Macroeconomic Dynamics in a Model of Goods, Labor and Credit

Market Frictions¹

Nicolas Petrosky-Nadeau‡  Etienne Wasmer‡
Tepper School of Business  Sciences Po Paris
Carnegie Mellon University

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Abstract

This paper shows that goods-market frictions drastically change the dynamics of the labor market, bridging the gap with the data both in terms of persistence and volatility. In a DSGE model with three imperfect markets - goods, labor and credit - we find that credit- and goods-market imperfections are substitutable in raising volatility. Goods-market frictions are however unique in generating persistence. The two key mechanisms generating autocorrelation in growth rates and the hump-shaped pattern in the response to productivity shocks are related to the goods market: i) countercyclical dynamics of goods market tightness and prices, which alter future profit flows and raise persistence and ii) procyclical search effort in the goods market, by either consumers, firms or both, raises both amplification and persistence. Expanding our knowledge of goods market frictions is thus needed for a full account of labor market dynamics.

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²Tepper School of Business, Carnegie Mellon University, 5000 Forbes Ave., Pittsburgh, PA, 15213. E-mail: npn@cmu.edu.
³Sciences Po, Paris, Departement d’Economie, 28 Rue des Saint-Pères, 75007 Paris, France. E-mail: etienne.wasmer@sciences-po.fr.
Introduction

The propagation of productivity shocks has been at the core of the real business cycle (RBC) research agenda since its inception. The need for large innovations to obtain realistic business cycle fluctuations, and the lack of any quantitatively significant endogenous persistence mechanism in the RBC model, has been emphasized in, respectively, King and Rebelo (1999) and Cogley and Nason (1995). The subsequent literature has focused on finding mechanisms that endogenously generate large and persistent responses to shocks.

The same issues and controversies arise when studying the cyclical dynamics of aggregate labor markets in the Mortensen-Pissarides search-and-matching model. The central variable in search models of the labor market, the ratio of job vacancies to unemployment, is very volatile in the data, and its response to a productivity shock peaks several quarters after the innovation. In a standard labor search model, however, this ratio reacts very little to productivity shocks and the peak response of the $v/u$ ratio is contemporaneous to the innovation. In response, a large body of research has developed amplifying mechanisms with some works focusing on the role of frictions in labor and credit markets.\(^1\) However, few of these papers have addressed the issue of persistence. Most important, there has been no systematic effort to identify and quantify the role of labor and credit frictions, let alone frictions in the goods market, in propagating shocks.

Our approach aims precisely to address this issue. We provide a model with three imperfect markets—goods, labor and credit—and find which market frictions are needed to match the cyclical properties of labor market variables in the data. Our modeling strategy represents each friction as a process matching two sides of the three markets, respectively, jobs, lending relationships and goods. Although this assumption is now well accepted in the labor and financial markets (e.g., Duffie et al., 2005), it is quite novel in modeling goods-market frictions, and indeed, it is only recently that this approach has proven useful in the macro-labor literature.\(^2\) The goods market was, however, the prototypical market in the early search literature (e.g., Diamond, 1982). Diamond's Nobel lecture (2011) emphasizes at length developments in search frictions in the goods market, and in particular their implications for unemployment.

The bilateral matching process leads to several appealing properties. As put forward in the macro-search literature (Mortensen and Pissarides, 1999, Rogerson and Shimer, 2011), the relative measure of supply and demand in each market captures the degree of market tightness: the familiar vacancy–unemployment ratio

\(^1\)For research on focusing on amplification in the labor market, see Shimer (2005) and Hall (2005), while Fujita and Ramey (2007) raise the issue of persistence. For financial frictions, we refer to the large literature on financial multipliers surveyed in Gertler and Kiyotaki (2013). Section 4 offers a detailed discussion of the relevant literature.

\(^2\)Recent papers include Bai et al. (2011), who model frictions in the goods market with a matching process between buyers and sellers, den Haan (2013), who models the friction on the seller’s side of the goods market with a focus on the dynamics of inventories, and Gourio and Rudanko (2013). We discuss this research below and in Section 4.
in the labor market, the ratio of prospecting consumers and products in the goods market, and the ratio of investment projects to creditors in the credit market. Further, it introduces endogenous mark-ups and turnover in each market. A frictionless view of markets is an oversimplification, and search and matching frictions offer a relevant characterization. In addition, a matching model of the goods market has interesting implications. For one, consumer search effort is procyclical. Consumers search more when they have higher income and when more firms supply goods, and the data speak clearly in favor of procyclicality. As Table 1 from the American Time Use Survey over the 2003-2011 period reports, the average time spent in a day on shopping activities declined during the last recession, with the exception for grocery shopping which remained stable.

Following up on our previous research (Petrosky-Nadeau and Wasmer, 2013) where credit and labor market imperfections were introduced in a dynamic general equilibrium model, we now develop a dynamic general equilibrium approach of frictional markets – goods, labor and credit – with free-entry. We then study how these frictions interact to propagate exogenous shocks to productivity. The calibration strategy matches a set of empirical moments in each market. This allows us to evaluate the qualitative and quantitative contribution of each market friction to the propagation of technology shocks and the cyclical dynamics of the labor market. That is, we decompose the role of each friction in generating amplification and persistence.

Our main insight is a surprising one. We find that goods-market frictions drastically change the qualitative and quantitative dynamics of the labor market. They amplify shocks just as credit market frictions. They are unique in being able to generate endogenous persistence and are thus better at bridging the gap

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This observation is true in the aggregate (see Table 1), even if there may also exist incentives for consumers to search more for lower price in recessions - a mechanism absent from our model with no price dispersion across identical goods. See Shi (2011) as well as the early contribution in Shi (1998) discussed in Section 4 for the cyclical implications of search in the goods market.

In our model, essential goods such as Food and Utility will be frictionless goods and reflect this cyclical pattern. See Appendix A for a description of the series.
with the data both in terms of persistence and volatility.

This contribute to the existing results in the literature, which faced the challenge of explaining persistence of labor market variables as measured by positive autocorrelations of growth rates and a hump-shaped pattern of impulse responses to productivity shocks. Our previous work (Petrosky-Nadeau and Wasmer, 2013), in line with a vast literature on financial market imperfections, was only able to significantly raise volatility by the presence of a financial multiplier arising from search frictions in the credit market but was not able to address the persistence challenge.

Persistence arises from the fact that the dynamics in the goods-market affect incentives to hire workers follow shocks to productivity. This translates into large and hump-shaped responses of labor-market tightness. During the first stages of an economic expansion, more firms enter the goods market relative to the change in the effective demand from consumers. This reduces the rate at which firms meet consumers in the goods market tightness, making it more difficult for a given product to meet a consumer and be sold. The negotiated price at which the goods are eventually sold thus declines due to consumers’ improved bargaining position. Even if the net impact on firms’ revenue – price multiplied by production – is positive, from the perspective of a firm deciding to hire a worker this moderates the incentives to create a vacancy at the beginning of an expansion as the additional production sells at a lower price. However, as the entry of firms in the goods market begins to slow down, the congestion in the goods market for firms eases. This process is stronger with endogenous consumer search effort. As consumers see rising incomes and a fall in prices, they raise their search effort in the goods market to reach their desired consumption level. This leads to a further easing of the matching rate in the goods markets from the perspective of the firms in the periods after the technology shock as there is relatively more demand from consumers than products competing for customers. As such, firms are more likely to find a consumer and sell at a higher price, so the incentives to recruit workers increase even as productivity is returning to trend. These mechanisms in the goods market combine to generate both amplification and persistence in the labor market. Labor market tightness peaks five quarters after the innovation, as in the data, and the model can match the empirical volatility of the $v-u$ ratio. Propagation arises from the fact that the economic value of hiring a worker is tied to the dynamics of prices and congestion in the goods market. These mechanisms are absent from environments using the standard labor search model.

These properties of goods market frictions are robust to several alternative modeling strategies of the goods market. In particular, we show the results are robust to introducing endogenous search effort on both sides of the goods market, consumers and firms, separately and simultaneously, as well as to constant search effort. Furthermore, this additional amplification and persistence does not hinge on a particular
wage determination schedule, and in particular does not arise from wage rigidity.\footnote{We show this in a variety of ways. First, we implement a reduced-form wage rule, proposed by Blanchard and Gali (2010), in which the wage elasticity can be controlled directly. Second, we allow for Nash bargained wages.}

This paper is organized as follows. In Section 1, we develop the model. In Section 2, we calibrate the model to monthly U.S. data. In Section 3, we investigate the sources of propagation in detail. In Section 4, we review the literature and connect our model to the empirical evidence on goods-market frictions. Section 5 concludes.

1 An Economy with Goods-, Labor- and Credit-Market Frictions

We consider the case of a firm evaluating marginal investment projects. These projects first need to obtain financing on the credit market. A financed project is then managed so as to maximize the value to the firm and the creditor, and needs to hire a worker to produce a good. Thus far, we follow the structure in Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2013). However, the good cannot be sold until a consumer has been found. Once a match in the goods market is formed, consumers and producers bargain over the price.\footnote{Wasmer (2009) investigated a steady-state version of this economy with several differences, notably constant search effort and no dynamic implications. His focus was on the existence and uniqueness of the equilibrium. A result is that congestion in the goods market, the ratio of unmatched consumers to unmatched firms, is equal to one in the steady state. This property is proved, convenient for obtaining simple solutions, but does not hold out of the steady state. Here, the business cycle fluctuations of goods-market tightness are key for the results.} The creditor has a monopoly over the ability to allocate resources from one period to the next. There is no money in this economy in contrast with Berentsen et al. (2011) who study jointly a search–money framework with labor-market frictions and a focus on interest rates’ impact on unemployment. In particular, we assume that consumers do not have the ability to transfer wealth from one period to the next. However, this assumption is not binding as, with search frictions in the goods market, savings can be shown to be a dominated strategy (see Appendix F).

1.1 Financing Investment Projects

Time is discrete. In the \textbf{first stage}, an investment project is initially in need of a financial partner. This financing will cover the cost of recruiting a worker and the wage bill when the firm has not found a demand for its product. Prospecting on the credit market costs $\kappa_I$ units of effort per period of time. At time $t$, with probability $p_t$ the investment project finds a creditor, and with complementary probability it remains in this stage (denoted by $c$ for \textit{credit}). A creditor pays a per-period screening cost, $\kappa_B$, and meets a project with probability $\hat{p}_t$. The asset values of investment projects and creditors in this stage are denoted by $J_c$ and $B_c$, respectively. At meeting between creditor and project, both sides agree on the terms of a financial contract whereby the resulting costs of the project are financed by the creditor when the project’s revenue
is negative (in stages 2 and 3) and the project pays the banker when the revenue is positive (in stage 4). Going forward, we will be interested in the joint values to the creditor and the investment project, which we refer to as a firm. Let the value of a firm for each of the stages be denoted by $S_{j,t} = J_{j,t} + B_{j,t}$, with $j$ indicating the corresponding stage.\footnote{We relegate to Appendix B the Bellman equations for the entry of projects and creditors—that is, equations (B-1) to (B-4) and (B-5) to (B-8). The Bellman equations of the value of a firm (matched creditor and project) in each stage can be obtained by summing the corresponding equations.}

Now matched with a creditor, the project enters the second stage where it prospects on the labor market to hire a worker. It must pay a per-period cost $\gamma$ to maintain an active job vacancy. With probability $q_t$, the firm is successful in hiring a worker, with complementary probability, it remains in this stage (denoted by $l$ for labor). We denote by $S_l$ the asset value in this stage. The firm offers a wage $w_t$ to the worker as long as the firm is active.

In the third stage, now endowed with a worker, the firm could start producing $x_t$ units of output from this particular project and attempt to sell it on the goods market, but it has no customers. A consumer arrives with probability $\lambda_t$, and all production can be sold in the fourth stage because, as we will show later, matched consumers will absorb the entire production.\footnote{An alternative interpretation of $x_t$ is a quality shock affecting consumers’ utility when the good produced is indivisible.} Assuming that production involves an operating cost $\Omega$ over and above the wage and that the good cannot be stored, the firm chooses not to produce in this stage (denote by $g$ for goods market). The asset value of the firm in this stage is denoted $S_g$. Note that the creditor is still financing the firm by transferring the amount of resources necessary to pay the worker.

In the fourth and final stage, the firm is now matched with a consumer and its output $x_t$ is sold at price $P_t$. We assume that $x_t$ is a random, stationary process. With revenue $P_t x_t$, the firm pays the worker $w_t$, the operating cost $\Omega$, an amount $\rho_t$ to the creditor, and enjoys the difference.\footnote{Other papers in the literature consider sources other than productivity shocks as the driving force of fluctuations. See Bai et al. (2011) for demand shocks and Altig et al. (2011) for investment-specific shocks. We restrict our analysis to the more standard productivity shock as we are introducing new frictions in the model.} We denote this stage by $\pi$, standing for profit and use $S_\pi$ for its associated asset value. In addition, the consumer may stop consuming the particular good produced by this project with probability $\tau$, in which case the project returns to the previous stage $g$ to search for another consumer.

There are two additional types of separation. First, workers may choose to separate from the firm, which occurs each period with probability $s^L$. Second, the match between the investment project and its creditor may dissolve, leading to bankruptcy. These two separation events allow us to distinguish a small bankruptcy rate in the business sector, typically around $s^c = 1\%$ quarterly, from the larger the turnover rate of workers, here $s = s^c + (1 - s^c) s^L$.\footnote{Carlstrom and Fuerst (1997) provide a discussion of bankruptcy statistics.} 

Finally, as in Pissarides (2000), all profit opportunities are exhausted by new entrants such that the
value of the entry stages are always driven to zero. In the case of the credit market, this implies that 
\( J_{c,t} = B_{c,t} \equiv 0 \) at all times, which is also the continuation value following the credit destruction shock \( s^c \) that can occur at the end of every stage. This results in the following Bellman equations:

\[
S_{c,t} = 0 \iff \frac{\kappa_B}{p_t} + \frac{\kappa_I}{p_t} = S_{l,t} \tag{1}
\]

\[
S_{l,t} = -\gamma + \frac{1 - s^c}{1 + r} E_t \left[ q_t S_{g,t+1} + (1 - q_t) S_{l,t+1} \right] \tag{2}
\]

\[
S_{g,t} = -w_t + \frac{1 - s^c}{1 + r} E_t \left[ (1 - s^L) \left( (1 - \lambda_t) S_{\pi,t+1} + (1 - \lambda_t) S_{g,t+1} + s^K S_{l,t+1} \right) \right] \tag{3}
\]

\[
S_{\pi,t} = P_t x_t - w_t - \Omega + \frac{1 - s^c}{1 + r} E_t \left[ (1 - s^L) \left( (1 - \tau) S_{\pi,t+1} + \tau S_{g,t+1} + s^K S_{l,t+1} \right) \right] \tag{4}
\]

Equation (1) states that the value of a firm in the hiring stage is equal to the sum of capitalized search costs paid by each side in the previous credit market stage. This is driven to zero in the absence of credit-market frictions. The formulation the labor-market stage in equation (2) describes the value of a job vacancy as a flow cost \( \gamma \) and an expected gain from hiring a worker, valued at \( S_g \). \((1 - s^c)\) is the survival probability of the credit relationship with the complement probability that the match’s value is zero (dissolution of the match). The value of \( S_g \) in equation (3) takes into account the labor cost \( w_t \), and, conditional on no bankruptcy \((1 - s_c)\), the firm may or may not become profitable in the next period. This depends on meeting customers with probability \( \lambda_t \), and whether the worker remains with the firm with probability \((1 - s^L)\). If the worker leaves, the firm returns to the earlier stage \( l \). As we will discuss in detail, the presence of a frictional goods market fundamentally alters the dynamics of \( S_g \) compared to the standard framework (e.g., Pissarides, 2000, Shimer, 2005) through the dynamics of the goods-market meeting rate, \( \lambda_t \), and the price, \( P_t \). Finally, in stage \( \pi \), equation (4) shows that the match produces revenue \( P_t x_t \), pays a wage and other production costs, and may return to earlier stages depending on the occurrence of consumer change of taste, labor turnover or bankruptcy. The repayment \( \rho_t \) does not appear in the last equation, as it is merely a transfer between the project and the creditor.

1.2 Search and Matching in the Goods Markets

1.2.1 Matching in the Goods Market

Consumers may spend a disposable income \( Y^d \), which we define below, on either an essential good (serving as a numeraire), \( c_0 \), or a preferred manufactured good, \( c_1 \). Consuming the later first requires searching in the goods market. When a consumer is matched with a manufacturing firm, it purchases the production, \( x_t \), at a unit price \( P_t \). The remaining income is spent on the essential good, which is supplied as a transfer.
of resource across individuals.\footnote{To avoid any further complication, we assume that the numeraire is produced by a technology without labor. The extension of our framework along the lines of Berentsen et al. (2011) with money-search would permit richer interpretations.}

At any point in time in this economy there are matched and unmatched consumers. Normalizing the mass of consumers to 1, we denote these shares by \( C_{1,t} \) and \( C_{0,t} \), respectively. In equilibrium, these will be the fractions of disposable household income allocated to either category of goods. Unmatched consumers \( C_{0,t} \), exert an average search effort, \( \bar{e}_{c,t} \), to find unmatched goods, \( N_{g,t} \), through a process summarized by a constant return-to-scale function, \( M_G(\bar{e}_{c,t}C_{0,t},N_{g,t}) \). \( \bar{e}_{c,t}C_{0,t} \) can be thought of as the effective demand for new goods. Following Pissarides (2000), the aggregate meeting rates between consumers and firms are given by

\[
\lambda_t = \frac{M_G(\bar{e}_{c,t}C_{0,t},N_{g,t})}{N_{g,t}} = \lambda(\xi_t, \bar{e}_{c,t}) \quad \text{with} \quad \lambda'(\xi_t) > 0
\]

\[
\tilde{\lambda}_t = \frac{M_G(\bar{e}_{c,t}C_{0,t},N_{g,t})}{C_{0,t}} = \tilde{\lambda}(\xi_t, \bar{e}_{c,t}) \quad \text{with} \quad \tilde{\lambda}'(\xi_t) < 0,
\]

where \( \xi_t = C_{0,t}/N_{g,t} \) is tightness in the goods market (from the consumers’ point of view). \( \tilde{\lambda}_t \), the probability that an unmatched consumer finds a suitable firm from which to buy goods, is decreasing in goods-market tightness. Conversely, the greater \( \xi_t \), the greater the demand from consumers relative to the goods waiting to find consumers and the shorter the producer’s search. This is precisely what creates the feedback from the goods market to the labor market: the returns to hiring a worker are greater when it is easier to find customers.

### 1.2.2 Disposable Income

Total net profits in this economy, \( \Pi_t \), are the sum of profit flows to projects and creditors. This corresponds to

\[
\Pi_t = (P_t x_t - \Omega)N_{\pi,t} - w_t N_t - \gamma N_{l,t} - \kappa B_{c,t},
\]

where \( N_t = N_{\pi,t} + N_{g,t} \) is the sum of the number of firms matched with a consumer, \( N_{\pi,t} \), and of the number of firms in stage \( g \), \( N_{g,t} \), (that is, matched with a creditor and a worker but not with a consumer). The number of firms prospecting for workers in stage \( l \), the labor market, is \( N_{l,t} \), and the number of creditors screening projects in stage \( c \) is \( B_{c,t} \). In the equation above, the first term is revenue generated by firms in stage 4, net of operating costs. The second term represents wage payments in the economy. The remaining terms represent the creditor’s outlays during the first stages due to search costs in labor and credit markets.
These profits net of search costs are pooled and distributed as lump sums to workers. The mass 1 of workers, the unemployed and employed, therefore receive $\Pi_t$ per person and per period as a transfer. Further, resources are pooled across categories of workers, as in Merz (1995) and Andolfatto (1996). The average disposable income of a representative consumer is $\Pi_t + N_tw_t$. The timing of the transmission of income from firm to consumer within a period corresponds to the simplest possible assumption, and also the most natural.\(^{12}\)

1.2.3 Value Functions for Consumers

Individuals want to consume manufactured goods, but may not buy them before prospecting on the goods market. Let us denote by $D_0,t$ and $D_1,t$ the values for a consumer of being unmatched and matched, respectively. The generic utility of consuming both goods is denoted by $U(c_1,c_0)$. Unmatched consumers search for a good at an effort cost $\sigma(e_{c,i})$, with $\sigma'(e_{c,i}) > 0$, $\sigma''(e_i) \geq 0$ and $\sigma(0) = 0$, with elasticity with respect to effort $\eta_\sigma > 0$. They perceive their search effort as influencing their effective finding rate, $e_{c,i,t} \tilde{\lambda}_t / \bar{e}_{c,t}$, where $\bar{e}_{c,t}$ was defined above as aggregate consumer search effort. Consequently, we have

\[
D_{0,t} = U(0,c_{0,t}) - \sigma(e_{c,i,t}) + \frac{1}{1 + \rho} E_t \left[ \frac{e_{c,i,t} \tilde{\lambda}_t}{e_{c,t}} D_{1,t+1} + \left( 1 - \frac{e_{c,i,t} \tilde{\lambda}_t}{e_{c,t}} \right) D_{0,t+1} \right]
\]

\[
D_{1,t} = U(c_{1,t},c_{0,t}) + \frac{1 - s^c}{1 + \rho} E_t \left[ (1 - s^L) \left( \tau D_{0,t+1} + (1 - \tau) D_{1,t+1} \right) + \frac{s^c + (1 - s^c) s^L}{1 + \rho} E_t D_{0,t+1} \right].
\]

Assuming the manufactured good has greater marginal utility, matched consumers will always absorb $x_t$ units of good 1 at the expense $P_t x_t$ and then expend what is left, $Y^d_t - P_t x_t$, on $c_{0,t}$. In other words, the utility in the first equation (unmatched consumer) is

\[
U(0,Y^d_t),
\]

and the utility in the second equation (matched consumer) will be

\[
U(x_t,Y^d_t - P_t x_t).
\]

\(^{12}\)Alternative specifications, where the disposable income in $t$ would be generated through profits and sales from period $t-1$ would affect the dynamics and add a source of persistence, which we avoid here by our specification. The dynamic path we obtain is thus not artificially slowed.
Finally, going forward, we will assume a linear form for consumer utility:

$$U(c_{1,t}, c_{0,t}) = \Phi c_{1,t} + c_{0,t}$$

with the assumption that $\Phi \geq 1$.

### 1.2.4 Optimal Search Effort

The optimal individual search effort is given by a condition equating the marginal cost of effort to the discounted, expected benefit yielded by that marginal unit of effort:

$$\bar{e}_{c,t} \sigma'(\bar{e}_{c,t}) = \frac{\tilde{\lambda}_t}{1 + r} E_t \left[ (D_{1,t+1} - D_{0,t+1}) \right]. \quad (7)$$

Equation (7) implies that consumer search effort is increasing in the expected capital gain from consuming the manufactured good, and it follows that all consumers exert the same effort:

$$e^*_{c,t} = \bar{e}_{c,t}.$$

Combining the first-order condition above and the asset-value equations for consumers, as detailed in Appendix C, we have

$$\sigma'(\bar{e}_{c,t}) = \frac{\tilde{\lambda}_t}{1 + r} E_t \sum_{i=0}^{\infty} \psi^i \left[ \left( \frac{\Phi}{P_{t+1+i}} - 1 \right) Y^{d}_{t+1+i} + (1 - \eta_d) \sigma(\bar{e}_{c,t+1+i}) \right], \quad (8)$$

where $\psi \equiv (1 - \tau) (1 - s^L) (1 - s^c) / (1 + r)$ is a discount factor. This expression reveals that the level of effort will depend on expected disposable income and the dynamics of the price $P$. A drop in the bargained price raises the incentive to exert search effort in the goods market, as does an expected increase in future income.

### 1.2.5 Determining the Dynamics of the Goods Surplus and Price

Consistent with the search literature, we postulate that the price $P_t$ is bargained between a consumer and a firm. The total surplus to the consumption relationship is $G_t = (S_{\pi,t} - S_{g,t}) + (D_{1,t} - D_{0,t})$. The good’s price is determined as $P_t = \arg\max (S_{\pi,t} - S_{g,t})^{1-\delta} (D_{1,t} - D_{0,t})^\delta$, where $\delta \in (0, 1)$ is the share of the goods surplus, $G_t$, going to the consumer. This results in the sharing rule

$$(1 - \delta) (D_{1,t} - D_{0,t}) = \delta (S_{\pi,t} - S_{g,t}), \quad (9)$$
from which we obtain the negotiated price rule:

\[ P_t x_t = (1 - \delta) \left[ \Phi_t x_t + (1 - \eta \sigma) \sigma(\bar{e}_{c,t}) \right] + \delta \Omega + (1 - \delta) \lambda_t (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [\delta G_{t+1}] . \]  

(10)

This rule emphasizes the forward-looking aspect of price determination: today’s price is increasing in the expectation of tomorrow’s surplus in the goods market.\(^{13}\) The latter effect on price depends on current goods-market congestion through the meeting rate, \( \lambda_t \): A greater the effective demand on the consumer side relative to the supply of unmatched goods \( \bar{N}_g \) or more market tightness \( \xi_t \) engenders a higher price and hence profits for firms. This effect captures the incidence of greater competition for consumers in goods markets on prices.

In addition, equation (10) states that the revenue accruing to the firm is increasing in the marginal utility, \( \Phi_t \), from consuming good \( c_1 \). The worker is ready to pay a higher price to consume. Since we assume convex costs—that is, an elasticity \( \eta > 1 \)—the effect of consumer search effort is negative. This is a pure surplus effect: when effort increases, the consumer’s surplus increases ceteris paribus because it negatively affects the value of consumption search, \( D_{0,t} \). Now, as the consumption of good 1 is inelastic, an increase in the price reduces the consumption of \( c_0 \) one to one. Hence, for a given bargaining strength \( \delta \), there must be a negative association between price and effort. This turns out to be an important qualitative effect, generating counter cyclicity of the price per unit of output. It arises from the existence of goods-market frictions and bargaining over the price, and is absent from a frictionless goods market.

1.3 Matching in Other Markets

1.3.1 Matching and the Division of Rents in the Credit Market

The matching rates \( p_t \) and \( \hat{p}_t \) are made mutually consistent by the existence of a matching function \( M_C(\mathcal{B}_{c,t}, \mathcal{N}_{c,t}) \), where \( \mathcal{B}_{c,t} \) and \( \mathcal{N}_{c,t} \) are, respectively, the number of creditors and projects in stage \( c \). This function is assumed to have constant returns to scale. Hence, denoting by \( \phi_t \) the ratio \( \mathcal{N}_{c,t}/\mathcal{B}_{c,t} \), which reflects tightness of the credit market from the projects’ point of view, we have

\[ p_t = \frac{M_C(\mathcal{B}_{c,t}, \mathcal{N}_{c,t})}{\mathcal{N}_{c}} = p(\phi_t) \text{ with } p'(\phi_t) < 0 \] 

(11)

\[ \hat{p}_t = \phi_t p(\phi_t) \text{ with } \hat{p}'(\phi_t) > 0 . \] 

(12)

The rents from implementing a project, \( S_{t,t} \), are divided by bargaining about \( \rho \), upon meeting. Calling

\(^{13}\)The details of the derivation for this and subsequent equations are provided in Appendix C.
\( \beta \in (0, 1) \) the bargaining power of the creditor, the Nash bargaining condition,

\[
(1 - \beta) B_{l,t} = \beta J_{l,t},
\]  

states that with \( \beta = 1 \) the creditor receives all the surplus. Note that the rule for \( \rho \) is determined at the time of the meeting but paid a few periods after the negotiation, when the firm becomes profitable. We assume that there is no commitment problem, as in Wasmer and Weil (2004), so that any new realization of aggregate productivity will not undo the financial contract and there is no renegotiation.

The equilibrium value of \( \phi_t \), denoted by \( \phi^* \), is obtained by combining (B-1), (B-5) and (13), with the definition of \( \hat{p} \) in (12):

\[
\phi^* = \frac{\kappa_B}{\kappa_I} \frac{1 - \beta}{\beta} \sqrt{t}.
\]  

(14)

Free-entry of both creditors and projects to credit markets implies a constant credit-market tightness over time, even out of the steady state. Going forward, all the information pertaining to the credit market is contained in the total transaction costs paid by both firms and creditors in stage \( c \),

\[
K \equiv \frac{\kappa_B}{\phi^* p(\phi^*)} + \frac{\kappa_I}{p(\phi^*)},
\]  

(15)

where \( K \) is simply a constant of the parameters given the equilibrium value of \( \phi \) determined in equation (14).

1.3.2 Matching in the Labor Market and Wage Determination

We assume that matching in the labor market is governed by a function \( M_L(N_{l,t}, u_t) \), where \( u_t \) is the rate of unemployment and the total number of unemployed workers since the labor force is normalized to 1. \( N_{l,t} \), already defined as the number of firms in stage \( l \), is also the number of “vacancies,” \( V_t \). The function is assumed to have constant returns to scale. Hence, the rate at which firms fill vacancies is a function of the ratio \( N_{l,t}/u_t = \theta_t \), a measure of the tightness of the labor market. This vacancy filling rate, \( q(\theta_t) \), is given by

\[
q(\theta_t) = \frac{M_L(N_{l,t}, u_t)}{N_{l,t}} \text{ with } q'(\theta_t) < 0.
\]

Conversely, the rate at which the unemployed find a job is

\[
\frac{M_L(N_{l,t}, u_t)}{u_t} = \theta_t q(\theta_t) = f(\theta_t) \text{ with } f'(\theta_t) > 0.
\]
Once employed, workers earn a wage $w_t$. We follow two approaches for the wage. First, for simplicity, we assume a wage rule that takes the functional form

$$w_t = \chi_w(P_t x_t)^{\eta_w},$$

(16)

where $\eta_w$ is the elasticity of wages to the marginal product of labor, $P_t x_t$. This simple rule, used in Blanchard and Galí (2010), allows us to focus on the role played by the elasticity of wages to productivity for propagation. That is, for a given elasticity of wages, we can evaluate the propagation of shocks to the economy, and in particular the labor market, coming from frictional goods and credit markets.

In the spirit of search models, one may want to have a wage schedule as the outcome of Nash-bargaining between the firm and the worker. This is the second approach in which there is not one, but two wage schedules depending on whether or not the firm is currently selling its production. This will add analytical complications, but makes only a small quantitative difference compared to the simple wage rule. The wage rule in this case is

$$w_t = \begin{cases} 
\arg\max (S_{g,t} - S_{t,t})^{1-\alpha} (W_{g,t} - U_t)^{\alpha} & \text{if the firm is in stage } g \\
\arg\max (S_{\pi,t} - S_{t,t})^{1-\alpha} (W_{\pi,t} - U_t)^{\alpha} & \text{if the firm is in stage } \pi,
\end{cases}$$

(17)

where $W_g$ and $W_{\pi}$ are the asset values of employment to a worker, and $U$ is the value of unemployment.

These Bellman equations, along with the derivations of the wage rules, are presented in Appendix D. The resulting wage rules are

$$w_t = \begin{cases} 
\alpha \theta_t (\gamma + \frac{r+\delta}{1+r} K) - \alpha \left(1 + sL \frac{1-\alpha}{1+r}\right) K + (1-\alpha)b & \text{in stage } g \\
\alpha \left(P_t x_t - \Omega - \left[1 + sL \frac{1-\alpha}{1+r}\right] K\right) + \alpha \theta_t (\gamma + \frac{r+\delta}{1+r} K) + (1-\alpha)b & \text{in stage } \pi.
\end{cases}$$

(18)

In the numerical exercise, we will implement the two wage rules and show that this makes no difference in the results, both qualitatively and quantitatively.

### 1.4 Stocks of Consumers, Employment and Unemployment

Having stipulated the transition rates for all agents in the economy, we can now write the laws of motion for the stocks of consumers, firms and, consequently, employment. Potential consumers, $C_0$, become consumers the period after meeting a producer, and a fraction $0 < \tau < 1$ of current consumers separate from their

14A second complication, which we do not fully address here and leave to future work, arises from the number of bargaining parties. This can lead to several complexities depending on the assumptions on timing and bargaining structure.
product only to return to the pool of potential consumers the following period. The stocks of consumers in the goods market therefore evolve according to stock-flow equations, fully reported in Appendix E, where we also report the laws of motion for $N_g, N_\pi$ and unemployment.

### 1.5 Equilibrium

The equilibrium is a set of policy and value functions for the consumers \( \{ D_{0,t}, D_{1,t}, \bar{e}_{c,t} \} \) and firms \( \{ S_{c,t}, S_{l,t}, S_{g,t}, S_{\pi,t} \} \); a set of prices in goods, labor and credit; and stocks and measures of tightness in the markets for goods, labor and credit \( \{ B_{c,t}, N_{c,t}, N_{l,t}, N_{g,t}, N_{\pi,t}, C_{0,t}, C_{1,t}, u_t \} \) and \( \{ \xi_t, \theta_t, \phi \} \) such that:

1. Consumers’ value follows functions (5) and (6), with the search-effort optimality condition (7).
2. The value for firms follows (1) to (4) with free entry in the credit market.
3. Prices in the goods, labor and credit markets are determined by Nash bargaining given by conditions (9), (18) and (13)
4. Stock in the goods and labor markets follow conditions (E-10) to (E-15)

### 2 Calibration Strategy

We begin by detailing our calibration strategy. In the next section, we discuss and quantify the sources of propagation that arise in our model by computing Hodrick-Prescott (H.-P.) filtered second moments and impulse-response functions.\(^{15}\) This section also compares the roles of the different market frictions for the propagation of shocks for the dynamics of the labor market. In addition we present robustness results with respect to the modeling and parameters of the goods market, and to the specification of the wage rule.

The basic unit of time is a month. The process for productivity is assumed to be an AR(1) in logs with persistence of $\rho_x = 0.95^{1/3} = 0.983$, as in Gertler and Trigari (2009). We then calibrate its conditional volatility, $\sigma_x$, to be 0.007 to match the standard deviation of 0.013 for labor productivity in the data.\(^{16}\) The matching functions in the labor, goods and credit markets take the functional form proposed in den Haan et al. (2000). All parameter values are reported in Table 2.

**Credit market parameters:** $r, s^C, \beta, \nu_C, \kappa_I, \kappa_B$

The risk free rate $r$ is set to an annualized 4%. The separation rate $s^C$ is set for a 1% quarterly firm exit rate, as per the evidence reported in Carlstrom and Fuerst (1997). The remaining parameters follow

---

\(^{15}\)The models in this section are solved using a projection algorithm as in Petrosky-Nadeau and Zhang (2013a). As a result of the nonlinearity of the model we target simulation moments in our calibration rather than using steady state relationships. The second moments are computed by taking the averages of the monthly simulated data. The impulse responses of the monthly model are converted to quarters in the same way.

\(^{16}\)Seasonally adjusted real average output per person in the nonfarm business sector from the Bureau of Labor Statistics.
Table 2: Baseline Monthly Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Sources or Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Labor market:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>job-separation rate</td>
<td>$s^L_0$ 0.043</td>
<td>Davis et al. (2006)</td>
</tr>
<tr>
<td>matching curvature</td>
<td>$\nu_L 1.25$</td>
<td>Den Haan et al. (2000)</td>
</tr>
<tr>
<td>vacancy cost</td>
<td>$\gamma 0.14$</td>
<td>Silva and Toledo (2007)</td>
</tr>
<tr>
<td>wage elasticity</td>
<td>$\eta_w 0.65$</td>
<td>Wage elasticity</td>
</tr>
<tr>
<td>wage level parameter</td>
<td>$\chi_w 0.80$</td>
<td>Wage to productivity ratio</td>
</tr>
<tr>
<td><strong>Goods market:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>goods exit rate</td>
<td>$\tau 0.01$</td>
<td>Broda and Weinstein (2010)</td>
</tr>
<tr>
<td>matching curvature</td>
<td>$\nu_G 1.40$</td>
<td>Goods market transition rate</td>
</tr>
<tr>
<td>cost function level parameter</td>
<td>$\chi_\sigma 0.5$</td>
<td>American Time Use Survey</td>
</tr>
<tr>
<td>cost function elasticity</td>
<td>$\eta_\sigma 2$</td>
<td>Quadratic cost</td>
</tr>
<tr>
<td>consumer bargaining weight</td>
<td>$\delta 0.30$</td>
<td>Share of expenditure on essential good</td>
</tr>
<tr>
<td>marginal utility of $c_1$</td>
<td>$\Phi 1.15$</td>
<td>Price Markup</td>
</tr>
<tr>
<td>production cost</td>
<td>$\Omega 0$</td>
<td></td>
</tr>
<tr>
<td><strong>Credit market:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>separation rate</td>
<td>$s^C 0.01/3$</td>
<td>Bernanke et al (1999)</td>
</tr>
<tr>
<td>bank bargaining weight</td>
<td>$\beta 0.62$</td>
<td>Petrosky-Nadeau and Wasmer (2013)</td>
</tr>
<tr>
<td>search costs</td>
<td>$\kappa_B = \kappa_I 0.1$</td>
<td>Petrosky-Nadeau and Wasmer (2013)</td>
</tr>
<tr>
<td>risk-free rate</td>
<td>$r 0.01/3$</td>
<td>3 month U.S. T-bill</td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>persistence parameter</td>
<td>$\rho_x 0.95^{1/3}$</td>
<td>B.L.S labor productivity</td>
</tr>
<tr>
<td>standard deviation</td>
<td>$\sigma_x 0.007$</td>
<td>B.L.S. labor productivity</td>
</tr>
</tbody>
</table>

The five parameters related to the labor market, $s^L, \nu_L, \gamma, \chi_w$ and $\eta_w$, are set as follows. The rate of labor separation $s^L$ is set such that the aggregate rate of job separation, $s^C + (1 - s^C)s^L$, equals 0.045 per month, consistent with the estimate based on JOLTS data in Davis et al. (2010). Given a value of $s^C = 0.01/3$, we set $s^L = (0.045 - s^C) / (1 - s^C) = 0.043$. For the curvature parameter in the matching function $\mathcal{N}(U) / (\mathcal{N}(U^L + U^L)^{1/\nu_L}$, we follow den Haan et al. (2000) set it to target an average job filling rate.

The financial sector’s share of GDP in the model is defined $\Sigma_t = [(1 - u_t) \rho_t - \gamma N_{t,t} - B_{t,t} \kappa_B] / \{P_t (1 - u_t) + C_{0,t}\}$. 

The calibration strategy in Petrosky-Nadeau and Wasmer (2013), to which we refer for details. That is, $K$ is set for a target unemployment rate of 5.80%, the average rate in the U.S. post-war sample. From the fact that the share of the financial sector in GDP in the model is a strictly increasing function of the bargaining weight $\beta$, we set $\beta = 0.6$ to target a 2.5% share in U.S. data. The curvature of the credit matching function $\mathcal{N}(U) / (\mathcal{N}^{\nu_C} + U^{\nu_C})^{1/\nu_C}$ is set such that the average duration of search in the credit market by creditors is four months, resulting in $\nu_C = 1.35$. Finally, search flow costs are obtained as $\kappa_I = \beta K_p(\phi)$ and $\kappa_B = (1 - \beta) K_p(\phi)$. 

**Labor market parameters:** $s^L, \nu_L, \gamma, \chi_w$ and $\eta_w$
\( q(\theta) \) of 0.40 per month. This results in a value \( \nu_L = 1.25 \), which is close to the value in the referenced work. The unit vacancy costs are set to \( \gamma = 0.14 \) to be consistent with evidence in Silva and Toledo (2009) that the average cost of recruiting is about 40% of a monthly wage. Finally, the parameters of the wage rule are set as follows. The level parameter \( \chi_w \) is set to 0.80 or such that the wage equals 0.75 of labor productivity on average. The elasticity parameter is set to \( \eta_w = 0.65 \) such that the elasticity of wages to changes in productivity in the model is about 0.60. This elastic wage is in the upper range of estimates on U.S. aggregate data. Hagedorn and Manovskii (2008), for instance, estimate an elasticity of 0.49, while Haefke et al. (2013) argue for a value of 0.79.

**Goods market parameters:** \( \tau, \nu_G, \chi_\sigma, \eta_\sigma, \delta, \Phi, \Omega \).

Broda and Weinstein (2010) report a four-year product exit rate of 0.46, and a median 1 year exit rate of 0.24. From the model equivalent rate, \( s^c + (1 - s^c)(1 - s^L)\tau \), targeting the mid-range of these exits rates implies a monthly goods separation rate due to changes in tastes of \( \tau = 0.01 \). Broda and Weinstein (2010) also report a rate of product entry of 0.25 on an annualized basis. This implies an average transition rate for consumers in the goods market of \( \tilde{\lambda} = 0.20 \), which we target with the curvature parameter in the goods market matching function \( e \zeta_0 N' g / \left( (e \zeta_0)^{\nu_G} + N'^{\nu_G} \right) \). This results in \( \nu_G = 1.40 \). We rely on the Bureau of Labor Statistics’ time-use survey calibrate the search costs in the goods market. This survey reports that households spend on average half an hour a day purchasing goods and services (0.4 hour for men, 0.6 hour for women). Of course, this is not necessarily time spent searching and comparing goods before making a choice. Nor does it include travel related to these activities. Assuming an individual works on average seven hours a day, spread over a week, the cost of time searching in the goods market corresponds to approximately 7% of wage income. That is, we target an average value for \( \sigma(e)/w \simeq 0.07 \). Assuming a cost function \( \sigma(e) = (\chi_\sigma \eta_\sigma) e^{\nu_\sigma} \), we obtain \( \chi_\sigma = 0.50 \). Our baseline parameterization specifies the costs as quadratic with \( \eta_\sigma = 2 \), and we investigate the sensitivity of the results to the degree of curvature later. We target an expenditure share of the essential good in order to determine share of the goods market surplus accruing to the consumer, \( \delta \). Data from the Household Consumption Expenditure Survey reveal that average annual expenditure on food consumed at home, plus utilities, over the period 1984 to 2009 amounts to 10 to 15% of total annual expenditures. The share in the model is defined as \( C_0 / [PC_1 + C_0] \), where the expenditures are weighted by the fraction of unmatched and matched consumers. This results in \( \delta = 0.30 \). The marginal utility \( \Phi \) is set to 1.15 for an average markup over marginal cost of 10%, in the lower-range of values reported in Basu and Fernald (1997) and Nekarda and Ramey (2011). The cost of

---

18At a steady state the annual entry rate of 0.25 implies a monthly consumer finding rate in the goods market of \( \tilde{\lambda} = 0.25C_1 \tau / \left[ C_0 \left( 1 - (1 - \tau)^{12} \right) \right] = 0.20 \) given an average share of matched consumers of \( C_1 = 0.90 \) that results from our calibration. We verify in simulations that the average product exit rate is consistent with the empirical evidence reported above.

19We define this markup in the model as \( \mathcal{P}_t \sigma_{x,t} / \left[ N_{g,t} w_t + N_{x,t} (w_t + \Omega) \right] \).
production parameter is assumed negligible and set to \( \Omega = 0 \), avoiding any amplification that could arise from the introduction of an additional fixed element in the profit flow during the sales stage.

3 The Sources of Propagation

3.1 Theoretical Insights

Fluctuations in the labor market are driven by the decision of firms to create jobs. Thus, a job-creation condition, an equation relating labor-market tightness to the expected value of a hired worker, lies at the heart of propagation. Combining equations (1) and (2), and calling \( o_t = \frac{(1-q_t)(r+s^c)}{q_t(1+r)} \), a quantitatively negligible term, this job-creation condition is\(^{20}\)

\[
\frac{K(\phi^*)(1+o_t)}{q(\theta_t)} + \frac{\gamma}{q(\theta_t)} = \frac{1 - s^c}{1+r} E_t S_{g,t+1}
\]

(19)

which equates the average cost of creating a job – the left-hand side, equal to the financial costs properly discounted, \( K \), and the expected costs of search on the labor market, \( \gamma/q(\theta_t) \) – to the discounted expected value of a worker to the firm in the goods market stage (the right-hand side).

A few words of comparison with the canonical labor search model are warranted here. First, the costs of financial intermediation enter the left-hand side of the equation and place a lower bound on the value of a “vacancy” to a firm. Absent credit-market frictions, the average cost of creation depends on the flow cost of a vacancy, \( \gamma \), and congestion on the labor market. Second, the expected value on the right-hand side corresponds to the ability to produce and sell a good once a consumer has been located. Under frictionless goods markets, the right-hand side is simply the value of the profit stage. Thus, the current model nests the canonical search model when \( K \) tends to zero and the goods-market friction is removed.

The relationship between labor-market tightness and the expected value of a filled vacancy to a firm becomes very transparent when looking at a log-linear approximation of this job-creation condition around the deterministic steady state:

\[
\hat{\theta}_t = \frac{1}{\eta_L} \times \frac{S_q}{S_q - K} \times E_t \hat{S}_{g,t+1}
\]

(20)

where \( \eta_L \) is the elasticity of the job-filling rate with respect to labor-market tightness and “hatted” variables

\(^{20}\)With our calibration values \( o_t \) is approximately equal to 0.01.
indicate proportional deviations from the steady state. Over and above the amplification of changes in $S_g$ from frictions in the labor market, measured as the inverse of the elasticity of the labor-matching function, credit-market frictions create an amplifying factor of $\frac{S_g}{S_g-K}$. This financial accelerator is decreasing in the firm’s surplus to hiring a worker, $S_g - K$. Note that frictions in the goods market will also affect the value of $S_g$ and provide amplification through the known channel of reducing the firm’s surplus, $S_g - K$. However the amplification from the goods market in the second term in the right hand side $S_g/(S_g-K)$ only occurs when $K > 0$. Thus, the main additional and novel effects of goods-market frictions on the dynamics of the labor market work through their impact on the dynamics of the expected value of a filled vacancy, $\mathbb{E}_t \hat{S}_{g,t+1}$.

Two features of the goods market fundamentally change the dynamics: i) the likelihood of reaching the profit stage in the period after hiring the worker, $\lambda_t$, which will affect the value of $\mathbb{E}_t \hat{S}_{g,t+1}$, and ii) the expected profit flow, which depends on the price of the goods, $P_t$. In order to see this more clearly, consider the value of a filled vacancy in a standard Mortensen-Pissarides matching model where the goods market is perfect: $S_{g,t}^{MP} = x_t - w_t + \left(1 - \alpha \frac{1}{1+r}\right) \mathbb{E}_t S_{g,t+1}^{MP}$. From the recursive nature of $S_{g,t}^{MP}$, all that matters for the dynamics of labor-market tightness under perfect goods markets is the expected path of the net profit flow, $x_t - w_t$.

### 3.2 The Response of Labor Market Tightness to Technology Shocks: Amplification and Persistence

We begin by describing the dynamics of the model’s response to productivity innovations in order to illustrate the various determinants of amplification and persistence. We will next compare the baseline responses and second moments to alternative models and specifications with a particular focus on the role of frictions in each market.

Figure 1 plots the impulse response to a 1% shock to technology $x$ of labor-market tightness (first panel). The second panel (top right) reports the impulse response of the expected values of a filled vacancy, $\mathbb{E}_t \hat{S}_{g,t+1}$. Figure 1 also reports the response of the goods-market meeting rate for firms, $\lambda$, (third panel) and price, $P$, (fourth panel) that enter the job creation condition (19), along with consumer search effort (fifth panel) and goods market tightness (sixth panel) in order to shed light on the dynamics introduced by goods-market frictions. The expected value of a filled vacancy peaks five periods after the realization of the shock, generating an inverted U-shape for the response of labor-market tightness. This is a feature of the data emphasized in Fujita and Ramey (2007) and absent in the standard labor search model.\(^{21}\)

We now focus on the impulse responses of the meeting rate in the goods market and the price by

\(^{21}\)Shi (1998)’s approach of search in the goods market is able to replicate the hump shape pattern of the data, although in his case it is in response to a shock to the growth rate of money supply.
examining the bottom panels of Figure 1. These are the key aspects of the goods market that create the intertemporal linkages between the labor and goods markets, and are the novel features of our model with respect to the literature. Firms expect a drop in next period’s consumption tightness and the likelihood of finding a consumer for their goods after the realization of the technology shock. This arises from an inflow of new goods on the market greater than the change in effective consumer demand, or equivalently from a drop in the tightness of the goods market, $\xi = C_0/N_g$. However, this partially is undone by the rise in consumer search effort. Indeed, as shown in equation (8), an increase in the expected surplus value of the consumption match, due to the rise in the consumer goods-meeting rate, $\tilde{\lambda}$, and a decline in the price $P$, will raise consumers’ search effort to get a share of this surplus.

The rise in effort by consumers and decline in goods-market tightness have a second effect. They lead to a drop in the negotiated price $P$. This effect acts to limit the incentives to create jobs in the immediate period following the technology shock. In the subsequent periods, consumers increase their search effort in the goods market further in response to the drop in prices and the rise in their disposable income. This improves the position of firms in price negotiations such that the price $P$ is increasing even as technology is returning to trend. The evolution of the goods market thus generates increasing incentives to hire workers even as technology is returning to trend, allowing labor-market tightness to increase for several quarters after the innovation.
3.3 The Respective Roles of the Different Market Frictions

We explore how the propagation properties of frictions in the goods market contrast with propagation in the canonical labor search model and from the amplification due to a financial accelerator in models with credit market imperfections. The first three columns of Table 3 report the standard deviations of HP filtered data and model labor market variables of unemployment, $u$, job vacancies $V = N_1$, and labor market tightness $\theta$. This measure will establish a degree of amplification of productivity shocks. The next four columns report the first and second order autocorrelation of the growth rate in job vacancies $V$ and labor market tightness $\theta$. This measure will establish the degree of persistence generated by market frictions. The final column reports the correlation between unemployment and vacancies.

The first row of Table 3 reports the moments from U.S. quarterly data over the period 1951:I to 2012:IV. Labor market variables display large fluctuations over the business cycle in the data, a well know fact, and are very persistent at the first and second lags. This persistence in the data has received less attention. The subsequent rows present the results for four different models: i) the baseline model with endogenous consumer search effort (model CLG); ii) a model with a frictional labor market and perfect credit and goods markets (model L); iii) a model with frictional credit and labor markets and perfect goods market (model CL) and; iv) a model with labor and goods market frictions and perfect credit markets (model GL). These alternative economies are calibrated to match the same average unemployment rate as the baseline model by adjusting the flow cost of vacancies $\gamma$ in the GL and L models, and by adjusting $\gamma$ and the credit market costs $K$ in proportion CL model.

The model with frictional goods, labor and credit markets generates standard deviation of unemployment, vacancies and labor-market tightness of 0.09, 0.16 and 0.24, respectively. This degree of volatility is comparable to that observed in the data. Removing the friction in the credit and goods market, as the following rows of Table 3 attests, reduces the standard deviations. In particular, removing both credit and goods market frictions (row ‘Labor frictions’) brings the standard deviation of unemployment down to 0.02 and the standard deviation of labor market tightness to 0.04. From this reference model, adding credit market imperfections (row ‘Credit-Labor frictions’) adds a factor of 3 to 4 the volatility: the standard deviation of unemployment reaches 0.06 and the standard deviation of labor market tightness reaches 0.16 (as in Petrosky-Nadeau and Wasmer, 2013). Similarly, from this reference model ‘Labor’, adding goods market imperfections (row ‘Goods-Labor frictions’) adds a factor 4 to 5 to the volatility: the standard deviation of unemployment reaches 0.07 and the standard deviation of labor market tightness reaches 0.20. The combination of both goods and credit market friction thus generates a sizable degree of amplification. The volatility of unemployment increases by a factor of 4.5, job vacancies by a factor of 5 and labor market...
tightness by a factor of 6. Hence, frictions in credit and goods markets are similar and complementary in generating amplification.

The columns reporting the measures of persistence establish the uniqueness of goods market frictions in propagating shocks to productivity compared with credit market frictions. The model without frictional labor and credit markets, the canonical labor search environment (model L), generates autocorrelation at the first and second lag of job vacancies of -0.12 and -0.06, respectively. This is sharply improved upon in the baseline model with credit and goods market frictions as the autocorrelations increase to 0.09 and -0.01. Moreover, only frictions in the goods market can induce this persistence as the results from model CL and GL in the last two rows of Table 3 attest. The model with goods and labor market frictions has a coefficient of autocorrelation of 0.05 at the first lag. The model with credit and labor market frictions in contrast has a coefficient of -0.06 at the first lag.
Table 3: Labor Market Second Moments

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Autocorr. $\Delta V_t$</th>
<th>Autocorr. $\Delta \theta_t$</th>
<th>$\text{Corr}(U,V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U$</td>
<td>$V$</td>
<td>$\theta$</td>
<td>Lag 1</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.13</td>
<td>0.14</td>
<td>0.27</td>
<td>0.68</td>
</tr>
<tr>
<td>Baseline model Credit-Labor-Good frictions (CLG)</td>
<td>0.09</td>
<td>0.16</td>
<td>0.24</td>
<td>0.09</td>
</tr>
<tr>
<td>Labor frictions (L)</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>-0.12</td>
</tr>
<tr>
<td>Credit-Labor frictions (CL)</td>
<td>0.06</td>
<td>0.10</td>
<td>0.16</td>
<td>-0.06</td>
</tr>
<tr>
<td>Goods-Labor frictions (GL)</td>
<td>0.07</td>
<td>0.13</td>
<td>0.20</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Data sources: B.L.S. and Petrosky-Nadeau and Zhang (2013b)

Table 4: Labor market second moments: Extensions

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Autocorr. $\Delta V_t$</th>
<th>Autocorr. $\Delta \theta_t$</th>
<th>$\text{Corr}(U,V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U$</td>
<td>$V$</td>
<td>$\theta$</td>
<td>Lag 1</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.13</td>
<td>0.14</td>
<td>0.27</td>
<td>0.68</td>
</tr>
<tr>
<td>Variant 1. Baseline (consumer effort)</td>
<td>0.09</td>
<td>0.16</td>
<td>0.24</td>
<td>0.09</td>
</tr>
<tr>
<td>Variant 2. Firm effort (advertising for goods)</td>
<td>0.08</td>
<td>0.24</td>
<td>0.28</td>
<td>0.11</td>
</tr>
<tr>
<td>Variant 3. Two sided effort (consumer and firm)</td>
<td>0.11</td>
<td>0.22</td>
<td>0.27</td>
<td>0.12</td>
</tr>
<tr>
<td>Variant 4. Constant consumer effort</td>
<td>0.07</td>
<td>0.21</td>
<td>0.23</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Figure 2 compares the responses of labor market tightness $\theta_t$ to a productivity shock in our four comparison models. The CLG model generates amplification and persistence that is very apparent in the hump-shape pattern of the response. Indeed, the response of labor market tightness peaks 5 quarters after the innovation, consistent with the empirical evidence for the U.S. in Fujita and Ramey (2007). The GL models loses some amplification but, significantly, retains some persistence in the response. The hump-shaped pattern is clear, although slightly less pronounced. Finally, the CL model’s response is similar in magnitude to the GL model yet loses all the persistence. The peak response is contemporaneous to the innovation. The L model fails in terms of both amplification and persistence. This emphasizes the unique feature of frictions in the goods market, namely the ability to generate endogenous persistence in labor market variables.

Summarizing, goods market frictions contribute most to improving the qualitative and quantitative dynamics of labor market variables. The model with search frictions in the goods market features a lagged response to productivity shocks. In what follows we investigate further the sources and robustness of goods market frictions to generate persistence.

3.4 Robustness: Alternative Specifications for Goods Market Frictions

Alternative specifications of goods market frictions lead to similar results and conclusions. In the benchmark model we introduced endogenous consumer search effort, which was procyclical, and consistent with the search behavior exhibited in the introduction. It is however possible to consider effort on either side of the goods market, individually or simultaneously, as well as constant search effort in the goods market. That is, we can introduce advertising effort by firms, consider simultaneous consumer search effort and advertising effort, or no time variation in either margin. We explore each of these cases in this subsection, and present the details of the derivations Appendix G.\footnote{As rightly pointed out by a referee, there is a total of six intensive search margins in the model. Four more search margins could have been introduced: in the credit market, search effort of creditors and new investment projects. In the labor market, job advertising by firms and search effort by the unemployed. We can show that three out of the four intensive margins are actually invariant to the cycle due to free entry assumptions. Only the last one, search effort by the unemployed, generates procyclical search behavior, arising from the convexity in the costs of job search for workers. Although this is likely to increase amplification of productivity shocks, it is not going to raise persistence and has received extensive attention in the literature (Pissarides, 2000). As such, we focus on the alternative intensive search margins in the goods market detailed in this Section.}

The model with advertising effort $e_{A,t}$ and no consumer search is the symmetrical of the model with consumer effort. The transitions in the goods market are now: $\bar{\lambda}_t = M_G(C_{0,t}, e_{A,t}, N_{g,t})/C_{0,t}$ and $\lambda_t = M_G(C_{0,t}, e_{A,t}, N_{g,t})/N_{g,t}$, and the cost of advertising effort is specified as $\sigma_{A,t} = (\chi_A/\eta_A) e_{A,t}^{\eta_A}$ with $\chi_A > 0$.
and $\eta_A = 2$. The first order condition for advertisement effort is

$$\sigma'_A(e_{A,t}) = \frac{\lambda_t}{e_{A,t}} (1 - s^L) \left( \frac{1 - s^c}{1 + \bar{\delta}} \right) \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}]$$

(21)

The search efforts of both sides of the goods market are procyclical as they follow the marginal expected surplus from the goods market match. This procyclicality of advertising is empirically confirmed by Hall (2012) the U.S. Finally, the resulting price equation is:

$$P_t x_t = (1 - \delta) [v(x_t)] + \delta [\Omega - (1 - \eta_A)\sigma_A(e_{A,t})]$$

It is possible to allow for endogenous search effort on both sides of the goods market. In this case, combining the optimality conditions for the intensive search margins (8) and (21), we have:

$$\frac{\bar{e}_{A,t}\sigma'_A(e_{A,t})}{\bar{e}_{c,t}\sigma'_c(e_{c,t})} = \left( \frac{1 - \delta}{\delta} \right) (1 - s^L) (1 - s^c) \xi_t$$

Since effort of one side increases the returns to effort of the other side, we have a strategic complementarity arising from bilateral search effort. This can potentially increase the amplification of productivity shocks relative to the baseline model of Section 1. This is quantitatively evaluated in Table 4. Variant 1 is the baseline model with only consumers’ effort. Variant 2 is the dual with no consumer effort but advertising effort, and where we have set the level parameter $\chi_A = 1.5$ to target a ratio of advertising costs to GDP of 2% as documented in Tremblay (2012). The volatility and degree of persistence are quite similar to the baseline variant 1. As expected, variant 3 with bilateral search effort has some additional volatility and persistence. Quite interestingly, shutting down both endogenous search margins, as in variant 4 with a constant consumer effort, does not eliminate amplification and persistence. Both remain quite high, and more than the alternative models without goods market frictions L and CL.

### 3.5 Sensitivity to Other Parameters and Specifications

Table 5 reproduces the main moments and compares them with alternative curvature to the search cost function. In particular, instead of a quadratic cost function with $\eta_\sigma = 2$, smaller values of the elasticity (1.75) have a marginal effect on volatility and persistence, while a larger elasticity (2.25) only marginally reduces the volatility (standard deviation of labor market tightness) and slightly raises the autocorrelation of first differences (the persistence). The intuition of the higher volatility comes from inspection of equation (8). The larger $\eta_\sigma$, the steeper the marginal cost in the left hand side. The level of effort thus reacts less to changes in the right hand side, which captures the surplus from the consumption relation. Overall, the
model and our conclusions are quite stable with respect to various values of $\eta_\sigma$ around 2.

We also explore the role of matching parameters. Alternative values of the matching curvature leave the main moments almost unchanged: both amplification and persistence outcomes are preserved in the range of 1.1 to 1.6. Finally, since the rigidity of wages to changes in labor productivity have an important role in amplifying technology shocks in labor-market search models (see Hall, 2005, Shimer, 2005), we need to explore alternative wage determination scenarios. We compare the business-cycle statistics of the baseline model CLG solved with the Nash-bargained wages defined in equation (18).\textsuperscript{23} Nash-bargained wages do not change the results with respect to either volatility or persistence. The business-cycle second moments in Table 5 indicate that the degree of amplification is comparable to the baseline model.

\textsuperscript{23}In parameterizing the CLG model with Nash wages, with set $b = 0.71$ as in Hall and Milgrom (2008) and the bargaining weight $\alpha = 0.15$ to target a wage elasticity of 0.70.
Table 5: Labor market second moment: sensitivity analysis

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Autocorr. $\Delta V_t$</th>
<th>Autocorr. $\Delta \theta_t$</th>
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<td>Lag 1</td>
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<tr>
<td>U.S. Data</td>
<td>0.13</td>
<td>0.14</td>
<td>0.27</td>
<td>0.68</td>
</tr>
<tr>
<td>Baseline (CLG, $\eta = 2$, $\nu = 1.5$)</td>
<td>0.09</td>
<td>0.16</td>
<td>0.24</td>
<td>0.09</td>
</tr>
<tr>
<td>Alternative search cost elasticity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta = 1.75$</td>
<td>0.08</td>
<td>0.17</td>
<td>0.24</td>
<td>0.09</td>
</tr>
<tr>
<td>$\eta = 2.25$</td>
<td>0.09</td>
<td>0.15</td>
<td>0.22</td>
<td>0.10</td>
</tr>
<tr>
<td>Alternative matching curvature</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 1.1$</td>
<td>0.08</td>
<td>0.16</td>
<td>0.23</td>
<td>0.08</td>
</tr>
<tr>
<td>$\nu = 1.6$</td>
<td>0.10</td>
<td>0.17</td>
<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>Baseline (CLG) and Nash wage</td>
<td>0.10</td>
<td>0.17</td>
<td>0.22</td>
<td>0.09</td>
</tr>
</tbody>
</table>
4 Relation to the Literature

Early research into propagation in business-cycle models focused on the labor market, either increasing the elasticity of labor supply – e.g., models of indivisible labor in Hansen (1985) and Rogerson (1988)—or introducing a market friction in the form of wage rigidity (e.g., Taylor, 1980, Christiano et al., 2005). The importance of the latter in amplifying the response of the demand for labor to changes in productivity has received renewed attention in search models of equilibrium unemployment as a means of addressing the lack of volatility in job vacancies and unemployment. In this sense, our work is related, first, to that of Shimer (2005), which fully articulates the lack of amplification in the search model of equilibrium unemployment. Second, it is related to the work of Fujita and Ramey (2007) who document the persistence in the empirical response of labor-market tightness to a productivity shock, and propose treating vacancies as a stock variable to generate sluggishness in the response to shocks.

The role of credit markets in amplifying exogenous shocks to economies and the existence of a financial accelerator has been emphasized in such papers as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). We take into account the potential importance of frictional credit markets by introducing a financial accelerator of the type explored in Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2013). These papers considered the static and dynamic properties of the financial accelerator that arises from the interaction of frictional labor and credit markets.

Our main novelty here is to develop a model in which the introduction of goods-market imperfections generates additional insights into the sources of macroeconomic dynamics when allowed to interact with a frictional labor market. It has, of course, long been recognized that non-clearing or departures from Walrasian equilibria in goods markets can generate additional unemployment. Several waves of research have attempted to put this intuition into models (see the survey in Benassy, 1993). This previous literature has mostly been centered around the idea of price rigidities leading to excess supply of (or demand for) goods, which in turn generate inefficient outcomes in the labor market. In our paper, expected profits, and hence labor demand, depend on the ability of the supply and demand in the goods market to meet, over and above the role of prices. In this we are continuing a tradition of the search literature with an explicit focus on frictions in the goods market (e.g., Diamond, 1971, 1982), and more recently in the conclusions drawn in Diamond 2011: “For addressing unemployment, there are clear needs to incorporate credit markets and (non-Walrasian) output markets and to include nominal thinking and nominal contracting as well as a larger role for current income”.

Goods markets are indeed characterized by the presence of rents and product turnover that lend themselves to be modeled as search-and-matching markets. The earliest paper related to ours is Shi (1998). As
noted above, his approach of search in the goods market is able to generate the hump shape pattern of the data, which is another confirmation of the importance of search in the goods market. His model is more in the tradition of monetary economics, incorporates money demand and the velocity of money, and the economy reacts to shocks to the growth rate of money. In addition, contrary to our model, the seller’s surplus is countercyclical.

Price mark-ups over marginal cost are prevalent in goods markets and remain a core feature of New Keynesian models of the macroeconomy. A certain degree of market power permits firms to charge mark-ups in the range of 11% to 40%, according to Basu and Fernald (1997) and Nekarda and Ramey (2011), respectively. Price mark-ups can also indicate the presence of frictions in locating consumers, of having to establish and maintain a distribution network. Retail trade, for instance, amounts to 5.8% of aggregate GDP in the U.S., suggesting a costly allocation of final goods to consumers operated by intermediaries. Moreover, Foster et al. (2008) present evidence for frictions in accessing a distribution network or reaching consumers in that new firms face lower demand than comparable older firms, and they are willing to charge a lower price. Note that this same evidence also suggests the presence of costs for consumers to acquire information about a producer. These frictions lead to time and costs for both sides of the goods market in searching before acquiring or beginning to consume a good for the first time.

Over and above these observations, new data on household goods transactions paint a portrait of the goods market akin to the flows of gross and net creation and destruction in the labor and credit markets. Following Davis and Haltiwanger’s (1990, 1992) seminal contributions for labor markets, Dell’Ariccia and Garibaldi (2005) have measured creation and destruction in the U.S. loans market, while, most recently, Broda and Weinstein (2010) have carefully documented the magnitude of flows of entry and exit of goods in a typical household’s consumption basket. The authors built and used a unique data set of 700,000 products with bar codes purchased by 55,000 households. The covered sectors amount to 40% of all expenditures on goods in the CPI. Relevant to our approach, they find large flows of entries and exits of "products" in a typical consumer’s consumption basket, actually four times more than is found in labor markets. Moreover, a large share of product turnover happens within firms. Over their 9 year sample period, 1994–2003, the product entry rate, defined as the number of new product codes divided by the stock, is estimated to be 78%. The product exit rate, defined as the number of disappearing product codes over the stock, 72%. Further, net product creation is strongly procyclical and primarily driven by creation rather than destruction, which is weakly countercyclical. This suggests that high demand leads to the introduction of new goods, reminiscent of the implementation cycles present in the work of Shleifer (1986).

These empirical findings point to the direction theory should take, inspiring the details of the modeling

---

24 Similar estimates of mark-ups are found in Bils (1987) and Rotemberg and Woodford (1999).
and its interpretation. Throughout the paper, we followed the birth of a "product line,” its development and finally its death due in part to technological obsolescence, in part to changes in consumer tastes. The empirical moments just documented were the basis of the calibration strategy when we evaluated the model.

Last but certainly not least, the closest paper to ours is Bai et al. (2011). The authors independently developed a model with a process of matching between consumers and firms in the goods market in a DSGE framework. They use the same convincing argument that buying goods is an active process involving costs, which should, therefore, be part of a macroeconomic model. Their focus is on the impact of demand shocks in goods markets, and in particular their implications for the identification of technology shocks within the RBC paradigm. In contrast, ours is on the propagation of productivity shocks to the cyclical dynamics of the labor market. In their model, a competitive search environment with price posting, differentiated markets are indexed by the price and market tightness. As a result, across markets, prices are higher when goods are easier to find. Our model introduces search effort and price bargaining: search effort increases when future expected surplus is higher, which has implications in terms of the fit with the data. These ingredients turn out to be necessary to replicate the observed persistence in U.S. time series. Another independent work by Gourio and Rudanko (2013) focuses on frictions in the goods market with a focus on “customer acquisition” as a search friction in the goods market. Their paper investigates the level and volatility of firm level variables such as investment and sales and calculate a Tobin’s q. Finally, den Haan (2013) provides a detailed theory of inventory in line with search frictions in the goods market, a dimension that we ignored here but that deserves some more developments.

5 Conclusion

This paper investigates the significance of goods-market frictions for the dynamics of the labor market. The model with search frictions in the goods market features an amplified and persistence response to productivity shocks. This arises from new features of the dynamics of prices, goods market congestions and consumer search effort absent from models with perfect goods markets. Goods market frictions leads to rich intertemporal linkages between the labor and goods markets.

During the first stages of an economic expansion, more firms enter the goods market relative to the change in the effective demand from consumers. This reduces the rate at which firms meet consumers int the goods market tightness, making it more difficult for a given product to meet a consumer and be sold. The negotiated price at which the goods are eventually sold thus declines due to consumers’ improved bargaining position. Even if the net impact on firms’ revenue – price multiplied by production – is positive, from the perspective of a firm deciding to hire a worker, this moderates the incentive to create a vacancy at
the beginning of an expansion. However, as the entry of firms in the goods market begins to slow down, the congestion in the goods market for firms eases. This process is stronger with endogenous consumer search effort. Propagation in the model features both endogenous amplification and persistence, contrary to the standard labor search model and a large class of its extensions. Moreover, this unique aspect of goods market frictions is robust to introducing endogenous search effort on both sides of the market, consumers and firms.

Finally, our model of a frictional economy is a natural framework for introducing additional sources of shocks, in particular demand shocks and money as in Bai et al. (2011) and Berentsen et al. (2011). This is left to future work.

References


Appendix

Not for Publication

A Data Used in Calibration

In order to calculate the financial sector’s share of aggregate value added, we rely on the BEA’s industry value-added tables available at http://www.bea.gov/industry/io_histannual.htm. Because of i) changes in industry labels in 1987 and ii) a change of classifications in 1998, we report the exact series used for each time period:

- 1947 to 1987: we sum the value added of “banking” (code 60), “Credit agencies other than banks” (code 61) and “Security and commodity brokers” (code 62).

- 1987 to 1998: we sum the value added of “Depositary institutions” (code 60), “Non-depositary institutions” (code 61) and “Security and commodity brokers” (code 62).

- 1998 to 2009: we sum the value added of “Federal Reserve banks, credit intermediation, and related activities” (codes 521 and 522), “Securities, commodity contracts, and investments” (code 523) and “Funds, trusts, and other financial vehicles” (code 525).

The share sum is obtained by dividing by the corresponding year’s aggregate GDP from the same value-added tables.

Data on consumption expenditure is obtained from BLS series CXUTE000201 on total average annual expenditures, series CXUFH000201 for food consumed at home and series CXUUT000101 for utilities, fuels, and public services, covering the years 1984–2009.

Data for Search effort of Consumers is obtained from the BLS, according to which the category “Purchasing goods and services” includes purchases of consumer goods, professional and personal care services, household services, and government services. Consumer purchases include most purchases and rentals of consumer goods, regardless of the mode or place of purchase or rental (in person, via telephone, over the Internet, at home, or in a store). Gasoline, grocery, other food purchases, and all other shopping are further broken out in subcategories. Time spent obtaining, receiving, and purchasing professional and personal care services provided by someone else also is classified in this category. Professional services include childcare, financial services and banking, legal services, medical and adult care services, real estate services, and veterinary services. Personal care services include day spas, hair salons and barbershops, nail salons,
and tanning salons. Activities classified here include time spent paying, meeting with, or talking to service providers, as well as time spent receiving the service or waiting to receive the service. Time spent arranging for and purchasing household services provided by someone else also is classified here. Household services include housecleaning; cooking; lawn care and landscaping; pet care; tailoring, laundering, and dry cleaning; vehicle maintenance and repairs; and home repairs, maintenance, and construction. This category also captures the time spent obtaining government services–such as applying for food stamps–and purchasing government- required licenses or paying fines or fees.”

**B  Bellman Equations of the Investment Project and Creditor**

Given the assumptions described in Section 2.1, the Bellman equations of the investment project, which faces a discount rate $r$ and assuming that transitions from the credit to the labor market stages occur within a single period, are

$$J_{c,t} = 0 = -\kappa_t + p_t J_{l,t}$$  \hspace{1cm} (B-1)

$$J_{l,t} = \frac{1 - s^c}{1 + r} \mathbb{E}_t [q_t J_{g,t+1} + (1 - q_t)J_{l,t+1}]$$  \hspace{1cm} (B-2)

$$J_{g,t} = \frac{1 - s^c}{1 + r} \mathbb{E}_t [(1 - s^L) [\lambda_t J_{\pi,t+1} + (1 - \lambda_t)J_{g,t+1}] + s^L J_{l,t+1}]$$  \hspace{1cm} (B-3)

$$J_{\pi,t} = \mathcal{P}_t x_t - w_t - \rho_t - \Omega$$  
$$\quad + \frac{1 - s^c}{1 + r} \mathbb{E}_t [(1 - s^L) [(1 - \tau)J_{\pi,t+1} + \tau J_{g,t+1}] + s^L J_{l,t+1}] .$$  \hspace{1cm} (B-4)

The corresponding Bellman equations for the creditor are

$$B_{c,t} = 0 = -\kappa_B + \hat{p}_t B_{l,t}$$  \hspace{1cm} (B-5)

$$B_{l,t} = -\gamma + \frac{1 - s^c}{1 + r} \mathbb{E}_t [q_t B_{g,t+1} + (1 - q_t)B_{l,t+1}]$$  \hspace{1cm} (B-6)

$$B_{g,t} = -w_t + \frac{1 - s^c}{1 + r} \mathbb{E}_t [(1 - s^L) [\lambda_t B_{\pi,t+1} + (1 - \lambda_t)B_{g,t+1}] + s^L B_{l,t+1}]$$  \hspace{1cm} (B-7)

$$B_{\pi,t} = \rho_t + \frac{1 - s^c}{1 + r} \mathbb{E}_t [(1 - s^L) [(1 - \tau)B_{\pi,t+1} + \tau B_{g,t+1}] + s^L B_{l,t+1}] .$$  \hspace{1cm} (B-8)
C Determinations of Price $\mathcal{P}_t$

Utility

We begin by assuming a quasilinear form for consumer utility:

$$U(c_{1,t}, c_{0,t}) = v(c_{1,t}) + c_{0,t}.$$ 

Given these preferences, we can determine the negotiated price for the good $c_1$ as the outcome of Nash bargaining over the consumption surplus. $G_t = (S_{\pi,t} - S_{g,t}) + (D_{1,t} - D_{0,t})$. $\delta \in (0, 1)$ will be the consumer’s bargaining weight. As a first step, we derive expressions for the surplus to the consumption relationship for each side of the market. We have a budget constraint per period (since we assume no savings):

$$\mathcal{P}_t c_{1,t} + c_{0,t} = Y^d_t.$$ 

Consumer Surplus

When matched, the consumer consumes what is available, that is

$$c_{1,t} = x_t.$$ 

Recall the Bellman equations for unmatched and matched consumers.

$$D_{0,t} = c_{0,t} - \sigma(e_{ci,t}) + \frac{1}{1 + r}E_t \left[ \frac{e_{ci,t} \lambda_{t}}{e_{ct}} D_{1,t+1} + \left( 1 - \frac{e_{ci,t} \lambda_{t}}{e_{ct}} \right) D_{0,t+1} \right]$$

$$= Y^d_t - \sigma(\bar{e}_{ct}) + \frac{1}{1 + r}E_t \left[ \bar{\lambda}_t D_{1,t+1} + (1 - \bar{\lambda}_t) D_{0,t+1} \right]$$

$$D_{1,t} = v(c_{1,t}) + Y^d_t - \mathcal{P}_t c_{1,t}$$

$$+ \left( \frac{1 - s^c}{1 + r} \right) E_t \left( 1 - s^L \right) \left[ \tau D_{0,t+1} + (1 - \tau) D_{1,t+1} \right] + \frac{s^L}{1 + r} D_{0,t+1}$$

$$= v(x_t) + Y^d_t - \mathcal{P}_t x_t + \left( \frac{1 - s^c}{1 + r} \right) E_t \left( 1 - s^L \right) \left[ \tau D_{0,t+1} + (1 - \tau) D_{1,t+1} \right]$$

$$+ \frac{s^c + (1 - s^c) s^L}{1 + r} E_t D_{0,t+1}$$
Then the surplus to a match for a consumer is

\[
D_{1,t} - D_{0,t} = v(x_t) - P_t x_t + \sigma(\tilde{e}_{ct}) + \tau (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [D_{0,t+1} - D_{1,t+1}]
+ (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t D_{1,t+1} + \frac{s^c + (1 - s^c)s^L}{1 + r} \mathbb{E}_t D_{0,t+1}
- \tilde{\lambda}_t \frac{1}{1 + r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}] - \frac{1}{1 + r} \mathbb{E}_t D_{0,t+1}
\]

From the optimal choice of effort, we had that

\[
\tilde{\lambda}_t \frac{1}{1 + r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}] = \tilde{e}_{ct} \sigma'(\tilde{e}_{ct}).
\]

Hence,

\[
D_{1,t} - D_{0,t} = v(x_t) - P_t x_t + \sigma(\tilde{e}_{ct}) - \tilde{e}_{ct} \sigma'(\tilde{e}_{ct}) + \tau (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [D_{0,t+1} - D_{1,t+1}]
+ \frac{(1 - s^c)(1 - s^L)}{1 + r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}]
\]

\[
D_{1,t} - D_{0,t} = v(x_t) - P_t x_t + \sigma(\tilde{e}_{ct}) - \tilde{e}_{ct} \sigma'(\tilde{e}_{ct}) + (1 - \tau) \frac{(1 - s^c)(1 - s^L)}{1 + r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}].
\]

**Firm Surplus**

Turning to the surplus of a consumption match for a firm, we have

\[
S_{\pi,t} - S_{g,t} = P_t x_t - \Omega
- (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [(1 - \tau)S_{\pi,t+1} + \tau S_{g,t+1} - \lambda S_{\pi,t+1} - (1 - \lambda_t)S_{g,t+1}]
\]

\[
S_{\pi,t} - S_{g,t} = P_t x_t - \Omega + (1 - \tau - \lambda_t) (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}].
\]

**Sharing Rule**

\[
P_t = \operatorname{argmax} (S_{\pi,t} - S_{g,t})^{1-\delta} (D_{1,t} - D_{0,t})^\delta.
\]

Since \( \partial (S_{\pi,t} - S_{g,t})/\partial P_t = x_t \) and \( \partial (D_{1,t} - D_{0,t})/\partial P_t = -x_t \),

the sharing rule is

\[
(1 - \delta) (D_{1,t} - D_{0,t}) = \delta (S_{\pi,t} - S_{g,t}).
\]

**Goods Surplus**

\[
G_t = (S_{\pi,t} - S_{g,t}) + (D_{1,t} - D_{0,t})
\]

\[
= v(x_t) - \Omega + (1 - \tau - \lambda_t) (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}]
+ \sigma(\tilde{e}_{ct}) - \tilde{e}_{ct} \sigma'(\tilde{e}_{ct}) + (1 - \tau) (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}]
\]

\[
G_t = v(x_t) - \Omega + \sigma(\tilde{e}_{ct}) - \tilde{e}_{ct} \sigma'(\tilde{e}_{ct})
+ (1 - \tau) (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t G_{t+1} - \lambda_t (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}]
\]

\[
G_t = v(x_t) - \Omega + \sigma(\tilde{e}_{ct}) - \tilde{e}_{ct} \sigma'(\tilde{e}_{ct})
+ (1 - \tau) (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t G_{t+1} - \lambda_t (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}]
\]

\[
38
\]
Under regular assumptions on the effort cost function \( \sigma(e_c) \), it will be the case that \( e_c \sigma'(e_c) = \eta_\sigma \sigma(e_c) \), where \( \eta_\sigma > 0 \) is the elasticity of the effort cost function.

\[
G_t = v(x_t) + (1 - \eta_\sigma)\sigma(\bar{e}_c) - \Omega + [(1 - \tau) - (1 - \delta)\lambda_t] (1 - s^L) \frac{1 - s^c}{1 + r} E_t G_{t+1}
\]

**Negotiated Price \( \mathcal{P} \)**

In order to solve for the price, we equate the firm surplus, \( S_{\pi,t} - S_{g,t} = (1 - \delta)G_t \), to the previous expression for the goods surplus:

\[
(1 - \delta)G_t = (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e_{ct}) - \Omega] + (1 - \delta) [(1 - \tau) - (1 - \delta)\lambda_t] (1 - s^L) \frac{1 - s^c}{1 + r} E_t G_{t+1}
\]

\[
= (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e_{ct}) - \Omega] + (1 - \delta) [(1 - \tau) - (1 - \delta)\lambda_t] (1 - s^L) \frac{1 - s^c}{1 + r} E_t G_{t+1}
\]

\[
= \mathcal{P}_t x_t - \Omega + (1 - \tau - \lambda_t) (1 - s^L) \frac{1 - s^c}{1 + r} E_t [S_{\pi,t+1} - S_{g,t+1}]
\]

\[
\mathcal{P}_t x_t = (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e_{ct})] + \delta \Omega + (1 - \delta)\lambda_t (1 - s^L) \frac{1 - s^c}{1 + r} E_t G_{t+1}
\]

\[
- (1 - \delta)^2 \lambda_t (1 - s^L) \frac{1 - s^c}{1 + r} E_t G_{t+1}
\]

\[
\mathcal{P}_t x_t = (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e_{ct})] + \delta \Omega + (1 - \delta)\lambda_t (1 - s^L) \frac{1 - s^c}{1 + r} E_t [\delta G_{t+1}] .
\]

Recall that \( \delta G_t = D_{1,t} - D_{0,t} \); that from the optimal choice of effort, \( \frac{1 - \delta}{1 + r} E_t [D_{1,t+1} - D_{0,t+1}] = \bar{e}_{ct} \sigma'(\bar{e}_{ct})/\tilde{\lambda}_t \); and that from the properties of the goods matching function, we have that \( \frac{\partial}{\partial \lambda_t} = \xi_t \). Therefore,

\[
\mathcal{P}_t x_t = (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e_{ct})] + (1 - s^c) (1 - s^L) \sigma'(e_{ct})\xi_t] + \delta \Omega.
\]

**Search Effort and Disposable Income**

From now on, assume that

\[
v(x_t) = \Phi x_t.
\]

Optimal search effort is determined by the conditions. The optimal individual search effort is simply given by a condition equating the marginal cost of effort to the discounted, expected benefit yielded by that marginal unit of effort:

\[
\bar{e}_{ct} \sigma'(\bar{e}_{ct}) = \frac{\tilde{\lambda}_t}{1 + r} E_t [D_{1,t+1} - D_{0,t+1}] .
\]
We use the recursivity of the expression

\[ D_{1,t} - D_{0,t} = (\Phi - \mathcal{P}_t) x_t + \sigma(\bar{e}_{ct}) - \bar{e}_{ct} \sigma'(\bar{e}_{ct}) + (1 - \tau) \left( 1 - s^L \right) \frac{1 - s^e}{1 + r} \mathbb{E}_t L^{-1} [D_{1,t} - D_{0,t}] \]

where \( L^{-1} \) is the forward operator. Defining \( \psi \equiv (1 - \tau) \left( 1 - s^L \right) \frac{1 - s^e}{1 + r} \) as a discount factor and \( \tilde{\sigma}(\bar{e}_{ct}) \equiv \sigma(\bar{e}_{ct}) - \bar{e}_{ct} \sigma'(\bar{e}_{ct}) \), we have

\[
D_{1,t} - D_{0,t} = (\Phi - \mathcal{P}_t) x_t + \tilde{\sigma}(\bar{e}_{ct}) + (1 - \tau)(1 - s^L) \frac{1 - s^e}{1 + r} \mathbb{E}_t L^{-1} [D_{1,t} - D_{0,t}]
\]

\[
[D_{1,t} - D_{0,t}]_1 - \psi \mathbb{E}_t L^{-1} = (\Phi - \mathcal{P}_t) x_t + \tilde{\sigma}(\bar{e}_{ct})
\]

\[
[D_{1,t} - D_{0,t}]_1 = \frac{1}{[1 - \psi \mathbb{E}_t L^{-1}]} [(\Phi - \mathcal{P}_t) x_t + \tilde{\sigma}(\bar{e}_{ct})]
\]

\[
[D_{1,t} - D_{0,t}]_1 = \mathbb{E}_t \sum_{i=0} \psi^i [(\Phi - \mathcal{P}_{t+i}) x_{t+i} + \tilde{\sigma}(\bar{e}_{ct+i})].
\]

We know that a matched consumer will spend all disposable income \( Y^d_i \) on good \( c_{1,t} \). Thus, income \( Y^d_i \) can purchase \( c_{1,t} = x_t = Y^d_i / \mathcal{P}_t \) goods and we have

\[
[D_{1,t} - D_{0,t}]_1 = \mathbb{E}_t \sum_{i=0} \psi^i \left[ \frac{Y^d_{t+i}}{\mathcal{P}_{t+i}} \right] \left[ (\Phi - \mathcal{P}_{t+i}) \frac{Y^d_{t+i}}{\mathcal{P}_{t+i}} + \tilde{\sigma}(\bar{e}_{ct+i}) \right]
\]

This expression clearly ties the surplus to a consumption relationship for the consumer to the future expected paths of disposable income and the price. Plugging this expression into the condition for optimal consumer search effort, we have

\[
\sigma'(\bar{e}_t) = \frac{\tilde{\lambda}_t}{1 + r} \mathbb{E}_t \sum_{i=0} \psi^i \left[ \frac{\Phi}{\mathcal{P}_{t+1+i}} - 1 \right] Y^d_{t+1+i} \tilde{\sigma}(\bar{e}_{ct+1+i})
\]

D Wage Bargaining

This section provides the details for the wage rule, which is the solution to

\[
w_t = \begin{cases} 
\text{argmax} \left( S_{g,t} - S_{l,t} \right)^{1-\alpha} (W_{g,t} - U_t)^\alpha & \text{if the firm is in stage } g \\
\text{argmax} \left( S_{\pi,t} - S_{l,t} \right)^{1-\alpha} (W_{\pi,t} - U_t)^\alpha & \text{if the firm is in stage } \pi,
\end{cases}
\]

where \( \alpha \in (0, 1) \) is the worker’s bargaining weight, \( U_t \) is the value to a worker of being in the unemployment state, and \( W^g_t \) and \( W^\pi_t \) are the values of being employed at firms in stages \( g \) and \( \pi \), respectively. These
value are

\[ W_{g,t} = w^g_t + (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [(1 - \lambda_t) W_{g,t+1} + \lambda_t W_{\pi,t+1}] + \frac{s^c + (1 - s^c) s^L}{1 + r} \mathbb{E}_t U_{t+1} \]

\[ W_{\pi,t} = w^\pi_t + (1 - s^L) \frac{1 - s^c}{1 + r} \mathbb{E}_t [(1 - \tau) W_{\pi,t+1} + \tau W_{g,t+1}] + \frac{s^c + (1 - s^c) s^L}{1 + r} \mathbb{E}_t U_{t+1} \]

\[ U_t = b + \frac{1}{1 + r} \mathbb{E}_t [f_t W_{g,t+1} + (1 - f_t) U_{t+1}] . \]

The sharing rules out of the Nash-bargaining problems are

\[ \alpha [S_{g,t} - S_{t,t}] = (1 - \alpha) [W_{g,t} - U_t] \]

\[ \alpha [S_{\pi,t} - S_{t,t}] = (1 - \alpha) [W_{\pi,t} - U_t] . \]

Note that \( \alpha [S_{\pi,t} - S_{g,t}] = (1 - \alpha) [W_{\pi,t} - W_{g,t}] \). Consider now the derivation of the wage rate in stage \( g \).

The labor surplus in this case, \( LS_g \), is

\[ LS_{g,t} = -b \frac{f(\theta_t)}{1 + r} \mathbb{E}_t [W_{g,t+1} - U_{t+1}] - (1 - s^c) \frac{(1 - s^L)}{1 + r} \mathbb{E}_t U_{t+1} + \frac{1 - s^c}{1 + r} s^L K - K \]

\[ + \frac{(1 - s^c)(1 - s^L)}{1 + r} \mathbb{E}_t [(1 - \lambda_t)(S_{g,t+1} + W_{g,t+1}) + \lambda_t (W_{\pi,t+1} + S_{\pi,t+1})] \]

\[ LS_{g,t} = -b \alpha \frac{f(\theta_t)}{1 + r} \mathbb{E}_t LS_{g,t+1} + \frac{(1 - s^c)(1 - s^L)}{1 + r} \mathbb{E}_t [\lambda_t LS_{\pi,t+1} + (1 - \lambda_t) LS_{g,t+1}] \]

\[ - K + s^L \frac{1 - s^c}{1 + r} K . \]

From the Bellman equation for the firm in stage \( g \), we also have

\[ (1 - \alpha) LS_{g,t} = -w^g_t - K + s^L \frac{1 - s^c}{1 + r} K \]

\[ + (1 - \alpha) \frac{(1 - s^c)(1 - s^L)}{1 + r} \mathbb{E}_t [\lambda_t LS_{\pi,t+1} + (1 - \lambda_t) LS_{g,t+1}] . \]

Combining these two equations, along with the result that \( (1 - \alpha) \mathbb{E}_t LS_{g,t+1}/(1 + r) = \left( \gamma + \frac{\gamma + \gamma K}{1 + r} \right) / q(\theta_t) \), yields the wage rule

\[ w^g_t = \alpha \theta_t \left( \gamma + \frac{r + s^c}{1 + r} K \right) - \alpha \left( \frac{1 + s^L}{1 + r} \right) K + (1 - \alpha) b. \]
Consider now the derivation of the wage rate in stage $\pi$. The labor surplus in this case is

$$LS_{\pi,t} = \mathcal{P}_t x_t - \Omega - b - K + s^L \frac{1-s^c}{1+r} K - \frac{\alpha f(\theta_t)}{1+r} \mathbb{E}_t LS_{g,t+1} + (1-s^c) \left(1-s^L\right) \mathbb{E}_t \left[(1-\tau)LS_{\pi,t+1} + \tau LS_{g,t+1}\right].$$

From the Bellman equation for the firms in stage $\pi$, we also have

$$(1-\alpha)LS_{\pi,t} = \mathcal{P}_t x_t - w^\pi_t - \Omega - K + s^L \frac{1-s^c}{1+r} K + (1-\alpha) \frac{(1-s^c)(1-s^L)}{1+r} \mathbb{E}_t \left[(1-\tau)LS_{\pi,t+1} + \tau LS_{g,t+1}\right].$$

Combining these two equations yields the wage rule in stage $\pi$:

$$w^\pi_t = \alpha \left(\mathcal{P}_t x_t - \Omega - \left[1 + s^L \frac{1-s^c}{1+r} \right] K\right) + (1-\alpha)b + \alpha \theta_t \left(\gamma + \frac{r + s^c}{1+r} K\right).$$

### E Stock-Flow Equations

We have the following laws of motion.

$$C_{0,t+1} = (1-\lambda_t)c_{0,t} + \left[s^c + (1-s^c) \left(s^L + (1-s^L) \tau\right)\right] c_{1,t} \quad \text{(E-10)}$$

$$C_{1,t+1} = (1-s^c) \left(1-s^L\right) \left(1-\tau\right) c_{1,t} + \lambda_t c_{0,t} \quad \text{(E-11)}$$

There is an inflow $q(\theta_t)N_{l,t}$ into the stock of firms searching in the goods market every period, where $N_{l,t}$ is the number of vacancies at time $t$. To this, $(1-s^c) \left(1-s^L\right) \tau N_{\pi,t}$ firms separated from consumers lead the stocks $N_{g}$ and $N_{\pi}$ to evolve according to

$$N_{g,t+1} = (1-s^c) \left(1-s^L\right) [(1-\lambda_t)N_{g,t} + \tau N_{\pi,t}] + q(\theta_t)N_{l,t} \quad \text{(E-12)}$$

$$N_{\pi,t+1} = (1-s^c) \left(1-s^L\right) [(1-\tau)N_{\pi,t} + \lambda_t N_{g,t}] \quad \text{(E-13)}.$$ 

Finally, the dynamics of aggregate unemployment and employment are then given by

$$u_{t+1} = \left[s^c + (1-s^c) s^L\right] (1-u_t) + (1-f(\theta_t)) u_t \quad \text{(E-14)}$$

$$1-u_t = N_{g,t} + N_{\pi,t}. \quad \text{(E-15)}$$
F Allowing for Savings or Borrowing

Case 1

We have a new budget constraint per period, with \( r_s \) as the rate of return on savings, assumed to be less than \( r \):

\[
P_t c_{1,t} + c_{0,t} + s_t = Y_t^d + s_{t-1}(1 + r_s).
\]

If the consumer expects a drop in price next period, he may not want to consume everything today in order to consume more of the numeraire next period. Hence, we have

\[
P_t c_{1,t} + s_t = P_t x_t.
\]

The Bellman equations for unmatched and matched consumers are

\[
D_{0,t} = c_{0,t} + s_{t-1}(1 + r_s) - \sigma(e_{ct}) + \frac{1}{1 + r} \mathbb{E}_t \left[ \lambda_t D_{1,t+1}(s_t) + (1 - \lambda_t) D_{0,t+1}(s_t) \right]
\]

\[
D_{1,t} = v(c_{1,t}) + Y_t^d - P_t c_{1,t} + s_{t-1}(1 + r_s)
\]

\[
+ \frac{1 - s_c}{1 + r} \mathbb{E}_t (1 - s_L)[\tau D_{0,t+1}(s_t) + (1 - \tau) D_{1,t+1}(s_t)]
\]

\[
+ \frac{s_c + (1 - s_c)s_L}{1 + r} \mathbb{E}_t D_{0,t+1}(s_t)
\]

\[
= v(x_t - s_t/P_t) + Y_t^d - P_t x_t + s_{t-1}(1 + r_s)
\]

\[
+ \frac{1 - s_c}{1 + r} \mathbb{E}_t (1 - s_L)[\tau D_{0,t+1}(s_t) + (1 - \tau) D_{1,t+1}(s_t)]
\]

\[
+ \frac{s_c + (1 - s_c)s_L}{1 + r} \mathbb{E}_t D_{0,t+1}(s_t).
\]

The solution for the optimal choice of savings is a trade-off between sacrificing consumption today, which costs

\[
\frac{v'(x_t - s_t/P_t)}{P_t},
\]

and increasing consumption opportunities tomorrow, which yields additional utility depending on the future state. In particular, in state 0, we have

\[
\frac{d\mathbb{E}_t D_{0,t+1}(s_t)}{ds_t} = 1 + r_s
\]
and
\[ \frac{dE_tD_{1,t+1}(s_t)}{ds_t} = 1 + r_s \]

since the consumer is constrained by the production tomorrow for next period’s consumption. Hence, without calculation, we know that the consumer does not want to save: What is gained tomorrow is only to consume more numeraire, which by definition costs a fixed price. So at best, the gain is some more numeraire tomorrow bringing \((1 + r_s)/(1 + r) < 1\) but sacrificing \(\frac{\nu(x_t-s_t/P_t)}{r_t}\), which is larger than 1. Therefore, the price must be below the marginal utility here.

Other Cases

1. A slightly different reasoning applies if the agent could store good 1 for a return to stage 0. In this case the agent may have an interest to smooth consumption. But good 1 was assumed to be non-storable.

2. By the same logic, one may want to know whether the agents could borrow in bot stages at rate \(r_b\), to consume more of the numeraire. Again, this is not possible given \((1 + r_b)/(1 + r) > 1\). The key insight here is that borrowing or savings is of no help because the agents are constrained to buy all the available goods.

3. Finally, a last possibility would be to have unconstrained agents, who would consume, when matched, both good 1 and the numeraire. In this case, they may want to reduce consumption of good 1 in periods of high prices and raise it in period of low price. However even in this case, this is not profitable: Postponing consumption of good 1 today entails a risk of loosing the good with probability \(\tau\). The gain is to have some more consumption of the good, by a marginal quantity equal to deflation. In the numerical exercise, this marginal quantity has to be discounted by \((1 - \tau)/(1 + r)\) and compared to deflation. Given that deflation is typically at most 0.5% quarterly in our simulation and that \(\tau\) is 1%, the consumer, in this case, would still not want to sacrifice consumption of good 1 today to buy more good 1 tomorrow.
G Search effort in the goods market

Denoting by $e_{A,t}$ and $e_{c,t}$ the respective effort of firms to sell (e.g. advertising) and consumer search effort, we have:

$$S_{g,t} = -w_t - \sigma_A(e_{A,t}) + \frac{1 - s^c}{1 + r} \mathbb{E}_t \left[ (1 - s^L) \left( 1 - \frac{s^c}{e_{A,t}} \right) S_{\pi,t+1} \right] + S_{\pi,t+1} \right]$$

$$D_{0,t} = U(0, c_{0,t}) - \sigma_c(e_{c,t}) + \frac{1}{1 + r} \mathbb{E}_t \left[ \tilde{\lambda}_t \tilde{e}_{c,t} D_{1,t+1} + \left( 1 - \frac{\tilde{\lambda}_t}{\tilde{e}_{c,t}} \right) D_{0,t+1} \right]$$

In this case we need to explicitly express the transition rate in the goods market as function of aggregate search effort by consumers and firms. Denote goods market tightness by $\xi_t = C_{0,t}/N_{g,t}$. We have:

$$\lambda_t = \frac{MG(e_{A,t}N_{g,t}, e_{c,t}C_{0,t})}{N_{g,t}} = \lambda(e_{A,t}, e_{c,t}, \xi_t)$$

$$\tilde{\lambda}_t = \frac{MG(e_{A,t}N_{g,t}, e_{c,t}C_{0,t})}{C_{0,t}} = \tilde{\lambda}(e_{A,t}, e_{c,t}, \xi_t)$$

and it follows that $\tilde{\lambda}_t = \lambda_t/\xi_t$. First order conditions lead to marginal efforts equal to the marginal expected surplus:

$$\sigma'_A(e_{A,t}) = \frac{\lambda_t}{\tilde{e}_{A,t}} \left( 1 - s^L \right) \left( 1 - \frac{s^c}{1 + r} \right) \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}]$$

$$\sigma'_c(e_{c,t}) = \frac{\tilde{\lambda}_t}{\tilde{e}_{c,t}} \frac{1}{1 + r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}]$$

Hence, as the marginal expected surplus is procyclical so are the search efforts of both sides of the goods market. This is empirically confirmed by Hall (2012). Further, combined with the price bargaining condition, with a bargaining weight of consumers $\delta$, we have:

$$\alpha_G e_{A,t} \sigma'_A(e_{A,t}) = (1 - \delta) e_{c,t} \sigma'_c(e_{c,t}) (1 - s^L) (1 - s^c) \xi_t$$

or

$$\frac{e_{A,t} \sigma'_A(e_{A,t})}{e_{c,t} \sigma'_c(e_{c,t})} = \left( \frac{1 - \delta}{\delta} \right) (1 - s^L) (1 - s^c) \xi_t$$

For instance, if the cost functions had the same elasticity, say quadratic, and we are in a symmetric equilibrium where all consumers and all firms provide, respectively, the same effort level, the effort of one side would be equiproportional to the effort in the other side. In the general case of convex but non identical costs functions, the effort of each side are strategic complement. To sum up, both search efforts in the goods market are individually procyclical. In addition, this procyclicallity is strengthened by the
strategic complementarity.