2.4.2. The shape of the after-tax income schedule under redistributive utilitarianism

Under the Assumptions of the previous section together with some regularity properties, the optimal after-tax schedules have a specific shape, illustrated on Figure 1. The figure represents the plan \((x, d)\) of agents’ characteristics. The curve \( R(x) \) delimits the set of workers, who lie below \([d < R(x) - s]\), from the unemployed who are above \([d > R(x) - s]\). \textit{Laissez faire} corresponds to the 45 degree line, \( R(x) - s = \omega \). Under Lemmas 2 and 3, the social weights of the workers decrease when one travels up along the 45 degree line. Ignoring the horizontal parts of the tax schedule, the first-order condition (7) holds, so that \( R(x) - s = \omega \). As a consequence, the after-tax income schedule \( R(x) \) crosses at most once, and from above, the \( \omega + s \) line. Two typical cases are shown in the Figure in which the support of the distribution of work opportunity costs has a positive lower bound \( \bar{\delta} \). The solid line represents a situation where the participation tax rate is negative for low productivities in the interval \([\underline{\delta}, \omega_m]\), the quantity \( p_E(\omega + s, s|\omega) \) being larger than \( \lambda \) in this

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\[ \int x u_c(x) h(x|\delta) = R - s, \omega) dx - \int x u_c(x) h(x|\delta \leq R - s, \omega) dx. \]

The desired result follows from Assumption 3 and the fact that \( u_c \) is decreasing in \( x \).

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\[ 3 \text{ This property holds in general, allowing for pooling; the argument follows the lines of the proof of Proposition 4 in Chone and Laroque (2011). Also the } R(x) \text{ curve may exhibit upward discontinuities in the region where it is above the } 45 \text{ degree line, a case not shown on Figure 1.} \]
interval. The dashed line has positive participation tax rates everywhere, and social
weights of the employees always smaller than the marginal cost of public funds. From
(6) which states that the overall average of the social weights is equal to the marginal
cost of public funds, this can only occur when the average social weight of the
unemployed agents is larger than \( \lambda \), the marginal cost of public funds.

2.5. Discussion and Possible Extensions

The above analysis shows that, by and large, taxation of income in the stationary life-cycle
model follows similar principles to that of the static model. The main difference comes
from the fact that current income in the static model is a direct measure of welfare while
here it is a noisy signal of the permanent income which the government wants to
redistribute. It is therefore likely that redistribution based on taxation of current income
is less efficient here, because of the reduced information contained in the signal.

There is no scope for improvement if, as postulated above, the only information
available to the government is the joint distribution of \((\omega, \delta, x)\). But better information,
together with a social willingness to condition the tax schedule on this information, will
enlarge the scope for redistribution.

A widely discussed possibility in the recent years is to condition taxes on age, (see,
e.g. Banks and Diamond, 2010; Blomquist and Micheletto, 2008, or Weinzierl, 2008).
Since age is an exogenous variable, under the assumption that the distributions of
characteristics \((\omega, \delta)\) conditional on \(x\) and age are given and known to the government,
the derivation of the optimal tax schedules by age is similar to what has been described
above, with a first-order condition (7) for each age, or age bracket.

The derivation of the properties of the optimal tax schedule has been facilitated by
the assumption of perfect financial markets and perfect insurance across the life cycle.
While the notations would be heavier and the computations more involved, a similar
first-order condition as equation (7) holds in an environment with imperfect insurance
and/or borrowing constraints. The complication is in figuring out the social weight
associated with a marginal increase in income: compared with the case studied here,
the marginal utility of income would vary over the life cycle instead of being constant,
being higher in periods where the credit constraint binds. Intuitively, it does not seem
to jeopardise the properties proved in Section 2.4.1, but this should be the subject of
future research.

The similarity of the static and life-cycle models noted above largely comes from a
property of the models, left implicit up to now, which is worth making explicit. The
determinants \((\omega, \delta)\) of the labour supply decision and of the associated income tax are
exogenous, independent of the government-chosen tax schedule. An agent cannot
modify her productivity profile when taxes become more progressive. Only consump-
tion and welfare depend on taxes. This completely ignores the role of taxes on human
capital accumulation, as recently studied for instance by Bohacek and Kapicka (2008).

3. The Taxation of Wealth

Taxing savings or wealth to redistribute welfare is natural provided that the agents with
the largest permanent incomes are also those with the largest savings. Contrary to
permanent income which only depends on the distribution of the present values of \((\omega, \delta)\) over the lifetime, savings is a function of the precise trajectory: indeed, for a given income distribution, the positive savings that obtains if income continuously decreases over the lifetime changes sign with increasing incomes. Taxation of wealth for a redistributive motive relies on a different type of information than current income taxation. I therefore go back to the formalism of the beginning of the article.

3.1. How Welfare Depends on the Wealth Tax

In the absence of a tax on wealth, with a zero interest rate, accumulated wealth or savings at age \(a\) is

\[
W_{2a} = \int_{t=0}^{a} \left\{ [R[\omega_x(t)] - \delta_x(t)] \ell(t) + s[1 - \ell(t)] - c(t) \right\} dt.
\]

Suppose now that saving bears a tax at constant rate \(s_k\) (this becomes a subsidy in case of borrowing, negative saving). The proceeds of the tax are distributed equally in a lump sum fashion, everyone in the economy receiving \(s_k\) at each instant. The formula becomes

\[
W_{2a} = \int_{t=0}^{a} \exp[-s_k(a - t)]\left\{ [R[\omega_x(t)] - \delta_x(t)] \ell(t) + s[1 - \ell(t)] + s_k - c(t) \right\} dt.
\]

The typical consumer chooses a consumption labour profile that maximises her lifetime utility

\[
\int_{0}^{\Lambda} u(c(a), z) da
\]

subject to the intertemporal budget constraint \(W_{2a} = 0\). With this time-separable formulation, labour supply is chosen so as to maximise income, and follows the rules of the previous section. In particular, it does not depend on wealth taxation: we can denote \(\ell_{zt} = \{R[\omega_x(t)] - \delta_x(t)\} \ell(t) + s[1 - \ell(t)]\) the income at age \(t\). In contrast, the consumption profile does depend on the wealth tax, that is, positive taxes induce agents to tilt consumption towards younger ages. The computation is reminiscent of that of the lifetime excess burden of an interest tax in Levhari and Sheshinski (1972).

Let \(C_{xa}(\tau_k, s_k)\) be the corresponding consumption function. The government optimal taxation programme consists in maximising

\[
U = \int_{x} \int_{a} u[C_{xa}(\tau_k, s_k), z] d\alpha dH(x)
\]

subject to the budget constraint

\[
A s_k - \tau_k \int_{x} \int_{a} W_{2a} da dH(x) = 0.
\]

It turns out that the derivative of the social objective at the no-tax point \(\tau_k = s_k = 0\) takes a very simple form. Note that at this point, consumption is constant over the life time, so that the marginal utility of consumption \(u_c(x)\) is independent of age. Then,
\[
\frac{dU}{dt_k} = -\text{cov} \left[ u_a(x), \int_0^A W_{za} \, da \right].
\] (8)

**Lemma 4.** With perfect financial markets, the derivative of social welfare with respect to the tax rate on wealth at the no-tax point is equal to the opposite of the covariance of the social weights with the lifetime base of a wealth tax for agents of different types.

**Proof.** We first make explicit the conditions associated with the individual behavior of the agents of type \( x \). To alleviate notations, let \( u_{za} \) denote the marginal utility of consumption of type \( x \) at age \( a \). The first-order conditions from consumer optimisation over the intertemporal budget constraint, marginal utility proportional to the tax factor, are

\[
u_{za} \exp[\tau_k(A - a)] = u_{z0} \exp(\tau_kA) = u_{zA}.
\]

Also, differentiating the intertemporal budget constraint \( W_{zA} = 0 \) with respect to \( \tau_k \) and \( s_k \) gives

\[
\int_{a=0}^A \left\{ -\exp[-\tau_k(A - a)] \frac{\partial C_{za}}{\partial \tau_k} - (A - a) \exp[-\tau_k(A - a)](i_{za} + s_k - c_a) \right\} \, da = 0 \quad \text{(9)}
\]

and

\[
\int_{a=0}^A \exp[-\tau_k(A - a)] \left[ \frac{\partial C_{za}}{\partial s_k} - 1 \right] \, da = 0. \quad \text{(10)}
\]

We first work on (9). Through an integration by parts:

\[
\int_{a=0}^A W_{za} \, da = (aW_{za})_0^A - \int_{a=0}^A a \exp[-\tau_k(A - a)](i_{za} + s_k - c_a) \, da,
\]

or

\[
\int_{a=0}^A W_{za} \, da = -\int_{0}^A a \exp[-\tau_k(A - a)](i_{za} + s_k - c_a) \, da.
\]

Therefore, using the fact that \( W_{zA} = 0 \), (9) can be rewritten as

\[
\int_{a=0}^A \exp[-\tau_k(A - a)] \frac{\partial C_{za}}{\partial \tau_k} \, da + \int_{a=0}^A W_{za} \, da = 0.
\]

Multiplying through by \( u_{zA} \) yields

\[
\int_{a=0}^A u_{za} \frac{\partial C_{za}}{\partial \tau_k} \, da + u_{zA} \int_{a=0}^A W_{za} \, da = 0. \quad \text{(11)}
\]

The same multiplication applied to (10) gives

\[
\int_{a=0}^A u_{za} \frac{\partial C_{za}}{\partial s_k} \, da = Au_{zA}. \quad \text{(12)}
\]

Using (11) and (12), the derivative with respect to the tax rate on wealth of the government objective
\[
\frac{dU}{d\tau_k} = \int_x \int_a u_{aa} \left( \frac{\partial C_{za}}{\partial \tau_k} + \frac{\partial C_{za}}{\partial s_k} \right) da \, dH(z),
\]
simplifies into
\[
\frac{dU}{d\tau_k} = -\int_x u_{Aa} \int_{a=0}^{A} W_{za} \, da \, dH(z) + A \int_x u_{Aa} \, dH(z) \frac{ds_k}{d\tau_k}.
\]

There remains to evaluate the derivative of government revenue with respect to the rate of tax on wealth, that is, \( \frac{ds_k}{d\tau_k} \). It is well defined at the no-tax point \( \tau_k = s_k = 0 \). Indeed, at this point, the government budget constraint yields through the implicit function theorem
\[
\frac{ds_k}{d\tau_k} = \frac{1}{A} \int_x \int_a W_{za} \, da \, dH(z).
\]
Substituting yields the desired result.

A similar, more complicated, formula holds when there are borrowing constraints.\(^4\)

### 3.2. A Numerical Illustration

To have a rough idea of the orders of magnitude involved in (8), I consider a stripped down example with two types of agents. I abstract from income taxes, and take as after-tax income profiles\(^5\) the deterministic profiles estimated for men on the US PSID, for the two categories, less educated (high school graduate or below) and more educated (above high school), by Meghir and Pistaferri (2004), private communication from Luigi Pistaferri. A quadratic fit gives a yearly income for ages between 20 and 65, for the less educated type
\[
\omega_l(a) = -2.80 + 0.19a - 0.0014a^2,
\]
and for the more educated
\[
\omega_h(a) = -4.24 + 0.26a - 0.0008a^2.
\]

After 65 years, I complete the profile with 15 years of retirement without labour income. Lifetime utility is the sum of the logarithms of consumption from age 20 to 80, and social welfare is the sum of the utilities of the two types.

With perfect financial markets, the aggregate savings of the less and more educated types are, respectively, 362 and 309. The difference is small, in the direction of a wealth subsidy (!), and the optimum tax rate is zero (see Figure 2). With borrowing

\(^4\) Then, the marginal utility of consumption \( u_c(z) \) typically changes during life, being constant only on the age intervals where the borrowing constraint does not bind. This does not allow to simplify the formula, as when marginal utility of consumption was constant over time:
\[
\frac{dU}{d\tau_k} = -\int_x \int_{a=0}^{A} u_{za} \, da \left[ W_{za} - \int_x \int_{a=0}^{A} W_{za} \, da \, dH(z) \right] da \, dH(z).
\]

\(^5\) As noted in Section 3.1, under time separable preferences, labour supply does not change when a wealth tax is introduced. This allows to implement this exercise.

constraints, they become, respectively, 588 and 830. The more educated segment of the population accumulates large savings for retirement, in comparison with the less educated. Taxing these savings, contrary to what is seen in developed countries where savings in view of retirement are often subsidised, is redistributive. The optimum tax rate on wealth is 0.9%, as shown on Figure 2.

Figure 3 shows the wage and consumption time path of the two types under borrowing constraints. Wages are the dashed lines, increasing with a parabolic shape below age 65, zero above 65. Consumption without tax on wealth follows the wage schedule until the borrowing constraint becomes lax, and is constant afterwards: it is represented by a line with alternating small and large dashes. Applying the tax of 0.9% without operating the lump sum transfers yields the thin solid line, increasing the age at which the borrowing constraint ceases binding and inducing a declining consumption level overtime after that. The lump sum transfers move this consumption path upwards, on the bold solid lines. Taxation increases the low educated log lifetime utility by 0.69, while that of the high education type barely decreases of 0.06, for a government budget which amounts to 4% of the aggregate wage bill.

4. On Taxation of Consumer Expenditures

The observation of both current income and the evolution of wealth allows the government to estimate consumer expenditure, $c + \delta \ell$, through the budget identity. This makes it possible to substitute taxes on income and wealth with a tax on consumers expenditures,\(^6\) as advocated in the famous Meade (1978) report. Of course, there is a

\(^6\) Richard Blundell urged me to investigate this question.

twist with respect to the simplest interpretations of Meade: the government does not observe consumption, but only expenditure, which prevents it to attain the first best.

To setup the optimal expenditure tax problem, first consider the situation of a general non-linear tax scheme: to buy a quantity of good $c$, the consumer must pay $C[c]$, where $C$ is a non-negative increasing function. The typical consumer programme then is

$$\max \int_0^A u[c(a), \omega] d\omega,$$

$$\int_0^A \{ C[c(a) + \delta_z(a)] - \rho_z(a) \} d\omega \leq 0. \quad (3)$$

Contrary to what happened with the tax on wealth, we see that the tax on consumer expenditures typically distorts both labour supply and the consumption path. Indeed, comparing the terms in the budget set for $\ell$ equal to 0 or 1, we see that the consumer is willing to work whenever

$$\omega - \delta \geq C[c + \delta] - C[c].$$

The consumer works whenever his income is larger than the sum required to finance the pecuniary cost of going to work, given the tax system. The consumer problem looks rather hard to solve in general for a non-linear tax function!

However, a linear tax function, say $C[c] = ac - b$, is easy to handle. The (after-tax) cost of going to work becomes $\omega \delta$, and the constant consumption level of consumer $z$ is given by

$$aC(z) = b + \int_0^{\omega(1+\alpha)} (\omega - \delta) f(\delta | \omega, z) g(\omega | z) d\delta d\omega.$$

The government then chooses the parameters $a$ and $b$ to maximise welfare

$$\int_z u[C(z), z] dH(z).$$

subject to the budget constraint

$$\int_x [aC(x) - b] \, dH(x) \geq \int_x C(x) \, dH(x).$$

5. Conclusion

This article has studied the shape of optimal taxation in an intertemporal overlapping generation model. The decision to supply labour is extensive, either to work full time or to stay unemployed. The participants in the economy compare the financial incentive with work provided by the government, that is, the difference between after-tax income at work and subsistence income when unemployed, with their (privately known) opportunity cost of work.

The tax authority has a distributive objective of lifetime welfare, which it can implement through a small number of memoryless tax instruments: a non-linear income tax based on current income, a linear tax on wealth, and a linear tax on consumption.

We give a set of assumptions under which the income tax may involve labour subsidies to low productivity types, while high incomes are taxed as in the static model of extensive labour supply. A major difference, however, in the intertemporal setup lies in the fact that the tax base, current income, is here a noisier signal of the objective, lifetime utility, than in the static framework.

There is therefore scope for a tax on wealth. This is in contrast with a large part of the literature which focuses on efficiency issues, and concludes in favour of a zero tax on capital. Here, the redistributive motive leads to a positive wealth tax whenever the individuals with the highest lifetime utility are also the ones who have the highest savings over their lives.

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References


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