The Cyclic Volatility of Labor Markets under Frictional Financial Markets†

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We provide a dynamic extension of an economy with search on credit and labor markets (Wasmer and Weil 2004). Financial frictions create volatility. They add an additional, almost acyclical, entry cost to procyclical job creation costs, thus increasing the elasticity of labor market tightness to productivity shocks by a factor of five to eight, compared to a matching economy with perfect financial markets. We characterize a dynamic financial multiplier that is increasing in total financial costs and minimized under a credit market Hosios-Pissarides rule. Financial frictions are an element of the solution to the volatility puzzle. (JEL C78, E24, E32, E44, G21, J63)

An increasing body of research has looked into the ability of the conventional matching model to replicate US statistics regarding the volatility of job vacancies, unemployment, and their ratio (called labor market tightness), in response to productivity shocks. Cole and Rogerson (1999) and Shimer (2005) have investigated the cyclical properties of search matching models following Pissarides (1985) and Mortensen and Pissarides (1994). Shimer’s main finding is that the elasticity of labor market tightness to productivity shocks is around 20 in the data, and around 1 in a calibration of the model. Mortensen and Nagypal (2007) argue that a large part of fluctuations in the unemployment/vacancy ratio is not due to productivity shocks, and Pissarides (2009) retains a value of 7.56. Several improvements have been proposed, both structural, such as wage rigidity (Hall 2005) and on-the-job search (Mortensen and Nagypal 2007), and to the parameterization. In the latter case, the small labor surplus assumption of Hagedorn and Manovskii (2008), firms make small profits that are more responsive to productivity shocks, leading the labor market to be overall more volatile.1

1More precisely, Hagedorn and Manovskii (2008) impose a calibrated value of nonemployment utility that is close to market productivity, within only a few percentage points, and very low values for the bargaining power of workers.

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One line of research that has so far been ignored is the existence of credit market imperfections. Indeed, it has been known for a while that credit market imperfections generate additional volatility in models of the business cycle. Early papers, such as Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and subsequent papers, such as Bernanke and Gertler 1995; Bernanke, Gertler and Gilchrist 1996; among others, have emphasized the amplification role of credit markets and the existence of a financial accelerator. Although part of this literature is centered on the credit channel of monetary policy, the elements that give rise to amplification may be relevant for the study of cyclical fluctuations in labor markets. At the microeconomic level, there exists a body of empirical research on the effects of financial frictions on firm employment decisions. Beginning with Sharpe (1994) using US manufacturing data, more financially constrained firms have a more cyclical labor force. Specifically, employment at these firms is more responsive to demand shocks. More recently, Benmelech, Bergman, and Seru (2011) have shown that financial constraints affect firm-level employment decisions, and Caggese and Cunat (2008), in an estimated model, have shown that the volatility of total employment is significantly higher for financially constrained firms than for financially unconstrained ones.

In this paper, we pursue this logic, starting from the simple, steady state, setup of Wasmer and Weil (2004), who introduce financial imperfections in a Mortensen-Pissarides economy with two matching functions—one in the labor market, and one in the credit market.\(^2\) We provide a dynamic, stochastic extension of this model, reinterpreting the firm as an entity managing several marginal investment projects, each requiring labor and financing from a banker on a frictional credit market. In this interpretation, the relevant financial costs affect all firms, not only new firms.\(^3\)

Our paper has four contributions. First, we derive a dynamic financial multiplier, the analogue to the static context in Wasmer and Weil (2004), and clearly identify the manner in which financial imperfections raise the elasticity of labor market tightness to productivity shocks. In the standard search and matching model with no financial market imperfections, the increase in the vacancy-unemployment ratio following a positive productivity shock causes a rise in the duration of search for firms to fill a vacancy. This increase in costs limits the incentive for firms to post vacancies. The introduction of financial frictions gives rise to an additional entry cost for firms, the cost of accessing finance. In equilibrium, any improvement in expected productivity, and therefore profits, attracts firms and banks in fixed proportions, such that credit market tightness is always constant and the cost of accessing finance for firms is almost acyclical. As a result, the total cost of creating a job is less sensitive to changes in labor market tightness and the associated congestion effects. Job creation responds more to changes in productivity, thereby amplifying business cycle fluctuations. Here, we join Pissarides (2009) in arguing that part of

\(^2\) Petrosky-Nadeau (2009) considers the case of external financing of period operating expenses, including recruiting costs, on imperfect credit markets. The imperfection takes the form of an agency problem between borrower and lender and, as in this paper, financial frictions amplify the business cycle of labor market variables.

\(^3\) The closest paper related to ours is an unpublished manuscript by Nicoletti and Pierrard (2006). Their work allows for two types of firms, one of which must borrow on a search frictional credit market. They focus on the implications for cross-correlations between real quantities, such as output and consumption, and, while not emphasizing the feedback from credit to labor markets, they do discuss the existence of a financial multiplier.
the solution to the labor market volatility puzzle requires that hiring costs must be partly nonproportional to congestion in the labor market.\footnote{This mechanism differs from the extensive literature on the role of credit market frictions for business cycles. The mechanism is not based on a collateral constraint, as in Kiyotaki and Moore (1997), nor on countercyclical agency costs, as in Bernanke and Gertler (1989).}

Second, we show that there exists a link between the size of the financial multiplier and an efficiency rule in the credit market. Indeed, the social optimum is achieved when the bargaining power of firms vis-à-vis banks is equal to the elasticity of the credit finding rate of firms with respect to credit market tightness (the so-called Hosios-Pissarides condition, Pissarides 1990). When this happens, congestion externalities from the matching process are internalized by rent sharing. We show that this condition also implies that search costs in the credit market are minimized, and so is the financial multiplier. Relaxing the condition leads to a larger financial multiplier because, away from that condition, the crowding-out effects in financial markets are higher. This arises from a greater negative externality in the matching process, which is less socially efficient, leading to greater overall financial costs in the economy that raise the constant part in the entry costs of firms. As explained above, this further raises volatility. We show that the model can match or even overshoot the elasticity of labor market tightness to productivity shocks observed in the data.

Third, we illustrate these results under a set of parameter values that allows us to match the share of the financial sector in GDP in the United States. These parameters deviate substantially from the credit market Hosios-Pissarides rule. In our baseline, with an elasticity of the matching function in the credit market of 0.5, we find a relative bargaining power of banks relative to firms of 0.68. As such, the model implies a financial multiplier of 5.5, and an elasticity of labor market tightness to productivity shocks of 10. We also explore a set of alternative parameters in the labor and credit markets, illustrating that the financial multiplier can vary between 5 and 8, and the elasticity of labor market tightness can reach 20, all the while restricting the size of the financial sector. Moreover, we show that the magnitude of the financial multiplier is stable across different targeted sizes of the financial sector.

Fourth, this result is obtained while restricting the economic costs of financial intermediation within an empirically plausible range, and wages at two-thirds of labor productivity. Thus, amplification of the v-u ratio to productivity here does not rely on the small labor surplus assumption of Hagedorn and Manovskii (2008). Financial market imperfections enable us to relax the small labor surplus assumption in order to match the elasticity of market tightness to productivity found in the data, a desirable implication given the concerns raised by assuming small profits to labor for the behavior of the search model to changes in policy parameters (e.g., Costain and Reiter 2008). Obtaining an elasticity of the v-u ratio of 20, as retained by Shimer (2005), requires a financial multiplier of 8.84. This can be achieved by reducing the bargaining power of workers close to the calibrated value retained by Hagedorn and Manovskii (2008). However, the model still does not require the assumption of a small labor surplus as wages are kept at two-thirds of labor productivity, and the resulting elasticity of the unemployment rate to unemployment benefits is very low.
The rest of the paper is organized as follows. In Section I, we provide the setup of our dynamic model and derive its entry condition. In Section II, we derive the elasticity of labor market tightness to productivity shocks and show how the Hosios-Pissarides rule in the credit market affects the volatility of the labor market. In Section III, we describe the parameterization strategy and illustrate how deviating away from Hosios-Pissarides substantially raises the elasticity of labor market tightness in Section IV. In Section V, we conclude that financial frictions are an element of the solution to the volatility puzzle, as they imply job creation costs that are partly nonproportional to congestion in the labor market.

I. An Economy with Credit and Labor Market Frictions

A. Setup

Time is discrete. There are three types of agents: new projects in need of capital, banks with no ability to produce, and workers with no capital and no investment project. The timing of events for new projects is as follows. First, a project requires a “banker” in order to finance the installation costs and recruiting a worker. This search process costs $e$ units of effort per period. Search is successful with probability $p_t$. The newly formed production unit is referred to as a firm, but it is in fact a marginal project that found financing. This entity then goes, in a second stage, to the labor market. The bank finances the vacancy posting cost $\gamma$ to attract workers (the so-called recruitment costs) for the firm. This search process succeeds with probability $q_t$. The firm is then able, in the third stage, to produce and sell in the goods market, which generates a flow profit $y_t - w_t - \Psi_t$, where $y_t$ is the marginal product, $w_t$ is the wage, and $\Psi_t$ is the flow repayment to the bank (determined through bargaining). Call $r$ the per-period discount rate, which is constant due to the assumption of risk neutral agents.

Labor productivity is assumed to follow a stationary AR(1) process, $y_t = \rho y_{t-1} + \nu_t$, where $0 < \rho_y < 1$, and $\nu_t$ is white noise. We assume for convenience that at a new project meeting a banker begins the recruiting process within the period. A successful meeting between a firm and worker begins production the following period. Firms are subject to destruction shocks with exogenous probability $s$, which also entail the destruction of the associated job.

The asset values of the project in the three stages described above are denoted by $E_{j,t}$, with $j = c, l, g$, standing, respectively, for the credit, labor, and goods markets, the market in which the project is operating. We also assume free entry on the credit market, that is $E_{c,t} \equiv 0$ for all $t$. We therefore have the following Bellman equations:

1. $E_{c,t} = -e + p_t E_{l,t} + (1 - p_t) \frac{1}{1 + r} E_{c,t+1}$
2. $E_{l,t} = \frac{1}{1 + r} \mathbb{E}_t [q_t E_{g,t+1} + (1 - q_t) E_{l,t+1}]$
3. $E_{g,t} = y_t - w_t - \Psi_t + \frac{1}{1 + r} \mathbb{E}_t [s E_{c,t+1} + (1 - s) E_{g,t+1}]$
where $\mathbb{E}_t$ is an expectations operator over productivity. In the second line, the hiring cost $\gamma$ does not show up because it is financed by the bank.

Symmetrically, the bank’s asset values are denoted by $B_{j,t}$, $j = c, l, o$ for each of the stages. In the first stage, the bank screens projects. In the second stage, it finances the posting of a vacancy; and in the third stage, it enjoys the repayment $\Psi_t$ from the project. We also assume free entry on the bank side of the credit market implying $B_{c,t} = 0 \forall t$. We denote by $\kappa$ the screening cost per unit of time incurred by banks in the first stage, and by $\phi_t$, the probability with which a bank finds a project to be financed. We have:

$$B_{c,t} = -\kappa + \bar{p}_t B_{l,t} + \left(1 - \bar{p}_t\right) \frac{1}{1 + r} \mathbb{E}_t B_{c,t+1}$$

$$B_{l,t} = -\gamma + \frac{1}{1 + r} \mathbb{E}_t \left[ q_t B_{g,t+1} + (1 - q_t) B_{l,t+1} \right]$$

$$B_{g,t} = \Psi_t + \frac{1}{1 + r} \mathbb{E}_t \left[ s B_{c,t+1} + (1 - s) B_{g,t+1} \right].$$

B. Matching

Matching in the labor market is denoted by $M_t(\mathcal{V}_t, u_t)$, where $u_t$ is the rate of unemployment and the total number of unemployed workers since the labor force is normalized to 1. $\mathcal{V}_t$ is the number of “vacancies,” that is, the number of firms in stage $l$. The function is also assumed to be constant returns to scale, hence, the rate at which firms fill vacancies is a function of the ratio $\mathcal{V}_t/u_t = \theta_t$, or the tightness of the labor market. We have:

$$q(\theta_t) = \frac{M_t(\mathcal{V}_t, u_t)}{\mathcal{V}_t} \text{ with } q'(\theta_t) < 0.$$ 

On the credit market, the matching rates $p_t$ and $\bar{p}_t$ are made mutually consistent by the existence of a matching function $M_t(B_t, \mathcal{E}_t)$, where $B_t$ and $\mathcal{E}_t$ are, respectively, the number of bankers and of projects in stage $c$. This function is assumed to have constant returns to scale. We denote by $\phi_t$ the ratio $\mathcal{E}_t/B_t$, which is a reflection of the tension in the credit market and that we shall call credit market tightness from the point of view of new projects. Under the assumption of constant returns to scale, we have:

$$p_t = \frac{M_t(B_t, \mathcal{E}_t)}{\mathcal{E}_t} = p(\phi_t) \text{ with } p'(\phi_t) < 0.$$ 

$$\bar{p}_t = \phi_t p(\phi_t) \text{ with } \bar{p}'(\phi_t) > 0.$$
The first line states that matching with a bank is decreasing in credit market tightness. The second states that the rate at which banks match with a project is increasing in the relative abundance of investment projects.

C. Bargaining and Equilibrium in the Credit Market

After contact, the bank and the firm, owner of the investment project, engage in bargaining about $\Psi$, splitting the surplus $(B_{jt} + E_{jt})$. The solution to the generalized Nash bargaining problem implies the sharing rule at the time of the meeting:

$$ (1 - \beta)B_{jt} = \beta E_{jt}, $$

where $\beta \in (0, 1)$ is the bargaining power of the bank relative to the firm. With $\beta = 0$, the bank leaves all the surplus to the firm. Combining (1), (4), and (7), we obtain the equilibrium value of credit market tightness $\phi_t$. It turns out to be a constant in time, a function of parameters as in Wasmer and Weil (2004), in this dynamic generalization, and it is denoted by,

$$ \phi^* = \frac{1 - \beta}{\beta} \frac{\kappa}{e}. $$

The fact that this is constant is due to the assumption of double free entry on credit markets, by both banks and firms. Any improvement in expected productivity, and therefore profits, attracts firms and banks in fixed proportions, such that the ratio is always constant.\(^5\)

D. Equilibrium in the Labor Market and Repayment to Banks

Free entry in the credit market implies, from equations (1) and (4), constant values of being in the recruiting stage, $B_{jt} = \kappa/(\phi^* p(\phi^*))$ and $E_{jt} = e/p(\phi^*)$. In combination with the forward values of $B_{jt}$ and $E_{jt}$, in equations (2) and (5), we obtain a credit market free entry curve of new projects, denoted by (EE), and the free entry curve of banks, denoted by (BB).

$$ (EE): \frac{e}{p(\phi^*)} = \frac{q(\theta_t)}{1 + r} E_t [E_{\phi, t+1}] + \frac{(1 - q(\theta_t))}{1 + r} \frac{e}{p(\phi^*)} $$

$$ (BB): \frac{\kappa}{\phi^* p(\phi^*)} = -\gamma + \frac{q(\theta_t)}{1 + r} E_t [B_{\phi, t+1}] + \frac{(1 - q(\theta_t))}{1 + r} \frac{\kappa}{\phi^* p(\phi^*)}. $$

\(^5\) While access to finance is acyclical, the plausible alternative is for access to be procyclical. Easier access to finance in an expansion driven by increasing productivity would amplify the response of the economy. Conversely, restricted access to finance following a drop in productivity would generate a more severe recession. As such, having acyclical access to finance minimizes the magnitude of the financial accelerator.
These equations reflect entry conditions. Expected entry costs paid by projects or by banks on financial markets, over and above the costs of hiring labor, have to be equal to the properly discounted value of payoffs received by the different sides of the market.

We can combine them to obtain a single market equation, denoted by (CC), for equilibrium labor market tightness $\theta^*$:

$$(CC): \frac{e}{p(\phi^*)} + \frac{\kappa}{\phi^* p(\phi^*)} = \frac{q(\theta^*_t)}{r + q(\theta^*_t)} (1 + r) \left[ -\frac{\gamma}{q(\theta^*_t)} + \frac{1}{1 + r} \mathbb{E}_t S_{g,t+1} \right],$$

where $S_{g,t} = E_{g,t} + B_{g,t}$ is the joint value of an employed worker to an investment project and a bank. We prefer equation (CC) to describe a job creation condition for this double-matching economy because it naturally leads to the main intuition of our subsequent results and has a ready interpretation. The left-hand side is the total amount of search costs in financial markets, which we denote by,

$$K(\phi^*) \equiv \frac{e}{p(\phi^*)} + \frac{\kappa}{\phi^* p(\phi^*)}.$$

The right-hand side of (CC) is the expected value of the total dynamic surplus of the match resulting from the production process, net of hiring costs. With free entry in the labor market, as in the standard search and matching model, the bracketed term on the right-hand side is driven to zero. In this sense credit market frictions drive a wedge between the labor recruiting costs $\gamma/q(\theta)$ and the discount value of a filled vacancy to the firm.

We can reorganize the terms in equation (CC) by replacing the expected value of the surplus by a function of labor market tightness (see the online Appendix), and we obtain a traditional dynamic job creation condition (e.g., Merz 1995; Andolfatto 1996) adjusted for the presence of frictional credit markets:

$$\frac{\Gamma_t}{q(\theta^*_t)} = \frac{1}{1 + r} \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1 - s) \frac{\Gamma_{t+1}}{q(\theta^*_t)} \right],$$

where

$$\Gamma_t \equiv \gamma + K(\phi^*) \left( 1 - \frac{1}{1 + r} (1 - q(\theta^*_t)) \right),$$

are vacancy costs augmented for frictional credit markets.

It is worth noting two special cases. First, when $r = 0$, $\Gamma_t$ is simply the sum of all search costs in credit and labor markets, unadjusted for discounting. We obtain, in particular,

$$\frac{\Gamma_t}{q(\theta^*_t)} \equiv \frac{\gamma}{q(\theta^*_t)} + K(\phi^*).$$
The presence of frictional credit markets adds a new component to the costs of creating a job for firms that, in the special case $r = 0$, are independent of labor market tightness. Concurring with an insight already brought by Pissarides (2009), the economy will be more volatile as total job creation costs are less procyclical than if credit market imperfections were absent.\(^6\)

The second special case corresponds to perfect credit markets, that is, when $K(\phi^*) = 0$. In this case, $\Gamma_i$ boils down to $\gamma_i$, and the job creation condition reduces to

$$\frac{\gamma}{q(\theta^*_t)} = \frac{1}{1 + r} \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + (1 - s) \frac{\gamma}{q(\theta^*_{t+1})} \right],$$

where $\theta^*_t$ is equilibrium labor market tightness in the credit frictionless world of Pissarides (1985). Thus, compared to the world with perfect credit markets, frictional credit markets impose a lower, positive, limit on the value of a job vacancy to a firm. In particular, we can establish that in steady state $\theta^* < \theta^p$, as was shown in Wasmer and Weil (2004) and arises in Petrosky-Nadeau (2009).

The flow repayment to the bank, $\Psi_t$, which we derive next, varies with the state of the labor market and productivity at the time of the meeting. The value is solved using equation (7), where the values of the firm and the bank are replaced by their forward values (2), (3), (5), and (6). We obtain, after some steps exposed in the online Appendix and substituting in the job creation condition (10),\(^7\)

$$\mathbb{E}_t[\Psi_{t+1}] = \mathbb{E}_t[y_{t+1} - w_{t+1}]$$

$$- (1 - \beta)K(\phi^*) \mathbb{E}_t \left[ \frac{r + q(\theta^*_t)}{q(\theta^*_t)} \left( \frac{1 - s}{1 + r} \right) \frac{r + q(\theta^*_{t+1})}{q(\theta^*_{t+1})} \right].$$

This equation shows that the expected repayment is equal to the total expected gross profits minus a part of the entry costs $K(\phi^*)$. A higher bargaining share of the bank $\beta$ raises the expected repayment.\(^8\)

**II. Volatility and Constrained Efficiency**

We begin by illustrating the source of amplification in a simple setting with exogenous wages. This allows us to focus on the contribution of frictional credit markets by deriving a financial multiplier that is directly linked to the total cost of transactions on credit markets. The magnitude of this financial multiplier can then be

\(^6\)A stated in Pissarides (2009, 1341): ‘(...) a simple remodeling of the [matching] costs from proportional to partly fixed and partly proportional can increase the volatility of tightness and job finding, virtually matching the observed magnitudes, without violating wage flexibility.’

\(^7\)With intermediate step $\mathbb{E}_t[\Psi_{t+1}] = \beta \mathbb{E}_t[y_{t+1} - w_{t+1}] + (1 - \beta) \mathbb{E}_t[(1 + r)\gamma/q(\theta^*_t) - (1 - s)\gamma/q(\theta^*_{t+1})]$. The second term in brackets is always positive with our parameter values (Section III). It is positive in a neighborhood of $r = 0$ or in the neighborhood of the steady state.
related to deviations from a credit market Hosios-Pissarides constrained efficiency rule. Finally, we derive the elasticity of labor market tightness to productivity shocks for a model with endogenous wages.

A. Elasticity of $\theta_i$ to Productivity Shocks Under Fixed Wages

Assuming wages are exogenously fixed at $w_i = \bar{w}$, a log-linear approximation of equation (10) around a steady state allows us to express deviations in labor market tightness as follows (the derivation of this and the subsequent expressions of this section are available in the Appendix):

$$\hat{\theta}_i = \frac{1}{\eta} \frac{y - w}{y - \bar{w}} S_g \frac{S_g}{S_g - K(\phi^*)} \sum_{i=1}^{r+s} \left( \frac{1}{1+r} \right)^i \gamma_{t+i},$$

where hatted variables denote proportional deviations from the steady state, and $\eta$ is the absolute value of the elasticity of $q(\theta_i)$ to $\theta_i$. The first component $1/\eta$ of this elasticity is the amplification due to the existence of search frictions in the labor market. The second component $y/(y - \bar{w})$ is the gap between wages and the marginal product of labor. Under the small labor surplus assumption, the smaller the gap, the more responsive job creation is to productivity shocks. Finally, the third component, $S_g/(S_g - K(\phi^*))$, is the financial multiplier. The multiplier, as we will discuss, is related to the size of the joint surplus to opening a job vacancy, $S_g - K(\phi^*)$, a smaller surplus leading to greater responses of labor market tightness to changes in productivity.

Let us proceed further in deriving the elasticity of labor market tightness to productivity shocks both in the absence and presence of financial imperfections. In the absence of credit market frictions, the job creation condition in equation (13) means that deviations in labor market tightness appear as the discounted sum of deviations in future expected labor productivity. After intermediate steps, it follows that the elasticity of labor market tightness to a productivity shock in the Pissarides world with a fixed wage, denoted $\Lambda^p$, is $^9$

$$\Lambda^p = \frac{\partial \hat{\theta}^p_i}{\partial \nu_i} = \frac{1}{\eta} \frac{q(\theta^p) \rho_y}{\gamma ([1+r] - (1-s) \rho_y)}.$$

By the same steps, the elasticity of labor market tightness in the presence of credit frictions, $\Lambda$, is given by

$$\Lambda = \frac{\partial \hat{\theta}^*}{\partial \nu_i} = \frac{q(\theta^*) \rho_y}{\eta \gamma_r ([1+r] - (1-s) \rho_y)},$$

$^9$ We have $\hat{\theta}^p_i = (q(\theta^p)/\gamma ([1+r] \eta)) \sum_{i=0}^{\infty} \gamma^i ((1-s)/\rho_y) (\hat{\gamma}^i_{t+i})$. Given the assumption that productivity follows an AR(1) process, this can be rewritten more simply as $\hat{\theta}^p_i = (q(\theta^p)/\eta \gamma ([1+r])) \sum_{i=0}^{\infty} \rho^{i+1} \nu_r$. The approach we follow here, examining the response of an endogenous variable to exogenous innovations, is frequently employed in the business cycle literature. For instance, Dotsey and King (2006) use the similar log-linear approximations and transformations to express the impact response of aggregate output to a monetary policy shock.
where \( \tilde{\gamma}(r) \equiv [\gamma + K(\phi')(r/(1+r))] > \tilde{\gamma}(0) = \gamma \) is a measure of total frictional costs in both credit and labor markets. Consequently, the financial multiplier, or the increment in the responsiveness of labor market tightness to a productivity shock from the introduction of frictional credit markets in this dynamic setting, can be defined as the ratio of these two quantities:

\[
M_f^D = \frac{\Lambda}{\Lambda^p} = \frac{q(\theta^*)}{q(\theta^p)} \frac{\tilde{\gamma}(0)}{\tilde{\gamma}(r)}.
\]

When \( r \) goes to zero, this formula is the same as the multiplier identified in Wasmer and Weil (2004). This multiplier is increasing in the gap between equilibrium labor market tightness with frictional credit markets, \( \theta^* \) and \( \theta^p \), obtained without credit frictions. In other words, the financial multiplier is a function of the distortions induced by frictional credit markets.\(^{10}\)

**B. A Hosios-Pissarides Condition for the Credit Market**

Frictions in the credit market lead to inefficiencies. In the labor literature, a second best efficiency condition has been derived by Hosios (1990) and Pissarides (1990). In short, this condition states that when the bargaining share of one side equals its elasticity in the matching function, agents’ entry decisions internalize the congestion externality they create onto the other agents. A similar rule will be shown here for the credit market. Further, minimizing this distortion—that is, reaching the constrained efficient allocation—also minimizes the financial multiplier \( M_f^D \). We first calculate the constrained efficiency condition for financial markets in a double-matching economy.

The social welfare function at time \( t \) is a discounted sum of output in all future periods \( t + i, i = 0, 1, \ldots \), added to value of nonemployment utility denoted by \( \zeta \), net of all search costs for labor \( \gamma \theta_{t+i} u_{t+i} \) and financial contracts \( \kappa B_{t+i} + e E_{t+i} \), in this double-matching economy is:

\[
\Omega_t = \sum_{i=0}^{\infty} \left( \frac{1}{1 + r} \right)^{t+i} [y_{t+i} (1 - u_{t+i}) + \zeta u_{t+i} - \gamma \theta_{t+i} u_{t+i} - \kappa B_{t+i} - e E_{t+i}],
\]

where \( \gamma \theta_{t+i} u_{t+i} \) is the number of firms prospecting in the labor market. The mass of prospecting banks and projects in the credit market (\( B \) and \( E \)) can be determined from defining period vacancies as unfilled preceding period vacancies and new entries from the credit market:

\[
\mathcal{V}_t = \mathcal{V}_{t-1} (1 - q(\theta_{t-1})) + \mathcal{E}_t p(\phi_t) = \mathcal{V}_{t-1} (1 - q(\theta_{t-1})) + B_t \phi_t p(\phi_t).
\]

\(^{10}\)Given that \( q(\theta^*)/q(\theta^p) \) is above 1 but that \( \tilde{\gamma}(0)/\tilde{\gamma}(r) \) may be below 1, there is a theoretical possibility that, far away from \( r = 0 \), the multiplier is actually smaller than 1. However, in a neighborhood of \( r = 0 \), this does not happen and in all our calibrations it will always be above 2.
Consequently, the social planner’s program can be expressed as

\[
\max_{\{u_t, \theta_t, \phi_t\}} \Omega_t = \mathbb{E}_t \sum_{i=0}^{\infty} \left( \frac{1}{1 + r} \right)^{t+i} \left[ y_{t+i} (1 - u_{t+i}) + zu_{t+i} - \gamma \theta_{t+i} u_{t+i} \right] - \left( \frac{\kappa}{\phi_{t+i} p(\phi_{t+i})} + \frac{e}{p(\phi_{t+i})} \right) (\theta_{t+i} u_{t+i} - (1 - q(\theta_{t+i})) \theta_{t+i-1} u_{t+i-1})
\]

\[
\text{s.t. } u_{t+i+1} = u_{t+i} (1 - \theta_{t+i} q(\theta_{t+i})) + (1 - u_{t+i})s,
\]

and is therefore separable in \( \phi_t \) on the one hand, and in labor market variables \( u_t, \theta_t \) on the other hand.\(^\text{11}\) The optimal choice of \( \phi_t \) amounts to minimizing total search costs in the credit market \( K(\phi_t) = \kappa/(\phi_t p(\phi_t)) + e/p(\phi_t) \):

\[
\frac{\partial \Omega_t}{\partial \phi_t} = (\theta_t u_t - (1 - q(\theta_{t-1})) \theta_{t-1} u_{t-1}) \frac{\partial}{\partial \phi_t} K(\phi_t) = 0
\]

\[
\Leftrightarrow \phi_{\text{opt}}^{*} = \frac{1 - \varepsilon}{\varepsilon} \frac{\kappa}{e},
\]

where \( \varepsilon = -\phi p'(\phi)/p(\phi) \) is the elasticity of the finding rate of new projects on credit markets. Hence, since \( \frac{\partial^2}{\partial \beta^2} K(\phi^*) > 0 \), the socially optimal value of credit market tightness is the one that minimizes search costs on credit markets. The Hosios-Pissarides condition also minimizes the gap between financial friction. The calibration in Wasmer and Weil (2004), by setting \( \beta = \varepsilon \), thus implied a minimized static financial multiplier of \( M_f^S = 1.74 \). Away from this parameterization, one has a larger financial multiplier. We summarize this in a Proposition.

\(^{11}\) This is because the term \( \kappa/(\phi_{t+i} p(\phi_{t+i})) + e/p(\phi_{t+i}) \) must be minimized in order obtain the maximal value of \( \Omega_t \). Since whatever the value of \( \theta \) and \( u \), the next term is positive in the steady state. For this term to be negative, implausibly large fluctuations in \( u \) and \( \theta \) must occur between two consecutive periods.

Another interpretation is as follows. One can verify that the Hosios-Pissarides condition is the one that minimizes entry costs in the credit market with respect to the bargaining power (and not with respect to \( \phi \) as above). To do so, recall that the left-hand side of job creation condition (CC) is a function of \( \beta \) and \( \varepsilon \) denoted by \( K(\beta, \varepsilon) \). As the right-hand side is increasing in \( \theta \), it is enough to show that \( K(\beta, \varepsilon) \) is minimized in \( \beta = \varepsilon \). Before doing so, we can use two intermediate steps. First, note that \( K(\beta, \varepsilon) = e/(p(\phi^*) (1 - \beta)) \) from equation (EE) divided by \( (1 - \beta) \).

Second, we have \( \frac{\partial \phi^*}{\partial \beta} = -\kappa/(\beta^2 \varepsilon) \), hence \( \frac{\partial^2 K}{\partial \beta^2} = -\frac{\partial \phi^*}{\partial \beta} e^2 p^2(\phi^*) (1 - \beta) + e/(p(\phi^*) (1 - \beta)^2) = 0 \Leftrightarrow \varepsilon = \beta \).
PROPOSITION: The financial multiplier is minimized at $\beta = \epsilon$, and can be arbitrarily large: as the value of $\beta$ reaches either 0 or 1, the multiplier tends toward infinity.

This is a variant of the small surplus assumption. When one side receives an arbitrarily small surplus, the economy becomes infinitely reactive to small shocks given the complementarity between banks and projects in the matching function. Figure 1 provides a graphic illustration of this result in which the elasticity of labor market tightness to productivity shocks increases symmetrically as the bargaining weight $\beta$ deviates from the elasticity of the matching function $\epsilon$, set here to 0.5.\(^\text{13}\)

C. Elasticity of $\theta_t$ to Productivity Shocks Under Endogenous Wages

We now extend our results to a setting with endogenous wages, assuming that the wage is determined by bargaining between a worker and a firm. Since we define the firm as the investment project-banker block, the bank is not excluded from the wage negotiations, and instead forms a block with the firm in order to negotiate with the worker (a block-bargaining assumption).\(^\text{14}\)

\(^{13}\) Figure 1 is provided for a set of parameters that we discuss below and the assumption of a fixed wage. The general shape of the elasticity of the $v-u$ ratio to productivity shocks as the credit market parameters $\beta$ and $\epsilon$ deviate from the Hosios-Pissarides condition does not depend on the specific parameterization.

\(^{14}\) There are two interrelated reasons for this choice. The first one is that the alternative in which the firm bargains alone with its worker leads to complex strategic interactions illustrated in Wasmer and Weil (2004). In this case, the firm and the bank, meeting prior the wage negotiation, wish to raise the debt of the firm above what is needed in order to reduce the size of total surplus to be shared between the firm and the worker at a later time. Hence, wages are driven down to the reservation wage of workers and do not vary with the firm’s productivity, which is counterfactual. This leads to the second reason, which is that we want our endogenous wage extension to be comparable to the classical wage solution in the labor search literature (which turns out to be the case when we make the block-bargaining assumption) in order to compare the volatility in the model to other elasticities found in the literature.
Define the values of unemployment \( (U_t) \) and employment \( (W_t) \) for a worker as

\[
U_t = z + \frac{1}{1 + r} \mathbb{E}_t [f(\theta_t)W_{t+1} + (1 - f(\theta_t))U_{t+1}]
\]

\[
W_t = w_t + \frac{1}{1 + r} \mathbb{E}_t [(1 - s)W_{t+1} + s U_{t+1}],
\]

where \( z \) is the value of nonemployment activities, and \( f(\theta) = \theta q(\theta) \) is the value of the job finding rate. The wage is the solution to

\[
\arg \max (W_t - U_t)^\alpha (S_{s_t})^{1-\alpha}.
\]

Beginning with an environment without credit frictions, the outcome of Nash bargaining over the surplus of the labor relationship is a rule \( w_{t}^P, nb = \alpha(y_t + \gamma \theta_{t}^P, nb) + (1 - \alpha)z \), where \( \alpha \in (0, 1) \) is the bargaining power of workers vis-à-vis the firm, and the superscript \((P, nb)\) refers the to credit frictionless environment with Nash bargained wages. Introducing this wage rule in our earlier derivations and relating movements in labor market tightness to expected deviations of productivity from trend, yields an elasticity of labor market tightness to productivity shocks given by

\[
\Lambda_{P, nb} = \frac{\partial \theta_{t}^P, nb}{\partial \nu_t} = \frac{q(\theta_{t}^P, nb)(1 - \alpha)\rho_y}{\eta \gamma (1 + r) - \gamma [\eta (1 - s) - \alpha f(\theta_{t}^P, nb)]\rho_y}.
\]

Compared to the elasticity when wages are fixed in (15), only a share \((1 - \alpha)\) of the rise in productivity accrues to the firm. In addition, the rise in equilibrium labor market tightness following a positive productivity shock improves the outside option of the worker and his bargaining position in wage determination. This appears in the denominator as the term \( \alpha f(\theta_{t}^P, nb) \), further reducing the elasticity of labor market tightness to productivity shocks.

Turning now to the case of frictional credit markets, the worker-firm negotiated wage must satisfy a sharing rule \( \alpha S_{s_t} = (1 - \alpha)(W_t - U_t) \). The resulting wage

\[
w_t = \alpha[y_t + \Gamma \theta_{t}^*, nb] + (1 - \alpha)z
\]

differs from the Pissarides wage by the coefficient \( \Gamma_t \) on market tightness. Recall that \( \Gamma_t \equiv \gamma + K(\phi^*)(1 - (1 - q(\theta_{t}^*, nb))/(1 + r)) \) are the set-up costs augmented for frictional credit markets. With \( r = 0 \), this term becomes \( \gamma + q(\theta_{t}^*, nb)K(\phi^*) \).

\[\text{We first define the log-deviation around the steady state as } \begin{align*}
\theta_{t}^{P, nb} &= (q(\theta^{P, nb})(1 - \alpha)) / (\eta \gamma (1 + r)) \\
\mathbb{E}_t \sum_{k=0}^{\infty} \Theta \gamma^{k+1} \end{align*}\]

where the second term in \( \Theta = ((1 - s)/(1 + r)) - \alpha \theta_{t}^{P, nb} q(\theta^{P, nb})/(\eta (1 + r)) \) reflects the share of the change in productivity accruing to the worker through the wage. As a comparison, note that if \( \rho_t = 1 \), this is the elasticity obtained when comparing steady states, or to a permanent productivity shock, as in Shimer (2005), i.e., \( \epsilon_{\theta_{t}} = (1 - \alpha)/(\gamma [r (r + s)/q(\phi^*) + \alpha \theta^*]) \).
Therefore, as compared to a Pissarides economy in which wages depend linearly on labor market tightness with a term in $\gamma \theta$, wages now have a new procyclical term in $\theta^*_t q(\theta^*_t, nb) K(\phi^*)$. An increase in labor market tightness improves the worker’s bargaining position in wage negotiations by weakening the value of the firm’s outside option over and above the recruiting costs $\gamma$. Thus, as compared to the fixed wage model, the endogenous wage model will generate a lower volatility of hiring decisions. As compared to the endogenous wage economy without financial market imperfections, this economy will have a tendency to have more volatile wages. However, the main channel discussed in the introduction, the fact that entry costs are less procyclical, will undo this wage volatility effect, and our calibration will show higher elasticities for the model with financial market imperfections.

Finally, the elasticity of labor market tightness to a productivity shock under frictional credit markets and with an endogenous wage is

$$\Lambda^{nb} = \frac{\partial \hat{\theta}^*_t}{\partial \nu_t} q(\theta^*, nb) (1 - \alpha) \rho_y,$$

$$\gamma \bar{\gamma}(r)(1 + r) - [\eta \bar{\gamma}(r)(1 + s) - \alpha f(\theta^*, nb)(\bar{\gamma}(r) + (1 - \eta) \bar{\kappa})] \rho_y,$$

where $\bar{\kappa} \equiv K(\phi^*) q(\theta^*, nb)/(1 + r)$. Consequently, the dynamic financial multiplier when the wage is endogenous is given by

$$M_f^{P, nb} = \frac{q(\theta^*, nb)}{q(\theta^*, nb)} \times \frac{\eta \bar{\gamma} - \gamma \rho_y \eta (1 - s) - \alpha f(\theta^*, nb)}{\eta \bar{\gamma}(r) - \bar{\gamma}(r) \rho_y \eta (1 - s) - \alpha f(\theta^*, nb) \left[ 1 + (1 - \eta) K(\phi^*) q(\theta^*, nb) / \bar{\gamma}(r) \right]}.$$

Just as in the previous case with a fixed wage, the financial multiplier tends toward 1 as costs on credit markets, $K(\phi^*)$, tends toward zero.

### III. Parameterization

In this section, we pursue the objective of providing a set of parameters to illustrate the main points. We choose parameters to match the data where possible, and investigate a range of parameters where the data is scarce. We begin by detailing the determination of parameters related to the labor market before discussing our approach to pinning down the values for parameters of the credit market.

The basic unit of time is a month. The matching function in the labor market is assumed to take the usual form $M_l^{\nu, u} = \chi^{1 - \eta} u^{\eta}$, where $\chi > 0$ is a level parameter. We set $\eta = 0.6$, drawing on the survey of estimates of the labor matching function in Petrongolo and Pissarides (2001) and advocated by Brügemann (2008). We set the job separation rate, $s$, to 0.03, corresponding to the monthly value reported for the United States in Davis, Faberman, and Haltiwanger (2006) based
on the Jobs Openings and Labor Turnover Survey (JOLTS). We target an unemployment rate of 7 percent, as in Gertler and Trigari (2009), which implies a job finding rate \( f = s(1 - u)/u \). We target a job filling rate \( q = 0.4 \) based on the estimates for the United States in den Haan, Ramey and Watson (2000).\(^{16}\) The constant returns of the matching function imply that labor market tightness is \( \theta = f/q \), and steady-state job vacancies can be calculated as \( \mathcal{V} = \theta u \), given our target for the unemployment rate.

Silva and Toledo (2007) estimate costs related to recruiting a worker of 3.6 percent of the wage rate, or \( \gamma \mathcal{V}/w = 0.036 \). In combination with a target for a labor share of national income, \( w/y \), of two-thirds (Golin 2002), the unit vacancy costs is calculated as \( \gamma = 0.036 w/\mathcal{V} \).\(^{17}\)

The economy’s job creation condition then determines the value of \( \Gamma \) as
\[
\Gamma = q(y - w)/(r + s),
\]
where the variable \( \Gamma \) is defined as \( \Gamma = \gamma + K(r + q)/(1 + r) \). Consequently, we can compute \( K = (\Gamma - \gamma)(1 + r)/(r + q) \). Recall the wage equation \( w = \alpha[y + \Gamma \theta] + (1 - \alpha)z \). Assuming \( z = 0.4 \), as in Shimer (2005), the bargaining weight \( \alpha \) that satisfies our targeted wage is
\[
\alpha = \frac{w - z}{y + \Gamma \theta - z}.
\]

The specification of the credit market requires choosing five parameters: the costs of prospecting on credit markets, \( \kappa \) and \( \epsilon \); the bargaining weight \( \beta \); and the parameters of the credit matching function, \( M_e(B, E) = \zeta e^{1 - \epsilon} B^\epsilon \), a level \( \zeta > 0 \), and elasticity \( 0 < \epsilon < 1 \). For lack of direct evidence, we set the elasticity of the credit market matching function to \( \epsilon = 0.5 \) and assume a symmetry in the cost of prospecting in the credit market, i.e., \( \kappa = e \). Our strategy is then to use an observed size of the financial sector to determine the weight \( \beta \), constrain the level parameter \( \zeta \) by limiting the duration of search in the credit market, and the search cost \( \kappa \) is determined by the previously calculated costs \( K \).

Our main target for the credit market is the financial sector’s share of aggregate value added, which in the model is
\[
\Sigma = \frac{(1 - u) \Psi - \gamma \mathcal{V} - B \kappa}{1 - u},
\]
where the first term in the numerator represents total bank gross profits (i.e., \( \Psi \) times the number of banks in the profit state, which is the number of producing firms \( 1 - u \)); the second term represents the negative cash flows of banks financing vacancies times the number of job vacancies \( \mathcal{V} \); and the last term represents the costs of financial intermediation paid by banks. Note that we assumed the costs \( e \) paid by

\(^{16}\) Table A.3 of the online Appendix shows that the results are almost invariant to the chosen value for the job filling rate.

\(^{17}\) This target for the wage rate places the model in a region where amplification of labor market tightness will not arise from having assumed a small labor surplus. This choice is further justified by the fact that, since revenues are shared between banks, firms, and workers, the model is richer than a simple worker-firm model. Reimbursement to banks is a proxy for the cost of accessing capital. The remaining third goes to the owner of the illiquid and liquid capital, i.e., the firm and the banker. We think of the firm and banker as being the joint owners of installed capital that can produce a good with a unit of labor.
entrepreneurs did not enter GDP, as they are effort costs. The denominator is total production at \( y = 1 \).

With details provided in the online Appendix, the financial sector’s share \( \Sigma \) can be shown to be an increasing function of the bank’s bargaining weight \( \beta \):

\[
\Sigma(\beta) = \frac{(1 - u) \left[ \beta (y - w) + (1 - \beta) \frac{\gamma}{q} (r + s) \right] - (\gamma + q \beta K) \nu}{1 - u}.
\]

Given our targets and parameter values so far, this allows us to pin down \( \beta \) for a given value of \( \Sigma \) and compute credit market tightness as \( \phi = (\kappa / e)(1 - \beta) / \beta = (1 - \beta) / \beta \).

Calculating the empirical counterpart to \( \Sigma \) is not straightforward. Recall that our model captures the entire cycle of a new, marginal, production project and the costs of external finance. The costs of financial frictions therefore encompass both firm’s creation and the development in existing firms of new projects and the associated new hires. The value added of the corporate financial sector should therefore be used as our target. We calculate the financial sector’s value added as a share of US GDP using the industry value added tables provided by the Bureau of Economic Analysis (BEA). Over the period 1985–2002, this represents approximately 7.4 percent of GDP, increasing over the period to 9 percent in 2002. To this fraction, we subtract the share in GDP of household financial services and insurance from the National Income and Product Accounts tables. This averaged to 4.9 percent of GDP over the period in question. Therefore, the value of \( \Sigma \) we match in our numerical exercise is 2.5 percent, and would range from 1.73 percent in 1985 to 3.5 percent in 2002. As reported in Table 1, this results in a bank’s bargaining weight \( \beta = 0.68 \). In order to provide robustness checks of both the main results and the suggested parameter values for the credit market, we also provide results for a range of sizes of the financial sector \( \Sigma \) from 1.5 percent to 3.5 percent. However, at this stage, we acknowledge that more research is needed in order provide a more accurate description of the financial costs of job creation.

The functional form for the matching function in the credit market implies meeting rates \( p(\phi) = \zeta \phi^{-\epsilon} \) and \( \phi p(\phi) = \zeta \phi^{1-\epsilon} \). We target an average duration of search in the credit market for banks of four months, i.e., \( 1 / \phi p(\phi) = 4 \). This pins down the level parameter in the credit market matching function as \( \zeta = 1 / \left[ 4 \phi^{1-\epsilon} \right] \). We consider this a relatively short duration of search, and note that it does not affect the results with respect to amplification. Finally, the search costs are obtained as \( \kappa = \beta K \phi p(\phi) \) and \( e = (1 - \beta) K p(\phi) \), completing our parameterization of the credit market.

The magnitude of the response of labor market tightness to a productivity shock depends on the persistence of the process for labor productivity (see Section III). We draw from the existing literature and set the persistence coefficient \( \rho_y \) to 0.95\(^{1/3} \) (see, Merz 1995; Andolfatto 1996; den Haan, Ramey, and Watson 2000; Gertler and

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PETROSKY-NADEAU AND WASMER: THE VOLATILITY OF LABOR MARKETS UNDER FRICTIONAL FINANCIAL MARKETS

IV. Results

A. Baseline Results

Our baseline parameterization of the model implies a substantial degree of amplification due to credit market frictions. The elasticity of labor market tightness to productivity shocks, as reported in Table 1, is equal to 9.77, with a financial multiplier $M_f = 5.55$. In order to illustrate the degree of amplification that can arise from credit market frictions in this environment, we consider the effects of moving away from Hosios in the credit market. As we have shown in Section III, this increases the financial multiplier $M_f$ and elasticity of labor market tightness to productivity shocks, $\Lambda$. In the second column of Table 1, we arbitrarily increase the value of the bank’s share, $\beta$, further away from the elasticity of the matching function in the credit market, $\epsilon$, keeping all other parameters fixed to their baseline values. Increasing $\beta$ from 0.68 to 0.84 indeed results in a significantly larger financial multiplier of 8.24, and the elasticity of labor market tightness to productivity shocks increases to 14.67. As this increases the distortions caused by credit market frictions, the unemployment rate reaches 11 percent, and the share of the financial section in GDP rises to 3.5 percent.

Note: The Costain-Reiter elasticity of $u$ to $z$ with Hagedorn-Manovskii parameters is 14.3.

Trigari 2009; among others). The risk-free rate is set to 4 percent annually, corresponding to a 3-month US Treasury bill.
In order to further investigate the potential magnitude of the financial multiplier, we adjust the value of nonemployment $z$ in order to match elasticity of labor market tightness to productivity shocks of 15 and 20, keeping other parameters and targets for the labor and credit markets constant. The results are reported in the third and fourth columns of Table 1, respectively. The columns report that only the value of the worker’s bargaining weight is affected by this change in order to maintain the target for the wage rate. Matching an elasticity of 15 requires $z = 0.49$ and an elasticity of 20 requires $z = 0.53$. In the first case, the financial multiplier increases to 7.22, and in the second, it increases to 8.84, indicating a strong interaction between credit and labor market frictions. To be precise, the magnitude of the financial multiplier is increasing in the degree of wage rigidity. At the extreme of a fixed wage, reported in the last column, the financial multiplier reaches a value of $M^D_f = 11.86$.

In the last row of Table 1, we calculate the elasticity of unemployment to changes in unemployment benefits. Costain and Reiter (2008) found this elasticity to be much too large, around 14.3, for the parameter values implied by Hagedorn and Manovskii’s (2008) calibration. This led to the criticism that it is difficult to match both the elasticity of labor market tightness to productivity and the elasticity of unemployment to policy parameters, such as the replacement ratio. By contrast, at an elasticity of labor market tightness to productivity shocks of 20, the model’s elasticity of unemployment to the policy parameter $z$ is much lower, 2.70 instead of 14.3, while it is only 1.17 for the baseline parameterization.

**B. Robustness to Changes in the Cost of External Finance**

In this subsection, we investigate the robustness of our conclusions to varying the targeted share of the financial sector from 1 percentage point below to 1 percentage point above its mean value, 2.5 percent of GDP. The results are reported in Table 2 (with full details in Table A.1 and A.2 of the online Appendix).

For the model to match a share of the financial sector of 1.5 percent, the implied bargaining weight $\beta$ is 0.37. In order to match a share of $\Sigma = 3.5$ percent, $\beta = 0.98$. The resulting financial multiplier is the same in both cases and equal to its value under the baseline parameterization. The same is true of the elasticity of labor market tightness to productivity shocks. The next column for each scenario increases the distortions in the credit market, keeping labor market parameters constant. When $\Sigma = 1.5$ percent, we reduce the bank’s bargaining weight to 0.18. When $\Sigma = 3.5$ percent, we reduce the elasticity of the matching function in the credit market $\epsilon$ to 0.45. In each case, the financial multiplier increases to approximately 8, and the increased distortion increases the unemployment rate to 11 percent.

The following two columns show that to match greater elasticities of labor market tightness requires similar parameter values for $\alpha$ and $z$, as in the baseline case. Our conclusion is that although the share of the financial sector is a key determinant of frictions in our parameterization, the model adjusts to this target in changing the value of $\beta$ while affecting other results relatively little.
Financial imperfections raise the elasticity of labor market tightness to productivity shocks by a factor $M_f^D$ called the financial multiplier. It is possible to generate a plausibly large elasticity of labor market tightness to productivity shocks without relying on the small labor surplus assumption, even with endogenous wages, if one relaxes the Hosios-Pissarides rule in the credit market. Under the assumption of a large enough difference between the bargaining power of banks vis-à-vis firms ($\beta$) with the elasticity of the rate at which new projects meet bankers with respect to credit market tightness ($\varepsilon$), one can obtain an elasticity around 20 or even larger. Our results are in fact a generalization of the small labor surplus assumption—when the credit market is either very tight or very slack for firms, one side of the market has a very small total surplus to entering the relationship. Consequently, the entry of that side of the credit market is restricted, and even small productivity shocks can generate large responses on the restricted side of the market. Here, the small surplus is on firms in the credit-search stage.

The financial sector introduces a new element to hiring costs that are not proportional to the procyclical average duration of filling a vacancy. This is consistent with Pissarides (2009) who pleads in favor of adding a nonproportional part to hiring costs as a means of generating amplification in this class of models. Our paper thus provides an interpretation of this fixed part in entry costs linked to financial market imperfections. Our conclusion is that macroeconomic modeling is improved, both qualitatively and quantitatively, with the introduction of financial market imperfections, and our model is a step in this direction.

Finally, the model featured acyclical access to finance as credit market tightness is a function of financial parameters. This arises from the free entry of both
firms and banks in response to a productivity shock, leading to an equiproportional change in both the demand and supply of credit and leaving the financial equilibrium unchanged. This may be a limit of the model, or alternatively suggest some new directions. In our setup, a financial boom or recession may be modeled as a shock on intermediation costs, in particular, the screening costs of banks $\kappa$ or the cost of accessing credit $\epsilon$. A third possibility is a shock affecting the relative bargaining power of banks relative to firms $\beta$. In these three examples, the cost of finance would become cyclical and may allow for a quantitative investigation of the recent financial recession. This is not explored here but is a way toward future research.

**APPENDIX I**

**A. Credit Market Tightness**

Free entry on both sides of the credit market, along with Nash bargaining over the surplus of a credit relationship, results in a time invariant credit market tightness. To show this, note first that we had under free entry,

$$B_{lt} = \frac{\kappa}{\phi_t p(\phi_t)}; \text{ and } E_{lt} = \frac{\epsilon}{p(\phi_t)}.$$

Denoting the banker’s bargaining weight by $\beta \in (0, 1)$, defining the credit relationship surplus as $S_{c,t} = (E_{lt} - E_{ct}) + (B_{lt} - B_{ct})$, and finally using the free entry conditions $E_{ct} = 0$ and $E_{ct} = 0$, we obtain $E_{lt}/B_{lt} = (1 - \beta)/\beta$, and therefore $\phi^* = (1 - \beta)\kappa/(\beta\epsilon)$.

**B. Deriving a Job Creation Condition**

It will be convenient at this stage to express the joint value of recruiting a worker to a banker and a project as $S_{lt} = E_{lt} + B_{lt}$, which corresponds to the surplus from the credit relationship, as:

$$\frac{\epsilon}{p(\phi^*)} + \frac{\kappa}{\phi^* p(\phi^*)} = -\gamma + q_t \frac{1}{1 + r} E_t[E_{g,t+1} + B_{g,t+1}] + (1 - q_t) \frac{1}{1 + r} \left[ \frac{\epsilon}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} \right],$$

or the expression for the (CC) curve in the text:

$$\frac{\epsilon}{p(\phi^*)} + \frac{\kappa}{\phi^* p(\phi^*)} = \frac{q(\phi^*)}{r + q(\phi^*)} (1 + r) \left[ -\frac{\gamma}{q(\theta^*_t)} + \frac{1}{1 + r} E_t S_{g,t+1} \right].$$

Define total costs on the credit market as $K(\phi^*) = \epsilon/p(\phi^*) + \kappa/(\phi^* p(\phi^*))$ and $\Gamma_t \equiv \gamma + K(\phi^*)(1 - (1 - q_t)/(1 + r))$, then

$$\frac{\Gamma_t}{q_t} = \frac{1}{1 + r} E_t[E_{g,t+1} + B_{g,t+1}].$$
Using the Bellman equations for the new project and banker during production to define \([E_{g,t} + B_{g,t}] = S_{g,t} = y_t - w_t + (1 - s)E_t[S_{g,t+1}]/(1 + r)\), we obtain a job creation condition in the presence of frictional credit and labor markets:

\[
\frac{\Gamma_t}{q_t} = \frac{1}{1 + r} E_t \left[ y_{t+1} - w_{t+1} + \left(1 - s\right)\frac{\Gamma_{t+1}}{q_{t+1}} \right].
\]

**C. Repayment \(\Psi\)**

This section provides the details in deriving the rental rate \(\Psi\). The sharing rule under Nash bargaining implied \(B_{l,t} = \beta S_{C,t}\) and \(E_{l,t} = (1 - \beta)S_{C,t}\). Expanding on the former:

\[
-\gamma + \frac{1}{1 + r} E_t[q_t B_{g,t+1} + (1 - q_t)B_{l,t+1}] = -\beta \gamma + \beta q_t \frac{1}{1 + r} E_t[E_{g,t+1} + B_{g,t+1}]
\]

\[
+ \beta (1 - q_t) \frac{1}{1 + r} E_t[E_{l,t+1} + B_{l,t+1}].
\]

Rearranging terms,

\[
E_t B_{g,t+1} + \frac{(1 - q_t)}{q_t} E_t B_{l,t+1} = (1 - \beta) \frac{\gamma(1 + r)}{q_t}
\]

\[
+ \beta E_t \left[ E_{g,t+1} + B_{g,t+1} \right] + \frac{(1 - q_t)}{q_t} \left[ E_{l,t+1} + B_{l,t+1} \right]
\]

\[
E_t \left[ \Psi_{t+1} + \frac{1 - s}{1 + r} B_{g,t+2} \right] = (1 - \beta) \frac{\gamma(1 + r)}{q_t}
\]

\[
+ \beta \left[ y_{t+1} - w_{t+1} + (1 - s)\frac{1}{1 + r}[B_{g,t+2} + E_{g,t+2}] \right]
\]

\[
+ \beta \frac{(1 - q_t)}{q_t} E_t[E_{l,t+1} + B_{l,t+1}] - \frac{(1 - q_t)}{q_t} E_t B_{l,t+1}
\]

Since \(B_{l,t} = \beta[E_{l,t} + B_{l,t}]\), \(E_t B_{g,t+1} = (1 - \beta)\gamma(1 + r)/q_t + \beta E_t[E_{g,t+1} + B_{g,t+1}]\), or

\[
E_t[1 - \beta)B_{g,t+1} - \beta E_{g,t+1}] = (1 - \beta) \frac{\gamma(1 + r)}{q_t},
\]
then

\[
\mathbb{E}_t\left[\Psi_{t+1} + \frac{1-s}{1+r} B_{g,t+2}\right] = (1-\beta) \frac{\gamma(1+r)}{q_t} \\
+ \beta \mathbb{E}_t\left[y_{t+1} - w_{t+1} + \frac{1-s}{1+r} [B_{g,t+2} + E_{g,t+2}]\right],
\]

and

\[
\mathbb{E}_t[\Psi_{t+1}] = \beta \mathbb{E}_t[y_{t+1} - w_{t+1}] - (1-\beta) \mathbb{E}_t\left[\frac{(1+r)\gamma}{q(\theta_t)} - \frac{(1-s)\gamma}{q(\theta_{t+1})}\right].
\]

Finally, from equation (10) in the text, \(\mathbb{E}_t[(1+r)\gamma/q(\theta_t) - (1-s)\gamma/q(\theta_{t+1})] = \mathbb{E}_t[y_{t+1} - w_{t+1}] + K(\phi^*) \mathbb{E}_t[(r + q(\theta_t))/q(\theta_t) - (1-s)(r + q(\theta_{t+1}))/q(\theta_{t+1}) \times (1+r)]\), we have that

\[
\mathbb{E}_t[\Psi_{t+1}] = \mathbb{E}_t[y_{t+1} - w_{t+1}] - (1-\beta)K(\phi^*) \mathbb{E}_t\left[\frac{r + q(\theta_t)}{q(\theta_t)} - \frac{(1-s)r + q(\theta_{t+1})}{q(\theta_{t+1})}\right].
\]

D. Workers and Wages

We assume that the wage is negotiated between a worker-firm pair, with surplus \(S^L_t = S_{g,t} + W_t - U_t\), and satisfies the sharing rule \(\alpha S_{g,t} = (1-\alpha)(W_t - U_t)\), where \(\alpha \in (0,1)\) is the Nash bargaining weight of workers. Applying this sharing rule to the worker-firm surplus, first we have

\[
S^L_t = y_t - w_t + (1-s)\frac{1}{1+r} \mathbb{E}_t S_{g,t+1} + w_t + \frac{1}{1+r} \mathbb{E}_t[(1-s)W_{t+1} + sU_{t+1}] \\
- z - \frac{1}{1+r} \mathbb{E}_t[\theta_t q_t W_{t+1} - (1-\theta_t q_t) U_{t+1}]
\]

\[
S^L_t = y_t - z + \frac{1-s}{1+r} \mathbb{E}_t [S_{g,t+1} + W_{t+1} - U_{t+1}] - \theta_t q_t \frac{1}{1+r} \mathbb{E}_t [W_{t+1} - U_{t+1}]
\]

\[
S^L_t = w_t - z + (1-s)\frac{1}{1+r} \mathbb{E}_t S^L_{t+1} - \alpha \theta_t q_t \frac{1}{1+r} \mathbb{E}_t S^L_{t+1},
\]

and second, using \(S_{g,t} = (1-\alpha)S^L_t\) and \(\frac{\Gamma_t}{\psi_t} = \frac{1}{1+r} \mathbb{E}_t (1-\alpha)S_{g,t+1}\),

\[
y_t - w_t + (1-s)\frac{1}{1+r} \mathbb{E}_t S_{g,t+1} = (1-\alpha)\left(y_t - z + (1-s)\frac{1}{1+r} \mathbb{E}_t S^L_{t+1}\right) - \alpha \theta_t \Gamma_t.
\]

Rearranging terms yields the wage rule under frictional labor and credit markets:

\[
w_t = \alpha(y_t + \theta_t \Gamma_t) + (1-\alpha)z.
\]


APPENDIX II
DERIVING THE ELASTICITY OF MARKET TIGHTNESS TO A PRODUCTIVITY SHOCK

A. Derivation of Expression for $\hat{\theta}_i$ in Section IIIA

This section details the steps in deriving the following expression for proportional deviations of labor market tightness from the deterministic steady state, assuming a fixed wage $\bar{w}$:

$$\hat{\theta}_i = \frac{1}{\eta} y - \frac{S_g}{S_g - K(\phi^*)} \mathbb{E}_t \left( r + s \sum_{i=1}^{\infty} \frac{1-s}{1+r} y_{t+i} \right),$$

where hatted variables denote proportional deviations from the steady state, and $\eta$ is the elasticity of $q(\theta)$ to $\theta$.

We begin with the job creation condition expressed as,

$$\frac{\Gamma_t}{q(\theta)} = \frac{1}{1+r} \mathbb{E}_t S_{g,t+1}.$$ 

Since $\Gamma_t = \gamma + Kr/(1+r) + q(\theta)K/(1+r)$, and $q(\theta) = \chi \theta^{-\eta}$, a log-linear approximation of this equation around the steady state is given by

$$\eta \frac{rK}{1+r} \hat{\theta}_t = \frac{S_g}{1+r} \mathbb{E}_t \hat{S}_{g,t+1}.$$ 

Note that

$$\frac{\gamma + rK}{1+r} = \frac{\gamma + rK}{1+r} + q(\theta) \frac{K}{1+r} - \frac{K}{1+r} = \frac{\Gamma}{q(\theta)} - \frac{K}{1+r},$$

such that the previous expression can be rearranged as

$$\hat{\theta}_t = \frac{1}{\eta} \left( \frac{1}{\frac{\Gamma}{q(\theta)} - \frac{K}{1+r}} \right) \frac{S_g}{1+r} \mathbb{E}_t \hat{S}_{g,t+1}.$$ 

Since we have, in a steady state, that $\Gamma/q(\theta) = S_g/(1+r)$, we therefore have

$$\hat{\theta}_t = \frac{1}{\eta} \left( \frac{1+r}{S_g - K} \right) \frac{S_g}{1+r} \mathbb{E}_t \hat{S}_{g,t+1}$$

$$= \frac{1}{\eta} \left( \frac{S_g}{S_g - K} \right) \mathbb{E}_t \hat{S}_{g,t+1}.$$
The final step involves developing the expression for $\mathbb{E}_t \hat{S}_{g,t+1}$. The value of a filled vacancy to a firm, assuming a fixed wage, is given by

$$S_{g,t} = y_t - \bar{w} + (1 - s) \frac{1}{1 + r} \mathbb{E}_t [S_{g,t+1}].$$

A log-linear approximation of this equation yields

$$S_{g,t} \hat{S}_{g,t} = \hat{y}_t + (1 - s) \frac{S_{g,t}}{1 + r} \mathbb{E}_t [\hat{S}_{g,t+1}]$$

$$S_{g,t} \hat{S}_{g,t} \left( 1 - \frac{1 - s}{1 + r} \mathbb{E}_t L^{-1} \right) = \hat{y}_t$$

$$S_{g,t} \hat{S}_{g,t} = \mathbb{E}_t \sum_{i=0}^{\infty} \left( \frac{1 - s}{1 + r} \hat{y}_{t+i} \right).$$

Also, in a steady state, we have that $S_g = (y - w)(1 + r)/(r + s)$, such that

$$\hat{S}_{g,t} = \frac{y}{y - \bar{w}} \frac{r + s}{S_g - K} \frac{1}{1 + r} \sum_{i=0}^{\infty} \left( \frac{1 - s}{1 + r} \hat{y}_{t+i} \right).$$

Substituting this expression in the previous one for the proportional deviation in labor market tightness, we obtain the expression in Section 3.1:

$$\hat{\theta}_t = \frac{1}{\eta} \frac{y}{y - \bar{w}} \frac{S_g}{S_g - K} \frac{r + s}{1 + r} \mathbb{E}_t \sum_{i=0}^{\infty} \left( \frac{1 - s}{1 + r} \hat{y}_{t+i} \right).$$

B. Elasticity of Labor Market Tightness

Define period profit flows as $\Pi = y - w$. Taking log-linear deviations of the job creation condition

$$\frac{\gamma}{q(\theta^P_t)} = \frac{1}{1 + r} \mathbb{E}_t \left[ \Pi_{t+1} + (1 - s) \frac{\gamma}{q(\theta^P_{t+1})} \right]$$

around a stationary steady state yields

$$\eta \frac{\gamma(1 + r)}{q(\theta^P_t)} \hat{\theta}_t^P = \Pi \mathbb{E}_t \hat{\Pi}_{t+1} + \eta \frac{\gamma(1 - s)}{q(\theta^P_t)} \mathbb{E}_t \hat{\theta}_{t+1}^P$$

$$\eta \hat{\theta}_t^P = \frac{q(\theta^P_t) \Pi}{\eta \gamma(1 + r)} \mathbb{E}_t \hat{\Pi}_{t+1} + \frac{1 - s}{1 + r} \mathbb{E}_t \hat{\theta}_{t+1}^P.$$
Making use of a forward operator, \( x_{t+1} = L^{-1}x_t \), we have that
\[
[1 - (1 - s)\mathbb{E}_t L^{-1}/(1 + r)] \hat{\theta}_t^p = q(\theta^p)\Pi \gamma (1 + r) \mathbb{E}_t \hat{\Pi}_{t+1},
\]
and can express current deviations of labor market tightness as
\[
\hat{\theta}_t^p = \frac{q(\theta^p)\Pi}{\eta \gamma (1 + r)} \mathbb{E}_t \sum_{i=0}^{\infty} \left( \frac{1 - s}{1 + r} \right)^i \hat{\Pi}_{t+i+1}.
\]

**Fixed Wage—Pissarides.**—Under the assumption of a fixed wage \( w_t = \bar{w} \), proportional deviation in profits are equal to \( \hat{\Pi}_t = \hat{y}_t/\Pi \), and the previous expression for deviations in labor market tightness becomes:
\[
\hat{\theta}_t^p = \frac{1}{\eta} \frac{q(\theta^p)}{\gamma (1 + r)} \mathbb{E}_t \sum_{i=0}^{\infty} \left( \frac{1 - s}{1 + r} \right)^i (y_{t+i+1}).
\]
Given the assumption that productivity follows an AR(1) process, this is simply \( \hat{\theta}_t^p = q(\theta^p)/(\eta \gamma (1 + r)) \sum_{i=0}^{\infty} \rho_i^p y_t \). It follows that the elasticity of labor market tightness to a productivity shock in the Pissarides world with a fixed wage, denoted \( \Lambda^p \), is
\[
\Lambda^p = \frac{\partial \hat{\theta}_t^p}{\partial \nu_t} = \frac{1}{\eta} \frac{q(\theta^p) \rho_t}{\gamma [(1 + r) - (1 - s)\rho_t]}.
\]

**Fixed Wage—Financial Friction.**—Recall that the job creation condition in this setting is
\[
\frac{\Gamma_t}{q_t} = \frac{1}{1 + r} \mathbb{E}_t \left[ y_{t+1} - \bar{w} + (1 - s) \frac{\Gamma_{t+1}}{q_{t+1}} \right],
\]
with \( \Gamma_t \equiv \gamma + K(\theta^p)(1 - (1 - q_t)/(1 + r)) \) and, again, \( q(\theta) = \chi \theta^{-\eta} \). Taking the log-linear deviations around a stationary steady state, we have
\[
\frac{\eta(1 + r)}{q(\theta)} \left[ \gamma + K(\phi^*) \left( \frac{r}{1 + r} \right) \right] \hat{\theta}_t = \mathbb{E}_t \hat{\nu}_{t+1} + \frac{\eta(1 - s)}{q(\theta)} \left[ \gamma + K(\phi^*) \left( \frac{r}{1 + r} \right) \mathbb{E}_t \right] \hat{\theta}_{t+1}.
\]
Calling \( \gamma(r) \equiv [\gamma + K(\phi^*)r/(1 + r)] \) with \( \gamma(0) = \gamma \), we have
\[
\hat{\theta}_t = \frac{q(\theta)}{\eta \gamma(r) \times (1 + r)} \mathbb{E}_t \hat{\nu}_{t+1} + \frac{1 - s}{1 + r} \mathbb{E}_t \hat{\theta}_{t+1}.
\]
Making use of the forward operator, we have that \[ \left[ 1 - (1 - s)E_t L^{-1} / (1 + r) \right] \hat{\theta}_t = (q(\theta) / (\eta \gamma(r)(1 + r)))E_t \hat{y}_{t+1}, \] and can express the current deviations of labor market tightness as the discounted expected future path of productivity:

\[
\hat{\theta}_t = \frac{q(\theta)}{\eta \gamma(r) \times (1 + r)} E_t \sum_{i=0}^{\infty} \left( \frac{1 - s}{1 + r} \right)^i \hat{y}_{t+1+i}.
\]

If productivity follows the same AR(1) process, then \( \hat{\theta}_t = q(\theta) / (\eta \gamma(r) \times (1 + r)) \times E_t \sum_{i=0}^{\infty} ((1 - s)/(1 + r))^{i+1} \nu_t \), or \( \hat{\theta}_t = q(\theta) / (\eta \gamma(r) \times (1 + r))(\rho_s / (1 - 1/(1 + r) \rho_s)) \nu_t \), and the expression for the elasticity of labor market tightness to productivity shocks under frictional credit markets and a fixed wage is:

\[
\frac{\partial \hat{\theta}_t}{\partial \nu_t} = \frac{q(\theta) \rho_s}{\eta \gamma(r) \times [(1 + r) - (1 - s)] \rho_s}.
\]

**Flexible Wage—Pissarides.**—Using the definition of the wage \( w_t = \alpha \left( y_t + \gamma \theta_t \right) + (1 - \alpha)z \), we can further express the deviations of labor market tightness as a discounted sum of the expected future path of productivity:

\[
\hat{\theta}^{P, nb}_t = \frac{q(\theta^{P, nb})(1 - \alpha)}{\eta \gamma \times (1 + r)} E_t \sum_{i=0}^{\infty} \xi^i \hat{y}_{t+1+i},
\]

where \( \xi = ((1 - s)/(1 + r)) - \alpha \theta^{P, nb} q(\theta^{P, nb}) / (\eta(1 + r)). \) Assuming that productivity follows an AR(1) with persistence parameter \( 0 < \rho_s < 1 \), and innovation \( \nu_t \) as white noise, then,

\[
\hat{\theta}^{P, nb}_t = \frac{q(\theta^{P, nb})(1 - \alpha)}{\eta \gamma \times (1 + r)} \sum_{i=0}^{\infty} \xi^i \rho_s^{i+1} \nu_t,
\]

so that \( \hat{\theta}^{P, nb}_t = q(\theta^{P, nb})(1 - \alpha) / (\eta \gamma(1 + r)(1 - \xi \rho_s)) \rho_s \nu_t \), and the elasticity of labor market tightness to productivity shocks is

\[
\frac{\partial \hat{\theta}^{P, nb}_t}{\partial \nu_t} = \frac{(1 - \alpha)q(\theta^{P, nb}) \rho_s}{\eta \gamma \times (1 + r) - \gamma \times [\eta(1 - s) - \alpha f(\theta^{P, nb})] \rho_s}.
\]

**Flexible Wage—Financial Friction.**—Given the wage rule \( w_t = \alpha \left[ y_t + \Gamma \beta_t^{*, nb} \right] + (1 - \alpha)z \) derived above, the job creation condition can be written as

\[
\frac{\Gamma_t}{q_t} = \frac{1}{1 + r} \left[ E_t (1 - \alpha)(y_{t+1} - z) - \alpha \Gamma_{t+1} \theta_t^{*, nb} + (1 - s) \frac{\Gamma_{t+1}}{q_{t+1}} \right].
\]
The following preparatory steps are useful. First, recall that \( \Gamma_t = \bar{\gamma}(r) + K(\phi^*) \times q(\theta^*_{t \to t})/(1 + r) \), such that the job creation condition is expressed as a function of labor market tightness and productivity. Then, we take the log-linear deviations of the job creation condition around a stationary steady state:

\[
\frac{\eta(1 + r)}{q(\theta)} \bar{\gamma}(r) \hat{\theta}^*_{t \to t} = (1 - \alpha) \mathbb{E}_t \hat{\gamma}_{t+1} - \alpha \left( \bar{\gamma}(r) \theta^*_{t \to t} + \frac{K(\phi^*)}{1 + r} (1 - \eta) f(\theta^*_{t \to t}) \right) \mathbb{E}_t \hat{\theta}^*_{t+1} + \frac{\eta(1 + s)}{q(\theta^*_{t \to t})} \bar{\gamma}(r) \mathbb{E}_t \hat{\theta}^*_{t+1}
\]

\[
\theta^*_{t \to t} = \frac{(1 - \alpha) q(\theta^*_{t \to t})}{\eta \bar{\gamma}(r) \times (1 + r)} \mathbb{E}_t \hat{\gamma}_{t+1} + \left[ \frac{(1 + s)}{(1 + r)} - \frac{\alpha q(\theta^*_{t \to t})}{\eta \bar{\gamma}(r) \times (1 + r)} \left( \bar{\gamma}(r) \theta^*_{t \to t} + \frac{K(\phi^*)}{1 + r} (1 - \eta) f(\theta^*_{t \to t}) \right) \right] \mathbb{E}_t \hat{\theta}^*_{t+1}.
\]

Denoting

\[
\Phi \equiv \left[ \frac{(1 + s)}{(1 + r)} - \frac{\alpha q(\theta^*_{t \to t})}{\eta \bar{\gamma}(r) \times (1 + r)} \left( \bar{\gamma}(r) \theta^*_{t \to t} + \frac{K(\phi^*)}{1 + r} (1 - \eta) f(\theta^*_{t \to t}) \right) \right]
\]

for the moment, we then follow similar steps by first obtaining deviations of labor market tightness as a discounted sum of expected future deviations of productivity:

\[
\hat{\theta}^*_{t \to t} = \frac{(1 - \alpha) q(\theta^*_{t \to t})}{\eta \bar{\gamma}(r) \times (1 + r)} \mathbb{E}_t \sum_{i=0}^{\infty} \Phi^i \hat{\gamma}_{i+1+i}
\]

and, making use of the specification for labor productivity,

\[
\hat{\theta}^*_{t \to t} = \frac{(1 - \alpha) q(\theta^*_{t \to t})}{\eta \bar{\gamma}(r) \times (1 + r)} \mathbb{E}_t \sum_{i=0}^{\infty} \Phi^i \rho_{\gamma}^{i+1} \nu_{i+1}
\]

Finally, \( \hat{\theta}^*_{t \to t} = \frac{(1 - \alpha) q(\theta^*_{t \to t})}{\eta \bar{\gamma}(r) \times (1 + r)} \left( \frac{\rho_{\gamma}}{1 - \Phi_{\nu}} \right) \nu_{i+1} \) and the elasticity of labor market tightness to productivity shocks is

\[
\frac{\partial \hat{\theta}^*_{t \to t}}{\partial \nu_{\gamma}} = \frac{(1 - \alpha) q(\theta^*_{t \to t}) \rho_{\gamma}}{\eta \bar{\gamma}(r) \times (1 + r) - \left[ \eta \bar{\gamma}(r) \times (1 + s) - \alpha f(\theta^*_{t \to t}) \left( \bar{\gamma}(r) + (1 - \eta) \tilde{\kappa} \right) \right] \rho_{\gamma}}
\]

where \( \tilde{\kappa} \equiv K(\phi^*) q(\theta^*_{t \to t})/(1 + r) \).
REFERENCES


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