Drivers of Labor Market Duality: Employment Protection and Screening Motives

Elisa Guglielminetti*
Jamil Nur†

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Abstract
This paper theoretically and empirically investigates the legal and economic origins and consequences of a dual labor market. Its segmented structure separates permanent contracts—open-ended, protected by lay-off costs—from temporary ones—with lower protection but narrower scope, convertible into permanent upon expiration. We develop a novel approach addressing the role played by restrictions on both contract types in the endogenous emergence of firms’ sorting. In our economy, legislation limits the use of fixed-term positions by imposing a cost inversely proportional to the firm’s permanent workforce. In facts, several countries set quotas on the use of temporary contracts. Firms may hire under temporary or permanent contracts. The former can be extended once, up to a regulated maximum length; the latter are subject to a probationary period. The productivity of the match is revealed at the expiration of the fixed-term or the trial period. At this moment, the firm decides either to keep the worker under a full permanent contract, or to let her leave at no cost. Thus, the first contract acts as a screening device. Our contribution is threefold: i) the cost structure endogenizes contract duality; ii) the use of temporary contracts is no longer exclusively driven by dismissal costs, but also by screening motives; iii) productivity differentials and the threshold for firing and conversions emerge endogenously. The model, calibrated on Italian data, finds that higher firing costs push firms to employ more fixed-term workers. The effects of a liberalization of temporary contracts depend on the protection on permanent ones: i) it decreases open-ended contracts, especially when firing costs are high; ii) increases vacancies and reduces unemployment, but only when firing costs are high; iii) lowers average productivity among the unemployed and increases conversions, less so in presence of high firing costs. We test these predictions on a large matched employer-employee Italian administrative dataset. We study conversions around a EU reform (2001) that dramatically reduces quantitative and qualitative constraints on the use of standard temporary contracts. Preliminary results, using firms’ size as a proxy for firing costs’ intensity, confirm our theoretical findings.

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*Sciences Po and La Sapienza University of Rome. E-mail: elisa.guglielminetti@sciencespo.fr
†Sciences Po. Corresponding Author. E-mail: jamil.nur@sciencespo.fr
1 Introduction

This paper develops a novel approach to study the endogenous emergence of a dual market. In Several European countries, the workforce is employed under a segmented structure, separating permanent (open-ended) from temporary (fixed-term) contracts. The latter are characterized by lower protection, as measured by the OECD’s EPL index\(^1\), but legislation often limits their scope. In the past 20 years however, the introduction of flexicurity policies has widely lifted such restrictions. For instance, between 1990 and 2013, the share of temporary jobs on total dependent employment has risen from 4 to 13% in Italy, from 9 to 15% in France, while averaging 31% in Spain.

Empirically, fixed-term contracts are often preferred to permanent ones. First, they do not entail any job-termination costs. Furthermore, when the contract length is particularly short, temporary workers do not count in the organic for trade union regulation. Second, they act as a buffer stock to adjust the workforce to changing economic conditions. Finally, they may constitute a screening device when the productivity of the match is unknown. This paper investigates the latter motive to develop and test a theory of firms learning. The search of full information, the strength of employment protection legislation, and the degree of job flexibility interact to shape the endogenous emergence of a dual labor market.

In our economy, firms hire under temporary or permanent contracts. Fixed-term contracts have limited duration; once their cumulative length expires, they are either converted or terminated at no cost. Legislation limits their use by imposing a cost inversely proportional to the firm’s permanent workforce. Indeed, several European and non-European countries impose quotas on the use of temporary contracts. Open-ended contracts are unlimited, but entail an initial probationary period. At the end of the trial period, firms can either confirm workers in their permanent positions or costlessly let them go. In the first case, firms will incur in a fixed cost in the event of a lay-

\(^1\)The OECD computes an aggregate measure of employment protection legislation (EPL), rated on a (0-6) scale. EPR represents the index for regular (open-ended) contracts, rating legislation on individual and collective dismissals. EPT is the corresponding index for standard fixed-term positions, and contracts stipulated by temporary work agencies. EPT measures valid cases for use, the maximum number of successive contracts and maximum cumulated duration.
The match’s productivity is unknown to the employer upon hiring. Revelation only occurs at the expiration of the fixed-term contract or of the probationary period. In this respect, the first contract form acts as a screening device, allowing us to derive a productivity threshold for conversions and lay-offs. We focus on three endogenous outcomes: the equilibrium conversion rates, and the distributions of new hires and of employed workers. The steady-state model allows us to overcome three limits in the literature: i) contract duality becomes an endogenous outcome of the model; ii) the use of temporary contracts is no longer exclusively driven by dismissal costs, but also by screening motives; iii) productivity differentials and the threshold for conversions emerge endogenously.

The Italian labor market is a suitable case of study. Sustained high rates of unemployment and negative business cycles spurred several reforms at the end of the '90s. The introduction of temporary work agencies (TWA), new contract forms, and less restrictions on standard fixed-term contracts considerably increased the fraction of temporary positions (Figure 1). This surge is mainly driven by the youngest age group (15-24), as their share more than doubled in the past 15 years. In this respect, a stronger reaction by young workers upholds the existence of a screening motive for temporary contracts.

We build on these facts to analyse the interaction between firms learning, and labor protection. We calibrate the model on Italian data and solve it to obtain a benchmark solution. We then study the policy implications of lower firing costs for permanent contracts, and lifted restriction on temporary ones. As expected, higher firing costs push firms to employ more fixed-term workers. However, the effects of a liberalization of temporary contracts depend on the protection on permanent ones: a) it decreases open-ended positions, especially when firing costs are high; b) increases vacancies and reduces unemployment, but only when firing costs are high; c) lowers average productivity among the unemployed and increases conversions, less so in presence of high firing costs.

The literature on labor market duality has mostly focused on the impact of temporary work liberalizations and firing costs’ reforms on unemployment. Most studies (see, for example, Blanchard and Landier (2002), Cahuc and Postel-Vinay (2002), Cahuc, Charlot and Malherbet (2012)) find an ambiguous net effect on job creation and destruction. Few contributions instead have studied
the use of temporary contracts as a screening device. Fixed-term jobs may act as a stepping stone or a dead-end (see, e.g., Casquel and Cunyat (2008), Guell and Petrongolo (2007)), reveal match productivity (see, e.g., Nagypal (2007)) and reduce the negative welfare effect of labor regulation (see, e.g., Faccini (2013)). However, the joint effect of learning and employment protection in the emergence of a two-tier market remains unexplored. The analysis of this relation represents our first contribution. A further difficulty in this literature pertains to the required conditions for the contemporaneous presence of permanent and temporary contracts. Indeed, when a job type presents relative cost advantages, firms should only employ workers by that contract form. A discriminatory element is then needed to guarantee the existence of a pooling equilibrium. Previous contributions relied on exogenous firms’ sorting. For instance, Berton and Garibaldi (2012) assume the existence of separate markets for permanent and fixed-term positions. Fialho (2014) assumes a lower TFP for temporary contracts. Our innovative approach assumes the existence of a non-linear signing cost for temporary contracts, inversely proportional to the firm’s permanent workforce. The constraint endogenizes firms’ decisions on the optimal ratio of fixed-term jobs. This is our second contribution.
Figure 1: Italy: Share of Temporary Employment by Age Groups

Source: Aggregate data from the Italian Labor Force Survey (LFS); OECD, Employment Protection Legislation (EPL) index; St. Louis Federal reserve, Recession Index.

Note: The OECD computes an aggregate measure of employment protection, rated on a (0-6) scale. EPR represents the index for regular (open-ended) contracts, rating legislation on individual and collective dismissals. EPT is the corresponding index for standard fixed-term positions, and contracts stipulated by temporary work agencies. EPT measures valid cases for use, the maximum number of successive contracts and maximum cumulated duration.

The LFS consider temporary workers: i) persons with a seasonable job; ii) persons engaged by an employment agency or business and hired out to a third party for the carrying out of a "work mission;" iii) persons with specific training contracts.

The vertical dotted lines mark labor reforms: a) Treu law (1997) introduced temporary work agencies (TWA) in Italy; b) EU directive (1999 but enforced in 2001) lifted qualitative and quantitative restrictions on standard temporary contracts; c) Biagi law further liberalized TWA jobs, and introduced new contract forms; d) Fornero law lowered firing costs for regular (permanent) contracts.
2 Model

2.1 The setting

The economy is populated by a continuum of firms indexed by \( j \) and a continuum of workers, indexed by \( i \). In a symmetric equilibrium, all agents adopt the same strategy; we can thus simplify the notation by omitting the indexes \( j \) and \( i \). Workers inelastically supply labor:

\[
\int_0^1 L(i) \, di = L = 1,
\]

where \( L \) stands for labor force. We abstract from population growth and we assume \( L \) constant.

\[
\int_0^1 N_t(i) \, di = N_t \text{ is aggregate employment at time } t \text{ and } u_t = 1 - N_t \text{ is aggregate unemployment.}
\]

In a stationary environment (as we assume in what follows), we can neglect the time index.

Firms produce using only labor, which is hired on a frictional labor market. We assume that the realized number of matches is the outcome of a Cobb-Douglas technology, which depends on the number of vacancies \((V_t)\) and searchers \((u_t)\):

\[
M_t(V_t, u_t) = \chi V_t^{\eta} u_t^{1-\eta}.
\]

The probability that a firm matches with a worker is \( q(\theta) = \frac{M_t(V_t, u_t)}{V_t} \). The job-seeker’s probability of being hired is \( f(\theta) = \frac{M_t(V_t, u_t)}{u_t} \). Labor market tightness is defined as \( \theta_t = \frac{V_t}{u_t} \). It is easy to show that \( q(\theta) \) is a decreasing function of \( \theta \), while \( f(\theta) \) is an increasing function of \( \theta \). Furthermore, there exists the following relationship:

\[
f(\theta) = \theta q(\theta).
\]

When a firm meets a worker, it can choose between two contractual arrangements: fixed-term (denoted by \( F \)) or permanent contracts (denoted by \( P \)). The fraction of hirings stipulated under permanent contracts is an endogenous outcome of the model \((\tilde{p})\). Short-term contracts can be converted into permanent but cannot be renewed. It follows that the (endogenous) proportion of permanent contracts in the working population \((p)\) can be higher than \(\tilde{p} \). To fix notation:

\[
N = N^F + N^P = (1 - p)N + pN
\]

\[
M = M^F + M^P = (1 - \tilde{p})M + \tilde{p}M
\]

Upon meeting, the match-specific productivity \( y_i \) is drawn from the continuous distribution \( g(y) \) defined over the support \([y_l, y_u]\). The productivity distribution \( g(y) \) is common knowledge, while the match-specific realization is not observed neither by firms nor by workers upon hiring. To
keep the model simple, we do not include the learning process explicitly\(^2\): we assume productivity becomes known after a given time the match has been created. We start by assuming constant productivity over the length of the productive relationship.

2.2 Contracts

We assume that the hiring market is not segmented. Firms and workers search for a match in the same market, irrespective of the contractual arrangement they prefer. Once the contact has been established, firms choose whether to offer a fixed-term or a permanent contract. We assume that both offers beat the worker’s outside option. The worker thus accepts whatever offer she receives.

We view this assumption as a realistic feature of a slack labor market, such as the current situation in countries like Italy and Spain. In economic downturns job seekers are less picky and they are likely to accept also job offers which provide them lower utility \(^3\). Moreover, we are interested in the endogenous emergence of a dual labor market for screening motives. As such, we prefer not to introduce other mechanisms that would drive selection into the two types of contracts, as in Berton and Garibaldi (2012).

**Fixed-term contracts**

There exists only one type of fixed-term (temporary) contract, whose duration is exogenously established and equal to \(T^F\). At the moment of stipulating a fixed-term contract, firms have to pay a cost \(c(p)\), where \(p = \frac{N^P}{N}\) is the proportion of permanent contracts in total employment at the firm. This cost represents the limitations in the use of temporary contracts set by the legislators of many European countries. We assume \(c'(p) < 0\): the higher the permanent workforce at the firm, the lower the cost of creating temporary jobs. This assumption is mainly motivated by the Italian legislation, which impose quantitative limits on the amount of temporary contracts as a percentage

\(^2\)For formal models of learning in the context of a dual labor market see Nagypal (2002) and Faccini (2014).

\(^3\)One way to think about this issue is the following. Suppose fixed-term contract provide the worker lower utility for any reason. Then, she would not accept a fixed-term job if she has high probability of finding a permanent one in the subsequent weeks. However, when unemployment is high the probability of finding a better match is significantly reduced, pushing the worker to accept a temporary job.
We assume $c(p) = e^{(1-p) - 1}$ with $p_{\text{min}} = 0.5$. For the sake of presentation we cap the cost function to 10 for $p < p_{\text{min}}$. $p_{\text{min}}$ stands for the minimum fraction of permanent workforce on total employment at the firm required by the law.

of the permanent workforce\(^4\). An example of the cost function we adopt is plotted in Figure 2. Other contributions in the literature postulate an exogenous threshold that prevents fixed-term contracts to crowd out permanent jobs. We prefer this formulation because we think to it as more realistic. Moreover, it allows us to study the endogenous emergence of a dual labor market.

Once the firm has stipulated a fixed-term contract, she must keep the worker until its natural end (i.e. until $T_F$), except if the worker voluntary quits. Quits happen at the exogenous rate $s_F$. Workers are paid wages $w^F$: because the match-specific productivity is not observed in the early stage of the productive relationship, all temporary jobs pay the same wage.

We postulate that the productivity is fully revealed at the time of expiration of the contracts.

\(^4\)In other legislations, temporary jobs need to fulfill criteria which can be harder to meet the lower the number of permanent jobs at the firm. In France, for instance, the use of temporary contracts (CDD, contrats à durée déterminée) is limited to one of these three circumstances: i) replacement of a permanent worker who is temporarily absent; ii) temporary increase in the economic activity; iii) seasonal job. It is not allowed to sign CDDs for jobs that guarantee the normal operation of the firm. This norm clearly limits the firm’s use of temporary contracts and more so the higher the share of temporary contracts already in place.
This moment come, the firm can decide either to keep the worker under a permanent contract (without probationary period: see below for further details) or to let her leave at no cost. Renewals are not allowed\(^5\).

**Permanent contracts**

Permanent contracts are open-ended and start with a probationary period during which firms can observe the worker and learn her productivity in the match. This is a common provision contained in the law which is generally neglected in other theoretical works on this topic. The length of trial period is exogenous and set to \( T^P \leq T^F \). As it is the case for fixed-term contracts, the match-specific productivity of a permanent job under trial is not observed an all workers receive the same wage \( w^{P_0} \). However, the productivity gets fully revealed by the end of the probationary period. At expiration, the firm chooses either to let the worker leave at no cost or continue the relationship under more costly firing rules. Furthermore, the wage is renegotiated to let it depend on the observed productivity: we indicate it as \( w^P(y) \).

Continuing permanent contracts (i.e. those that are converted either from fixed-term contracts or from probationary periods) are hit by i.i.d. random shocks at Poisson rate \( \mu \). Contrary to the rest of the literature, we assume that the shocks add to the intrinsic match quality \((y)\). This preserves the motive for screening and has interesting implications on the pattern of layoffs. If the new productivity of the match is too low, the firm may prefer to fire the worker; in this case, the law obliges her to pay a fixed cost \( K \). \( K \) represents the cost of legal procedures related to the firing of permanent workers; it can be interpreted as the model counterpart of the Employment Protection legislation on Regular contracts (EPR). As such, \( K \) is not paid to the worker but it’s a pure waste. As shown by Lazear (1990), firing costs can be entirely internalized by the wage bargaining process.

\(^5\)In many countries, only 2 or 3 renewals are allowed. We think to \( T^F \) as the average duration of subsequent fixed-term contracts up to the impossibility of renewal.
if they take the form of severance payments\textsuperscript{6}.

\section*{2.3 The firms}

In this section we characterize the value functions of the firm in the different states.

Firms post vacancies at unit cost $\kappa$. The vacancy is filled at rate $q(\theta)$, which negatively depends on the labor market tightness ($\theta$).

Denote with $E(J)$ the expected value of a productive match. The value of an unfilled vacancy is thus expressed by the following Bellman equation:

$$rE(J^V) = -\kappa + q(\theta)(E(J) - E(J^V))$$  \hfill (1)

The value of a converted permanent contract producing $y_i$ is given by:

$$rJ^P(y_i) = y_i - w^P(y_i) + s^P(E(J^V) - J^P(y_i))$$

$$+ \mu \int_{\xi}^{\bar{\xi}} \max \left[ J^P(y_i + \varepsilon) - J^P(y_i); -K - J^P(y_i) \right] dF(\varepsilon) \hfill (2)$$

Continuing permanent jobs are hit by additive i.i.d. random shocks at Poisson rate $\mu$. The shocks are drawn from c.d.f $F(\varepsilon)$ defined over the support $[\xi, \bar{\xi}]$. When a shock occurs, firms decide whether to keep the job in place at the new productivity or to lay off the worker by paying the firing cost $K$.

The value of a the probationary period of a new permanent contract is:

\textsuperscript{6}In many countries, EPR also establishes a compensation to be paid to the fired worker. It seems unlikely that firms are able to transfer all these costs to the worker: evidence in this sense for the Italian case is provided by Leonardi and Pica (2007). We will thus argue that the calibration of $K$ must indeed take into account law provisions regarding the payments received by the worker.
\[ r \mathbb{E}(J^{Pb}) = \left( 1 - e^{-(r+s^P)T^P} \right) \left( \bar{y} - w^{Pb} \right) + s^P \left[ \left( 1 - e^{-(r+s^P)T^P} \right) \mathbb{E}(J^V) - \mathbb{E}(J^{Pb}) \right] \\
+ (r + s^P)e^{-(r+s^P)T^P} \int_{y_u}^{y_u} \left[ \rho^{Pb,P}(y) J^{Pb}(y) + (1 - \rho^{Pb,P}(y)) \mathbb{E}(J^V) \right] dG(y) \quad (3) \]

where \( \bar{y} = \int_{y_u}^{y_u} y \, dG(y) \) is the expected productivity of the match. To accentuate the analogy with temporary contracts, we neglect the possibility of firing during the probationary period.

Notice the difference with eq. (2). The value of a new permanent can be defined only in expectation, because the match-specific productivity is still unknown. Because the trial period has a known limited duration \( (T^P) \) we need to multiply the net gain of the match by the first bracketed term. At rate \( s^P \) the match exogenously terminates and the vacancy becomes again unfilled. At the expiration of the trial period, the productivity of the match is revealed. The match is continued with endogenous probability \( \rho^{Pb,P}(y) \), thus generating the value \( J^{Pb}(y) \) to the firm, or severed. For future reference, we can define the aggregate conversion rate as \( \check{p} = \int_{y_u}^{y_u} \check{p}(y) dG(y) \).

Finally, the value of a fixed-term contract is:

\[ r \mathbb{E}(J^F) = \left( 1 - e^{-(r+s^F)T^F} \right) \left( \bar{y} - w^F \right) + s^F \left[ \left( 1 - e^{-(r+s^F)T^F} \right) \mathbb{E}(J^V) - \mathbb{E}(J^F) \right] \\
+ (r + s^F)e^{-(r+s^F)T^F} \int_{y_u}^{y_u} \left[ \rho^{F,P}(y) J^{P}(y) + (1 - \rho^{F,P}(y)) \mathbb{E}(J^V) \right] dG(y) \quad (4) \]

Eq. (3) and (4) are very similar. However, differences may arise in: i) workers' quit rates \((s^F, s^{Pb})\); ii) length of the contracts \((T^F, T^P)\); iii) wages \((w^F, w^{Pb})\). Moreover, equations (2)-(4) represent the values of the contracts to the firm once they become productive; at stipulation further costs imposed by the legislator influence the choice of the contract type.

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\(^7\)The finite horizon of probationary periods and temporary contracts causes non-stationarity in the value of each productive match stipulated under either of the two contracts. In fact, the value of the match depends on the remaining contractual spell. However, firms take decisions only at the moment of stipulation and are not concerned about subsequent changes in the value of the match until expiration. Equations (3) and (4) represent values at the moment of the creation of a new relationship.
2.4 The workers

Now we can introduce the workers’ value functions, equivalent to equations (1)-(4) in the previous section.

\[ r_U = b + \theta q(\theta) \left[ \hat{p}(W^F - U) + (1 - \hat{p})(W^P - U) \right] \quad (5) \]

\[ r_{WP}(y_i) = w^P(y_i) + s^P (U - W^P(y_i)) \]

\[ + \mu \int_\varepsilon^\infty \max \{W^P(y_i + \varepsilon) - W^P(y_i); U - W^P(y_i)\} dF(\varepsilon) \quad (6) \]

\[ r_{W^P_0} = (1 - e^{-(r + s^P)T^P})w^P_0 + s^P \left[ (1 - e^{-(r + s^P)T^P})U - W^P_0 \right] \]

\[ + (r + s^P)e^{-(r + s^P)T^P} \int_{y_0}^{y_u} \left[ \rho^{P_0,P}(y)W^P(y) + (1 - \rho^{P_0,P}(y))U \right] dG(y) \quad (7) \]

\[ r_{W^F} = (1 - e^{-(r + s^F)T^F})w^F + s^F \left[ (1 - e^{-(r + s^F)T^F})U - W^F \right] \]

\[ + (r + s^F)e^{-(r + s^F)T^F} \int_{y_0}^{y_u} \left[ \rho^{F,P}(y)W^P(y) + (1 - \rho^{F,P}(y))U \right] dG(y) \quad (8) \]

where \( b \) represents unemployment benefits.

2.5 Wage setting

As it is customary in search models, we assume that wages are Nash bargained between the firm and the worker. The splitting of the surplus thus obeys the following rule:

\[ (W^c - U) = \beta S^c \]

\[ J^c = (1 - \beta)S^c \quad c = F, P_0, P \quad (9) \]

where \( S^c = W^c - U + J^c \) is the joint surplus of the productive relationship stipulated under contract \( c \), \( \beta \) is the workers’ bargaining power and we have imposed the free entry condition. Combining (9)
with (1) and (5) allows us to express the value of unemployment as follows:

\[ rU = b + \theta \frac{\beta}{1 - \beta} [\kappa + q(\theta)(1 - \bar{p})c(p)] \]  

(10)

After some algebra we obtain:

\[ w^F(y_i) = \beta \left[ y_i + (r + s_R)K \right] + (1 - \beta) \left[ b + \theta \frac{\beta}{1 - \beta} [\kappa + q(\theta)(1 - \bar{p})c(p)] \right] \]  

(11)

\[ w^{P_0} = w^{F} = \beta \bar{y} + (1 - \beta) \left[ b + \theta \frac{\beta}{1 - \beta} [\kappa + q(\theta)(1 - \bar{p})c(p)] \right] \]  

(12)

Notice that eq. (11) implies that the wage is a positive function of the firing costs. As emphasized by other authors (see Bentolilla et al. (2010)), the cost imposed by the law in case of layoff can be used by the worker as a threat on the employer in the bargaining game. Equation (12) shows that the wage during the trial period and under a fixed-term contract are equal, depending only on the expected productivity of the match and the value of unemployment.

### 2.6 Job creation and conversions

Competition among firms drive the value of a vacancy to zero. Imposing this standard free entry condition on eq. (1) yields

\[ \mathbb{E}(J) = \frac{\kappa}{q(\theta)} \]  

(13)

Firms optimize over \( V, \bar{p} \) and \( \hat{p}(y) \). It can be shown that the optimality conditions read as \(^8\):

---

\(^8\)Explicit formulas for the derivatives are relegated to the Appendix.
\( \frac{K}{q(\theta)} = (1 - \tilde{p}) \left[ \mathbb{E}(J^F) - c(p) \right] + \tilde{p} \mathbb{E}(J^F_0) \)  

(14)

\( \mathbb{E}(J^F) - c(p) = \mathbb{E}(J^F_0) - (1 - \tilde{p}) c'(p) \frac{\partial p}{\partial \tilde{p}} \),  

\( 0 \leq \tilde{p} \leq 1 \)  

(15)

Eq. (14) is the job creating condition (JCC). Firms post vacancies up to the point where the real cost (lhs) equals the expected return of the productive match (rhs). Notice that the expected value of a productive match depends on the contractual arrangement the firm is willing to offer (\( \tilde{p} \)), which has to be determined jointly.

Eq. (15) says that firms choose \( \tilde{p} \) to equalize the marginal returns of fixed term and permanent contracts. An additional temporary contracts provides expected value \( \mathbb{E}(J^F) \) but the firm must bear the cost \( c(p) \) to set it. On the other side, a new permanent contract provides value \( \mathbb{E}(J^F_0) \). In addition, a new permanent contract marginally relaxes the EPL on temporary hirings (remember that \( c'(p) < 0 \), so that the last term on the rhs is positive).

Provided that \( \tilde{p} \in (0, 1) \), equation 16 implies:

\[
\rho^{F, P}(y_i) = \begin{cases} 
1 & \text{if } \left( e^{-sF_T} g(y_i) J^P(y_i) - c'(p) \frac{\partial p}{\partial \rho^{F, P}(y_i)} \right) > 0 \\
0 & \text{if } \left( e^{-sF_T} g(y_i) J^P(y_i) - c'(p) \frac{\partial p}{\partial \rho^{F, P}(y_i)} \right) < 0 \\
(0, 1) & \text{otherwise} 
\end{cases} 
\]

(16)

In words, firms are willing to convert temporary positions into permanent only if they provide a positive surplus. The value of a converted contract is augmented by the advantage of reducing...
the cost on hiring temporary workers. The choice of conversion can be expressed more intuitively in terms of productivity. There exists a productivity level $y_P$, such that firms only convert matches featuring higher productivity.

$$
\rho_{F,P}(y_i) = \begin{cases} 
1 & \text{if } y_i \geq y_P \\
0 & \text{if } y_i < y_P 
\end{cases} \quad y_i \in [y_l, y_u]
$$

where $y_P$ is such that

$$
e^{-s_F \tau_F} g(y_P) J_P(y_P) - c'(p) \frac{\partial p}{\partial \rho_{F,P}(y_P)} = 0.
$$

A similar procedure allows to determine $y_P^P$ such that

$$
e^{-s_P \tau_P} g(y_P^P) J_P(y_P^P) - (1 - \bar{p}) c'(p) \frac{\partial p}{\partial \rho_{P,P}(y_P^P)} = 0.
$$

Developing the derivatives, one can show that the thresholds of productivity for conversions from temporary or probationary contracts are equal ($y_P = y_P^P = y_P^P$). These results are summarized by the following lemma:

**Lemma 1.** There exists a productivity threshold $y_P$ such that any match of productivity $y_i$ stipulated under a temporary contract or in probationary period is converted to an open-ended contract iff $y_i > y_P$ and is terminated otherwise. The conversion threshold satisfies

$$
J_P(y_P) - c'(p) \frac{(1 - p)(1 - \bar{p}) q(\theta) V}{N(s_F + \tilde{s}(y_P^F))} = 0
$$

Figure 3 represents the solution to (18) as function of $\tilde{p}$. The figure shows that the higher is $\tilde{p}$, the more productive the match must be to be converted to permanent. The intuition for this results is that at low values of $\tilde{p}$ the cost of creating temporary jobs is higher. As a consequence, the firm tries to mitigate this cost by converting more contracts. Conversely, the conversion rate (measured on the right vertical axis) is declining in $\tilde{p}$.

2.7 Job destruction

In our model, there are three possibilities for a match to be severed: i) worker’s quit at exogenous separation rates $s_F$ and $s_P$ for fixed-term and permanent contracts, respectively; ii) expirations of temporary contracts and trial periods; ii) layoffs of permanent workers. Before defining an aggregate
Figure 3: Conversion threshold ($\tilde{y}^P$) and conversion rate ($\tilde{p}$) \footnote{$\tilde{y}^P$ is represented on the left axis, while $\tilde{p}$ is measured on the right vertical axis. The exercise is conducted by taking as given $\theta = 3$ and $\chi = 0.05$. The rest of the calibration is the same as in the benchmark solution.}

measure of job destruction, let us study the firing problem in continuing open-ended jobs. When a shock $\varepsilon$ occurs, the firm-worker pair compares the value of the ongoing relationship with their outside option, which is represented by the match surplus

$$S^P(y_i) = J^P(y_i) + K + W^P(y_i) - U \quad (19)$$

Call $\varepsilon^d$ the shock such that the firm-worker pair is indifferent between continuing to produce and breaking the match. Then, $\varepsilon^d$ is the solution to $S^P(y_i + \varepsilon^d) = 0$. Doing the computation leads to an implicit formula for the firing threshold:

$$\varepsilon^d + \mu \int_{\varepsilon^d}^{\varepsilon} S^P_f(y_i + \varepsilon)dF(\varepsilon) - rU + (r + s^P)K + y_i = 0 \quad (20)$$

It is apparent from eq. (20) that the firing threshold crucially depends on the intrinsic match quality and the firing costs. For what follows, it is convenient to highlight the dependence on $y_i$, thus writing $\varepsilon^d(y_i)$. We summarize these results in the following lemma:
Figure 4: Job destruction threshold and composite separation rate \(^a\)

\(^a\)The exercise is conducted by taking as given the following variables: \(\theta = 3, \chi = 0.05, \bar{p} = 0.5, y^P = 1\). The rest of the calibration is the same as in the benchmark solution.

**Lemma 2.** There exists an \(\varepsilon^d(y_i)\) such that \(S^P(y_i + \varepsilon^d(y_i)) = 0\). Any match of quality \(y_i\) hit by a shock \(\varepsilon\) is continued iff \(\varepsilon > \varepsilon^d(y_i)\) and severed otherwise. Moreover:

- \(\varepsilon^d < 0\)
- \(\frac{\partial \varepsilon^d}{\partial y_i} = - \left[ 1 + \frac{\mu(1-F(\varepsilon^d))}{r+s^P+\mu} \right] \)
- \(\frac{\partial \varepsilon^d}{\partial K} = -(r+s^P) \left[ 1 + \frac{\mu(1-F(\varepsilon^d))}{r+s^P+\mu} \right] \)

For future convenience, we can define the firing rate as \(\dot{s}(y_i) = \mu F(\varepsilon^d(y_i))\). This can be interpreted as the arrival rate of a shock sufficiently bad to destroy the match. By combining Lemma 2 with the monotonicity of the c.d.f. we can state

**Lemma 3.** Define the firing rate as: \(\dot{s}(y_i) = \mu F(\varepsilon^d(y_i))\). Then, the firing rate is a decreasing function of \(y_i\).

Notice that the firing decision does not take into account variations of \(c(p)\). As we showed in Section 2.6, firms manage the size of the permanent workforce \((p)\) through systematic choices on \(\bar{p}\) and the conversion rates \(\rho^{F,P}\) and \(\rho^{F_0,P}\), given the expected firing function.
We can now summarize all the separations occurring in a given period in a composite separation rate which depends on the productivity distribution (because the conversion and the firing rate vary with \( y \)) and on the share of hirings stipulated as temporary (\( \tilde{p} \)):

\[
\bar{s}(\tilde{p}) = \left[ \frac{(1 - \tilde{p})(1 - e^{-s_F^F})}{s_F^F} + \frac{\tilde{p}(1 - e^{-s_P^P})}{s_P^P} + (1 - \tilde{p})e^{-s_F^F} \int_{y_l}^{y_u} \frac{\rho_{F,F}(y)}{s^F + s(y)} dG(y) + \tilde{p}e^{-s_P^P} \int_{y_l}^{y_u} \frac{\rho_{P,P}(y)}{s^P + s(y)} dG(y) \right]^{-1}
\]  

(21)

Eq. (21) is derived from the flow-balance equations which are introduced in the following section.

We provide a graphical representation of Lemma 2 in Figure 4, left panel: the job destruction threshold is a negative function of match productivity and is negatively affected by the firing costs. The right panel plots the composite separation rate defined in eq. (21): the negative slope is given by the firing rate.

2.8 Flow-balance equations

Since we are interested in steady state equilibria, we require inflows and outflows to balance for each labor sub-market.

\[
(N^F) \quad s_F^F \int_{y_l}^{y_u} N^F(y)dy + q(\theta)V(1 - \tilde{p})e^{-s_F^F} = q(\theta)V(1 - \tilde{p})
\]

(22)

\[
(N^P) \quad s_P^P \int_{y_l}^{y_u} N^P(y)dy + q(\theta)V\tilde{p}e^{-s_P^P} = q(\theta)V\tilde{p}
\]

(23)

\[
(N^F(y_i)) \quad (s_F^F + \tilde{s}(y_i))N^F(y_i) = q(\theta)Vg(y_i) \left[ (1 - \tilde{p})e^{-s_F^F} \rho_{F,F}(y_i) + \tilde{p}e^{-s_P^P} \rho_{P,P}(y_i) \right]
\]

\[\forall y_i \in [y_l, y_u]\]

(24)

Equations 22-24 imply that inflows in each type of contract (rhs) equal outflows (lhs). The requirement is stricter for converted contracts, since entries and exits need to balance for each productivity level. This difference is motivated by the information structure of our setup, where match productivity is known only for continuing permanent contracts. In equations (22) and (23), outflows are represented by quits and expirations; inflows are new matches. In eq. (24) outflows are
represented by separations (either quits or fires); inflows are conversions from fixed-term contracts or from trial periods.

By combining the flow-balance equations, we get a modified version of the well-known Beveridge curve:

\[ u = \frac{\bar{s}(\rho)}{\bar{s}(\rho) + f(\theta)} \]  

(25)

where \( \bar{s} \) is defined in eq. (21).

2.9 Equilibrium

The results presented above can be collected in the following definition of stationary equilibrium.

Definition 1. An equilibrium is a triple of scalars \((\theta, \bar{p}, w^F = w^{P F})\) and a quadruple of functions \((w^P(y), \rho^{F,P}(y), \rho^{P F,P}(y), \bar{s}(y))\), such that

1. The flow-balance equations (22)-(24) holds.

2. The value of all newly created productive matches is constant over time. Formally: \( \dot{J}^c(y) = \dot{W}^c(y) = 0 \quad \forall \quad c = P_0, P, F; \quad y \in [y_l, y_u] \).

3. Free entry holds: \( J^V = 0 \).

4. Firms maximize their expected payoff.

5. Wages are established through Nash bargaining.

Point 2 of Definition 1 was already implicit in the definition of the Bellman equations provided above, which were assumed to be time-invariant. Points 3 and 4 imply that \( \bar{p}, \theta \) and \( \rho \) are the solutions to the optimality conditions (14)-(17). Finally, the last point implies that the equilibrium wages are given by equations (11)-(12).
Table 1: Benchmark calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>discount rate (annual)</td>
<td>0.20</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>vacancy posting cost</td>
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</tr>
<tr>
<td>$\chi$</td>
<td>matching efficiency</td>
<td>0.04</td>
</tr>
<tr>
<td>$\eta$</td>
<td>elasticity of the matching function w.r.t $V$</td>
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</tr>
<tr>
<td>$s^F, s^P$</td>
<td>separation rates</td>
<td>0.01</td>
</tr>
<tr>
<td>$T^F$</td>
<td>duration of temporary contracts (years)</td>
<td>1.5</td>
</tr>
<tr>
<td>$T^P$</td>
<td>duration of permanent contracts (years)</td>
<td>1</td>
</tr>
<tr>
<td>$c_0$</td>
<td>constant in $c(p)$</td>
<td>0.00</td>
</tr>
<tr>
<td>$p_{\min}$</td>
<td>min proportion of perm workforce</td>
<td>0.5</td>
</tr>
<tr>
<td>$y_l$</td>
<td>lower bound of the prod. distribution</td>
<td>0.5</td>
</tr>
<tr>
<td>$y_u$</td>
<td>upper bound of the prod. distribution</td>
<td>0.6</td>
</tr>
<tr>
<td>$K$</td>
<td>firing costs</td>
<td>6</td>
</tr>
</tbody>
</table>

2.10 Calibration and solution

3 Renewals

In this section we extend the model to allow for two important features of the labor market institutions, namely the possibility of renewal of a temporary match and the choice of the length of the fixed-term contract. We assume that firms that stipulate a new temporary contract choose its length ($T^F$); then they decide whether to convert it into a second temporary contract, into a permanent one or to let the worker leave at no cost. In case the firm opts for the renewal of a temporary contract, it chooses again its length ($T^F$); at expiration, it can convert the relationship into permanent or let the worker leave at no cost (like in the benchmark setting). Renewals are admissible only once and there is a maximum length of the cumulated fixed-term contracts that we denote with $\bar{T}^F$. For simplicity, we do not introduce explicitly a learning process; we rather assume that there is a given period $\tilde{t}$ from the start of the job which is required to learn the match quality. More details about the extended model can be found in Appendix B.1. The following propositions summarize the main implications of the extended model. The relative proofs are relegated to Appendix ??.
**Proposition 1.** The optimal length of the first temporary contract is equal to the (exogenous) time needed to learn the match-specific productivity:

\[(T^{F_1})^* = \tilde{t}\]

The intuition behind Proposition 1 is quite simple: firms use the first temporary contract as a screening device to learn about the quality of the match. Once they have obtained this information, they can exploit it by upgrading only those matches whose productivity turns out to be high enough and dismiss all the others. Since they can do so with a new fixed-term contract which is subject to the same institutional provisions as the first one, there is no reason to keep in place \(F_1\) after the screening period has expired.

**Proposition 2.** There exist productivity thresholds \(y_{F_1,F_2}, y_P\) and a productivity value \(\tilde{y}\) such that

a) If \(\tilde{y} < y_P < y_{F_1,F_2}\) no renewals of fixed-term contracts take place and only those matches with productivity \(\geq y_P\) are converted from \(F_1\) to \(P\).

b) If \(y_{F_1,F_2} < y_P < \tilde{y} > y_u\) there are no direct conversions from the first temporary contract to permanent. All matches of productivity \(y_i \geq y_{F_1,F_2}\) are offered a second temporary contract. Among these, those which overcome \(y_P\) are then offered a permanent contract.

c) If \(y_{F_1,F_2} < y_P < \tilde{y} < y_u\), both renewals and conversions from \(F_1\) to \(P\) take place. Moreover, some of the matches in \(F_2\) are converted to permanent, too.

Proposition 2 states that, depending on the parameterization and regarding the continuation of the first temporary contract, different equilibria may arise. In the first two, only one type of continuation, either renewal or conversion, emerges; in the third one both co-exist, instead.

**Proposition 3.** The optimal length of the second temporary contract (provided that firms find optimal to stipulate one) is equal to the (exogenous) maximum length allowed by the law:

\[(T^{F_2})^* = T^F - \tilde{t}\]
Proposition 3 is intuitive. If the firm has decided to stipulate a second temporary contract, it means that this arrangement has been preferred against the alternatives of dismissing the worker or offering her a permanent contract. Since the productivity of the match is invariant over time, there is no reason the firm would want to sever the match until it is forced to do so by the law provision.
References


A Derivatives

The optimality conditions (14)-(17) require the computation of several derivatives, which we report here.

\[ \frac{\partial p}{\partial \tilde{p}} = \frac{1}{N} \left( (1-p) \frac{\partial (N_{P_0} + N^P)}{\partial \tilde{p}} - p \frac{\partial N^F}{\partial \tilde{p}} \right) = \frac{q(\theta)V}{N} \left\{ (1-p) \left[ -e^{-sF_TF} \int_{y_0}^{y_1} \frac{e^{F_P(y)}}{s^{P+y(s)}(y)} dG(y) + e^{-sF_TF} \int_{y_0}^{y_1} \frac{e^{F_P(y)}}{s^{P+y(s)}(y)} dG(y) \right] \right\} \geq 0 \]

\[ \frac{\partial p}{\partial \rho_{P, P}(y_i)} = \frac{(1-p)q(\theta)V(1-\tilde{p})q(y_i)e^{-sF_TF}}{N(s^P + \bar{s}(y_i))} \geq 0 \]

\[ \frac{\partial p}{\partial \rho_{P_0, P}(y_i)} = \frac{(1-p)q(\theta)V\tilde{p}q(y_i)e^{-sF_TF}}{N(s^P + \bar{s}(y_i))} \geq 0 \]

Notice that all derivatives are \( \geq 0 \): a rise in either the proportion of new permanent or conversions unambiguously determine an increase in the proportion of permanent contracts on total employment. Notice that in case a) an increase in \( \tilde{p} \) implies an increased number of new permanent contracts but also less conversions from temporary to permanent. However, the amount of new permanents outweighs the loss in conversions.
B Renewals

B.1 Extended model

With respect to the benchmark model, the possibility of renewals modifies the continuation value of the first temporary contract (contract $F_1$) and introduces the renewed fixed-term job (contract $F_2$). Their respective values to the firm are as follows:

$$rE(J^{F_1}) = \left(1 - e^{-(r+s^F)T^F}\right) \left(y_1 - w^{F_1}\right) + s^F \left[ \left(1 - e^{-(r+s^F)T^F}\right) E(J^V) - E(J^{F_1}) \right]$$

$$+ (r + s^F)e^{-(r+s^F)T^F} \int_{y_i}^{y_u} \left[ \rho^{F_1,P}(y) J^{P}(y) + (1 - \rho^{F_1,P}(y))\rho^{F_1,F_2}(y) J^{F_2}(y) \right] dG(y) \quad (26)$$

$$rJ^{F_2}(y_i) = \left(1 - e^{-(r+s^F)T^F}\right) \left(y_i - w^{F}(y_i)\right) + s^F \left[ \left(1 - e^{-(r+s^F)T^F}\right) E(J^V) - J^{F_2}(y_i) \right]$$

$$+ (r + s^F)e^{-(r+s^F)T^F} \left[ \rho^{F_2,P}(y_i) J^{P}(y_i) + (1 - \rho^{F_2,P}(y_i))E(J^{V}) \right] \quad (27)$$

Consistently with the wage setting mechanism introduced in Section the wage in the contract $F_1$ is the same in any match and it is set considering the average productivity. The wage in contract $F_2$ does depend on the idiosyncratic match productivity instead. The possibilities of evolution of one contract into another one are more variegated and they are indicated as follows:

- $\rho^{F_1,P}(y_i) = 1$ if a match of productivity $y_i$ stipulated under contract $F_1$ is converted to permanent. It is 0 otherwise.

- $\rho^{F_2,P}(y_i) = 1$ if a match of productivity $y_i$ stipulated under contract $F_2$ is converted to permanent. It is 0 otherwise.

- $\rho^{F_1,F_2}(y_i) = 1$ if a match of productivity $y_i$ stipulated under contract $F_1$ is renewed and continues under contract $F_2$. It is 0 otherwise.
\( \rho_{P_{0}, P(y_{i})} = 1 \) if a match of productivity \( y_{i} \) in a trial period is confirmed and continues under a permanent contract covered by firing costs. It is 0 otherwise.

**Proof of Proposition 1**

The purpose of this section is to prove that the optimal length of the first temporary contract (denoted as \( (T^{F_{1}})^{*} \)) is equal to the period required to learn the match specific productivity (\( \tilde{t} \)). We articulate the proof into two steps.

**Step 1**: prove that \( (T^{F_{1}})^{*} \geq \tilde{t} \).

In case firms are not interested in converting the first contract, it is easy to show that they want to keep the match as much as possible, provided that its expected value is positive (which is always true, otherwise there would be no interest in posting the vacancy).

Conversely, suppose the first temporary contract is such that the firm finds convenient either to renew it or to convert it into permanent at expiration. If contract \( F_{1} \) lasts less than \( \tilde{t} \), the firm has no additional information about the match quality; on the contrary, when it lasts more than \( \tilde{t} \) positive discrimination is possible. We can thus compare the expected values of the second contract under these two scenarios.

Denote with \( E(J_{c}) \) the expected value of a second contract (where \( c \) stands either for \( F_{2} \) or for converted \( P \)) with known idiosyncratic match productivity and with \( E(\tilde{J}_{c}) \) its the expected value when \( y_{i} \) is not observed. The wages which are associated to these two situations are \( E(w^{c}) \) and \( \tilde{w}^{c} \), respectively. Knowing the match productivity, firms are able to promote to the next contract only the most productive workers: let assume that they find profitable to convert only those matches whose productivity is higher than \( y^{c} \). We can then compute:

\[
E(J_{c}) - E(\tilde{J}_{c}) = \frac{1 - e^{-(r + s^{c})T^{c}}}{r + s^{c}} \left[ \int_{y^{c}}^{y_{u}} (y - w^{c}(y))dG(y) - (\bar{y} - \tilde{w}^{c}) \right]
\]

where \( T^{c} \) is the length of the second contract and \( s^{c} \) is the associated total separation rate. Combining with eq. (12) we obtain
\[ E(J^c) - E(\tilde{J}^c) = \frac{(1 - \beta) \left( 1 - e^{-(r+s^c)T^c} \right)}{r + s^c} \left[ \int_{y_c}^{y_u} ydG(y) - \bar{y} \right] \geq 0 \]  

Eq. (29) shows that, for any length of the second contract, firms prefer a match with known match quality. Then, the firm that wants to engage in a new temporary relationship finds optimal to offer a first contract long enough to learn about the match quality.

Formally, \((T^{F_1})^* \geq \bar{t}\), Q.E.D.

**Step 2**: prove that \((T^{F_1})^* \leq \bar{t}\).

We need to consider the possible outcomes of the first contract:

a) Renewal to \(F_2\): in this case, we need to show that the firm wants to switch to the new contract as soon as it has the information on the match quality. Given that contracts \(F_1\) and \(F_2\) are identical under any other respect, the comparison is the same as in eq. (29). We have already shown that, indeed, firms want to select the more productive matches as soon as they have this information. At most, they are indifferent to switching when \(y^c \leq y_l\).

b) Conversion to \(P\): in this case, the formula (29) does not directly apply because the two contracts are different. However, if firms prefer to convert the contract to \(P\) rather than going through \(F_2\), it means that it is more profitable for them to do so. From the first point, we already know that \(F_2\) after screening it is always preferred to \(F_1\). Then, by transitivity, the conversion to \(P\) as soon as the match productivity becomes observed.

c) No extension of \(F_1\): those matches whose productivity is revealed to be lower than any threshold either for renewal or for conversion are terminated at \(T^{F_1}\). These are the cases in which the value of the match is negative (the low productivity does not compensate the wage) and therefore lower than the value of a new vacancy \((J^v = 0\) for the free-entry condition). It follows that the firm prefer to terminate the match as soon as possible and go back to the market.

In any of the three cases considered above, \((T^{F_1})^* \leq \bar{t}\), Q.E.D.
The combination of Step 1 and Step 2 prove that

\[(T^{F_1})^* = \tilde{t}\] 
Q.E.D.

Proof of Proposition 2

In the extended model there are four possibilities for a contract to be upgraded: i) from the first temporary contract to permanent, \(\rho^{F_1, P}\); ii) from the second temporary contract to permanent, \(\rho^{F_2, P}\); iii) from the trial period to continuing permanent, \(\rho^{P_0, P}\); iv) renewal of a fixed-term contract, \(\rho^{F_1, F_2}\). In the first three cases, the new contract is always a permanent one; therefore, the threshold for upgrading is common to all these situations and equal to \(y^P\), implicitly defined in eq. (18).

Now, denote with \(y^{F_1, F_2}\) the threshold for renewal of a fixed-term contract. \(y^{F_1, F_2}\) solves

\[J^{F_2}(y^{F_1, F_2}) = 0\]

Cases i) and iv) are mutually exclusive: either \(F_1\) is converted directly to permanent or is renewed. To assess which is the optimal choice, a simple comparison of the thresholds is not sufficient; indeed, the relative values of the two contracts changes with match quality because the firing rate is a decreasing function of \(y\). In other words, it may well be that \(y^{F_1, F_2} < y^P\), thus suggesting that firms prefer to renew a fixed-term contract rather than upgrading it to permanent; however, at higher productivity values the value provided by the permanent contract is augmented because of the lower firing rate and may thus be preferred to \(F_2\).

Formally, indicate with \(\tilde{y}\) the productivity value such that the firm is indifferent between switching from \(F_1\) to \(P\) or from \(F_1\) to \(F_2\). Then, \(\tilde{y}\) solves

\[J^{F_2}(\tilde{y}) = J^P(\tilde{y}) + \Omega(\tilde{y})\]  
(30)

where \(\Omega(\tilde{y}) = -c'(\tilde{p})\frac{\Omega - p}{(1 - \tilde{p})q^{(0)}\tilde{v}}\) is the marginal gain of relaxing \(c(p)\).

We can then contemplate three cases:
1) $\tilde{y} < y^P < y^{F_1,F_2}$. Then,

$$
\begin{align*}
\begin{cases}
\rho_{F_1,P}(y_i) = 0; & \rho_{F_1,F_2}(y_i) = 0; & \rho_{F_2,P}(y_i) = \text{ind.} & \forall y_i \in [y_1,y^P) \\
\rho_{F_1,P}(y_i) = 1; & \rho_{F_1,F_2}(y_i) = 1; & \rho_{F_2,P}(y_i) = 1 & \forall y_i \in [y^P,y_u]
\end{cases}
\end{align*}
$$

This corresponds to a scenario in which the firing costs are not extremely high (or the probability of paying them is very low) and they are more than compensated by the marginal gain of relaxing the cost of stipulating new temporary contracts. Therefore, no renewals are observed, because firms always prefer to directly offer a permanent contract to those workers whose productivity is higher than $y_P$.

2) $y^{F_1,F_2} < y^P < \tilde{y} > y_u$. Then,

$$
\begin{align*}
\begin{cases}
\rho_{F_1,P}(y_i) = 0; & \rho_{F_1,F_2}(y_i) = 0; & \rho_{F_2,P}(y_i) = \text{ind.} & \forall y_i \in [y_1,y^{F_1,F_2}) \\
\rho_{F_1,P}(y_i) = 0; & \rho_{F_1,F_2}(y_i) = 1; & \rho_{F_2,P}(y_i) = 0 & \forall y_i \in [y^{F_1,F_2},y^P) \\
\rho_{F_1,P}(y_i) = 0; & \rho_{F_1,F_2}(y_i) = 1; & \rho_{F_2,P}(y_i) = 1 & \forall y_i \in [y^P,y_u] \\
\rho_{F_1,P}(y_i) = 0; & \rho_{F_1,F_2}(y_i) = 1; & \rho_{F_2,P}(y_i) = 0 & \forall y_i \in [\tilde{y},y_u]
\end{cases}
\end{align*}
$$

In this configuration, firms always prefer to renew a fixed-term contract, provided that the match is good enough (that is $> y^{F_1,F_2}$). There is no room for direct conversion from $F_1$ to $P$.

3) $y^{F_1,F_2} < y^P < \tilde{y} < y_u$. Then,

$$
\begin{align*}
\begin{cases}
\rho_{F_1,P}(y_i) = 0; & \rho_{F_1,F_2}(y_i) = 0; & \rho_{F_2,P}(y_i) = \text{ind.} & \forall y_i \in [y_1,y^{F_1,F_2}) \\
\rho_{F_1,P}(y_i) = 0; & \rho_{F_1,F_2}(y_i) = 1; & \rho_{F_2,P}(y_i) = 0 & \forall y_i \in [y^{F_1,F_2},y^P) \\
\rho_{F_1,P}(y_i) = 0; & \rho_{F_1,F_2}(y_i) = 1; & \rho_{F_2,P}(y_i) = 1 & \forall y_i \in [y^P,\tilde{y}) \\
\rho_{F_1,P}(y_i) = 1; & \rho_{F_1,F_2}(y_i) = 0; & \rho_{F_2,P}(y_i) = \text{ind.} & \forall y_i \in [\tilde{y},y_u]
\end{cases}
\end{align*}
$$

This scenario is similar to the previous point, with the difference that there is a value $\tilde{y} < y_u$
for which it becomes convenient to directly convert an $F_1$ to $P$, without going through $F_2$. This happens because, as productivity heightens, the firing costs become negligible and the marginal gain in relaxing $c(p)$ weights more.

**Proof of Proposition 3**

First notice that the choice of $T^{F_2}$ is relevant iff the firm has decided to renew a fixed-term contract. This implies that, using the classification used in Proposition 2, we need to consider only cases 2) and 3), in which renewals do exist. Moreover, we can restrict the attention to those matches which are effectively interested by the renewal, that is $y_i \in [y^{F_1,F_2}, \tilde{y})$. We can now derive eq. (27) w.r.t $T^{F_2}$:

$$\frac{\partial J^{F_2}(y_i)}{\partial T^{F_2}} = e^{-(r+sF)T^{F_2}} [y_i - w^{F_2}(y_i) - (r + sF)\rho^{F_2,F}(y_i)J^P(y_i)]$$

(31)

Two possibilities may emerge:

a) If $y_i \in [y^{F_1,F_2}, y^P)$, $\rho^{F_2,F}(y_i) = 0$. In this case, eq. (31) is always positive. It follows that firms choose the maximum length allowed.

b) If $y_i \in [y^P, \tilde{y})$, $\rho^{F_2,P}(y_i) = 1$. Therefore, we need to study the sign of the bracketed term in eq. (31). After some algebra, eq. (31) can e rewritten as follows:

$$\frac{\partial J^{F_2}(y_i)}{\partial T^{F_2}} = \frac{e^{-(r+sF)T^{F_2}}}{r + sP + \mu} \Lambda(y_i)$$

(32)

where $\Lambda(y_i) = (sP + \mu - sF)(1-\beta)(y_i-rU)-(r+sF) \left[ K (\tilde{s}(y_i) + \beta (+sF)) + \frac{\mu(1-\beta)}{r+sF + \mu} \int_{\varepsilon_u}^{\varepsilon} (1 - F(\varepsilon))d\varepsilon \right]$. Additionally, developing eq. (30) yields:

$$\Lambda(\tilde{y}) = \frac{(r + sF)(r + sP + \mu)}{1 - e^{-(r+sF)T^{F_2}}} \Omega(\tilde{y}) > 0$$

(33)

Assuming that $sF$ is not too small compared to $sP + \mu$, $\Lambda'(y) < 0$. Given that we situate in the case in which $y_i \leq \tilde{y}$, it follows that $\Lambda(y_i) > 0$. Therefore the firm chooses the length of the second temporary contract as the highest possible.
For both cases we thus have that

\[(T^{F_2})^* = T^F - T^{F_1}\]  
Q.E.D.