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Barter for price discrimination

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Abstract

We study barter as a discriminatory instrument in oligopoly with asymmetric information. Buyers (producers of final goods) differ in the quality of their products. Sellers (producers of inputs) use barter as a screening device: the higher quality buyers pay in cash while the lower quality ones pay in kind. Barter, identified with non-monetary contracts that give a seller control over a buyer's output, emerges in equilibrium even in the absence of financial constraints.

There is a positive relationship between market concentration and the level of barter. Barter disappears as the market becomes more competitive. Barter and no-barter equilibria coexist for a range of market structures.

JEL classification: D43; L13; P42

Keywords: Barter; Price discrimination; Oligopoly

1. Introduction

Monetary economics predicts that money should crowd out barter as a medium of exchange. The superiority of money is established in various general equilibrium settings with asymmetric information and/or random matching (e.g. Kiyotaki and Wright, 1989; Banerjee and Maskin, 1996). However, barter continues to be used in the trade between OECD and developing economies (Marin and Schnitzer, 1999) and is growing within OECD economies. The barter exchanges in the US reached a value of 10 billion dollars in 1998 (Economist, 2000). Moreover, in several transition economies, barter has effectively overtaken money as the major means of exchange in the second half of 1990s; barter

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accounted for 30% to 70% of inter-firm transactions in Russia (Aukutzionek, 1998; Seabright, 2000).[1]

The International Reciprocal Trade Association, the leading association of barter companies in OECD countries, offers (IRTA, 2001) two common sense explanations of barter: (i) financial constraints: 'barter is a relatively inexpensive method of finance', and (ii) spare capacity: 'to take advantage of barter, a firm must have slow-moving or non-performing assets to exchange, or spare capacity to take on additional sales'. The former argument is quite intuitive, the latter is less so. If a firm sells some of its goods for cash and the rest for barter at different relative prices, then the firm effectively engages in price discrimination. It is not clear, however, why in order to discriminate firm has to use barter rather than cash contracts (e.g. discounts on 'slowly moving' goods).

The paper presents a model of imperfect competition in which barter contracts indeed enhance firm's ability to price discriminate. The argument is based on asymmetric information: the quality of the good involved in barter payments is better known to its producer. Barter contracts may, therefore, be used as a screening device. The firms that produce output of higher quality prefer to keep it and pay the supplier in cash while the firms with low quality output keep cash and pay in kind. This self-selection, in turn, allows the supplier to benefit from barter. Were there no barter, the volume of trade would be inefficiently low due to imperfect competition; some customers willing to pay a price above the marginal cost would not be served. Barter allows serving the lower quality customers without sacrificing the profits from the high quality ones who prefer to pay in cash.

The issue of barter as a price discriminatory device has been addressed in the literature. Caves (1974) and Caves and Marin (1992) show that the price discrimination is responsible for the widespread use of countertrade in trade between OECD and developing countries.[2] The model of Caves (1974), however, is applicable only to international trade when customers are exogenously separated and first or third-degree price discrimination is possible. In contrast, this paper develops a model of second-degree price discrimination.

Prendergast and Stole (1998, 1999) Ellingsen (1998) and Marin and Schnitzer (1999) show, in different settings, that barter emerges as a means of segmenting markets in the presence of asymmetric information or contractual incompleteness, (bilateral) monopoly, and liquidity constraints. The presence of liquidity constraints is crucial to all these models. Ellingsen (1998) shows that barter helps to separate buyers whenever liquidity constraints do not allow firms to discriminate through money. Prendergast and Stole (1998) prove that in their setting, barter emerges only in the presence of liquidity constraints. Marin and Schnitzer (1999) explicitly refer to liquidity constraints as to one of the two major building blocks of their model.

Our contribution to the existing literature is two-fold. First, we show that barter may emerge as a means of price discrimination even if there are no liquidity constraints.

[1] In the work of Banerjee and Maskin (1996), barter may prevail in an equilibrium with high inflation. In Russia, however, the growth of barter was observed after the inflation was brought down.

[2] Ellingsen and Stole (1996) suggest that international barter may act as a commitment device not to engage in unilateral imports. Magenheim and Murrell (1988) put forward yet another reason to use barter for price discrimination: in a repeated game, barter helps not to reveal the seller's type to future customers.

Moreover, to demonstrate that barter, even if it is extremely inefficient, can emerge in the presence of market power we introduce additional costs of barter: lack of the double coincidence of wants and imperfect divisibility. Second, we analyze partial equilibrium in the case of oligopoly rather than monopoly. The analysis of strategic interaction among sellers does produce new insights. Self-selection of buyers gives rise to the emergence of the 'cash demand externality' that one seller's choice imposes on the others. If a firm sells a greater share of its output for cash, cash prices go down, and the most efficient barter customers switch to cash payments. With these customers leaving the barter economy, the average quality of in-kind payments deteriorates and the firm's competitors have more incentives to sell for cash. The cash demand externality leads to multiple equilibria which, in turn, may explain why barter has been so widespread in Russia but not in the other countries.

The paper proceeds as follows. Section 2 lays out a model and examines equilibria in the cases of monopoly and oligopoly. Section 3 illustrates the theoretical findings using empirical evidence on barter in Russia. Section 4 concludes.

2. The model

In this section, we study a model of barter as a screening device for price discrimination. Subsection 2.1 starts with a standard model of monopoly serving a continuum of buyers. Subsection 2.2 introduces barter contracts, Subsection 2.3 extends the analysis to the case of oligopoly.

2.1. The setting

Monopoly seller S with the constant marginal costs of production $c \in [0, 1]$ supplies an input to a continuum of buyers (industrial firms). Each buyer B has access to a linear technology with the maximum capacity of one unit which transforms $q \in [0, 1]$ units of the input into q units of output. Each unit of output is worth v to the buyer. B's productivity (type) v is her private information and is distributed on [0, 1] with a c.d.f. $F(v)$. The distribution function $F(v)$ is common knowledge. B's ex-ante outside option is zero.

The timing is as follows. The seller S offers a menu of contracts, each buyer B learns her type v and chooses a contract, the contract is signed and input delivery occurs. Then buyers produce output, and the quality of each buyer's output v is observed by all agents.

The timing has two implications that are crucial for the following analysis. First, while the buyer's type is private information at the time of purchasing the input, it is revealed later. Effectively, we assume that the type is realized during production, so that there is no information asymmetry at the time of selling the output.

Second, the input cannot be resold by one buyer to another buyer: once purchased, it can only be used in the production. This assumption is common for all price discrimination models. It is related to the Myerson–Satterthwaite theorem: since at the time of potential resale, the parties' valuations of the input are private information, there arise transaction costs in the secondary market. Also, there may be technological barriers to resale: e.g. natural gas, electricity, heating, transportation (or other) services cannot be easily resold.

Denote by $G(v)$ the average value of output given it is below v:

$$G(v) = \int_0^v x\mathrm{d}F(x) \Big/ \int_0^v \mathrm{d}F(x) \tag{1}$$

Assumption A1. The density $f(v)=F'(v)$ is continuous, positive, and non-increasing for all $v\in[0, 1]$. $v-G(v)$ is an increasing function of v. The hazard rate $f(v)/(1-F(v))$ is a non-decreasing function of v.

The assumption ensures that the second-order conditions are satisfied for all optimization problems throughout the paper. In the absence of the assumption, most results hold, but the analysis becomes more involved. Assumption A1 is satisfied whenever the distribution is sufficiently close to uniform. In particular, for the uniform distribution $F(v)=v$, $f(v)=1$, $G(v)=v/2$, $v-G(v)=v/2$, and $f(v)/(1-F(v))=1/(1-v)$.

The first best outcome is to supply one unit of the input to the buyers with $v\geq c$ and shut down all the others. This outcome would be implemented if the input market were perfectly competitive. The price of the input would be equal to its marginal cost c so that only buyers with $v\geq c$ would buy the input and produce. Aggregate social welfare would be $W^* = \int_c^1 (v-c)f(v)\mathrm{d}v$.

In the second best outcome, the seller offers a menu of contracts $\{(p, q)\}$: 'buy $q\in[0, 1]$ units of input for the monetary payment of p'. The Revelation Principle allows restating S's problem as follows. The seller chooses a menu of contracts $\{(p(v), q(v))\}$, $v\in[0, 1]$ to maximize

$$\int_0^1 (p(v) - cq(v))f(v)\mathrm{d}v$$

subject to incentive compatibility constraints

$$vq(v) - p(v) \geq vq(v') - p(v') \text{ for all } (v, v')\in[0,1]\times[0,1],$$

and individual rationality constraints

$$vq(v) - p(v) \geq 0 \text{ for all } v\in[0,1].$$

The standard analysis of this adverse selection problem yields

$$q(v) = \arg\max_{q\in[0,1]} q\left[v - c - \frac{1-F(v)}{f(v)}\right].$$

Thus, the seller offers only two contracts $\{(p^{\mathrm{m}}, 1), (0, 0)\}$. The price p^{m} solves

$$p^{\mathrm{m}} - c = \frac{1-F(p^{\mathrm{m}})}{f(p^{\mathrm{m}})}. \tag{2}$$

Assumption A1 implies that $p^{\mathrm{m}}\in[c, 1]$ is unique. Only buyers with $v\geq p^{\mathrm{m}}$ buy the input and produce. The deadweight loss

$$\int_c^{p^{\mathrm{m}}} (v-c)f(v)\mathrm{d}v \tag{3}$$

results because buyers with $v\in[c, p^{\mathrm{m}})$, who could potentially add value, do not produce.

Remark 1. The equilibrium is essentially the textbook case of a non-discriminating monopoly serving the market with the demand $D(p) = 1 - F(p)$. Monopoly does not discriminate in equilibrium because both cost and utility are linear in quantity, and all agents are risk-neutral.

2.2. Barter as a means of price discrimination

To show that barter can emerge in the presence of market power, even if it is extremely inefficient, we introduce all possible costs of barter. Liquidity constraints that may make money inferior to barter are also not imposed.

The first drawback of barter is the need for the double coincidence of wants. We assume that the seller values the buyer's output less than the buyer herself. A unit of buyer v's product is worth only αv to the seller, where $\alpha \in (0, 1)$. It implies that the seller has an inferior technology for re-selling or using the buyer's product, or there are storage or transportation costs borne by the seller. The cost of barter $1 - \alpha$ may also be interpreted as the probability that there is no double coincidence of wants and S has to throw the in-kind payment away.

Another disadvantage of barter is that, unlike money, it is not perfectly divisible. For the sake of simplicity, we assume the extreme degree of indivisibility and allow only contracts with $b \in \{0, 1\}$. This assumption is a modelling shortcut for the increasing returns in the barter exchange. The legal, storage, and transportation costs per unit of barter decrease with the amount bartered, therefore, exchanging small portions of the good may be prohibitively costly.

Suppose now that the seller can offer the buyers a menu of triplets $\{(p, b, q)\}$: 'buy $q \in [0, 1]$ units of input for the monetary payment of p and the in-kind payment $b \leq q'$. The latter means that b out of q units produced are to be given back to the seller.

The timing is similar to that of Section 2.1 (see Fig. 1). If buyer of type v chooses contract (p, b, q), she gets $v(q - b) - p$ and the seller gets $p + \alpha v b - cq$. Using the Revelation Principle, the problem is restated as follows: the monopolist chooses a menu of triplets $\{(p(v), q(v), b(v))\}$, $v \in [0, 1]$, $b(v) \in \{0, 1\}$, $b(v) \leq q(v)$ to maximize

$$\int_0^1 (p(v) + \alpha v b(v) - cq(v)) f(v) \mathrm{d}v \tag{4}$$

subject to incentive compatibility constraints

$$v(q(v) - b(v)) - p(v) \geq v(q(v') - b(v')) - p(v') \text{ for all } (v, v') \in [0, 1] \times [0, 1], \tag{5}$$

and individual rationality constraints

$$v(q(v) - b(v)) - p(v) \geq 0 \text{ for all } v \in [0, 1]. \tag{6}$$

Denote by p^{mb} and p^* the solutions to

$$p^{\mathrm{mb}}(1 - \alpha) = \frac{1 - F(p^{\mathrm{mb}})}{f(p^{\mathrm{mb}})}, \tag{7}$$

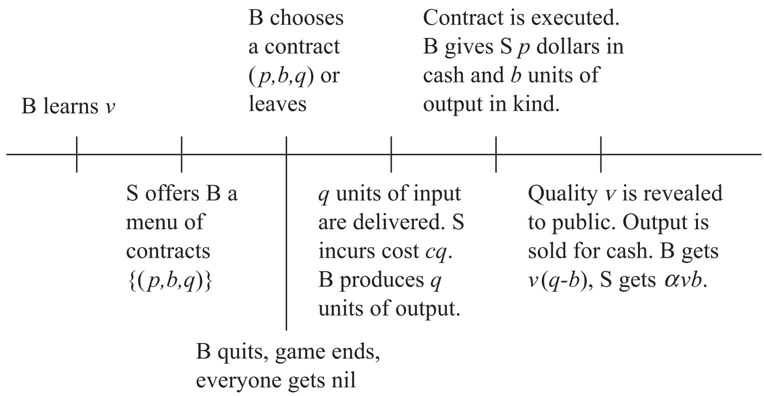

Fig. 1. The timing of the game in Subsection 2.2 (also applies to Subsection 2.1 if $b=0$).

and

$$\alpha G(p^*) = c, \tag{8}$$

respectively. Denote by $\bar{c}$ the value of c that solves

$$p^{\mathrm{mb}}(1 - F(p^{\mathrm{mb}})) + \alpha G(p^{\mathrm{mb}})F(p^{\mathrm{mb}}) - c = (p^{\mathrm{m}} - c)(1 - F(p^{\mathrm{m}})). \tag{9}$$

By definition, p^{mb}, p^* and $\bar{c}$ are unique. Also, $\bar{c}$ decreases with α: $d\bar{c}/d\alpha = G(p^{\mathrm{mb}})F(p^{\mathrm{mb}})/F(p^{m})>0$, and $\bar{c}$ goes to 0 as α approaches 0.

Lemma 1 and Proposition 1 below describe the structure of equilibria.

Lemma 1. *If a menu of contracts $\{(p(v), b(v), q(v))\}$, $v\in[0, 1]$, $b(v)\in\{0, 1\}$, $b(v) \leq q(v)$ satisfies the incentive compatibility (5) and participation constraints (6) then there exists $\bar{v}\in[0, 1]$ such that:*

(i) all buyers with $v<\bar{v}$ take the outside option or pay in kind;
(ii) all buyers with $v>\bar{v}$ pay in cash and $q(v)$ is non-decreasing in v for all $v\geq\bar{v}$.

Proof. In the Appendix A. □

Proposition 1. *The optimal menu of contracts is:*

(a) if $c>\bar{c}$, S does not use barter and offers the contracts $\{(p^m, 0, 1), (0, 0, 0)\}$;
(b) if $c<\bar{c}$, S uses barter and offers the contracts $\{(p^{mb}, 0, 1), (0, 1, 1), (0, 0, 0)\}$; in this case, $p^{mb}>p^m$, and $p^{mb}>p^*$.

Proof. In the Appendix A. □

In the rest of the subsection, we discuss the properties of and the intuition behind the barter equilibrium, i.e. concentrate on the case $c<\bar{c}$. When S chooses to use barter, the buyers with the higher valuations $v\geq p^{\mathrm{mb}}$ pay in cash while all the buyers with the lower valuations pay in kind. The 'barter customers' with the valuations $v<c$ who should not be served in the social optimum are pooled together with the efficient ones with the

valuations $v \in (c, p^{mb})$. And, if the gap between cash price and marginal cost is sufficiently large, serving this pool of customers is profitable for the seller. The average quality of the output is $G(p^{mb})$. Since $p^{mb} > p^*$, the in-kind revenues exceed marginal cost $\alpha G(p^{mb}) > \alpha G(p^*) = c$, and S receives positive profit from barter sales.

The condition $c < c$ also implies that in presence of barter contracts, the cash price is higher: $p^{mb} > p^{m}$. The intuition is simple: if there were no barter, increasing the monetary price would result in losing customers, while in the presence of barter, these customers do not leave the market, they switch to paying in kind and, in fact, improve the average quality of the barter payments.

Example. Consider a uniform distribution $f(v) = 1$. In this case, $\bar{c} = (1 - \alpha/2)^{-1/2} - 1$, $p^{mb} = 1/(2 - \alpha)$, $p^{m} = (1 + c)/2$, $p^* = 2c/\alpha$.

The welfare effect of barter is ambiguous. The deadweight loss in the equilibrium with barter $(1 - \alpha)G(p^{mb})F(p^{mb}) + (c - G(c))F(c)$ may be either larger or smaller than the deadweight loss without barter (Eq. (3)). Were barter prohibited, the monopoly seller would produce too little input, and some efficient buyers would close down. If, however, barter were allowed, the losses are not only due to lack of the double coincidence of wants (proportional to $1 - \alpha$), but also due to asymmetric information about the quality of in-kind payments. The average value of barter payments is greater than the input cost but some of the barter customers actually subtract value. This is the direct implication of the indivisibility of barter. Were barter payments perfectly divisible, the seller would be able to discriminate against inefficient buyers and only sell for barter to the buyers with $v > c/\alpha$ (see Proof of Proposition 1).

Remark 2. The model above can also be applied to a pure exchange setting, where v and αv are simply the values of a unit of the in-kind payments to the buyer and the seller, respectively. In this case, second-degree price discrimination through barter also results if v is the buyer's private information.

Remark 3. The barter menu is similar to a standard debt contract with a privately known value of the collateral. The debt contract states: 'S supplies q unit of input to B, B pays back p^{mb} in cash else S gets ownership of B's output'. Barter trade is essentially the (inefficient) liquidation which destroys $(1 - \alpha)$ of the collateral's value. Unlike the literature on debt contracts, we assume that there is no ex-post renegotiation (or that the renegotiation is very costly). The model with renegotiation, provided that the buyer has at least some bargaining power, would have a very similar equilibrium, except the elimination of the deadweight loss caused by the lack of double coincidence of wants.

2.3. Barter in oligopoly

Suppose there are N identical sellers with the marginal cost c. We study second-degree price discrimination in the Cournot oligopoly setting assuming that sellers determine how much to sell for cash and how much for barter taking into account the self-selection of buyers.

Each seller offers customers the following menu of contracts: a non-linear cash tariff $\{p(q), 0, q\}$ '*buy* $q \in [0, 1]$ units of input and pay $p(q)$ in cash' and a barter contract $(\bar{p}, 1, 1)$

'buy one unit of input and pay one unit of output and p in cash'. Each buyer compares three options: (a) the outside option that gives zero payoff, (b) the barter contract that gives $\bar{U} = -\bar{p}$, (c) the cash contract that gives $U(v) = \max_{q \in [0,1]} vq - p(q)$, and chooses one that maximizes her rent.

We define Cournot equilibrium in the way described by Oren et al., (1983) extending it to the setting in which the sellers can use both cash and barter contracts. In the linear case, as shown below, this model is reduced to a very simple game among the oligopolists.[3]

Denote by $v^*(q)$ the highest type that buys q units of input and pays in cash. The buyers' choice is the same as in the previous section; hence, Lemma 1 applies and $v^*(q)$ is an increasing function of q.

Each seller i is characterized by a function $T_i(q)$—the number of customers buying no more than q units for cash from i. By definition $\sum_{i=1}^{N} T_i(q) = F(v^*(q))$ for all $q>0$. $T_i(0)$ is the number of customers buying from i for barter. Each seller takes $T_j(q), j \neq i$ as given and chooses $p(q)$, p, and $T_i(0)$ to maximize profit

$$(\alpha G(v^*(0)) - c)(F(v^*(0)) - T_{-i}(q))T_i(0)I(\bar{p} \geq 0) + \int_0^1 (p(q) - cq)\mathrm{d}(F(v^*(q)) - T_{-i}(q)) \tag{10}$$

subject to the constraint that $v^*(q)$ is the inverse of the buyer's optimal response to $p(q), \bar{p}$. Here $T_{-i}(q) = \sum_{j \neq i} T_j(q)$, and $I(\bar{p} \geq 0)$ is the indicator function that equals 1 whenever $p \geq 0$ and is 0 otherwise. We look for symmetric equilibria where $T_i(q) = T_j(q)$ for all i, j, and q.

Lemma 2. *In any Cournot equilibrium, there are no buyers who buy $q \in (0, 1)$ for cash.*

Proof. In the Appendix A. □

Just as in the monopoly case, linear utility and cost functions rule out the intermediate quantities. This makes the contract menu very simple, buyers choose among three options: (i) buy one unit for cash, (ii) buy one unit for barter, (iii) do not buy at all.

The strategy $T_i(q)$ is now fully characterized by two numbers: $T_i(0)$ and $T_i(1)$. Each firm sells $y_i = T_i(1) - T_i(0)$ for cash at the market price $P = p(1) - p(0)$ and $z_i = T_i(0)$ for barter. In the equilibrium, the total quantity supplied to the cash market $Y = \sum_{i=1}^{N} y_i$ equals the quantity demanded $\int_P^1 f(v)\mathrm{d}v = 1 - F(P)$. The rest of the buyers $v < P$ are indifferent between paying in kind or not buying at all. The average quality of the barter payments is, therefore, $E(v|v<P) = G(P)$. Since the barter customers are indifferent between buying and not buying, we assume that whenever the total supply in the barter market $Z = \sum_{i=1}^{N} z_i$ is below $F(P)$, the demand is stochastically rationed so that the average quality of payments in kind remains $G(P)$.

[3] Ivaldi and Martimort (1994) and Stole (1995) model second-degree price discrimination under duopoly with imperfect substitutes, but these models are too complicated to study comparative statics with regard to change in the market structure.

The seller i takes other sellers' strategies y_j and z_j as given and maximizes

$$\pi(y_i, y_{-i}, z_i) = (P(y_i + y_{-i}) - c)y_i + (\alpha G(P(y_i + y_{-i})) - c)z_i \tag{11}$$

subject to

$$0 \le z_i \le F(P(y_i + y_{-i})) - z_{-i}, y_i \le 0. \tag{12}$$

Here $y_{-i} = \sum_{j \neq i} y_j, z_{-i} = \sum_{j \neq i} z_j$. The inverse demand function $P(Y)$ is given by $Y = 1 - F(P)$.

We classify equilibria by the presence of barter. Notice that firm i has an incentive to sell for barter whenever $\partial\pi/\partial z_i = \alpha G(P(Y)) - c \ge 0$, or $P(Y) \ge p^*$ (see Eq. (8)).

(1) "Barter" equilibria.

This is the case when $P(Y) > p^*$. The objective function (Eq. (11)) increases with z_i. Therefore, the sellers want to barter as much as possible $z_i = F(P) - z_{-i}$. The first order condition for y_i implies:[4]

$$y_i = f(P)[P - \alpha G(P) - \alpha(P - G(P))(F(P) - z_{-i})/F(P)].$$

Summing for $i = 1, \ldots, N$ and dividing by $f(P)$, we obtain the equation for equilibrium price:

$$(P - \alpha G(P))N - \alpha(P - G(P)) = \frac{1 - F(P)}{f(P)}. \tag{13}$$

Denote $p^{\mathrm{b}}(N)$ the price P that solves Eq. (13) for a given N. The necessary and sufficient condition for existence of a barter equilibrium is $p^{\mathrm{b}}(N) > p^*$. The total amount of barter sales is $Z = F(p^{\mathrm{b}}(N))$. The barter sales of individual sellers z_i must satisfy $\sum_{i=1}^{N} z_i = Z$. In the symmetric equilibrium $z_i = F(p^{\mathrm{b}})/N$ and $y_i = (1 - F(p^{\mathrm{b}}))/N$. There is also a continuum of asymmetric equilibria. In all equilibria, however, P, Y and Z are the same.

(2) "No-barter" equilibria.

If $P(Y) < p^*$, the sellers do not sell for barter, $z_i = 0$, and the first order condition for y_i implies $y_i = (P - c)f(P)$. Adding up and dividing by $f(P)$, we arrive at the conventional Cournot equilibrium:

$$(P - c)N = \frac{1 - F(P)}{f(P)}. \tag{14}$$

Similarly, denote $p^{\mathrm{nb}}(N)$ the price P which solves Eq. (14). The necessary and sufficient condition for existence of a no-barter equilibrium is $p^{\mathrm{nb}}(N) < p^*$. The total amount of barter sales is zero.

(3) "Rationed barter" equilibria.

[4] We have used the identity $G'(p) = (p - G(p))f(p)/F(p)$.

If $P(Y)=p^*$, the sellers are indifferent about how much to offer for barter. The first order condition for y_i implies $y_i=(p^*-c)f(p^*)-z_i(p^*-G(p^*))f(p^*)/F(p^*)$. Adding up, we obtain

$$Z/F(p^*) = [(p^* - c)N - (1 - F(p^*))/f(p^*)]/[\alpha(^* - G(p^*))]. \tag{15}$$

Barter sales of individual sellers z_i must satisfy $\sum_{i=1}^{N} z_i = Z$. The necessary and sufficient condition for the existence of a rationed-barter equilibrium is Eq. (12), i.e. $0 \le Z/F(p^*) \le 1$. These inequalities hold if and only if both inequalities $p^{\mathrm{b}}(N) \ge p^*$ and $p^{\mathrm{nb}}(N) \le p^*$ hold. Thus, the rationed barter equilibrium exists if and only if both "barter" and "no-barter" equilibria exist.

Let us denote N^{b} a solution to $p^{\mathrm{b}}(N)=p^*$ and N^{nb} a solution to $p^{\mathrm{nb}}(N)=p^*$.

Example. For the uniform distribution $f(p)\equiv 1$, $N^{\mathrm{nb}}=(1-2c/\alpha)/(2c/\alpha-c)$, $N^{\mathrm{b}}=(1-2c/\alpha+c)/(2c/\alpha-c)$.

Proposition 2. *Both N^b and N^{nb} exist and $N^b > N^{nb}$. The set of equilibria of the game above is as follows:*

1. *If $N < N^{nb}$ then there is a unique stable equilibrium which is a barter equilibrium.*
2. *If $N > N^b$ then there is a unique stable equilibrium which is a no-barter equilibrium.*
3. *If $N \in (N^{nb}, N^b)$ then there are three equilibria, two of which (barter and no-barter) are stable and one (rationed barter) is unstable.*
4. *If $N = N^b$ then there are two equilibria: a stable one (no-barter) and an unstable one (rationed barter).*
5. *If $N = N^{nb}$ then there are two equilibria: a stable one (barter) and an unstable one (rationed barter).*

Proof. In the Appendix A. □

Fig. 2 illustrates the structure of equilibria according to Proposition 2.

The intuition for the multiplicity of equilibria at $N \in (N^{\mathrm{nb}}, N^{\mathrm{b}})$ is as follows. Whenever one seller chooses to sell more for cash, she drives down the cash price of the input. The additional cash purchases are made by buyers who were the most efficient ones among those paying in kind. With these buyers switching from barter to cash, the average quality of payments in kind decreases. The other sellers will, therefore, have incentives to sell more for cash and less for barter as well.

The comparative statics with respect to market structure (total number of sellers N) are quite intuitive. First, in both barter and no-barter equilibria, prices decrease as the number of sellers increases. Second, given market structure, the cash price in the barter equilibrium is always greater than in the no-barter equilibrium. In barter equilibria, sellers charge higher prices because the marginal buyer, who would leave the market were there no barter, switches to barter and, therefore, contributes to profits from barter sales. Third, in the barter equilibria, the cash price should be above a certain level p^*; otherwise, the average quality of payments in kind is below marginal cost and barter is not profitable. Similarly, in the no-barter equilibria, price should be below p^*.

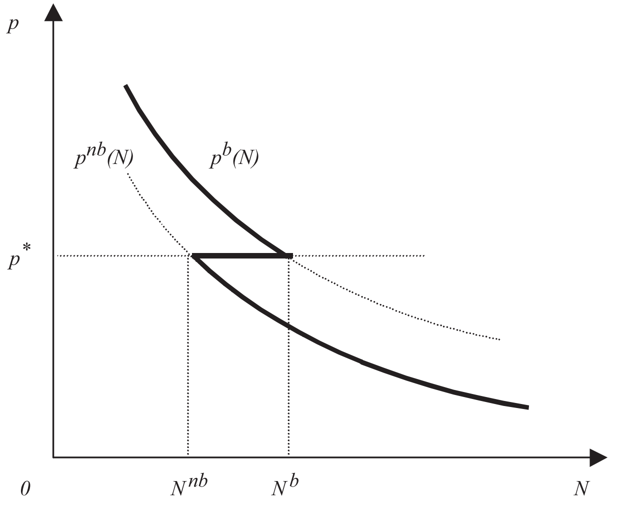

Fig. 2. Oligopoly price P as a function of the number of sellers N.

The model implies that there exists a positive relationship between the share of barter in sales $B = Z/(Z + Y)$ (Fig. 3) and market concentration $1/N$. In the highly concentrated markets $N < N^{nb}$, only barter equilibrium exists and $B = Z = F(p^{b}(N))$. Since $p^{b}(N)$ is a continuous decreasing function, B is a continuous decreasing function of N. As the number of sellers increases, the volume of barter trade goes down. Once the number of sellers becomes greater than N^{nb}, the no-barter equilibrium emerges. Further increase in competition narrows the basin of attraction of the barter equilibrium (the distance between barter and rationed barter equilibria decreases) and widens that of the no-barter equilib-

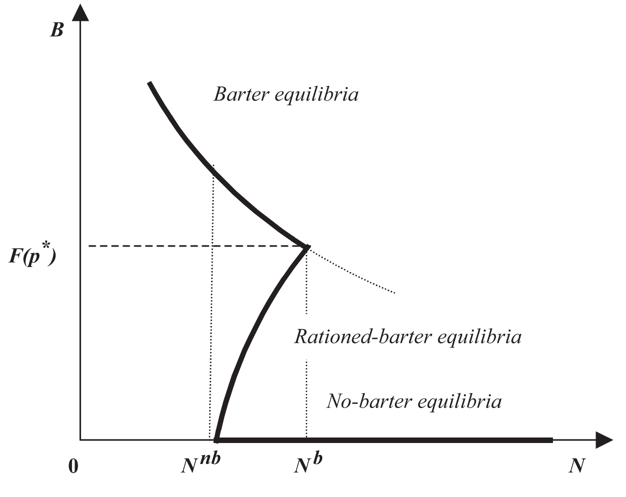

Fig. 3. Share of barter sales in total sales $B = Z/(Z + Y)$ as a function of the number of sellers N.

rium.[5] As the number of sellers goes from N^{nb} to N^{b}, the industry is more likely to switch from barter to no-barter equilibrium. Finally, as $N>N^{b}$, there is no barter.

The welfare properties of barter equilibria are similar to the case of monopoly. The barter equilibrium is not necessarily less efficient than the no-barter one. In the no-barter equilibria, some efficient buyers do not produce since the cash price is above the marginal cost. In the barter equilibria, all the buyers produce including the value-subtracting ones. In addition, there are transaction costs of barter $(1-\alpha)F(p^{b}(N))G(p^{b}(N))$. The social planner would have to compare the deadweight losses in the no-barter equilibrium (too many firms are shut down but the transaction costs are low) and the barter equilibrium (too few firms are shut down and transaction costs are high).

3. Empirical evidence

Russian demonetization of the second half of 1990s provides a good testing ground for the theory; the overall level and variation in the use of barter have been very large. The debate on Russian demonetization has centered around the financial constraints (the first explanation of barter by IRTA). Commander and Mumssen (1998) report that managers themselves explain the prevalence of barter by the liquidity squeeze due to the tight monetary policy. Barter may have also been used to hide revenues from shareholders and creditors, entrench and delay restructuring (formal model in Gaddy and Ickes, 2002, evidence in the work of Seabright, 2000, chapters 6 and 9). Other explanations include inefficiency of Russian fiscal federalism and the inability of the federal government to enforce the monopoly to issue money (Woodruff, 1999).

This paper argues that the spread of barter in Russia may also be related to the second motive for barter as suggested by IRTA ('spare capacities'). The model predicts that barter is more likely to occur in concentrated industries and decreases with competition. The evidence on barter in Russia seems to be consistent with this prediction. It is the natural monopolies that are engaged in barter the most (Gaddy and Ickes, 2002; Commander et al., 2002). In 1996–1997, Gazprom (the natural gas monopoly) and Unified Energy Systems (the electricity monopoly) reported cash receipts of as low as 15% to 20% total revenue (Pinto et al., 2000). The remainder of their revenues came in promissory notes, coal, metal, machinery and even jet fighters. All case studies discussed in the work of Woodruff (1999) refer to firms that are either national or regional monopolies.

While it is hard to argue that the Russian demonetization was caused by price discrimination, it turns out that the cross-sectional variation in the level of barter is related to market power. Below we use a unique firm-level dataset to show that even controlling for the common explanations the level of barter is indeed higher in more concentrated industries. The variation of barter across Russian industries in the mid-1990s is therefore generally consistent with the above model of barter as a means of price discrimination.

[5] In the rationed barter equilibria, $B=[1+(1-F(p^{*}))/Z]^{-1}$ is a continuous increasing hyperbolic function of N that connects points $(N^{nb}, 0)$ and $(N^{b}, F(p^{*}))$ in the (N,B) space.

3.1. Data and methodology

The empirical analysis uses three datasets: (i) surveys of managers of Russian industrial firms conducted since 1996 by Serguei Tsoukhlo at the Institute of Economies in Transition, Moscow;[6] (ii) the database of financial accounts of industrial firms compiled by Goskomstat (federal statistics agency); (iii) the Import Penetration Database compiled by the Russian European Center for Economic Policy.

The dependent variable B (the share of barter in sales) comes from the firm manager's answers to the question: "what share of your firm's sales was paid in kind?" The survey also included questions on the other means of payment, our variable covers barter only (excluding promissory notes). The average share of barter in sales is 39% and varies substantially across firms. It is distributed almost uniformly between 0 and 0.83. Only 10% of the sample have the share of barter in sales lower than 5%.

Our main independent variables are the concentration ratio CR4 (the share of the four largest firms in total sales of an industry) and import-adjusted concentration ratio $\text{CR4ia} = \text{CR4}^*(1 - \text{imp})$, where imp is the import penetration ratio. The ratios are calculated for five-digit industries (rough equivalent of US four-digit industries). In the sample, 120 (out of 450) industries are represented with 2.8 firms in each industry on average, with up to 19 firms in some industries.

The relationship between market concentration and barter may also be explained by transaction costs of barter ($1 - \alpha$ in terms of the model). First, per unit search, transportation, and storage costs may be smaller for larger firms. At the same time, market concentration is correlated with the average size of a firm in the industry. We control for size by adding the variable SIZE (the logarithm of annual sales in denominated rubles).

Second, transaction costs of barter are prohibitively high for retail customers. Hence, in consumer good industries, one should expect less barter. Also, these industries may be more competitive. Thus, the correlation between barter and market power may be explained by the lower level of barter in consumer good industries. To control for this effect, we introduce a dummy for consumer good industries CGI.

Third, the fewer players in the market for buyers' output, the lower the search costs of barter. Certainly, the numbers of players in the markets for the input and for the buyers' outputs may be correlated. This problem would be resolved via a panel regression, testing if an increase in concentration in the seller's market raises barter controlling for concentration in the buyers' markets. Unfortunately, the data on buyers are not available, and the dataset is limited to one year. As an imperfect substitute, we use 11 broad industry dummies—for the firms in the same broad industry, buyers are roughly the same. Also, the broad industry dummies help to control for technological characteristics that can influence costs of barter (e.g. per unit transportation costs). A similar role is played by regional dummies for Moscow and seven federal districts (Central Russia, South, North–West, Volga, Ural, Siberia and Far East). The base category is Central Russia excluding Moscow.

[6] See Russian Business Survey Bulletin (1996) for a detailed description of the survey.

We also control for other determinants of barter. First, to measure firm liquidity, we add the variable CASH—the ratio of stock of liquid assets (cash plus current accounts) to annual sales. Second, the variable EXPORT (the share of exports in sales) controls for sales to foreign customers.

The descriptive statistics and the correlation matrix are reported in Appendix B. The signs of pairwise correlations are intuitive. There is indeed more barter in more concentrated industries (whether adjusted for imports or not), in larger firms, and in those which sell less to foreign customers and consumers. Consumer goods industries are less concentrated. Average CR4 for consumer goods industries is 23% which is significantly lower than in the other industries (44%). Liquidity is not correlated with barter, or other variables except exports (exporting firms have more cash).

3.2. Estimations

Table 1 presents the results of the OLS regressions for share of barter in sales. Barter depends positively and significantly on concentration. The inclusion of broad industry dummies does not change the results. The effect of concentration is weaker if CR4 is used instead of CR4ia. This is partially explained by the difference in variances of CR4 and CR4ia.

The signs of other coefficients are intuitive. The insignificance of the CASH variable is not surprising. While Brana and Maurel (1999) find a significant relationship between liquidity and barter for a subset of their sample, most empirical studies of Seabright (2000) and Commander et al. (2002) find no significant relationship in cross-section OLS regressions. The effect of exports is negative but not very large (only 14–20%) which

Table 1
Results of OLS estimation for the share of barter in sales *B*

	Regression with CR4ia, industry dummies are not included	Regression with CR4ia, industry dummies are included	Regression with CR4, industry dummies are not included	Regression with CR4, industry dummies are included
CR4ia	0.22**(2.59)	0.23** (2.31)	–	–
CR4	–	–	0.13** (2.05)	0.12** (1.97)
SIZE	− 0.006 (0.53)	0.002 (0.21)	− 0.006 (0.55)	0.002 (0.19)
CGI	− 0.16*** (4.74)	− 0.11*** (2.97)	− 0.16*** (4.66)	− 0.10*** (2.91)
CASH	− 0.04 (0.56)	− 0.03 (0.53)	− 0.04 (0.54)	− 0.03 (0.52)
EXPORT	− 0.14** (2.33)	− 0.19*** (3.03)	− 0.15** (2.41)	− 0.20 (3.12)
Regional dummies	***	***	***	***
Industry dummies	–	*	–	*
CONST	0.45*** (2.75)	0.32* (1.90)	0.25*** (2.67)	0.32* (1.90)
N	337	337	337	337
R^2	0.29	0.33	0.28	0.32

Standard errors are estimated via the Huber–White procedure taking into account unobserved correlations within 5-digit industries.

* Significant at the 10% significance level.

** Significant at the 5% significance level.

*** Significant at the 1% significance level.

is again similar to the 0.19 obtained by Commander and Mumssen (1998). Firms in consumer good industries have 10–16% less barter. For brevity's sake, the coefficients for regional and industry dummies are not presented. There is 10% less barter in Moscow (relative to Central Russia), 11–13% more in the North Western, Volga, and Urals regions, 19% more barter in Siberia. The difference between Southern Russia, Far East and Central Russia is not significant. This implies that geography matters: distance to the thick and competitive central market (or, in case of the Far East, distance to the East Asia market) is important. The broad industry dummies are not significant. The results are robust to changes in specifications, using other proxies for size, and running separate regressions for subsamples with CGI = 1 and CGI = 0.

Thus, the empirical evidence from cross-section data is consistent with the predictions of the model even controlling for alternative explanations. However, two caveats are in order. First, panel data evidence would certainly be more convincing. There may exist unobservable firm-specific characteristics that influence willingness to barter (e.g. managers' "relational capital", Gaddy and Ickes, 2002). To make a strong empirical argument, one would have to prove that even controlling for the firm's fixed or random effects, change in competition leads to change in barter. The available data, however, do not allow us to perform this test.

Second, the model predicts that the relationship between barter and market concentration may be non-linear. Elsewhere (Guriev and Kvassov, 2000), we tested for a structural break and identified it at CR4 = 0.162 (or at CR4ia = 0.103). The nature of the structural break is consistent with the model. For the industries with concentration above the threshold level, competition reduces barter, the coefficient is significant but relatively small (in the range of 0.1). Once concentration is below the threshold, barter starts to fall rapidly with further increase in competition (coefficient becomes 10 times as large): firms switch from barter equilibria to ones without barter.

4. Concluding remarks

This paper shows that barter may be used as a discriminatory instrument in sufficiently concentrated markets with assymetric information. Imperfect competition is characterized by underproduction relative to the social optimum. The sellers, therefore, are willing to employ an additional channel of sales (barter) even if this channel is costly. The paper also demonstrates that there exists a relationship between the share of barter in sales and the market concentration. The only equilibrium at high levels of concentration is the barter equilibrium, and the only equilibrium at low levels of concentration is the no-barter equilibrium. Barter and no-barter equilibria coexist in the intermediate range of market concentration.

It is important to note that the model provides no clear ranking of the equilibria in terms of social welfare. The equilibrium without barter is characterized by underproduction, many efficient firms close down. In the barter equilibrium, on the other hand, all efficient firms produce but so do inefficient ones; besides, barter involves high transaction costs. The paper demonstrates that competition reduces barter and identifies the trade-offs this reduction involves.

A number of simplifying assumptions were made to keep the model tractable. We have assumed linear technology, risk neutrality, exogenous probability of double coincidence of wants, perfect substitution of oligopolists' products, extreme indivisibility of barter, etc. However, the model is robust to relaxing these assumptions: introduction of convex technology, for example, would result in price discrimination both with and without barter. But barter still represents an additional dimension for price discrimination and would, therefore, be used, only the equilibrium contracts become more complicated. This suggests that the insights of the model are not confined to the question of barter in transition or developing economics, but shed some new light on the theories of price discrimination under imperfect competition in general.

It is also worth emphasizing that throughout the paper, the term 'barter' is used as a shorthand for any non-monetary contract that involves the exchange of property rights. Thus, the developed model may be used to analyse such diverse phenomena as the debt contracting with asymmetric information about the value of collateral, or lease contracts where unobserved actions of one party change the value of the good for the other party.

5. Uncited reference

Brown et al., 1994

Acknowledgements

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Appendix A. Proofs

Proof of Lemma 1. Adding up the IC constraints (Eq. (5))

$$v''(q(v'') - b(v'')) - p(v'') \geq v''(q(v') - b(v')) - p(v'),$$
$$v'(q(v') - b(v')) - p(v') \geq v'(q(v'') - b(v'')) - p(v'').$$

for arbitrary $(v', v'')\in[0, 1]\times[0, 1]$ such that $v'<v''$ yields

$$(v''-v')\{(q(v'')-b(v''))-(q(v')-b(v'))\}\geq 0.$$

Therefore, $v''>v'$ implies that $q(v'')\geq b(v'')q(v')-b(v_0)$. In other words, the amount of output kept by the buyer $q(v)-b(v)$ is a monotonic function of v.

Because only barter contracts with $b(v)\in\{0, 1\}$ and $b(v)\leq q(v)$ are allowed, $q(v)-b(v)=0$ for the buyers who pay in cash and for the buyers who exercise outside option. For the buyers who pay in cash $q(v)-b(v)$ equals $q(v)>0$. Hence, there exists $\bar{v}$ such that buyers with $v<\bar{v}$ pay in kind (or stay out), and buyers with $v>\bar{v}$ pay in cash. Moreover, since $b(v)=0$ for the buyers who pay in cash, $q(v)$ is monotone non-decreasing function of v for all $v\geq\bar{v}$. □

Proof of Proposition 1. S may offer a menu of cash contracts $(p, q, 0)$ and one barter contract $(\bar{p}, 1, 1)$. The buyer of type v gets the rent $U(v)=v(q(v)-b(v))-p(v)$. Consider arbitrary $(v', v'')\in[0, 1]\times[0, 1]$ such that $v'<v''$. From the incentive compatibility constraints (Eq. (5)), it follows that

$$q(v')-b(v')\leq\frac{U(v'')-U(v')}{v''-v'}\leq q(v'')-b(v'').$$

Since $q(v)-b(v)$ is monotonic in v (Lemma 1), integration yields

$$U(v)=U(0)+\int_0^v[q(x)-b(x)]\mathrm{d}x. \tag{16}$$

Buyers who choose the barter contract get $-\bar{p}$. They will prefer it to the outside option if and only if $\bar{p}\leq 0$. The case with $\bar{p}>0$ (barter contract is proposed but is not chosen by any buyer) is equivalent to the model without barter and the optimal menu is $\{(p^{\mathrm{m}}, 0, 1), (0, 0, 0)\}$. Payoff to the seller is the seller is $(p^{\mathrm{m}}-c)(1-F(p^{\mathrm{m}}))$. If the seller offers a barter contract with $\bar{p}\leq 0$, then all buyers with $v<\bar{v}$ choose this contract and get $U(\bar{v})=-\bar{p}$.

Using Eq. (16) and $\int_{\bar{v}}^1 f(v)\mathrm{d}v\int_{\bar{v}}^v q(x)\mathrm{d}x=\int_{\bar{v}}^1(1-F(v))q(v)\mathrm{d}v$, S's problem is restated as follows. Seller chooses $\bar{p}\leq 0$, $\bar{v}\in(0, 1]$, and $q(v)\in[0, 1]$ to maximize

$$\bar{p}+\int_0^{\bar{v}}[\alpha v-c]f(v)\mathrm{d}v+\int_{\bar{v}}^1\left(v-c-\frac{1-F(v)}{f(v)}\right)q(v)f(v)\mathrm{d}v.$$

In the optimum, S sets $\bar{p}$ equal to zero and chooses

$$q(v)=\arg\max_{q\in[0,1]} q\left(v-c-\frac{1-F(v)}{f(v)}\right)\text{for all } v>\bar{v}$$

where $\bar{v}$ maximizes

$$\Pi(\bar{v})=(\alpha G(\bar{v})-c)F(\bar{v})+(\max\{\bar{v},p^{\mathrm{m}}\}-c)(1-F(\max\{\bar{v},p^{\mathrm{m}}\})) \tag{17}$$

If $\bar{v}<p^{\mathrm{m}}$ then $\mathrm{d}\Pi/\mathrm{d}\bar{v}=(\alpha\bar{v}-c)f(\bar{v})>0$ whenever $\bar{v}>c/\alpha$. If $\bar{v}>p^{\mathrm{m}}$ then $\mathrm{d}\Pi/\mathrm{d}\bar{v}=1-F(\bar{v})-(1-\alpha)\bar{v}f(\bar{v})>0$ whenever $\bar{v}<p^{\mathrm{mb}}$. Assumption A1 implies that p^{m} is always between c/α and p^{mb}. Indeed, $p^{\mathrm{m}}>p^{\mathrm{mb}}$ is equivalent to $(1-F(p^{\mathrm{m}}))/f(p^{\mathrm{m}})<(1-F(p^{\mathrm{mb}}))/f(p^{\mathrm{mb}})$ and, therefore, $p^{\mathrm{m}}-c<p^{\mathrm{mb}}(1-\alpha)<p^{\mathrm{m}}(1-\alpha)$ which implies $p^{\mathrm{m}}<c/\alpha$. Similar argument proves that $p^{\mathrm{m}}<p^{\mathrm{mb}}$ implies $p^{\mathrm{m}}>c/\alpha$. Therefore, the maximizer of Eq. (17) is either $\bar{v}=0$ or $\bar{v}=p^{\mathrm{mb}}$ with the latter possible only if $c/\alpha<p^{\mathrm{m}}<p^{\mathrm{mb}}$ and the payoff to the seller is $p^{\mathrm{mb}}(1-F(p^{\mathrm{mb}}))+\alpha G(p^{\mathrm{mb}})F(p^{\mathrm{mb}})-c$.

Hence, the optimal menu of contracts is either $\{(p^{\mathrm{mb}}, 0, 1), (0, 1, 1), (0, 0, 0)\}$ or $\{(p^{\mathrm{m}}, 0, 1), (0, 0, 0)\}$ whichever provides the seller with a higher payoff. The seller chooses to use barter if the left-hand side in Eq. (9) is greater than the right-hand side, which is the case whenever $c<\bar{c}$. Indeed, the left-hand side falls with c faster than the right-hand side.

Let us now prove that $c<\bar{c}$ implies $p^{\mathrm{mb}}>p^*$. Since the left-hand side of Eq. (9) is greater than the right-hand side,

$$(p^{\mathrm{mb}}-c)(1-F(p^{\mathrm{mb}}))+(\alpha G(p^{\mathrm{mb}})-c)F(p^{\mathrm{mb}})=\max_{p\in[0,\ 1]}[p(1-F(p))$$
$$+\alpha G(p)F(p)]-c>\max_{p}\{(p-c)(1-F(p))\}\geq(p^{\mathrm{mb}}-c)(1-F(p^{\mathrm{mb}})).$$

Therefore, $\alpha G(p^{\mathrm{mb}})-c>0$, and $p^{\mathrm{mb}}>p^*$.

Comment. If barter were perfectly divisible $b(v)\in[0, 1]$, the solution would be somewhat different. If $p^{\mathrm{mb}}<p^{\mathrm{m}}$, the equilibrium coincides with the monopoly equilibrium without barter. If $p^{\mathrm{mb}}>p^{\mathrm{m}}$, the seller is able to sort customers into three groups. The most efficient buyers pay cash price pmb, the buyers with intermediate productivity $v\in(c/\alpha, p^{\mathrm{mb}})$ pay in kind, and the rest of buyers is not served. In this equilibrium all buyers with $v\leq p^{\mathrm{mb}}$ receive zero rent and are indifferent between paying in kind or not producing at all. Above, we assumed that whenever indifferent, buyers choose to produce. To make buyers with $v<c/\alpha$ shut down and buyers with $v>c/\alpha$ produce, the seller offers some infinitesimal reward to the latter $(1-b(v)$ strictly positive but very small). Although in equilibrium, $b(v)$ is either 0 or very close to 1, perfect divisibility of barter is crucial for separating buyers with $v\in(0, c/\alpha)$ and $v\in(c/\alpha, p^{\mathrm{mb}})$. □

Proof of Lemma 2. The seller maximizes Eq. (10) by choosing three scalar numbers $T_i(0)$, $\bar{p}$, $p(0)$ and a function $p'(q)=\mathrm{d}p/\mathrm{d}q$, $q\in[0, 1]$. The optimal choice of $p'(q)$ does not allow for intermediate purchases $q\in(0, 1)$ for cash. Indeed, integration of the second term in Eq. (10) by parts yields

$$p(0)(1-T_{-i}(\bar{q})-F(v^*(0))+T_{-i}(0))+\int_0^{\bar{q}}(p'\ (q)-c)(1-T_{-i}(\bar{q}))$$
$$-F(v^*(q))+T_{-i}(q)\mathrm{d}q$$

where $\bar{q}$ is the quantity chosen by the buyers of the highest type $v=1$.

The first term in Eq. (10) does not depend on $p'(q)$, $q \in (0, 1)$. Therefore, the seller chooses

$$\int_0^{\bar{q}} [p'(q) - c)(1 - T_{-i}(\bar{q}) - F(v^*(q)) + T_{-i}(q)]\mathrm{d}q. \tag{18}$$

Buyers choose q solving $\max_{q \in [0,\ 1]} vq - p(q)$. Assume that there exist buyers that buy $q \in (0,\ 1)$ for cash. Then the first-order condition must hold $v = p'(q)$. Substituting $v^*(q) = p'(q)$ into Eq. (18), we find $p'(q) = \xi^*(q) = \arg\max_\xi (\xi - c)(1 - T_{-i}(\bar{q}) - F(\xi) + T_{-i}(q))$. The first-order condition is $(\xi^* - c)f(\xi^*) = 1 - T_{-i}(\bar{q}) - F(\xi^*) + T_{-i}(q)$. Using the symmetry condition $T_i(q) = T_j(q) = (1/N-1)T_{-i}(q) = (1/N)F(v^*(q))$, we obtain

$$\xi^* - c = \frac{1 - F(\xi^*)}{Nf(\xi^*)}.$$

Assumption A1 implies that such ξ^* exists and unique. It is crucial that ξ^* is the same for all q. Since $p'(q) = \xi^*$, the tariff is linear: $p(q) = p(0) + \xi^* q$. Therefore, all buyers with $v < \xi^*$ choose not to buy at all and all buyers with $v > \xi^*$ buy one unit ($q = 1$). The set of buyers who are indifferent $v = \xi^*$ is of measure zero. □

Proof of Proposition 2. The proof consists of four steps.

Step 1. $p^{\mathrm{b}}(N)$ and $p^{\mathrm{nb}}(N)$ are decreasing functions of N and $p^{\mathrm{b}}(N) > p^{\mathrm{nb}}(N)$ for all $N < N^{\mathrm{b}}$.

Solving Eq. (13) for N, we obtain

$$N = 1 + \ [(1 - F(P))/f(P) - (1 - \alpha)\ P]/[P - \alpha G(P)] \tag{19}$$

which is a decreasing function of P. Consequently, the inverse function $p^{\mathrm{b}}(N)$ is also decreasing. Since $p^{\mathrm{b}}(1) = p^{\mathrm{mb}} > p^*$ and $p^{\mathrm{b}}(\infty) = 0$, there exists a unique solution to $p^{\mathrm{b}}(N) = p^*$. Similarly, Eq. (14) implies $N = (1 - F(P))/[(P - c)f(P)]$ which is a decreasing function. Since $p^{\mathrm{nb}}(0) = 1 > p^*$ and $p^{\mathrm{b}}(\infty) = c < p^*$ there exists a unique solution to $p^{\mathrm{nb}}(N) = p^*$.

For all $N < N^{\mathrm{b}}$, we have $p^{\mathrm{b}}(N) > p^*$ and therefore $\alpha G(p^{\mathrm{nb}}(N)) > c$. Using Eqs. (13) and (14) for every N holds

$$\frac{1}{N} = \frac{(p^{\mathrm{nb}} - c)f(p^{\mathrm{nb}})}{1 - F(p^{\mathrm{nb}})} = \frac{(p^{\mathrm{b}} - c)f(p^{\mathrm{b}})}{1 - F(p^{\mathrm{b}})} - \frac{f(p^{\mathrm{b}}\ [(\alpha G(p^{\mathrm{b}}) - c) + \frac{\alpha}{N}(p^{\mathrm{b}} - G(p^{\mathrm{b}}))]}{1 - F(p^{\mathrm{b}})}$$

which implies $p^{\mathrm{nb}}(N) > p^{\mathrm{b}}(N)$.

Step 2. $N^{\mathrm{b}} > N^{\mathrm{nb}}$.

This follows directly from Step 1: both $p^{\mathrm{nb}}(N)$ and $p^{\mathrm{b}}(N)$ are continuous decreasing functions, $p^{\mathrm{nb}}(N) < p^{\mathrm{b}}(N)$ for all $N < N^{\mathrm{b}}$ and $p^{\mathrm{nb}}(N^{\mathrm{nb}}) = p^{\mathrm{b}}(N^{\mathrm{b}}) = p^*$.

Step 3. Existence of equilibria.

The barter equilibrium exists if and only if $p^{\mathrm{b}}(N^{\mathrm{b}}) \geq p^*$, i.e. $N \leq N^{\mathrm{b}}$. The no-barter equilibrium exists if and only if $p^{\mathrm{nb}}(N^{\mathrm{nb}}) \leq p^*$, i.e. $N \geq N^{\mathrm{nb}}$. The rationed barter equilibrium exists if and only if both barter and no-barter equilibria exist.

Step 4. Stability of equilibria.

Barter equilibrium at $N < N^{\mathrm{b}}$ and no-barter equilibrium at $N > N^{\mathrm{nb}}$ are stable. If there is no barter and one seller deviates by offering a positive amount of barter sales, other sellers have no incentives to deviate. If, in a barter equilibrium, one seller deviates by offering less barter, then other sellers's best response is to capture the unattended customers and restore total barter sales to $F(P)$. The rationed barter equilibrium is unstable. If one seller chooses to sell a little more for barter and little less for cash, the price in the cash market will increase making average quality of payments in kind $\alpha G(P)$ greater than marginal cost of production c. Then, all other sellers will want to sell for barter and the barter equilibrium will be reached. Similarly, if one seller decides to deviate from rationed barter equilibrium selling more for cash and less for barter, $\alpha G(P)$ will fall below c. Everyone will give up selling for barter leading to the no-barter equilibrium. □

Appendix B. Tables

Table B1
Definitions and descriptive statistics of variables

Variable	Description	Mean	S.D.	Min	Max
B	Share of barter in sales	0.39	0.24	0	0.83
CR4	Share of the four largest firms in total sales of an industry	0.39	0.27	0.04	1
CR4ia	Import-adjusted CR4	0.24	0.19	0	0.99
SIZE	Logarithm of annual sales in denominated rubles	17.8	1.3	14.6	22.3
CGI	Equals to 1 if firm belongs to consumer good industry	0.24		0	1
CASH	Ratio of stock of liquid assets (cash plus current accounts) to annual sales	0.02	0.10	0	1.64
EXPORT	Share of exports in sales	0.11	0.18	0	0.97
rCENTER	Equals to 1 if firm is located in Central Russia	0.35		0	1
rMSK	Moscow	0.14		0	1
rNW	North West Region	0.09		0	1
rSOUTH	Southern Russia	0.05		0	1
rVOLGA	Volga Region	0.20		0	1
rURAL	Urals Region	0.06		0	1
rSIB	Siberia	0.08		0	1
rFE	Far East Region	0.03		0	1
IND2	Equals to 1 if firm belongs to fuel industry	0.01		0	1
IND3	Ferrous metals	0.08		0	1
IND4	Non-ferrous metals	0.03		0	1
IND5	Chemical	0.12		0	1
IND6	Machinery	0.25		0	1
IND7	Pulp and woodworking	0.08		0	1
IND8	Construction materials	0.11		0	1
IND9	Textile	0.15		0	1
IND10	Food processing	0.15		0	1
IND11	Other	0.03		0	1

Table B2
Correlation matrix

	B	CR4	CR4ia	CASH	SIZE	EXPORT
B	1					
CR4	0.27***	1				
CR4ia	0.30***	0.80***	1			
CASH	0.004	0.06	0.04	1		
SIZE	0.11**	0.37***	0.29***	− 0.02	1	
EXPORT	0.0010.11	0.24***	0.14***	0.17***	0.25***	1
CGI	− 0.35***	− 0.33***	− 0.27***	− 0.06	− 0.14***	− 0.19***

** Significant at the 10% significance level.
*** Significant at the 1% significance level.

References

Aukutzionek, S., 1998. Industrial barter in Russia. Communist Economies and Economic Transformation 10 (2), 179–188.

Banerjee, A., Maskin, E., 1996. A Walrasian theory of money and barter. Quarterly Journal of Economics 111 (4), 955–1005.

Brana, S., Maurel, M., 1999. Barter in Russia: liquidity shortage versus lack of restructuring. CEPR Discussion paper, 2258.

Brown, A., Ickes, B. and Ryterman, R., 1994. The Myth of Monopoly: A New View of Industrial Structure in Russia. World Bank Policy Research Working Paper 1331.

Caves, R., 1974. The economics of reciprocity: theory and evidence on bilateral trading arrangements. In: Sellekaerts, W. (Ed.), International Trade and Finance: Essays in Honour of Jan Tinbergen. Macmillan, London, pp. 17–54.

Caves, R., Marin, D., 1992. Countertrade transactions: theory and evidence. Economic Journal 102, 1171–1183.

Commander, S. and Mumssen, C., 1998. Understanding Barter in Russia. EBRD Working Paper 37.

Commander, S., Dolinskaya, I., Mumssen, C., 2002. Determinants of barter in Russia: an empirical analysis. Journal of Development Economics 67 (2), 275–307.

Economist, C., 2000. Barter's Latest Comeback. Newspaper Ltd., London. Oct 19.

Ellingsen, T. 1998. Payments in Kind. Stockholm School of Economics Working Paper 244.

Ellingsen, T., Stole, L., 1996. Mandated countertrade as a strategic commitment. Journal of International Economics 40, 67–84.

Gaddy, C., Ickes, B., 2002. Russia's Virtual Economy. Brookings Institution Press, Washington, DC.

Guriev, S., Kvassov, D., 2000. Barter for price discrimination? CEPR Discussion Paper, 2449.

International Reciprocal Trade Association, 2001. Why business people barter? International Reciprocal Trade Association, Rochester, NY, p. 1 (http//:www.irta.net/factsheets/whybusinessesbarter.html).

Ivaldi, M., Martimort, D., 1994. Competition under nonlinear pricing. Annales d'Economie et de Statistique 34, 71–114.

Kiyotaki, N., Wright, R., 1989. On money as a medium of exchange. Journal of Political Economy 97, 927–954.

Magenheim, E., Murrell, P., 1988. How to haggle and stay firm: barter as mutual price discrimination. Economic Inquiry 26 (3), 449–460.

Marin, D., Schnitzer, M., 1999. Disorganization, financial squeeze and barter. CEPR Discussion Paper, 2245.

Oren, S., Smith, S., Wilson, R., 1983. Competitive nonlinear tariffs. Journal of Economic Theory 29, 49–71.

Pinto, B., Drebentsov, V., Morozov, A., 2000. Give macroeconomic stability and growth in Russia a chance. Economics of Transition 8 (2), 297–324.

Prendergast, C., Stole, L., 1998. Barter, Liquidity and Market Segmentation, Mimeo. Graduate School of Business, University of Chicago, Chicago, IL.

Prendergast, C., Stole, L., 1999. Restricting the means of exchange within organizations. European Economic Review 43, 1007–1019.

Russian Business Survey Bulletin, 1996. Various Issues. Institute for the Economy in Transition, Moscow.
Seabright, P. (Ed.), 2000. The Vanishing Ruble: Barter Networks and Non-Monetary Transactions in Post-Soviet Societies. Cambridge Univ. Press, Cambridge, UK, pp. 1–374.
Stole, L., 1995. Nonlinear pricing and oligopoly. Journal of Economics and Management Strategy 4 (4), 529–562.
Woodruff, D., 1999. Money Unmade: Barter and the Fate of Russian Capitalism. Cornell University Press, Ithaca.