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Incomplete Contracts with Cross-Investments

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Abstract

We study an incomplete contract model where both contracting parties can invest, and the investments have both self- and cross-effects. We analyze the performance of non-contingent contracts, message games, option contracts and property rights. We find that the first best is implemented if (i) the cross effects are negative or weaker than self-effects; (ii) the strength of cross-effects relative to self-effects is symmetric across parties. If either of these conditions is violated, even message contingent revelation mechanisms fail to provide efficient incentives. For this case, we obtain a number of results characterizing the second best. We find that property rights outperform contracts and partially relax the symmetry constraint. In either first best or second best, the stronger the cross-effects, the lower the value of contracting. The optimal allocation of property rights assigns ownership to the party with stronger cross-effects.

KEYWORDS: incomplete contracts, cooperative investments, property rights, option contracts

1 Introduction

An important function of contracts is to provide trading parties with the proper incentives for relationship-specific investments. If a contract makes payments and deliveries contingent on states of nature and on the quality of the good traded, both ex post trade outcome and ex ante investments are efficient. In the real world, however, it is impossible or prohibitively costly to write and enforce such a contract, so parties have to resort to incomplete contracts. A key feature of the incomplete contract models is ex ante uncertainty of ex post non-verifiable gains of trade that makes incomplete contracts vulnerable to renegotiation. Unless parties can commit not to renegotiate the contract, the hold-up problem arises. Hart and Moore (1988) have shown that this leads to underinvestment.

In this paper, we concentrate on a different problem related to contractual incompleteness, namely on providing incentives for relationship-specific investments with *cross-effects*.[1] Investments with cross-effects are very common in bilateral relationships. For example, a supplier can influence unverifiable quality of input which determines the value of the input to its buyers. An employee may invest in job-specific skills that will improve the quality of output but will be of little value outside the firm. Employers may provide employees with benefits and perquisites that are often employee-specific. Che and Hausch (1999) provide many other examples of cross-investments. In the collection of empirical studies of specific investments in a very broad range of industries (Masten, 1996), most cases involve cross-effects.

As shown in Che and Hausch (1999), it is very hard to provide efficient incentives once the investments are cooperative, i.e. involve positive cross-effects. Segal and Whinston (2002) suggest that the problem may be less severe when the cross-effects are negative. Indeed, if the cross-effect is negative, the challenge is over- rather than underinvestment. In order to reduce incentives to invest, parties can deliberately introduce hold-up by specifying trade below the ex post efficient level or even by not contracting ex ante at all. However, Segal and Whinston (2002) consider the setting where only one party invests. As we discuss below, this is an important restriction: providing incentives to one party affects the incentives of the other one. Indeed, suppose that parties' cross-effects have opposite signs. Then hold-up improves incentives of a party with a negative cross-effect (reducing its tendency to overinvest) but aggravates the inefficiency of incentives of the party with a positive cross-effect which already underinvests relative to the first best level.

In this paper, we consider a general framework where both parties can invest and the investments can have positive and negative cross- and self-effects. We believe that negative cross-effects are as common as positive ones; also, there is no reason to believe that the parties' cross-effects have the same sign and strength. Indeed, consider the conventional bilateral trade setting. Suppose that a seller can invest in a new technology that may influence her costs and a buyer's valuation of the good to be produced. By

[1] Che and Hausch (1999) refer to cross-investments as 'cooperative' investments. We prefer to use the term 'cross-investments' (MacLeod and Malcolmson, 1993) as we shall study, among others, the case of negative cross-effects where investments are 'anti-cooperative'. Some authors (e.g. Noldeke and Schmidt, 1995) also use the term 'investments with externalities.'

definition, the self-effect is positive (negative) whenever the new technology reduces (increases) cost, and the cross-effect is positive (negative) if the new technology produces a good of higher (lower) value to the buyer. If there were no cross-effects, the case of negative self-effects would be trivial — the seller would never purchase a technology that raises production costs. However, in the presence of cross-effects, a new technology may be worth investing if it involves either (i) positive self- and cross-effects, or (ii) negative cross-effects and positive self-effects, or (iii) negative self-effects and positive cross-effects. A cost-reducing technology may imply lower quality and therefore lower buyer's value (case (ii)); similarly, a new technology for producing higher quality may also involve higher operating costs (case (iii)). Hence, once the cross-effects are present, it is worth studying the case of negative cross-effects (ii) as well as the case of negative self-effects (iii). Moreover, if the buyer also makes a specific investment, there can be situations where cross-effects of the parties have different strength (relative to self-effects) and even opposite signs.

Our main research question is whether the first best can be implemented in this general setting through various contractual arrangements such as simple non-contingent contracts, general revelation mechanisms, option contracts and property rights. We show that providing efficient incentives for two investing parties is very difficult even if the cross-effects are negative. The first best is implemented only if the strength of cross-effects relative to self-effects is perfectly symmetric across parties (or, for some mechanisms, at least sufficiently symmetric).

In order to compare the performance of different mechanisms, we consider a simple framework of mutual "tax rates", i.e. shares of each party's ex post gains of trade to be paid as a "tax" to the other party. Such an arrangement is similar to the contract between two jointly operating buccaneer vessels in the late 17th century:

> "... These articles contained the common provisions that ... the vessel taking a prize should retain three fifths of its [the prize's] values, surrendering two fifths to its associate."
>
> *Rafael Sabatini, Captain Blood, chapter 13.*

Since the cross-effects are essentially externalities, "taxes" can indeed restore efficiency. We show that in the generic case there exists a unique pair of tax rates that provides the first best incentives to both parties. However, since the contracts are incomplete, the gains of trade cannot be directly contracted upon. Implementing an arbitrary pair of tax rates is therefore a difficult task. We decompose the problem into two stages. First, we establish a correspondence between incentives to be provided and the tax rates. Second, for each particular class of contractual arrangements we find the range of tax rates that can be implemented.

We start with a very simple model where parties can trade only one unit of good; also, we rule out the problem of renegotiation-proofness by assuming that trade is always efficient. Even in this setting, the analysis is not trivial. The first best can only be achieved when cross-effects are negative and perfectly (or at least sufficiently) symmetric. We then extend our analysis to the setting where parties can trade any positive quantity. The set of ex ante contracts is expanded to include those that specify trade levels well in excess of the ex post efficient volumes. It turns out, however, that even in this case, the first best is hard to implement. The only difference is that in addition to the case of negative cross-effects, the efficient incentives can also be provided if the cross-effects are positive but weaker than self-effects. The symmetry constraint remains important.

This paper also contributes to answering the question posed by Che and Hausch (1999): can property rights help to provide incentives for cross-investments?[2] We show that optimal allocation of property rights does outperform contracts. Property rights shift incentives from one party to another and therefore partially relax the symmetry constraint. We show that the optimal allocation of property rights implements the first best when cross-effects are negative and sufficiently symmetric, and study the second best when the first best cannot be achieved.

The paper is structured as follows. In Section 2, we introduce a simple model of bilateral trade where only one unit can be traded. We prove that in the presence of cross-effects contracting has no value if cross-effects are stronger than self-effects. We also calculate the optimal tax rates and show how they depend upon the signs and relative strength of cross-effects. In Section 3, we study whether the first best can be implemented through a general message contingent mechanism (Subsection 3.1), option contracts (3.2), and property rights (3.3). We also characterize the second best. Section 4 extends the analysis to a setting with continuous quantity. Section 5 concludes.

2 The model

2.1 The setting

Consider a simple bilateral trade setting. At time $t = 0$, two agents, a buyer B and a seller S, contemplate trading a unit of good that originally belongs to the seller. At time $t = 1/2$, the buyer and the seller make relationship-specific investments $\beta \geq 0$, $\sigma \geq 0$, respectively. Once made, the investments are sunk and have no value outside the relationship. At time $t = 1$, both parties observe the state of nature $\omega \in \Omega$, the seller's production cost $c(\omega, \beta, \sigma)$ and the buyer's valuation of the good $v(\omega, \beta, \sigma)$. Then the parties renegotiate whether to trade and at what price. If the trade occurs, the price p is determined either by a contract signed at time $t = 0$ or through bargaining at $t = 1$. The seller's payoff is $p - c(\omega, \beta, \sigma) - \sigma$ and the buyer's is $-p + v(\omega, \beta, \sigma) - \beta$. If no trade occurs, the payoffs are $-\sigma$ and $-\beta$, correspondingly. Parties choose their investment levels simultaneously and independently. The probability distribution of ω is common knowledge at $t = 0$. The distribution does not have mass points, the density function $f(\omega)$ is well-defined. The state set Ω is a path-connected subset of R^n.

We assume that the trade is always efficient:

$$c(\omega, \beta, \sigma) \leq v(\omega, \beta, \sigma) \text{ for all } \beta, \sigma \text{ and } \omega. \tag{1}$$

[2]This problem is also studied by Edlin and Hermalin (2000) but their setting is different. First, they study sequential rather than simultaneous investments, which makes it easier to design an optimal mechanism (Demsky and Sappington, 1991, Noldeke and Schmidt, 1998). Second, in their setting, the self- and cross-effects of investments cannot be explicitly defined: investments increase ex post returns to the owner of the asset. Hence, cross-effects are present only if the asset changes hands.

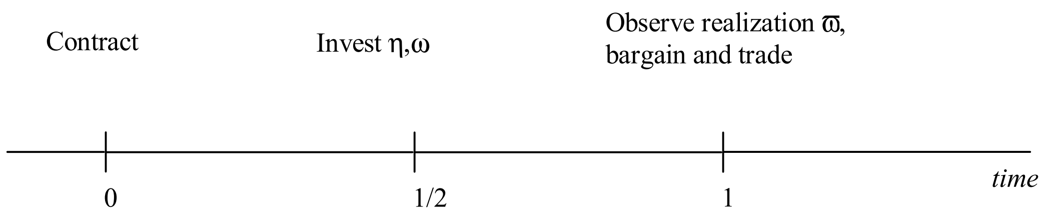

Figure 1: The timeline.

The total ex post gains of trade $w(\omega,\beta,\sigma) = v(\omega,\beta,\sigma) - c(\omega,\beta,\sigma)$ are always non-negative. The first-best choice of β,σ maximizes the total expected gains of trade $W(\beta,\sigma) = Ew(\omega,\beta,\sigma)$ minus the cost of investment:

$$(\beta^*,\sigma^*) = \arg\max_{\beta,\sigma\geq 0} \{W(\beta,\sigma) - \beta - \sigma\}. \tag{2}$$

Since trading is efficient, there exists a price $\bar{p}$ such that both parties' expected gains of trade are non-negative: $Ev(\omega,\beta^*,\sigma^*) - \bar{p} \geq 0$ and $S(\beta^*,\sigma^*) = \bar{p} - Ec(\omega,\beta^*,\sigma^*) \geq 0$. Let us introduce the gains of trade at the price $\bar{p}$:

$$b(\omega,\beta,\sigma) = v(\omega,\beta,\sigma) - \bar{p},\ \ s(\omega,\beta,\sigma) = \bar{p} - c(\omega,\beta,\sigma) \tag{3}$$

and the expected gains of trade: $B(\beta,\sigma) = Eb(\omega,\beta,\sigma)$; $S(\beta,\sigma) = Es(\omega,\beta,\sigma)$. Throughout the paper we assume that courts can verify delivery, so the contract specifying trade at price $\bar{p}$ is enforced even though in some states it may provide one party with a negative payoff (e.g. $b < 0$ or $s < 0$ for some ω).

The parties' gains of trade (b,s) are distributed on $\mathcal{B}\text{x}\mathcal{S}$, where $\mathcal{B} = \{b(\omega,\beta,\sigma) : \omega \in \Omega, \beta \geq 0, \sigma \geq 0\}$, $\mathcal{S} = \{s(\omega,\beta,\sigma) : \omega \in \Omega, \beta \geq 0, \sigma \geq 0\}$.

We will make the following technical assumptions: both $s(\omega,\beta,\sigma)$ and $b(\omega,\beta,\sigma)$ are strictly concave and continuously differentiable for $\beta \geq 0, \sigma \geq 0$ and twice continuously differentiable for all $\beta > 0, \sigma > 0$. Therefore $B(\beta,\sigma)$ and $S(\beta,\sigma)$ are strictly concave and continuously differentiable for $\beta \geq 0, \sigma \geq 0$ and twice continuously differentiable for all $\beta > 0, \sigma > 0$ i.e. $B_{\beta\beta} < 0$, $S_{\beta\beta} < 0$, $B_{\sigma\sigma} < 0$, $S_{\sigma\sigma} < 0$.[3] For simplicity's sake, we rule out complementarities: $B_{\beta\sigma} = S_{\beta\sigma} = 0$. Then, gains of trade $W(\beta,\sigma) = B(\beta,\sigma) + S(\beta,\sigma)$ and total ex ante welfare $W(\beta,\sigma) - \beta - \sigma$ are also concave. For the sake of simplicity we shall also assume Inada conditions: $W_\beta(0,\sigma) = W_\sigma(\beta,0) = W_\beta(0,\sigma) = W_\sigma(\beta,0) = \infty$ and $W_\beta(\infty,\sigma) = W_\beta(\infty,\sigma) = W_\sigma(\beta,\infty) = W_\sigma(\beta,\infty) = 0$. Therefore, the first-best choice of investment levels (β^*,σ^*) is unique, non-trivial and can be found from the first order conditions

$$W_\beta(\beta^*,\sigma^*) = 1,\ \ W_\sigma(\beta^*,\sigma^*) = 1. \tag{4}$$

[3]Most results below can be reformulated without concavity assumptions using strict monotone comparative statics techniques. For clarity's sake we shall use first-order conditions throughout the paper.

The total effect of the buyer's investment is the sum of self- and cross- effects: $W_\beta = B_\beta + S_\beta$. There can be three cases: (i) both self- and cross-effects are non-negative in the first-best: $B^*_\beta \equiv B_\beta(\beta^*, \sigma^*) \geq 0$, $S^*_\beta \equiv S_\beta(\beta^*, \sigma^*) \geq 0$; (ii) self-effect is positive while cross-effect is negative: $B^*_\beta > 0$, $S^*_\beta < 0$; (iii) self-effect is negative while the cross-effect is positive: $B^*_\beta < 0$, $S^*_\beta > 0$. To simplify, we assume below that the effects do not change signs, i.e. "negative cross-effects of buyer's investments" will mean $S_\beta > 0$ for *all* $\beta, \sigma \geq 0$ rather than only in the first best.

Throughout the paper we assume equal bargaining power.

Remark 1 *We have made two restrictive assumptions. First, the trade is binary, second, the trade is always efficient. Under these assumptions, parties cannot contract on quantities that are above the ex post efficient level of trade $q^* = 1$.*[4] *Also, the assumptions allow for writing a renegotiation-proof contract ('trade one unit at price $\overline{p}$') which rules out the hold-up problem. Both assumptions are relaxed in Section 4.*

2.2 Benchmarks

2.2.1 Null contract

Let us first describe the choice of investments at $t = 1/2$ if no arrangements are made ex ante (at $t = 0$). We will refer to this mechanism as the "null contract" (Che and Hausch, 1999, call it the "Williamsonian outcome").

At $t = 1$, the parties observe the state of nature and gains of trade $v(\omega, \beta, \sigma) - c(\omega, \beta, \sigma)$ and bargain on the division of the joint surplus. Equal allocation of bargaining power implies $p = (v + c)/2$. Anticipating this outcome, the buyer chooses β to maximize $\frac{1}{2}W(\beta, \sigma) - \beta$, and the seller chooses σ to maximize $\frac{1}{2}W(\beta, \sigma) - \sigma$. Under the assumptions above, the equilibrium levels of investments can be found from the first-order conditions

$$W_\beta = 2, \; W_\sigma = 2. \tag{5}$$

Comparing (5) to (4), we obtain that null contract is never efficient. Each party only gets half of the social returns to her investment, and has incentives to underinvest.

2.2.2 Non-contingent contract

Suppose that at $t = 0$ the parties sign a contract to trade at price $\bar{p}$ at time $t = 1$. Since $\bar{p}$ is observable and verifiable, the contract can be enforced. The contract is renegotiation-proof: since the trade is ex post efficient (1), renegotiation cannot increase joint surplus. The contract is still incomplete since it is not contingent on the payoff-relevant variables v and c. The incompleteness is crucial whenever cross-effects are present. Indeed, under the non-contingent contract, the buyer will choose β to maximize $B(\beta, \sigma) - \beta$ and the

[4] Certainly, in many applications the level of trade is indeed binary. Consider a situation where a certain service is either provided or not provided. Parties care only about the service's quality which is not verifiable.

seller will choose σ to maximize $S(\beta, \sigma) - \sigma$. The equilibrium investment levels are found from the first-order conditions:

$$B_\beta = 1, \ S_\sigma = 1. \tag{6}$$

If there were no cross-effects of investments, i.e. $B^*_\sigma = S^*_\beta = 0$, the conditions (6) would be the same as (4); the non-contingent contract would implement the first best. It is no longer true in the presence of cross-effects. Giving up all the change in v to the buyer, the seller does not care anymore about the effect of her investment on v. But is it more inefficient than the null contract? Intuitively, the advantage of the null contract is that it provides parties with non-trivial incentives to care about cross-effects. However, it expropriates half of the self-effects in favor of the other party. Hence, the null contract should be more efficient when the cross-effects are strong relative to the self-effects, and the non-contingent contract should be more efficient when the self-effects are stronger. Hence, if the cross-effects are strong enough, the parties prefer to scrap the contract although it is efficient and renegotiation-proof ex post.[5]

To summarize the above discussion, we establish the following Proposition.

Proposition 1 *Under the assumptions above the following is true.*

1. *The non-contingent contract is efficient if and only if there are no cross-effects* $B^*_\sigma = S^*_\beta = 0$.[6]

2. *The null contract is never efficient. However, if cross-effects are stronger than self-effects (*$S_\beta > B_\beta$, $B_\sigma > S_\sigma$ *for all* β *and* σ*), the null contract is more efficient than the non-contingent contract. If cross-effects are weaker but still positive* $B_\beta > S_\beta > 0$, $S_\sigma > B_\sigma > 0$ *then the null contract is less efficient than the non-contingent contract. If cross-effects and self-effects are of the same magnitude* $S_\beta = B_\beta$ *and* $B_\sigma = S_\sigma$ *then the null contract and the non-contingent contract are equally inefficient.*

Similarly to Che and Hausch (1999), we find that contracting has no value if cross-effects are sufficiently strong (in our setting, cross-effects must be stronger than self-effects). The intuition is straightforward: the non-contingent contract results in a more severe underinvestment than the null contract. The Proposition does not cover the case of negative cross-effects. In that case, the null contract results in underinvestment while the non-contingent contract provides incentives to overinvest, and it is hard to establish which outcome is less inefficient.

2.2.3 The optimal tax rates

In the presence of non-trivial cross-effects, the non-contingent contract is inefficient, since it does not provide parties with any share in other party's ex post gains. Suppose that

[5] In the risk-neutral framework, a more efficient outcome is unanimously preferred by both parties ex ante. They can redistribute the welfare gains through transfers at $t = 0$.

[6] The 'if' part has been shown in a more general framework by many authors. The 'only if' part has also been discussed in the literature (e.g. MacLeod and Malcolmson, 1993).

there exists a mechanism that gives α_b per cent of the seller's surplus to the buyer, and α_s per cent of the buyer's surplus to the seller. In this Subsection, we *assume* that such a mechanism can be implemented and solve for α_b and α_s that provide optimal incentives. In the next Section we show which pairs α_b and α_s can be implemented in different settings.

The rates α_b and α_s may be understood as "tax rates" (to offset the "externalities" due to cross-effects): the buyer's gains of trade B are being "taxed" at the rate α_s and the seller's — at the rate α_b. The budget is balanced: tax on one party is paid to the other one. The buyer's expected payoff is $(1-\alpha_s)B+\alpha_b S-\beta$ and the seller's is $(1-\alpha_b)S+\alpha_s B-\sigma$. Notice that at $\alpha_b=\alpha_s=1/2$ we obtain the null contract. The non-contingent contract is equivalent to $\alpha_b=\alpha_s=0$. The investments are determined by the first-order conditions:

$$(1-\alpha_s)B_\beta+\alpha_b S_\beta=1;\quad (1-\alpha_b)S_\sigma+\alpha_s B_\sigma=1. \tag{7}$$

Using these two equations, we can calculate the tax rates α_b and α_s that provide efficient incentives.

Lemma 1 *If $S^*_\beta+B^*_\sigma\neq 1$ then the tax rates α_b,α_s provide first best incentives if and only if*

$$\alpha_b=\frac{B^*_\sigma}{S^*_\beta+B^*_\sigma-1},\quad \alpha_s=\frac{S^*_\beta}{S^*_\beta+B^*_\sigma-1}. \tag{8}$$

If $S^*_\beta=B^*_\sigma=1/2$ then the first best is achieved if and only if $\alpha_b=\alpha_s=1/2$. If $S^*_\beta=1-B^*_\sigma\neq 1/2$ there are no α_b,α_s that provide efficient incentives.

The Lemma has a number implications that are important for the analysis below.

Corollary 1 *The formulae (8) define a mapping of (S^*_β,B^*_σ) to (α_b,α_s) with the following properties.*

1. *The mapping is reflexive: $B^*_\sigma=\frac{\alpha_b}{\alpha_b+\alpha_s-1}$, $S^*_\beta=\frac{\alpha_s}{\alpha_b+\alpha_s-1}$.*
2. *The mapping transforms the space of parameters (S^*_β,B^*_σ) to the space (α_b,α_s) as shown in Figure 2, in particular:*
   (a) *The tax rates α_b,α_s (8) satisfy $\alpha_b,\alpha_s\geq 0$, $\alpha_b+\alpha_s\leq 1$ if and only if both cross-effects are non-positive in the first-best: $S^*_\beta\leq 0$ and $B^*_\sigma\leq 0$ (domains G,H,I in Figure 2).*
   (b) *The tax rates α_b,α_s (8) satisfy $\alpha_b\in[0,1/2],\alpha_s\in[0,1/2]$ if and only if both cross-effects are non-positive and sufficiently symmetric $S^*_\beta\leq 0$, $B^*_\sigma\leq 0,\left|S^*_\beta-B^*_\sigma\right|\leq 1$ (domain H in Figure 2).*
3. *The tax rates α_b,α_s are perfectly symmetric $\alpha_b=\alpha_s$ if and only if the cross-effects are perfectly symmetric $S^*_\beta=B^*_\sigma$.*

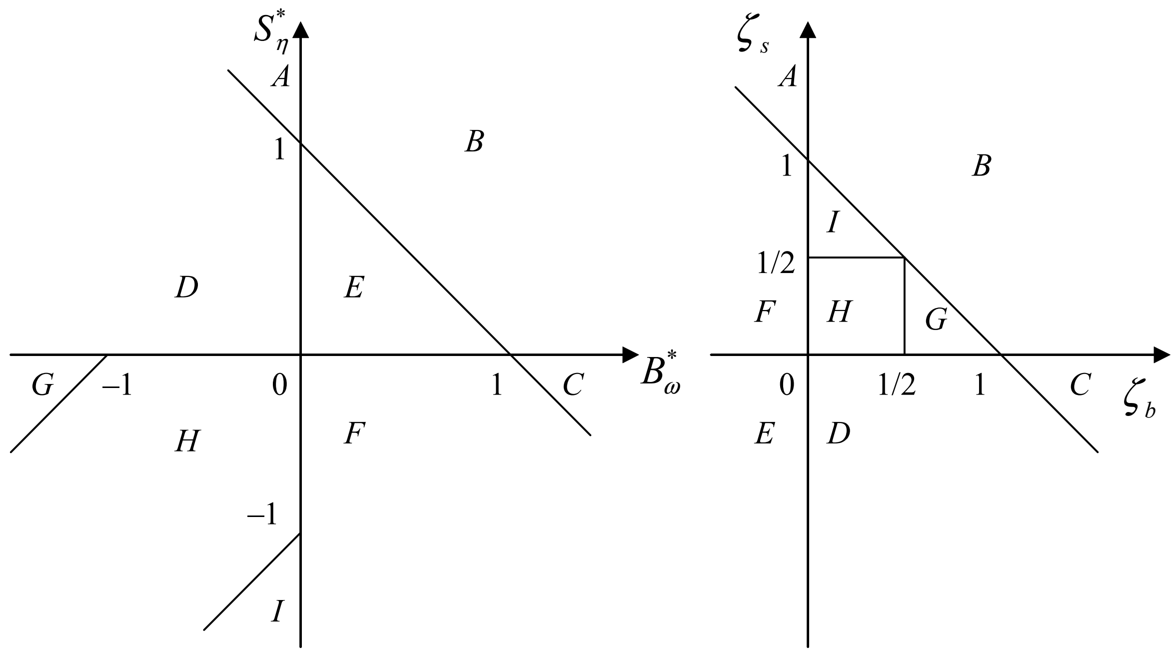

Figure 2: Optimal tax rates α_b, α_s as a function of cross-effects in the first best S^*_β, B^*_σ. The left-hand side graph partitions the space of parameters S^*_β and B^*_σ into nine domains $A, B, ..., I$. The right-hand side graph maps the partition onto the space of tax rates α_b, α_s.

The Corollary implies that (a) symmetry in cross-effects requires symmetry in tax rates, and (b) there is a relationship between sign and strength of cross-effects and those of the tax rates. Consider the case where cross-effects are on average weaker than self-effects $S^*_\beta + B^*_\sigma < B^*_\beta + S^*_\sigma$ (hence $S^*_\beta + B^*_\sigma < 1$, domains D-I in Fig.2). In domain D, the buyer's cross-effect is positive and seller's cross-effect is negative. The buyer receives a positive tax $\alpha_b > 0$ from the seller to restore incentives. The tax that the buyer pays on her own gain is negative $\alpha_s < 0$: the seller's investment reduces buyer's gain B, so the seller should pay for a negative externality.

Remark 2 *In this Section, we only consider* linear *taxes. The analysis can be generalized to include the optimal non-linear tax schedules. The only difference is that formulas (8) should describe* marginal *tax rates calculated at the equilibrium choice of investments.*

3 Optimal mechanisms

3.1 Message-contingent mechanisms

In this Section we study the performance of message-contingent mechanisms. At $t = 0$, parties sign an agreement to make payments and deliveries contingent on messages posted at $t = 1$. Messages are observable and verifiable. At $t = 1$, parties observe v and c and report $b = v - \bar{p}$ and $s = \bar{p} - c$.[7] We shall denote the buyer's reports of b and s by b^B, s^B, and the seller's reports by b^S, s^S, respectively.

Given b^B, s^B, b^S, s^S, the mechanism specifies payments of $T(b^B, s^B, b^S, s^S)$ by the buyer to the seller (T can be either positive or negative); with probability $X(b^B,s^B,b^S,s^S)$ the parties trade one unit at price $p(b^B, s^B, b^S, s^S)$ (on top of the payment of T); with probability $1 - X(b^B,s^B,b^S,s^S)$ parties do not trade. After the mechanism is played, parties can renegotiate.

We only consider budget-balanced mechanisms that include no payments to a third party. The first best could be easily implemented through unbalanced mechanisms, e.g. via subgame perfect implementation (see a survey in Moore, 1992) or even via a "shoot-them-all" game (similar to one in Maskin, 1999). If parties' reports on b, s differ, the parties pay large fines to the third party. If the reports match, parties get the payoffs $(1-\alpha_s)b^B + \alpha_b s^B$ and $(1-\alpha_b)s^B + \alpha_s b^B$, where α_b, α_s are given by (8). The efficiency of "shooting-them-all" is well-known; in particular, Che and Hausch (1999) show in Section IV that it helps to provide first best incentives for cooperative investment. The problem with the "shoot-them-all" mechanism (as well as with other non-balanced mechanisms) is that it is not collusion-proof: at least one party always prefers to bribe the third party.

The renegotiation occurs only if the mechanism prescribes no trade. Parties split the trade gains according to Nash bargaining solution. Hence, at $t = 1$, the buyer's and the seller's payoffs are, respectively:

$$\begin{aligned} U^B(b^B, s^B, b^S, s^S, b, s) &= X(b^B, s^B, b^S, s^S)[b + \bar{p} - p(b^B, s^B, b^S, s^S)] + \\ &+ \left(1 - X(b^B, s^B, b^S, s^S)\right)[b + s]/2 - T(b^B, s^B, b^S, s^S), \\ U^S(b^B, s^B, b^S, s^S, b, s) &= X(b^B, s^B, b^S, s^S)[s + p(b^B, s^B, b^S, s^S) - \bar{p}] + \\ &+ \left(1 - X(b^B, s^B, b^S, s^S)\right)[b + s]/2 + T(b^B, s^B, b^S, s^S) \end{aligned}$$

Definition 1 *A mechanism $\{X, p, T\}$ is said to implement a surplus division rule $K(\cdot,\cdot)$: $\mathcal{B}$x$\mathcal{S} \to \mathcal{R}$ if for every pair $b \in \mathcal{B}$, $s \in \mathcal{S}$ there exist such $\bar{b}^B, \bar{b}^S \in \mathcal{B}, \bar{s}^B, \bar{s}^S \in \mathcal{S}$ that $K(b, s) = U^B(\bar{b}^B, \bar{s}^B, \bar{b}^S, \bar{s}^S, b, s)$; $b + s - K(b, s) = U^S(\bar{b}^B, \bar{s}^B, \bar{b}^S, \bar{s}^S, b, s)$ and for all b^B, s^B, b^S, s^S the following inequalities hold*

$$\begin{aligned} U^B(\bar{b}^B, \bar{s}^B, \bar{b}^S, \bar{s}^S, b, s) &\geq U^B(b^B, s^B, \bar{b}^S, \bar{s}^S, b, s), \\ U^S(\bar{b}^B, \bar{s}^B, \bar{b}^S, \bar{s}^S, b, s) &\geq U^S(\bar{b}^B, \bar{s}^B, b^S, s^S, b, s). \end{aligned}$$

The mechanism design problem is now decomposed into (i) selecting a surplus division rule K as a function of true b and $s : (b, s) \to \{K(b, s), b + s - K(b, s)\}$ and (ii) choosing such $\{X, p, T\}$ that K is implementable via Nash equilibrium in the message game. In order to provide incentives for efficient investments, the function $K(b, s)$ should satisfy

$$\begin{aligned} \beta^* &\in \operatorname{Arg}\max_{\beta \geq 0} EK(b(\omega, \beta, \sigma^*), s(\omega, \beta, \sigma^*)) - \beta, \\ \sigma^* &\in \operatorname{Arg}\max_{\sigma \geq 0} W(\beta^*, \sigma) - EK(b(\omega, \beta^*, \sigma), s(\omega, \beta^*, \sigma)) - \sigma. \end{aligned}$$

[7] We describe the state in terms of parties gains of trade b, s rather than the valuations v, c to make notation symmetric.

Similarly to Moore (1992), one can easily prove that $K(b,s)$ is implementable if and only if it is implementable via truthful revelation. We shall restrict our analysis to the Nash equilibria with $\bar{b}^B = \bar{b}^S = b$, $\bar{s}^B = \bar{s}^S = s$.

Lemma 2 *A surplus division rule $K(b,s)$ is truthfully implementable only if for all $\Delta \geq 0$, $b \in \mathcal{B}$ and $s \in \mathcal{S}$*

$$\Delta/2 \leq K(b+\Delta, s) - K(b,s) \leq \Delta \tag{9}$$

$$0 \leq K(b, s+\Delta) - K(b,s) \leq \Delta/2 \tag{10}$$

$$K(b+\Delta, s+\Delta) - K(b,s) = \Delta \tag{11}$$

Proposition 2 *Under the assumptions above the following results hold:*

1. *If both cross-effects are symmetric and non-positive in the first best $B^*_\sigma = S^*_\beta \leq 0$, then there exists a mechanism that implements efficient choice of investments.*

2. *If either of the conditions below holds, no message-contingent mechanism can implement the first best.*[8]

   (a) *The cross-effect of the buyer's investment is positive in the first best in all states of nature i.e. $s_\beta(\omega, \beta^*, \sigma^*) > 0$ for all ω, and the total effect of buyer's investment does not change sign across states $w_\beta(\omega, \beta^*, \sigma^*) > 0$ for all ω.*

   (b) *The buyer's cross-effect is stronger than the seller's cross-effect and the buyer's self-effect is weaker than the seller's in the first best in all states of nature i.e. there exist such $\delta_1, \delta_2 > 0$ such that $s_\beta(\omega, \beta^*, \sigma^*) - b_\sigma(\omega, \beta) \geq \delta_1$ and $s_\sigma(\omega, \beta^*, \sigma^*) - b_\beta(\omega, \beta^*, \sigma^*) \geq \delta_2$, for all ω.*

3. *If cross-effects are positive and stronger than self-effects $s_\beta > b_\beta, b_\sigma > s_\sigma$ for all ω, β and σ, then the second best can be implemented through a null contract.*

4. *If cross-effects are positive but are weaker than self-effects $0 < s_\beta < b_\beta, 0 < b_\sigma < s_\sigma$ for all ω, β and σ, then the second best can be implemented through a non-contingent contract.*

The Proposition makes two negative statements. The first best cannot be achieved if cross-effects are either positive (condition 2a) or non-symmetric (condition 2b). To prove the statement 2a, we also need a technical assumption $w_\beta(\omega, \beta^*, \sigma^*) > 0$. It requires that the effect of investment on the total surplus is sufficiently uniformly distributed across states; on average, it is positive and equals one: $Ew_\beta = W^*_\beta = 1$.

Proposition 2 also establishes a relationship between the strength of cross-effects and the value of contracting. If cross-effects are positive but weaker than self-effects, then the availability of the non-contingent contract is valuable. Although the first best cannot be achieved, in the second best parties trade according to the non-contingent contract with positive probability. If the cross-effects are very strong (stronger than the self-effects), the second best message game never makes use of the non-contingent contract.

[8] For the brevity's sake, conditions (a) and (b) are formulated for the buyer's investment. Respective conditions can be stated for the seller's investment as well.

Remark 3 *The symmetry constraint is not important if only one party invests. Our assumptions require non-trivial investment by each party, so formally our analysis does not cover the case of one-sided investment. Still, it is clear how to provide incentives for such investment. Suppose that the buyer does not invest. Then one needs to implement* α_b, α_s *that solve only the second equation in (7). There are just enough degrees of freedom to find such tax rates even if the symmetry constraint* $\alpha_b = \alpha_s$ *is binding. The first best can therefore be implemented for any negative* B^*_σ *through* $\alpha_b = \alpha_s = \frac{B^*_\sigma}{2B^*_\sigma - 1}$.

3.2 Option contracts

In this Section, we study what tax rates can be implemented through simpler mechanisms such as option contracts. Noldeke and Schmidt (1995) have shown that option contracts solve the hold-up problem if investments do not have cross-effects. In concluding remarks, Noldeke and Schmidt argue that options should also provide efficient incentives if only one party's investment has cross-effects and doubt that option contracts can do well in case of two-sided cross-investments. The main distinction from their work is that we study self- and cross-investments made by two parties.

Consider the following arrangement. At time $t = 0$, the parties trade option contracts. The seller gets a right to sell the good to the buyer at time $t = 1$ at price p^s even if the buyer does not want to take it. This is the so called "take-or-pay" or "minimum bill" clause (Masten, 1996, provides many examples). In the meanwhile, the buyer gets a right to oblige the seller to deliver the good at time $t = 1$ at price $p^b > p^s$. At $t = 1$, parties bargain and conclude a spot contract keeping the option contracts in mind.

The option swap is equivalent to "minimum-maximum price limits" clauses in the contract or to a "flexible indexed contract" (i.e. the contract where parties agree on a verifiable price index and allow for certain upward and downward deviations from the index). These arrangements are very common in relationships between suppliers and customers in coal and petroleum coke industries (Joskow, 1987, 1990, and Goldberg and Erickson, 1987).

In the analysis of the option contracts, the structure of the bargaining solution becomes crucial. We contrast two approaches (mnemonic names as in to Binmore et al., 1989): "deal-me-out" (or the outside option principle) and "split-the-difference" (Nash bargaining solution with options as threatpoints). The difference between the two is similar to the difference between Hart's model of asset ownership and the one in Chiu (1998) and De Meza and Lockwood (1998). While Binmore et al. (1989) show that the deal-me-out solution performs better in the experiments, both bargaining solutions may occur in real life. Goldberg and Erickson (1987) argue that the deal-me-out model seems to be consistent with evidence on the petroleum coke contracts after 1973, while the model where options serve as disagreement points, seems to be more applicable to pre-1973 contracts. Below, we analyze the two bargaining solutions one by one.

3.2.1 Outside option principle ('deal-me-out')

Suppose that at $t = 1$ parties engage in an alternating offer game as described in Osborne and Rubinstein (1990); each offer takes a round, and there is a positive though very small discount rate ε per round (or a very small probability of an exogenous termination of bargaining). Each party can exercise her option at any moment during the game (if both party want to exercise their options simultaneously, one of them prevails with probability 1/2; this outcome does not occur in equilibrium). As described in Binmore et al. (1989), there is a unique subgame perfect equilibrium that converges to the following outcome as $\varepsilon \to 0$. Both parties know that if there were no option contracts, the price would be set at $p = (v + c)/2$. If this price is below p^s, the seller will not engage in bargaining but will simply exercise her option to sell at p^s right away. If $(v + c)/2 > p^b$ then the buyer will want to exercise her option and oblige the seller to sell the good at p^b.

The buyer's expected payoff is $B - 1/2 \int_{(v+c)/2\in[p^s,p^b]} (b - s)f(\omega)d\omega + \int_{(v+c)/2<p^s} (\bar{p} - p^s)f(\omega)d\omega - \int_{(v+c)/2>p^b} (p^b - \bar{p})f(\omega)d\omega$, the seller's is $S + 1/2 \int_{(v+c)/2\in[p^s,p^b]} (b - s)f(\omega)d\omega - \int_{(v+c)/2<p^s} (\bar{p} - p^s)f(\omega)d\omega + \int_{(v+c)/2>p^b} (p^b - \bar{p})f(\omega)d\omega$. The first-order conditions for the parties' investment levels are as follows:[9]

$$\begin{aligned} B_\beta + 1/2 \int_{(v+c)/2\in[p^s,p^b]} (-b_\beta + s_\beta)f(\omega)d\omega &= 1, \\ S_\sigma + 1/2 \int_{(v+c)/2\in[p^s,p^b]} (b_\sigma - s_\sigma)f(\omega)d\omega &= 1. \end{aligned}$$

In terms of incentives, this solution is similar to a combination of the non-contingent contract and the null contract. In some states of nature (whenever $(v + c)/2 \notin [p^s, p^b]$) parties trade at a fixed price (either p^s or p^b) and care only about self-effects. In other states, parties get their Nash bargaining solution payoffs $(v - c)/2$ and receive one half of the social return to investment. The option prices p^s, p^b determine the relative weights of the non-contingent contract and the null contract. If p^s and p^b are very close to each other, then the weight of the non-contingent contract is high and that of the null contract is low. If p^s is very small and p^b is very large, the choice of investments will be similar to the one under the null contract (5) rather than the non-contingent contract (6).

Proposition 3 *Let the shock ω be multiplicative, $v(\omega, \beta, \sigma) = \kappa(\omega)\bar{v}(\beta, \sigma)$ and $c(\omega, \beta, \sigma) = \kappa(\omega)\bar{c}(\beta, \sigma)$. Assume that ω is continuously distributed over $[0, 1]$ and $\kappa(\cdot)$ is a continuous increasing function. Then, for any (β, σ) that solve (7) for some $\alpha_b = \alpha_s \in [0, 1/2]$, there exist such p^s, p^b that the option swap (p^s, p^b) implements (β, σ). In particular, if cross-effects are non-positive and perfectly symmetric $B^*_\sigma = S^*_\beta \le 0$ then there exist such p^s, p^b that the option swap implements the first best.*

Therefore, if the outside option principle holds, option contracts can only implement the tax rates $\alpha_b = \alpha_s \in [0, 1/2]$. Although there are two parameters p^s, p^b, there is only one degree of freedom (at most one option can bind). Greater p^s as well as lower p^b increase both α_b and α_s simultaneously. One cannot reward cross-effects independently.

An option contract is effectively a message game where the outcome is contingent on the announcement of state of nature by one party. Hence it is not surprising that options

[9]The density is finite, hence the terms proportional to changes in the integration limits cancel out in the first order conditions. The effect of investments on the probability to make the option bind is negligible.

cannot outperform the general message contingent mechanism. It is rather remarkable though that if the shock is multiplicative, then options perform as well as the general mechanisms.

3.2.2 Nash bargaining solution ('split-the-difference')

The outside option principle predicts that the option contracts are actually exercised with a non-trivial probability $1 - Prob(p^s \leq (v + c)/2 \leq p^b)$. This is consistent with Goldberg and Erickson's (1987) account of the petroleum coke contracts after 1973 ("parties expected that the boundaries would routinely be reached"). However, before 1973, the minimum and maximum price limits "served a completely different function" (Goldberg and Erickson, 1987). In particular, the actual price was very rarely set equal to the limits specified in the contracts. In this Subsection we consider an alternative model of bargaining where the maximum and minimum limits are effectively used as threatpoints in renegotiation; the price does not reach the limits in equilibrium.

In order to explain how the split-the-difference rule can emerge as a subgame perfect equilibrium in a bargaining game, we consider an alternating offer game with the risk of breakdown and two options.[10] At $t = 1$ parties play an infinite horizon alternating offer game with a small probability of termination ε in each round. There is no discounting. In each round, one party makes an offer to trade at certain price p. If the other party agrees, trade at this price occurs and the game ends. If the other party disagrees, a random shock arrives. With probability ε the bargaining breaks down, with probability $1-\varepsilon$ the game continues. The breakdown of negotiations can be interpreted as the urgent need of a party to attend to other issues. Once this party leaves, the other party can choose either not to trade or to exercise her option.

Unlike the setting in the previous subsection, options can be exercised not only during the game but also after exogenous termination. This effectively turns options into disagreement outcomes. The split-the-difference solution is based on the assumption that making an offer takes time, while accepting or rejecting an offer and exercising an option take no time. Technically, making an offer is writing a contract while exercising an option is essentially accepting a contract that has already been written. Exercising an option is therefore similar to accepting an offer; it is less time-consuming and does not require participation of the other party. This is why if the negotiations break down, there is no time to make another counteroffer, but there is time to exercise the option contract.

Straightforward calculations prove that there is a unique subgame perfect equilibrium which converges to $p = \left(\min\{v, p^b\} + \max\{c, p^s\}\right)/2$ as $\varepsilon \to 0$. The trade occurs in the first round with equilibrium price p being always between p^s and p^b.

The intuition is as follows: when the buyer quits, the seller can either choose not to trade leaving both parties with trivial payoffs or to exercise her option and get $p^s - c$. Similarly, when the seller has to quit, the buyer chooses between $v - p^b$ and zero. Each party can exercise her option contract at any time, but she does not until the other party leaves. is based on the assumption that making an offer takes time, while

[10] There may be other settings that generate the same bargaining solution. E.g. analysis of inside options (Muthoo, 1999) also results in the split-the-difference solution.

accepting/rejecting an offer or exercising an option takes no time.

In equilibrium, the buyer gets $v - \left(\min\{v, p^b\} + \max\{c, p^s\}\right)/2$ and the seller gets $\left(\min\{v, p^b\} + \max\{c, p^s\}\right)/2 - c$. The parties' choice of investments is given by

$$\begin{aligned} B_\beta - 1/2 \int_{v \le p^b} b_\beta f(\omega) d\omega + 1/2 \int_{p^s \le c} s_\beta f(\omega) d\omega &= 1, \\ S_\sigma + 1/2 \int_{v \le p^b} b_\sigma f(\omega) d\omega - 1/2 \int_{p^s \le c} s_\sigma f(\omega) d\omega &= 1. \end{aligned} \tag{12}$$

These conditions are very different from those derived under the outside option principle. The option prices p^s and p^b can be used to reward parties' cross-effects independently. An increase in p^b encourages both the buyer and the seller to care about buyer's utility v while a decrease in p^s provides incentives to invest in cost reduction. There are essentially two degrees of freedom.

Proposition 4 *Let the shock ω be separably multiplicative, i.e. $\omega = (\omega_1, \omega_2)$, $v(\omega, \beta, \sigma) = \lambda(\omega_1)\bar{v}(\beta, \sigma)$ and $c(\omega, \beta, \sigma) = \mu(\omega_2)\bar{c}(\beta, \sigma)$. Assume that ω_1, ω_2 are continuously (not necessarily independently) distributed over $[0, 1]$ and $\lambda(\cdot)$, $\mu(\cdot)$ are continuous increasing functions. Then for any (β, σ) that solve (7) for some $\alpha_b \in [0, 1/2]$, $\alpha_s \in [0, 1/2]$, there exist such p^s, p^b that the option swap (p^s, p^b) implements (β, σ). In particular, if the cross-effects are non-positive $B^*_\sigma \le 0$, $S^*_\beta \le 0$ and sufficiently symmetric $\left|B^*_\sigma - S^*_\beta\right| \le 1$ then there exist such p^s, p^b that the option swap implements the first best.*

In order to compare Propositions 3 and 4, let us assume $\omega_1 = \omega_2 = \omega$ and $\lambda(\omega) = \mu(\omega) = \kappa(\omega)$. In this case both Propositions hold and we indeed see that under the split-the-difference solution, the set of implementable tax rates is much broader: it is $[0, 1/2]$x$[0, 1/2]$ (domain H in the Figure 2) instead of $\alpha_b = \alpha_s \in [0, 1/2]$. There is also a similarity between the two results: in both cases, the optimal distance between the price limits p^b and p^s increases in the strength of cross-effects. If the cross-effects are absent, then $p^b = p^s$ attains the first best. Goldberg and Erickson (1987) report that $(p^b - p^s)/p^s$ varies from 30% to 100%, hence, cross-effects seem to be quite important in the petroleum coke industry.

Under the split-the-difference solution, options outperform the message-contingent mechanisms in Subsection 3.1 (which do not provide incentives for asymmetric cross-effects). In terms of Maskin and Moore (1999), the settings in subsections 3.1 and 3.2.2 are two different message games: although the mechanism design problem is the same, the renegotiation functions are different. The renegotiation function in 3.2.2 provides more degrees of freedom. In particular, parties can effectively manipulate ex post bargaining to achieve an asymmetric division of ex post surplus.

Remark 4 *While* two *option contracts are essential to provide two degrees of freedom, one does not need to use both* at the same time*. One can easily show that the pairs of tax rates $\alpha_b, \alpha_s \in [0, 1/2]$ such that $\alpha_b \le \alpha_s$ ($\alpha_s \ge \alpha_b$) can be implemented through the buyer's (the seller's) option only.*

3.3 Property rights

In this Subsection we consider a special case, where there is a alienable physical asset perfectly complementary to the relationship-specific investment. Allocation of property rights for such an asset can shift disagreement points in ex post bargaining. By reinforcing bargaining position of the asset owner at the expense of the other party, property rights can implement asymmetric tax rates.

Suppose that the buyer owns the asset at $t = 0$. Following Hart (1995) or Maskin and Tirole (1999), we assume that the buyer can threaten to leave the relationship and trade with an outside seller at $t = 1$. If the threat is implemented, S gets zero, while B gets utility of $v(\omega, \beta, \sigma) - c(\omega, 0, 0)$. Here $c(\omega, 0, 0)$ is the competitive price of an outside seller who has made no specific investments. Having this threat in mind, B and S bargain on the price p at $t = 1$. Equal bargaining power implies $p = (c(\omega, 0, 0) + c(\omega, \beta, \sigma))/2$.

For simplicity's sake we assume throughout this Section that $v(\omega, \beta, \sigma) - c(\omega, 0, 0) \geq 0$, and $c(\omega, 0, 0) \geq c(\omega, \beta, \sigma)$ (and, symmetrically, $v(\omega, 0, 0) - c(\omega, \beta, \sigma) \geq 0$ and $v(\omega, \beta, \sigma) \geq v(\omega, 0, 0)$) for all ω and for the first-best and second-best couples of β, σ. The first condition implies that the trade with an outside seller is still efficient; the gain from trade with the outside could also be described as the private value of self-consumption of the asset by B (Maskin and Tirole, 1999). The second condition requires that S is more efficient than the outside seller; this condition is less innocent. Since we consider negative cross-effects, the option of switching to an outside partner can be endogenously binding. For simplicity's sake, we rule this case out, even though studying the effect of this option on the parties' incentives is an interesting extention of our analysis.

Notice that we assume that cross-effects are embodied in the asset. If B keeps the asset and trades with an outsider, S's buyer-specific investment still affects B's valuation of the good. However, the cross-effects of B's investment which are specific to S are wasted: the cost of an outside seller is $c(\omega, 0, 0)$ rather than $c(\omega, \beta, 0)$.[11] E.g. consider the Fisher Body example (as discussed in Hart, 1995). Fishers invested in a physical technology for producing car bodies highly specific to General Motors car design; however, Fishers' skills to operate this technology were inalienable. Hence, if GM owned the physical assets but contracted with another car body producer, the value would still be $v(\omega, \beta, \sigma)$ while the cost would rise to $c(\omega, 0, 0) \geq c(\omega, \beta, \sigma)$.

At $t = 1/2$ the buyer expects to get $B + \frac{1}{2}S + \frac{1}{2}(\bar{p} - Ec(\omega, 0, 0))$, and the seller gets $\frac{1}{2}S - \frac{1}{2}(\bar{p} - Ec(\omega, 0, 0))$. The incentives to invest are therefore equivalent to the case with $\alpha_b = 1/2, \alpha_s = 0$ (compare to (7)).

Similarly, the seller's ownership is equivalent to $\alpha_b = 0, \alpha_s = 1/2$.

Parties can also agree to own the asset jointly. In this case, neither of them will be

[11] An alternative approach would be to assume that investments are specific to the investing party and bear fruit only if the investing party is present. In this case, the joint surplus from trading with an outside seller would be $v(\omega, \beta, 0) - c(\omega, \beta, 0)$ rather than $v(\omega, \beta, \sigma) - c(\omega, 0, 0)$. The choice between the two alternative settings is only relevant in the presence of cross-effects; it has not been addressed in the literature. We believe that our setting is more realistic; also, it is the one that is (implicitly) chosen by Rosenkrantz and Schmitz (1999). In their model, if one of the investing parties leaves the relationship, the total surplus (appropriated by the asset owner) still depends on her investment, even though it is below the joint surplus in the presence of both investing parties.

able to trade with outside parties (the other owner would veto outside deals), hence joint ownership is simply equivalent to the case without property rights.

Property rights matter as long as there is scope for ex post renegotiation. If parties sign the non-contingent renegotiation-proof contract ex ante, property rights become irrelevant. In this section we consider a general mechanism where parties ex ante on a verifiable partition of the state space Ω into subsets $\Omega^N, \Omega^C, \Omega^b$ and Ω^s such that $Prob\{\omega \in \Omega^C\} = \xi$, $Prob\{\omega \in \Omega^b\} = \zeta$, $Prob\{\omega \in \Omega^s\} = \eta$, and $Prob\{\omega \in \Omega^N\} = 1 - \xi - \zeta - \eta$. Whenever $\omega \in \Omega^N$, parties have no contract and bargain ex post (joint ownership without contracting); whenever $\omega \in \Omega^C$, parties can use the non-contingent contract $p = \overline{p}$; whenever $\omega \in \Omega^b$, the buyer gets ownership; and, whenever $\omega \in \Omega^s$, the ownership is assigned to the seller.

This combination of property rights and contracting can also be implemented through a mechanism similar to "stochastic ownership" (Hart, 1995). The parties agree that the asset is given to the buyer with probability ζ, given to the seller with probability η. With probability ξ, parties trade at the non-contingent $\overline{p}$. In the remaining contingencies, there is neither a contract nor property rights (joint ownership without contracting) — this outcome is equivalent to the null contract $\alpha_b = \alpha_s = 1/2$. The randomizing device is run after the investments are made.[12]

The choice of investment is given by

$$\left(1 - \frac{\eta}{2}\right) B_\beta + \frac{\zeta}{2} S_\beta = 1;\ \frac{\eta}{2} B_\sigma + \left(1 - \frac{\xi}{2}\right) S_\sigma = 1. \tag{13}$$

The incentives to invest are therefore equivalent to those under $\alpha_b = \zeta/2, \alpha_s = \eta/2$. Hence, contingent ownership can implement the tax rates $\alpha_b \in [0, 1/2], \alpha_s \in [0, 1/2]$. This means that the first best can be achieved if and only if the cross-effects are negative and sufficiently symmetric (part 2b of Corollary 1).

Proposition 5 *A combination of stochastic ownership, joint ownership (null contract) and the non-contingent contract achieves the first best if and only if the cross-effects are non-positive $B^*_\sigma \le 0$, $S^*_\beta \le 0$ and sufficiently symmetric $\left|B^*_\sigma - S^*_\beta\right| \le 1$.*

In terms of Fig. 2, stochastic ownership implements the first best only for the range of parameters H. For each pair of α_b, α_s within domain $H = [0, 1/2] \times [0, 1/2]$, one can find ζ, η, ξ such that $\alpha_b = \zeta/2$ and $\alpha_s = \eta/2$. Indeed, the area H is a convex combination of the non-contingent contract $(0, 0)$, the buyer's ownership $(1/2, 0)$, joint ownership without contracting $(1/2, 1/2)$, and the seller's ownership $(0, 1/2)$. Notice that the first best includes at most three rather than four arrangements. The buyer's (the seller's) ownership is not included in the optimal contract if $\alpha_b \le \alpha_s$ (if $\alpha_b \ge \alpha_s$), the

[12] Stochastic ownership can also be implemented via buyout options (Noldeke and Schmidt, 1998, or Maskin and Tirole, 1999). At time $t = 0$ the parties own the asset jointly. They sign a fixed-price contract but agree that it is only valid in some contingencies. It is agreed ex ante, that in other contingencies, parties can use buyout options: the seller has an option to buy the asset out at time $t = 4/6$ at price P_s, the buyer has an option to buy the asset out at time $t = 5/6$ at price P_b. Under certain assumptions on the nature of uncertainty, there exist such P_s and P_b that the choice of investment is given by (13).

non-contingent contract is never used if $\alpha_b + \alpha_s \geq 1/2$, and null contract is excluded if $\alpha_b + \alpha_s \leq 1/2$.

If the cross effects are negative, ownership is allocated away from the party with stronger negative cross-effects (according to Proposition 5, the greater $|S^*_\beta|$, the greater α_s, and the smaller ζ). Also, if both parties have stronger negative cross-effects (larger $|S^*_\beta|$ and $|B^*_\sigma|$), the joint ownership without contracting is used more and the non-contingent contract is used less often (both α_b and α_s increase).

If the first best cannot be achieved, the parties choose $\alpha_b, \alpha_s \in [0, 1/2] \times [0, 1/2]$ that maximize $W - \beta - \sigma$. The second best cannot belong to the interior of the square H, hence, the optimal arrangement is a combination of at most two outcomes: it is either a combination of vertical integration (ownership by one party) with the non-contingent contract, or a combination of vertical integration with joint ownership (without contracting); it can also be pure vertical integration, pure non-contingent contract (in this case property rights have no value), or joint ownership without contracting (neither contracting nor property rights have value).[13] The following Proposition characterizes some properties of the second best.

Proposition 6 *The following statements hold.*[14]

1. *If both cross-effects and self-effects are positive, and cross-effects are sufficiently weak $S_\beta > 0$, $B_\sigma > 0$, $B_\beta > 0$, $S_\sigma > 0$ and $S_\beta B_\sigma < B_\beta S_\sigma$ for all β, σ, then the second best is a combination of the non-contingent contract and vertical integration: $\alpha_b + \alpha_s < 1/2$.*

2. *If both cross-effects and self-effects are positive, and cross-effects are sufficiently strong $S_\beta > 0$, $B_\sigma > 0$, $B_\beta > 0$, $S_\sigma > 0$ and $S_\beta B_\sigma > B_\beta S_\sigma$ for all β, σ, then the second best is a combination of vertical integration and joint ownership*; *the non-contingent contract is never used $\alpha_b + \alpha_s > 1/2$.*

3. *If both self-effects are positive, the buyer's cross-effect is positive and the seller's cross-effect is non-positive $B_\beta > 0$, $S_\sigma > 0$, $S_\beta > 0 \geq B_\sigma$ for all β, σ, then the second best is either a combination of buyer's ownership and joint ownership without contracting, or a combination of buyer's ownership and the non-contingent contract; the seller never gets ownership $\alpha_b > \alpha_s$.*

4. *If both cross-effects are positive, the buyer's self-effect is positive and the seller's self-effect is non-positive $B_\beta > 0 \geq S_\sigma$, $S_\beta > 0, B_\sigma > 0$ for all β, σ, then the second best is a combination of buyer's ownership and joint ownership without contracting*

[13] This suggests an interesting empirical distinction between negative and positive cross-effects. In the situations where cross-effects are negative (and sufficiently symmetric), the optimal arrangement is more complex — it involves three different outcomes with positive probabilities. When at least one cross-effect is positive, the second best is a border solution and includes at most two outcomes: with some probability one party gets property rights, and in the remaining contingencies it is either non-contingent contract or null contract (joint ownership).

[14] Parts 3 and 4 refer to the buyer's investment. Similar statements for the seller's investment can be formulated and proven in a perfectly symmetric fashion.

$\alpha_b = 1/2$, $\alpha_s \in [0, 1/2]$. *If both cross-effects are positive, and both self-effects are non-positive, then the second best is pure joint ownership without contracting* $\alpha_b = \alpha_s = 1/2$.

These results are similar to those obtained for the first best. The non-contingent contract rewards self-effects, while the null contract rewards cross-effects. Whenever cross-effects are stronger than self-effects, the non-contingent contract is not used. Moreover, if self-effects are trivial or negative, joint ownership without contracting dominates all other arrangements; property rights have no value, nor does contracting.[15] If only one party has a positive cross-effect while the cross-effect of the other is trivial or negative, the optimal mechanism includes ownership of the party with positive cross-effect. Depending on the strength of the cross-effect, the optimal arrangement also includes either non-contingent contract or joint ownership.

Similarly to the option contracts under the split-the-difference bargaining solution, property rights allow shifting the disagreement points in the ex post bargaining. This allows property rights to outperform message-contingent games and to implement the first best if cross-effects are negative and sufficiently (but not necessarily perfectly) symmetric. It is crucial that property rights help adjusting the disagreement points but not the distribution of bargaining power. If bargaining power could be directly contracted upon (e.g. through design of ex post renegotiation, Aghion et al., 1994), then the first best would be implementable for any negative cross-effects; the symmetry constraint $\left|B^*_\sigma - S^*_\beta\right| \leq 1$ would be lifted. Indeed, arbitrary allocation of bargaining power would allow to implement all α_b, α_s such that $\alpha_b, \alpha_s \geq 0, \alpha_b + \alpha_s \leq 1$ (domains G, H, I in the Fig.2).

Remark 5 *The result that allocation of property rights achieves efficiency only if cross-effects are both negative and sufficiently* symmetric*, follows from the equal allocation of bargaining power. Suppose that bargaining splits the surplus between buyer and seller in proportion* $\gamma : 1-\gamma$. *Then the buyer's ownership results in* $\alpha_b = \gamma, \alpha_s = 0$, *the seller's ownership is equivalent to* $\alpha_b = 0, \alpha_s = 1-\gamma$, *and joint ownership (null contract) is simply* $\alpha_b = \gamma, \alpha_s = 1-\gamma$. *Hence, contingent ownership can implement the first best if and only if it can be achieved through* $\alpha_b \in [0, \gamma], \alpha_s \in [0, 1-\gamma]$. *Using (8), we find that the first best is achieved if and only if* $B^*_\sigma \leq 0$, $S^*_\beta \leq 0$ *and* $-\gamma \leq (1-\gamma)B^*_\sigma - \gamma S^*_\beta \leq 1-\gamma$.

4 Continuous quantity

The analysis above assumes that the binary quantity choice: "trade" vs. "no trade". This effectively restricts the contractible level of trade by one unit. Any quantity between 0 and 1 could be implemented through a combination of the null contract and the non-contingent contract; but parties cannot contract on quantities that are above one unit. In this Section, we consider a setting where parties can trade any positive quantity at $t = 1$.

[15] This result is similar to the analysis of know-how disclosure arrangements by Rosenkranz and Schmitz (1999) who show that importance of cross-effects makes joint ownership more efficient than other ownership structures.

More importantly, the quantity is verifiable ex post and can therefore be contracted upon at $t=0$. Although ex post renegotiation results in a quantity that maximizes the total gains of trade, the contracts on quantity provide parties with a wide choice of threatpoints (see Edlin and Reichelstein, 1996). It turns out that while the set of implementable outcomes is indeed broader, the main insights still hold.

To make the analysis tractable, we will assume additive separability: the buyer's utility of consuming q units of the good is $v(\omega,\beta,\sigma)q+\widetilde{v}(\omega,q)$, and the seller's production costs are $c(\omega,\beta,\sigma)q+\widetilde{c}(\omega,q)$. This form is similar to the ones considered in Edlin and Reichelstein (1996) and Segal and Whinston (2002). In both papers payoffs include yet another term that depends upon investments and state of nature ω. In our setting, this would make the analysis much more complex (Edlin and Reichelstein, 1996, do not allow for cross-effects; Segal and Whinston, 2002, consider one-sided investments).

We shall also assume that for all ω,β,σ the ex post efficient quantity

$$q^*(\omega,\beta,\sigma)\triangleq \arg\max_{q\geq 0}\left[v(\omega,\beta,\sigma)q+\widetilde{v}(\omega,q)-c(\omega,\beta,\sigma)q-\widetilde{c}(\omega,q)\right]$$

is finite. The first best choice of investment is given by (2) where the expected ex post surplus is $W(\beta,\sigma)=E\max_{q\geq 0}\left[v(\omega,\beta,\sigma)q+\widetilde{v}(\omega,q)-c(\omega,\beta,\sigma)q-\widetilde{c}(\omega,q)\right]$. To make the analysis comparable with that of the previous Section, we shall normalize quantity units so that $Eq^*(\omega,\beta^*,\sigma^*)=1$.

Introduction of quantity contracts does not change the results of Section 2.2.3 which determines the relationship between size and strength of cross-effects and the tax rates that implement the first best. However, a broader set of contracts may allow different mechanisms to implement a larger range of tax rates. Below, we shall consider these mechanisms one by one.

Non-contingent contract. A non-contingent contract obliges the seller to deliver $\overline{q}\geq 0$ units in exchange of the payment of $\overline{p}$. At $\overline{q}=0$, the non-contingent contract becomes the null contract.

The non-contingent contract is no longer renegotiation-proof: $\overline{q}\neq q^*(\omega,\beta,\sigma)$ for almost all ω. Hence it will be renegotiated to $q^*(\omega,\beta,\sigma)$, and the parties will split the increase in joint surplus. Straightforward calculations yield the following expected ex post payoffs for the buyer and the seller, respectively:

$$\frac{1}{2}W(\beta,\sigma)+\frac{1}{2}E\left[v(\omega,\beta,\sigma)\overline{q}+\widetilde{v}(\omega,\overline{q})+c(\omega,\beta,\sigma)\overline{q}+\widetilde{c}(\omega,\overline{q})\right]-\overline{p}$$
$$\frac{1}{2}W(\beta,\sigma)-\frac{1}{2}E\left[v(\omega,\beta,\sigma)\overline{q}+\widetilde{v}(\omega,\overline{q})+c(\omega,\beta,\sigma)\overline{q}+\widetilde{c}(\omega,\overline{q})\right]+\overline{p}$$

The choice of investment is therefore determined by

$$\frac{1+\overline{q}}{2}B_\beta+\frac{1-\overline{q}}{2}S_\beta=1;\ \frac{1+\overline{q}}{2}S_\sigma+\frac{1-\overline{q}}{2}B_\sigma=1.$$

In other words, a non-contingent contract $\overline{q}\geq 0$ can implement any α_b,α_s such that $\alpha_b=\alpha_s\leq 1/2$. Indeed, substituting $\alpha_b=\alpha_s=\frac{1-\overline{q}}{2}$,we obtain (7). It is important to emphasize that contracts on quantities in excess of the average optimal quantity $\overline{q}>Eq^*=1$ allow implementation of *negative* $\alpha_b = \alpha_s$ that correspond to positive perfectly symmetric cross-effects. By writing a contract for an inefficiently high quantity, parties provide each other with greater incentives to invest. It is not possible, however, to implement the first best if the cross-effects are stronger than self-effects (domain B in Fig.2); this would imply tax rates $\alpha_b = \alpha_s > 1/2$ which would require contracts with *negative* quantities.

Message games. Consider a general message-contingent mechanism: based on parties' reports on ω, β, σ, the mechanism prescribes quantity and price. Reconstructing the proof of Proposition 2, one can obtain the following results: the first best can be implemented if cross-effects are (i) symmetric and (ii) negative or weaker than cross-effects. If in all states of nature either (i) or (ii) does not hold, then the first best cannot be implemented.[16]

Option contracts. Similar results hold for the option contracts if we assume the "deal-me-out" solution. Options implement the first best only if cross-effects are negative or weaker than self-effects, and if cross-effects are perfectly symmetric.

If we assume the "split-the-difference" solution, then cross-effects must be sufficiently symmetric, but not necessarily perfectly symmetric. Indeed, consider a contract that provides the seller with an option to sell $\overline{q}$ at the price of p^s. The split-the-difference rule provides parties with the following expected ex post payoffs:

$$\begin{aligned}
&\frac{1}{2}W(\beta,\sigma) - \frac{1}{2}E\left[p^s - c(\omega,\beta,\sigma)\overline{q} - \widetilde{c}(\omega,\overline{q})\right]_+ ,\\
&\frac{1}{2}W(\beta,\sigma) + \ \frac{1}{2}E\left[p^s - c(\omega,\beta,\sigma)\overline{q} - \widetilde{c}(\omega,\overline{q})\right]_+ .
\end{aligned}$$

For simplicity, we shall again assume that the shock is multiplicative: $c(\omega,\beta,\sigma) = \mu(\omega)\overline{c}(\beta,\sigma)$. Then the first order conditions are as follows:

$$\frac{1}{2}(B_\beta + S_\beta) - \frac{1}{2}\eta\overline{q}S_\beta = 1;\ \frac{1}{2}(B_\beta + S_\beta) + \frac{1}{2}\eta\overline{q}S_\beta = 1, \tag{14}$$

where $\eta = Prob\{p^s > c(\omega,\beta,\sigma)\overline{q} + \widetilde{c}(\omega,\overline{q})\} \in [0,1]$ is the probability of the option being "in the money".

The formulas (14) are equivalent to (7) for $\alpha_b = (1-\eta\overline{q})/2$, $\alpha_s = 1/2$. The higher the option price p^s, the higher η; for any $\eta \in [0,1]$ there exists an option that implements η. Hence, through seller's options parties can implement any choice of $\alpha_b \leq 1/2, \alpha_s = 1/2$. Similarly, through call options (buyer's options to oblige the seller deliver certain quantity at a given price), one can implement $\alpha_b = 1/2, \alpha_s \leq 1/2$. By combining the two (i.e. including two options that are valid only with certain probability, e.g. in certain states of nature), parties can therefore implement any $\alpha_b \leq 1/2, \alpha_s \leq 1/2$. These tax rates provide efficient incentives whenever $\left(S^*_\beta + B^*_\sigma\right)/2 \leq 1/2$, and $\left|S^*_\beta - B^*_\sigma\right| \leq 1$. In other words, first, the parties' cross-effects should be negative or weaker *on average* than self-effects (either party's cross-effect can be stronger than her self-effect, but the sum of cross-effects should be less than the sum of self-effects). Second, the cross-effects should be sufficiently symmetric.

[16] The symmetry constraint is binding due to the lack of instruments: parties trade a scalar quantity of a single good while they need to provide incentives for two investment variables. However, the problem would hardly be solved even if parties traded several goods: the greater number of goods is traded, the higher dimensionality of investments in qualities of different goods.

Property rights. If there is a physical asset that is complementary to investments, allocation of property rights can also implement asymmetric α_b, α_s. Indeed, upstream (or downstream) integration helps implementing $\alpha_b = 1/2, \alpha_s = 0$ (or $\alpha_b = 1/2, \alpha_s = 0$). Hence for every surplus division rule α_b, α_s such that $\alpha_b \leq 1/2, \alpha_s \leq 1/2$, and $|\alpha_b - \alpha_s| < 1/2$, there exists a convex combination of vertical integration and a non-contingent contract that implements the rule. These rules provide efficient incentives whenever $\left|S_\beta^* - B_\sigma^*\right| \leq 1$, $S_\beta^* \leq 1/2$, and $B_\sigma^* \leq 1/2$. These constraints are more restrictive than those for the split-the-difference options: *each* cross-effect must be weaker than self-effect.

The above results can be summarized as the following

Proposition 7 *In the setting with continuous quantity, the first best can be implemented in the following cases.*

1. *Suppose that cross-effects are perfectly symmetric and are weaker than self-effects:* $S_\beta^* = B_\sigma^* < 1/2$. *Then a non-contingent contract* $\overline{q} = 1/(1 - 2S_\beta^*)$ *implements the first best.*

2. *If the split-the-difference solution applies, and the uncertainty is multiplicatively separable, the first best can be implemented through an option contract if cross-effects are negative or weaker* on average *than self-effects or negative* $\left(S_\beta^* + B_\sigma^*\right)/2 \leq 1/2$, *and the cross-effects are sufficiently symmetric* $\left|S_\beta^* - B_\sigma^*\right| \leq 1$.

3. *Suppose that there is a physical asset that is complementary to specific investments. The first best is implemented through allocation of property rights if the cross-effects are sufficiently symmetric* $\left|S_\beta^* - B_\sigma^*\right| \leq 1$, *and each cross-effect is either negative or weaker than self-effects:* $S_\beta^* \leq 1/2$, $B_\sigma^* \leq 1/2$.

5 Concluding remarks

We have studied a model of bilateral trade where contracts are incomplete, both parties invest, and investments may have both self- and cross-effects. We have calculated the "taxes" that parties have to pay to each other in order to provide efficient incentives and studied the ranges of tax rates that can be implemented via different contractual arrangements. The first best can only be achieved if cross-effects are (i) sufficiently weak or negative and (ii) symmetric (assuming equal bargaining power).

The intuition behind the first condition is straightforward. If cross-effects are negative, a non-contingent contract provides incentives for overinvestment relative to the social optimum. To reduce incentives, parties simply need to write a contract for a lower quantity (or to write no contract at all) ex ante; then ex post holdup and renegotiation adjust incentives down to the efficient level. If the cross-effects are positive, then parties need even greater incentives to invest than in the absence of cross-effects. To do so, parties need to write a contract for excessively high quantities. It turns out, however, that even contracts with very high quantities cannot implement first best if cross-effects are stronger than self-effects.

The second problem is the asymmetry of cross-effects. If the strength and sign of parties' cross-effects differ, a contract that encourages the first best investment by one party may provide inefficient incentives to the other. Suppose that one party's cross-effects are strong while the other's are weak or negative. Then for the former party to invest efficiently, the contract should include high quantity; while for the latter, it is best to have a contract with little or no trade.

In order to implement the first best, parties have to shift incentives from one party (e.g. the one with weak or negative cross-effects) to the other (the one with strong and positive cross-effects). This would certainly be feasible if parties could contract on reallocation of bargaining power. In this paper, however, we rule out such solutions as unrealistic and study the optimal choice of threatpoints. First, we consider option contracts. We find that for some bargaining solutions, option contracts can indeed relax the symmetry constraint. Second, in the special case where there is a specific physical asset, we study the optimal assignment of property rights. The reallocation of property rights also helps to relax the symmetry constraint. The properties of the first best allocation are intuitive: the property rights are more likely to be given to a party with positive cross-effects and are taken away from the party with negative cross-effects.

In addition to analyzing the first best, we also study the optimal allocation of property rights in the situations when the first best cannot be achieved. The second best arrangement is very simple. If cross-effects are weaker than self-effects, it includes vertical integration in some states and the non-contingent contract in others. The property rights are allocated to one party, but the parties also sign a non-contingent contract that is valid only in some contingencies. If cross-effects are strong, the optimal arrangement is different. It includes vertical integration in some states and joint ownership in others; contracting is never used. This is similar to the buyout options that are so common in venture capital contracts: the parties own the asset jointly but in some states ownership is transferred to one party (via equity options or via exit options). In the presence of cross-effects, joint ownership is no longer dominated by one party ownership. Whenever cross-effects are sufficiently strong, the optimal mechanism includes joint ownership with a positive probability. Moreover, for a certain range of parameters, joint ownership dominates all other arrangements; neither property rights, nor contracting have any value.

Our results extend the analysis of Che and Hausch (1999) and Segal and Whinston (2002) to a setting where both parties invest, and where cross-effects can differ in sign and intensity. First, the higher cross-effects, the lower the value of contracting (like in Che and Hausch) — hold-up is more important since it helps provide incentives for cross-effects. If cross-effects are weak or negative, the first best can be implemented (like in Segal and Whinston) but only if the cross-effects are symmetric. We show that property rights can provide incentives for the asymmetric cross-effects through strengthening the bargaining position of the party with stronger cross-effects. Our analysis re-emphasizes the difficulties of providing incentives in the setting with cooperative investments. While some mechanisms can help if cross-effects are negative or weak, there seems to be no solution if cross-effects are positive, and are stronger than self-effects.

Colophon

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Appendix: Proofs.

Proof of Proposition 1. The proof of part 1 is trivial. To prove 2, let us denote (β^C, σ^C) the choice of investments for the non-contingent Contract, and (β^N, σ^N) – the ones for the Null contract. We shall prove that if the cross-effects are positive and sufficiently strong, then (a) non-contingent contract results in underinvestment; (b) the underinvestment is more severe than the one under the null contract. Since the cross-effects are stronger than self-effects,

$$W_\beta(\beta^C, \sigma^C) = B_\beta(\beta^C, \sigma^C) + S_\beta(\beta^C, \sigma^C) > 2B_\beta(\beta^C, \sigma^C) > 2 = W_\beta(\beta^N, \sigma^N) > W_\beta(\beta^*, \sigma^*).$$

Since $W(\beta, \sigma)$ is concave, and $W_{\beta\sigma} = 0$, an increase in W_β requires a decrease in β. Indeed, $dW_\beta = W_{\beta\beta} d\beta$, and $W_{\beta\beta} < 0$. Hence $\beta^C < \beta^N < \beta^*$. Similarly, $W_\sigma^C > W_\sigma^*$, and $\sigma^C < \sigma^N < \sigma^*$. Since for all $\beta < \beta^*$, the ex ante social welfare $W - \beta - \sigma$ increases with β, and for all $\sigma < \sigma^*$ it increases with σ, the null contract is less inefficient than the non-contingent contract.

If the cross-effects are positive but weaker than self-effects, similar reasoning results in $\beta^N < \beta^C < \beta^*$ and $\sigma^N < \sigma^C < \sigma^*$. ■

Proof of Lemma 1. The proof is trivial: using (4) we obtain $B_\beta^* = 1 - S_\beta^*$ and $S_\sigma^* = 1 - B_\sigma^*$. Substituting into (7) and solving a system of two linear equations, we obtain (8). The system is not consistent if $S_\beta^* + B_\sigma^* = 1$ and $S_\beta^* \neq 1/2$. ■

Proof of Lemma 2. We shall prove that incentive compatibility constraints imply (9)-(11). Consider two arbitrary pairs $(b', s') \in \mathcal{B}\text{x}\mathcal{S}$ and $(b'', s'') \in \mathcal{B}\text{x}\mathcal{S}$. By definition, $U^B(b', s', b', s', b', s') \geq U^B(b'', s'', b', s', b', s')$ i.e.

$$\begin{aligned}&U^B(b', s', b', s', b', s') \geq (1 - X(b'', s'', b', s'))\,[b' + s']/2 + \\ &+ X(b'', s'', b', s')[b' + \bar{p} - p(b'', s'', b', s')] - T(b'', s'', b', s')\end{aligned}$$

On the other hand, $U^S(b'', s'', b', s', b'', s'') \leq U^S(b'', s'', b'', s'', b'', s'')$ which implies $U^B(b'', s'', b', s', b'', s'') \geq U^B(b'', s'', b'', s'', b'', s'')$ i.e.

$$\begin{aligned}&U^B(b'', s'', b'', s'', b'', s'') \leq (1 - X(b'', s'', b', s'))\,[b'' + s'']/2 + \\ &+ X(b'', s'', b', s')[b'' + \bar{p} - p(b'', s'', b', s')] - T(b'', s'', b', s').\end{aligned}$$

The two inequalities imply

$$K(b'', s'') - K(b', s') \leq \frac{b'' + s''}{2} - \frac{b' + s'}{2} + X(b'', s'', b', s') \left(\frac{b'' - s''}{2} - \frac{b' - s'}{2} \right). \quad (15)$$

Similarly, $U^B(b', s', b', s', b', s') \leq U^B(b', s', b'', s'', b', s')$ and $U^B(b', s', b'', s'', b'', s'') \leq U^B(b'', s'', b'', s'', b'', s'')$ imply $U^B(b'', s'', b'', s'', b'', s'') - U^B(b', s', b', s', b', s') \geq (1 - Y(b', s', b'', s''))(b'' - b') + X(b', s', b'', s'')(s'' - s')$. Hence,

$$K(b'', s'') - K(b', s') \geq \frac{b'' + s''}{2} - \frac{b' + s'}{2} + X(b', s', b'', s'') \left(\frac{b'' - s''}{2} - \frac{b' - s'}{2} \right). \quad (16)$$

Let us take $b'' = b' + \Delta$ and $s'' = s'$. Since $X \in [0,1]$, inequalities (15)-(16) imply (9). Similarly, by taking $b'' = b'$ and $s'' = s' + \Delta$, we obtain (10). By taking $b'' = b' + \Delta$ and $s'' = s' + \Delta$, we immediately arrive at (11). ■

PROOF OF PROPOSITION 2. The proof of the first part is trivial: one should set $X(b^B, s^B, b^S, s^S) = 1 - 2\alpha_b$, and $T(b^B, s^B, b^S, s^S) = 0$, where $\alpha_b = \alpha_s$ are given by (8). Since cross-effects are negative, $\alpha_b = \alpha_s \leq 1/2$, so $X \geq 0$ and $X \leq 1$ (Lemma 1).

Proof of 2a. Suppose that $s^*_\beta > 0$, $w^*_\beta > 0$ and the first best is achieved. This implies that β^* maximizes $EK(b(\omega, \beta, \sigma^*), s(\omega, \beta, \sigma^*)) - \beta$. Conditions (9)-(11) assure that K is a continuous function (though not necessarily differentiable). Thus, let us take two sequences β'_n, β''_n converging to β^* such that $\beta^* \in [\beta'_n, \beta''_n]$ and $EK(b(\omega, \beta'_n, \sigma^*), s(\omega, \beta'_n, \sigma^*))$ $-\beta'_n = EK(b(\omega, \beta''_n, \sigma^*), s(\omega, \beta''_n, \sigma^*)) - \beta''_n$. Denoting $\Delta\beta_n = \beta''_n - \beta'_n$, we obtain $EK(b(\omega, \beta''_n, \sigma^*), s(\omega, \beta''_n, \sigma^*)) - EK(b(\omega, \beta'_n, \sigma^*), s(\omega, \beta'_n, \sigma^*)) = \Delta\beta_n$.

Denote $b'_n = b(\omega, \beta'_n, \sigma^*)$, $s'_n = s(\omega, \beta'_n, \sigma^*)$, $b''_n = b(\omega, \beta''_n, \sigma^*)$, $s''_n = s(\omega, \beta''_n, \sigma^*)$. Then

$$\Delta\beta_n = E[K(b'_n + s'_n - s''_n, s''_n) - K(b'_n, s'_n)] + E[K(b''_n, s''_n) - K(b'_n + s'_n - s''_n, s''_n)] \quad (17)$$

Dividing and multiplying the first term by $(s''_n - s'_n)$ we obtain

$$E[K(b'_n + s'_n - s''_n, s''_n) - K(b'_n, s'_n)] = -\xi S^*_\beta \Delta\beta_n + o(\Delta\beta_n). \quad (18)$$

Here ξ is a weighted average of $[K(b'_n, s'_n) - K(b'_n + s'_n - s''_n, s''_n)]/(s''_n - s'_n)$ across states of nature. Since the cross-effect of buyer's investment is positive $s_\beta > 0$, we have $s''_n - s'_n = s_\beta \Delta\beta_n + o(\Delta\beta_n) > 0$ for sufficiently small $\Delta\beta_n$. Using (11) we obtain $\xi \in [0,1]$.

Similarly,

$$E[K(b''_n, s''_n) - K(b'_n + s'_n - s''_n, s''_n)] = \eta W^*_\beta \Delta\beta_n + o(\Delta\beta_n) = \eta\Delta\beta_n + o(\Delta\beta_n), \quad (19)$$

where η is a weighted average of $[K(b''_n, s''_n) - K(b'_n + s'_n - s''_n, s''_n)]/[(b''_n + s''_n) - (b'_n + s'_n)]$ across states of nature. Since $w_\beta(\omega, \beta^*, \sigma^*) > 0$, $b''_n + s''_n > b'_n + s'_n$ holds, and condition (9) implies $\eta \in [1/2, 1]$.

Substituting (18), (19) for the right-hand side in (17) and using Taylor expansion for $E\{(b^* + s^*) - (b + s)\}$ we obtain $1 = \eta - \xi S^*_\beta$. This can only be satisfied when $\xi = 0$ and $\eta = 1$ (otherwise $\eta - \xi S^*_\beta < 1$). But this is not possible: $\xi = 0$ requires $X = 0$ for all messages, so that η must be equal to $1/2$.

Proof of 2b. First, (9)-(11) implies that $K(b,s) = \frac{b+s}{2} + L\left(\frac{b-s}{2}\right)$, where $L(\cdot)$ is such that $L(\xi + \Delta) - L(\xi) \in [0, \Delta]$ for every ξ and $\Delta > 0$. The seller gets $b + s - K(b,s) = \frac{b+s}{2} - L\left(\frac{b-s}{2}\right)$. Suppose that the first best is achieved. Since β^* maximizes $EK - \beta = W/2 + EL - \beta$, a small change of $\Delta\beta = \xi$ in the buyer's investment should involve the change in the expected value of L by $\xi - \frac{1}{2}W^*_\beta \xi + o(\xi) = \frac{1}{2}\xi + o(\xi)$. Similarly, since σ^* maximizes $W/2 - EL - \sigma$, a small change of $\Delta\sigma = -\xi$ must change the expected value of L by $\xi - \frac{1}{2}W^*_\sigma \xi + o(\xi) = \frac{1}{2}\xi + o(\xi)$. However, since $b_\beta - s_\beta < b_\sigma - s_\sigma - (\delta_1 + \delta_2)$,this can only be the case if EL is constant in the neighborhood of the first best (up to a change of the order of $o(\Delta\beta) = o(\Delta\sigma)$). This, however, implies that parties incentives are equivalent to those under the null contract. Since the null contract is never efficient, this is not possible.

Proof of 3. Suppose that a mechanism X, p, T implements a surplus division rule $K(b,s)$. Let us denote β^0, σ^0 the investment levels chosen by the parties. We shall show that if in some states of nature $X > 0$ then the mechanism is dominated by the null contract. First, let us show that $W_\beta(\beta^0, \sigma^0) \geq 1$ and $W_\sigma(\beta^0, \sigma^0) \geq 1$ with at least one inequality being strict. Neither party overinvests and at least one underinvests. Therefore if parties were able to implement higher levels of investments, the change in social surplus would be positive: $d(W - \beta - \sigma) = (W_\beta - 1)d\beta + (W_\sigma - 1)d\sigma > 0$ for $d\beta > 0$ and $d\sigma > 0$.

By definition β^0 maximizes $EK(b(\omega, \beta, \sigma^0), s(\omega, \beta, \sigma^0)) - \beta$. Similarly to the Proof of Part 2a, we shall take an arbitrary sequence β_n' that converges to β^0 from below $\beta_n' < \beta^0$. Since K is a continuous function, for each $\beta_n' < \beta^0$ we can introduce another sequence β_n'' that converges to β^0 from the above $\beta_n'' > \beta^0$ and $EK(b(\omega, \beta_n', \sigma^0), s(\omega, \beta_n', \sigma^0)) - \beta_n' = EK(b(\omega, \beta_n'', \sigma^0), s(\omega, \beta_n'', \sigma^0)) - \beta_n''$. When $n \to \infty$, we have both $\beta_n' \to \beta^0$ and $\beta_n'' \to \beta^0$. Hence, $EK(b(\omega, \beta_n'', \sigma^0), s(\omega, \beta_n'', \sigma^0)) - EK(b(\omega, \beta_n', \sigma^0), s(\omega, \beta_n', \sigma^0)) = \Delta\beta_n$, where $\Delta\beta_n = \beta_n'' - \beta_n'$.

Denote $b_n' = b(\omega, \beta_n', \sigma^0)$, $s_n' = s(\omega, \beta_n', \sigma^0)$, $b_n'' = b(\omega, \beta_n'', \sigma^0)$, $s_n'' = s(\omega, \beta_n'', \sigma^0)$. Then

$$\Delta\beta_n = E[K(b_n' + s_n' - s_n'', s_n'') - K(b_n', s_n')] + E[K(b_n'', s_n'') - K(b_n' + s_n' - s_n'', s_n'')]\cdot$$

Since the cross-effect is positive, $s_n'' - s_n' > 0$ (for sufficiently large n). Conditions (9)-(11) imply that the first term is non-positive. Using (9), we can show that the second term equals $\eta W_\beta(\beta^0, \sigma^0)\Delta\beta + o(\Delta\beta_n)$ where $\eta \in [0,1]$. Therefore $W_\beta(\beta^0, \sigma^0) \geq 1$. Similarly, $W_\sigma(\beta^0, \sigma^0) \geq 1$. At least one of the inequalities must be strict; otherwise the mechanism would have achieved the first best which is not possible according to Part 1.

Let us consider the mechanism $\tilde{X}, \tilde{p}, \tilde{T}$ that implements the null contract: $\tilde{X} = 0, \tilde{p} = \overline{p}, \tilde{T} = 0, \tilde{K}(b,s) = (b+s)/2$. Let us prove that parties invest (weakly) more under $\tilde{K}$ than under K. We shall prove that for a given σ, the buyer has more incentives to invest under $\tilde{K}$. Indeed, let us fix σ and β' and consider $\beta'' > \beta'$ sufficiently close to β'. Compare the parties' payoffs for outcomes $(b', s') = (b(\omega, \beta', \sigma), s(\omega, \beta', \sigma))$ and $(b'', s'') = (b(\omega, \beta'', \sigma), s(\omega, \beta'', \sigma))$. The mechanism $\tilde{X}$, $\tilde{Y}$, $\tilde{T}^B$ allows to implement a surplus division rule such that $\tilde{K}(b'', s'') - \tilde{K}(b', s') > K(b'', s'') - K(b', s')$ for all $\beta' < \beta''$. Using, let us (15) for X, T, we obtain $K(b'', s'') - K(b', s') \leq \tilde{K}(b'', s'') - \tilde{K}(b', s') + X(b', s', b'', s'')[(s'' - s') - (b'' - b')]$ which is non-negative. Indeed, $(s'' - s') > (b'' - b')$ since cross-effects are stronger than self-effects $s_\beta > b_\beta$. Therefore under $\tilde{K}$, the buyer chooses a (weakly) greater β for a given σ. Since $b_{\beta\sigma} = 0$, the buyer's optimal choice does not depend on the σ. Hence, the buyer invests (weakly) more under the mechanism $\tilde{K}$ rather than under K. Similarly, we can prove that the seller chooses greater σ. Since both parties want to invest more, the change in welfare is non-negative.

The proof of the last statement of the Proposition is perfectly similar. ■

Proof of Proposition 3. Since the shock is multiplicative, for each $\varsigma \in [0,1]$ there exist p^b, p^s such that the investment levels solve

$$(1 - \varsigma/2)B_\beta + \varsigma S_\beta/2 = 1, \;\; (1 - \varsigma/2)S_\sigma + \varsigma B_\sigma/2 = 1. \tag{20}$$

Indeed, let us introduce $\omega_1(\beta, \sigma)$ and $\omega_2(\beta, \sigma)$ such that $\kappa(\omega_1(\beta, \sigma))(\bar{v}(\beta, \sigma) + \bar{c}(\beta, \sigma))/2 =$

p^s and $\kappa(\omega_2(\beta,\sigma))(\bar{v}(\beta,\sigma)+\bar{c}(\beta,\sigma))/2 = p^b$. The buyer's payoff is

$$\int_0^{\omega_1(\beta,\sigma)} (v-p^s)f(\omega)d\omega + \int_{\omega_1(\beta,\sigma)}^{\omega_2(\beta,\sigma)} \frac{v-c}{2} f(\omega)d\omega + \int_{\omega_2(\beta,\sigma)}^1 (v-p^b)f(\omega)d\omega - \beta.$$

The first-order condition is

$$B_\beta - \int_{\omega_1(\beta,\sigma)}^{\omega_2(\beta,\sigma)} \frac{v_\beta + c_\beta}{2} f(\omega)d\omega = B_\beta - \frac{\bar{v}_\beta + \bar{c}_\beta}{2} \int_{\omega_1(\beta,\sigma)}^{\omega_2(\beta,\sigma)} \kappa(\omega)f(\omega)d\omega = 1. \qquad (21)$$

Similarly, the seller's investment choice is given by

$$S_\sigma + \int_{\omega_1(\beta,\sigma)}^{\omega_2(\beta,\sigma)} \frac{v_\sigma + c_\sigma}{2} f(\omega)d\omega = S_\sigma + \frac{\bar{v}_\sigma + \bar{c}_\sigma}{2} \int_{\omega_1(\beta,\sigma)}^{\omega_2(\beta,\sigma)} \kappa(\omega)f(\omega)d\omega = 1.$$

Since ω_1 increases with p^s and ω_2 increases with p^b one can choose such p^b, p^s that

$$\int_{\omega_1(\beta,\sigma)}^{\omega_2(\beta,\sigma)} \kappa(\omega)f(\omega)d\omega = \varsigma \int_0^1 \kappa(\omega)f(\omega)d\omega$$

Under such p^b, p^s the f.o.c. become (20). Hence, for arbitrary $\alpha_b = \alpha_s \in [0,1/2]$ we can take $\varsigma = 2\alpha_s$ so that (20) coincide with (7).

The last statement of the proposition directly follows from the Corollary 1. ■

Proof of Proposition 4. The proof is similar to that of Proposition 3. Since the shock is separably multiplicative, for each $\xi \in [0,1]$ there exists p^b such that

$$\int_{v \le p^b} v_\beta f(\omega)d\omega = \xi B_\beta, \quad \int_{v \le p^b} v_\sigma f(\omega)d\omega = \xi B_\sigma.$$

Indeed, such p^b must solve $\int_{\lambda(\omega_1) \le p^b/\bar{v}(\beta,\sigma)} \lambda(\omega_1)f(\omega_1)d\omega_1 = \xi \int \lambda(\omega_1)f(\omega_1)d\omega_1$. For $\xi = 0$ this equation has a solution $p^b = \bar{p}$, for $\xi = 1$ there is a solution $p^b = \sup v$. Hence such p^b exists for all $\xi \in [0,1]$.

Similarly, we can prove that for each $\eta \in [0,1]$ there exists p^s such that

$$\int_{c \ge p^s} (-c_\beta)f(\omega_2)d\omega_2 = \eta S_\beta, \quad \int_{c \ge p^s} (-c_\sigma)f(\omega_2)d\omega_2 = \eta S_\sigma.$$

Given p^b, p^s, the buyer maximizes $E(v - (\min\{v,p^b\} + \max\{c,p^s\})/2) - \beta$ i.e.

$$B - \frac{1}{2}\int_{v>p^b} p^b f(\omega)d\omega - \frac{1}{2}\int_{v \le p^b} v f(\omega)d\omega - \frac{1}{2}\int_{c>p^s} c f(\omega)d\omega - \frac{1}{2}\int_{c \le p^s} p^s f(\omega)d\omega - \beta.$$

Since the term proportional to the change in the integration limits is infinitesimal, the first-order condition becomes $(1-\xi/2)B_\beta + \eta S_\beta/2 = 1$. Similarly, the seller's choice is given by the first order condition $(1-\eta/2)S_\sigma + \xi B_\sigma/2 = 1$.

For arbitrary $\alpha_b, \alpha_s \in [0,1/2]$ we can take $\xi = 2\alpha_s, \eta = 2\alpha_s$ so that the first order conditions coincide with (7).

The last statement of the proposition directly follows from the Corollary 1. ■

PROOF OF PROPOSITION 5. The proof directly follows from (8). ■

PROOF OF PROPOSITION 6. Let us introduce

$$J(\alpha_b, \alpha_s) = W(\beta(\alpha_b, \alpha_s), \sigma(\alpha_b, \alpha_s)) - \beta(\alpha_b, \alpha_s) - \sigma(\alpha_b, \alpha_s)$$

where $\beta(\alpha_b, \alpha_s), \sigma(\alpha_b, \alpha_s)$ solve (7) under given α_b, α_s. Then

$$\begin{aligned} \frac{\partial J}{\partial \alpha_b} &= \frac{(W_\beta - 1)S_\beta}{-[(1-\alpha_s)B_{\beta\beta} + \alpha_b S_{\beta\beta}]} - \frac{(W_\sigma - 1)S_\sigma}{-[(1-\alpha_b)S_{\sigma\sigma} + \alpha_s B_{\sigma\sigma}]}, \\ \frac{\partial J}{\partial \alpha_s} &= \frac{(W_\sigma - 1)B_\sigma}{-[(1-\alpha_b)S_{\sigma\sigma} + \alpha_s B_{\sigma\sigma}]} - \frac{(W_\beta - 1)B_\beta}{-[(1-\alpha_s)B_{\beta\beta} + \alpha_b S_{\beta\beta}]}. \end{aligned} \tag{22}$$

The second best choice of α_b, α_s maximizes $J(\alpha_b, \alpha_s)$ over the square $\alpha_b \in [0, 1/2]$, $\alpha_s \in [0, 1/2]$. Since the first best cannot be achieved, there cannot be an interior solution. Determining the signs of $\partial J/\partial \alpha_b$ and $\partial J/\partial \alpha_s$ is made easy by two observations: (i) denominators in (22) are non-negative, (ii) a positive cross-effect $S_\beta > 0$ implies underinvestment by the respective party: $W_\beta - 1 > 0$. The first fact follows from concavity. Let us prove the second one. Suppose that $W_\beta - 1 \leq 0$. Using (7) we obtain $0 \geq W_\beta - 1 = (1 - \alpha_b - B_\beta(1 - \alpha_b - \alpha_s))/\alpha_b$. Since the cross effect is positive, $B_\beta < W_\beta \leq 1$. Hence, the right-hand side is strictly greater than $\alpha_s/\alpha_b \geq 0$.

The first two statements deal with the dichotomy between the mechanisms with $\alpha_b + \alpha_s < 1/2$ (combination of vertical integration and the non-contingent contract) and $\alpha_b + \alpha_s > 1/2$ (combination of vertical integration and joint ownership). Suppose that the latter is the case. Then in the second best, both $\partial J/\partial \alpha_b, \partial J/\partial \alpha_s$ must be non-negative and one of them must be strictly positive. Then (22) implies $S_\beta/S_\sigma > B_\beta/B_\sigma$. Hence, whenever cross-effects are weak $S_\beta B_\sigma < B_\beta S_\sigma$, then $\alpha_b + \alpha_s > 1/2$ cannot be the case; the non-contingent contract is used with a non-trivial probability. Similarly, $\alpha_b + \alpha_s > 1/2$ implies $S_\beta/S_\sigma < B_\beta/B_\sigma$.

The proof of the third statement is also straightforward. If the seller's cross-effect is negative, then there can be three cases: (i) the seller underinvests $W_\sigma - 1 > 0$, then $\partial J/\partial \alpha_s < 0$, so the optimal contract is a combination of the non-contingent contract and buyer's ownership; (ii) the seller overinvests $W_\sigma - 1 < 0$, then $\partial J/\partial \alpha_b > 0$, the second best is a combination of buyer's ownership and joint ownership; (iii) the seller invests at the first best level $W_\sigma - 1 = 0$, then $\partial J/\partial \alpha_s < 0 < \partial J/\partial \alpha_b$, the second best is pure buyer's ownership.

If the seller's self-effect is non-positive, then (22) implies $\partial J/\partial \alpha_s > 0$. Similarly, if the buyer's self-effect is non-positive, then $\partial J/\partial \alpha_s > 0$. Hence if both self-effects are negative or zero, it is optimal to have pure joint ownership $\alpha_b = \alpha_s = 1/2$. ■

PROOF OF PROPOSITION 7. The proof directly follows from the discussion in Section 4 and Lemma 1. ■

REFERENCES

Aghion, Philippe, Mathias Dewatripont and Patrick Rey (1994). "Renegotiation Design with Unverifiable Information", *Econometrica*, 62, 257-82.

Binmore, Ken, Avner Shaked and John Sutton (1989). "An outside option experiment", *Quarterly Journal of Economics.* CIV (4), 753-770.

Che, Yeon-Koo and Donald B. Hausch (1999). "Cooperative investments and the value of contracting", *American Economic Review*, 89, 125-147.

Chiu, Y. Stephen (1998). "Noncooperative bargaining, hostages, and optimal ownership." *American Economic Review*, 88, 882-901.

Demsky, Joel and David Sappington (1991). "Resolving double moral hazard problems with buyout agreements", *Rand Journal of Economics*, 22 (2), 232-240.

De Meza, David and Ben Lockwood (1998). "Does asset ownership always motivate managers? Outside options and the property rights theory of the firm", *Quarterly Journal of Economics*, 113 (2), 361–386.

Edlin, Aaron and Benjamin Hermalin (2000). "Contract renegotiation and agency problems", *Journal of Law, Economics and Organization*, 16 (2), 395-423.

Edlin, Aaron and Stefan Reichelstein (1996). "Holdups, standard breach remedies and optimal investment", *American Economic Review*, 86, 478-501.

Goldberg, Victor P. and John R. Erickson (1987). "Quantity and price adjustment in long-term contracts: A case study of petroleum coke", *Journal of Law and Economics*, 30, 369-98.

Hart, Oliver (1995). Firms, contracts and financial structure. Clarendon Lectures in Economics. Oxford and New York: Oxford University Press, Clarendon Press.

Hart, Oliver and John Moore (1988). "Incomplete contracts and renegotiation", *Econometrica*, 56, 755-85.

Joskow, Paul (1987). "Contract duration and relationship-specific investments: empirical evidence from coal markets", *American Economic Review*, 77, 168-85.

Joskow, Paul (1990). "The performance of long-term contracts: further evidence from coal markets", *Rand Journal of Economics*, 21, 251-74.

MacLeod, W. Bentley and James Malcolmson (1993). "Investments, holdup, and the form of market contracts", *American Economic Review*, 83, 811-37.

Maskin, Eric (1999). "Nash equilibrium and welfare optimality", *Review of Economic Studies*, 66 (1), 23-38.

MASKIN, ERIC AND JOHN MOORE (1999). "Implementation and renegotiation", *Review of Economic Studies*, 66 (1), 39-56.

MASKIN, ERIC AND JEAN TIROLE (1999). "Two remarks on property rights", *Review of Economic Studies*, 66 (1), 139-149.

MASTEN, SCOTT E. (ED.) (1996). Case Studies in Contracting and Organization. NY, Oxford: Oxford University Press.

MOORE, JOHN (1992). "Implementation, contracts, and renegotiation in environments with complete information". In Laffont, Jean-Jacques (ed.), *Advances in economic theory: Sixth World Congress.* Vol. 1. Econometric Society Monographs, No. 20. Cambridge, New York and Melbourne: Cambridge University Press, 182-282.

MUTHOO, ABHINAY (1999). Bargaining theory with applications. Cambridge, New York and Melbourne: Cambridge University Press.

NOLDEKE, GEORG AND KLAUS M. SCHMIDT (1995). "Option contracts and renegotiation: a solution to the hold-up problem", *Rand Journal of Economics*, 26, 163-79.

NOLDEKE, GEORG AND KLAUS M. SCHMIDT (1998). "Sequential investments and options to own", *Rand Journal of Economics.* Vol. 29, No. 4, 633-653.

OSBORNE, MARTIN J. AND ARIEL RUBINSTEIN (1990). *Bargaining and Markets.* Academic Press.

ROSENKRANZ, STEPHANIE, AND PARTICK SCHMITZ (1999). "Know-how disclosure and incomplete contracts", *Economics Letters.* Vol. 63, 181-85.

SEGAL, ILYA, AND MICHAEL WHINSTON (2002). "The Mirrlees Approach to Mechanism Design with Renegotiation (with Applications to Hold-Up and Risk-Sharing)", *Econometrica*, 70 (1), 1-45.