Labor Market Volatility and Macroeconomic Shocks

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PRELIMINARY AND INCOMPLETE

Abstract

In this paper, the labor market puzzle (Shimer (2005)) has been reinvestigated. We address two issues which have received little attention in this context: i) the contribution of non-technological shocks; ii) the time-varying properties of the macroeconomic relationships and the volatility of the shocks. We conduct the analysis through a time-varying parameter VAR (TVPVAR) with stochastic volatility. We provide a structural interpretation by combining long-run and short run sign restrictions fully derived by a NK DSGE model. As regards job creation, We find that the responses to different shocks and the variance display considerable time variation, with the Great Inflation and the recent decade distinguishing from the rest of the sample. In the short run, the lion share of the variance is explained by cost-push and demand shocks. However, the importance of non-tech shocks for long-term volatility has dramatically dropped from the mid-'80s onwards.

Keywords: Labor market volatility, search and matching, structural Time Varying Parameters VAR, Bayesian estimation, Long-run restrictions, Sign restrictions.

JEL classification: C11, C32, E24, E32.

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1 Motivation

Since Shimer (2005)’s influential work, it has been widely recognized that the high volatility displayed by labor market data cannot be easily replicated by conventional search and matching frameworks. Following his observation, researchers in recent years have provided numerous plausible solutions to explain this empirical finding. However, many of those proposals usually suffer from two major caveats. The first caveat is the widespread tendency in considering technological shocks as the unique driving force of economic fluctuations. Moreover, many works focus on the magnitude of the conditional elasticity while neglecting to verify that the sign is consistent with the empirical observations. The second caveat is that less attention has been paid to the time-varying features of the labor market and the time-varying volatilities of the underlying shocks. In this work we try contributing to the literature by addressing these two caveats. We do that by combining an empirical methodology which allows us to study the time-varying properties of the data and a theoretical setup to guide our identification strategy.

Shimer (2005) observed that the conventional search and matching models cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to a shock of plausible magnitude. He documented that in the United States the volatility of the vacancy-unemployment ratio is almost 20 times as large as the volatility of average labor productivity. The reason is that a shock being able to change the average labor productivity is mainly absorbed by a wage increase, thus dampening firms’ incentives to enhance recruitment. This observation has been validated for other countries such as Japan (Esteban-Pretel et al. (2011)) and for a set of OECD countries (Amaral and Tasci (2013)). The ubiquitous presence of this puzzle has led to a great amount of research on the topic, which is not possible to extensively review here\(^1\).

Numerous proposals have been put forward by researchers. Amongst them, three of are worth mentioning: i) real wage rigidity; ii) the small labor surplus assumption and iii) fixed component in the vacancy costs. Mechanisms which prevents full wage renegotiation greatly improve the performance of the basic model, in that the firms’ surplus is allowed to respond more to productivity movements. Models embedding wage staggering have been proposed by Hall (2005, 2006) and Gertler and Trigari (2009) among others. Another

\(^1\)See Mortensen and Nagypal (2005) for a survey.
concurrent explanation hinges on a different calibration strategy, which assigns a higher value to non-market activities (Hagedorn and Manovskii (2008)). This implies a small size of accounting profits which become more elastic to changes in productivity. Finally, Pissarides (2009) propose to incorporate a fixed component in hiring costs to reduce the proportionality between them and the labor market tightness and foster firm’s response to productivity shocks.

These proposals usually suffer from two major shortcomings. First, most of the above mentioned contributions and many others in the field assume that the main or unique source of fluctuations is represented by technological shocks. Mortensen and Nagypál (2007) and Barnichon (2010, 2012) cast doubts on this approach. They highlight the possibility of non-technological shocks contributing to the observed volatility. Ravn and Simonelli (2007) provide some evidence on the importance of monetary policy shock in explaining the volatility of the labor market in the context of a large, constant SVAR. Focusing only on one shock is also contrary to the common practice in macroeconomics: widely used models such as Christiano et al. (2005) and Smets and Wouters (2007) need to incorporate different driving impulses when they are confronted to the data. Moreover, while the magnitude of the elasticity of labor market variables to different types of shocks has gained lot of attention, Barnichon (2010, 2012) and Balleer (2012) insist on the importance of studying the sign of the elasticity conditional on the type of shock. Importantly, they find that technological shocks generally determine a rise in unemployment, contrary to what implied by standard DMP models.

The second point we want to address in this paper regards the time-varying properties of the labor market and a few structural shocks. It has been well documented that macroeconomic shocks have time-varying volatilities (Primiceri (2005), Justiniano and Primiceri (2008)). However the impact of such time-varying volatilities on the labor market has not been investigated thoroughly. To the best of our knowledge, only Barnichon (2010) and Benati and Lubik (2014) perform a similar exercise. Barnichon (2010) studies the correlation between unemployment and labor productivity and finds a substantial change in the mid-’80s. Benati and Lubik (2014) investigate the time-varying properties of the Beveridge curve. Authors document that evidence point towards similarities and differences between the Great Recession and Volcker disinflation.

We contribute to the ongoing literature by addressing the two caveats. First, we estimate a time-varying parameter VAR (hereafter TVPVAR) in the time series of interest (GDP
growth, inflation, real interest rate, vacancy) and we provide reduced-form evidence on the time pattern of volatility in job creation. Second, we provide a structural interpretation by building a NK DSGE model enriched by a search and matching framework and a large set of shocks. The framework we adopt allows us to overcome some limitations of previous works though maintaining tractability and comparability by abstracting from capital accumulation and labor force participation. Our identification structure is robust to a wide range of parameterizations; moreover, we have also conducted an identification test to make sure that shocks are identified given the choice of our time series. We are thus able to study the impact of numerous shocks in a more realistic environment which features price stickiness and real wage rigidity à la Hall (2005). We then combine long-run restrictions on technological shocks and model-implied short-run sign restrictions to structurally identify the TVPVAR. This allows us to disentangle the contribution of different shocks to the labor market volatility throughout business cycles.

2 Methodology

2.1 A Bayesian-Time Varying Parameter VAR with Stochastic Volatility

Following Benati and Lubik (2012, 2014), we specify a time-varying parameter VAR(p) model. \( Y_t = [\Delta y_t, \pi_t, r_t, v_t] \) is the vector which collects the time series of interest, where \( \Delta y_t \) is the real GDP growth computed as log difference of real GDP, \( \pi_t \) is inflation computed as the log difference of GDP deflator, \( r_t \) is the real interest rate computed as the difference between the 3 months Treasury bill rate and inflation and \( v_t \) is the vacancy rate, that is composite Help-Wanted-Index (HWI) calculated by Barnichon (2010) and normalized by the size of the labor force\(^2\). The TVP-VAR(p) takes the following form:

\( \Delta y_t = \sum_{i=1}^{p} A_i \Delta y_{t-i} + \sum_{i=1}^{p} B_i \pi_{t-i} + \sum_{i=1}^{p} C_i r_{t-i} + \sum_{i=1}^{p} D_i v_{t-i} + \epsilon_t \)

\( \epsilon_t \) is a residual term.

\(^2\)The 3 months Treasury bill rate is preferred to the federal funds rate because it is available for a longer period of time. Moreover, we transform it to quarterly frequency to make it consistent with our inflation measure. We prefer to include the real interest rate rather than the nominal one to ensure stationarity in the VAR.
\[ Y_t = B_{0,t} + B_{1,t}Y_{t-1} + \ldots + B_{p,t}Y_{t-k} + \epsilon_t \equiv X'_t \theta_t + \epsilon_t \quad (1) \]

\[ X'(t) = I_n \otimes [1, Y'_{t-1}, \ldots Y'_{t-k}] \]

where $\otimes$ is the Kronecker product and $I_n$ is the identity matrix of order $n$.

As it is customary in the VARs literature\(^3\), we set the lag order $k = 2$. We then collect the VAR’s time-varying coefficients at time $t$ - that is, the elements of the matrices $B_{0,t}, B_{1,t}, \ldots B_{p,t}$ - in the vector $\theta_t$ and we postulate that they evolve according to:

\[ p(\theta_t | \theta_{t-1}, Q) = I(\theta_t) f(\theta_t | \theta_{t-1}, Q) \quad (2) \]

with $I(\theta_t)$ being an indicator function that rejects the unstable draws, thus enforcing stationarity on the VAR. Following Primiceri (2005), $f(\theta_t | \theta_{t-1}, Q)$ is given by:

\[ \theta_t = \theta_{t-1} + \eta_t \quad (3) \]

with $\eta_t$ following a normal distribution of mean zero and variance-covariance matrix $Q$. The VAR’s reduced-form innovations are assumed to be zero-mean and normally distributed. We factor the time-varying covariance matrix $\Omega_t$ as:

\[ \text{Var}(\epsilon_t) \equiv \Omega_t = A_t^{-1} H_t \left(A_t^{-1}\right)' \quad (4) \]

where the matrices $H_t$ and $A_t$ are defined as following:

\[
H_t = \begin{bmatrix}
h_{1,t} & 0 & 0 & 0 \\
0 & h_{2,t} & 0 & 0 \\
0 & 0 & h_{3,t} & 0 \\
0 & 0 & 0 & h_{4,t}
\end{bmatrix}
\]

\[
A_t = \begin{bmatrix}
1 & 0 & 0 & 0 \\
\alpha_{21,t} & 1 & 0 & 0 \\
\alpha_{31,t} & \alpha_{32,t} & 1 & 0 \\
\alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1
\end{bmatrix}
\]

The $h_{i,t}$ are assumed to evolve as independent geometric random walks:

\(^3\)See Cogley and Sargent (2001, 2005), Primiceri (2005) and Benati and Mumtaz (2007), among others.
\[
\ln (h_{i,t}) = \ln (h_{i,t-1}) + \nu_{i,t} \quad \forall \; i = 1, \ldots, 4
\] (6)

As in Primiceri (2005), we collect the non-zero and non-unity elements of the matrix \(A_t\) in the vector \(\alpha_t = [\alpha_{21,t}, \alpha_{31,t}, \ldots, \alpha_{41,t}]'\) and we assume that it evolves as driftless random walk:

\[
\alpha_t = \alpha_{t-1} + \tau_t
\] (7)

We assume the vector \(\left[u_t', \eta_t', \tau_t', \nu_t'\right]'\) to be distributed as:

\[
\begin{bmatrix}
  u_t \\
  \eta_t \\
  \tau_t \\
  \nu_t
\end{bmatrix}
\sim N(0, V)	ext{, with } V = \begin{bmatrix}
  I_4 & 0 & 0 & 0 \\
  0 & Q & 0 & 0 \\
  0 & 0 & S & 0 \\
  0 & 0 & 0 & Z
\end{bmatrix}\text{ and } Z = \begin{bmatrix}
  \sigma_1^2 & 0 & 0 & 0 \\
  0 & \sigma_2^2 & 0 & 0 \\
  0 & 0 & \sigma_3^2 & 0 \\
  0 & 0 & 0 & \sigma_4^2
\end{bmatrix}
\] (8)

where \(u_t\) is such that \(\epsilon = A_t^{-1}H^{1/2}u_t\). Since model is already heavily parameterized, we impose block-diagonal structure on \(V\) for parsimony. Moreover, allowing for a completely generic correlation structure among different sources of uncertainty would preclude any structural interpretation of the innovations. Finally, as an additional simplifying assumption, a block-diagonal structure for \(S\) is adopted:

\[
S \equiv \text{Var}(\tau_t) = \text{Var}(\tau_t) = \begin{bmatrix}
  S_1 & 0_{1\times 2} & 0_{1\times 3} \\
  0_{2\times 1} & S_2 & 0_{2\times 3} \\
  0_{3\times 1} & 0_{3\times 2} & S_3
\end{bmatrix}
\] (9)

with \(S_1 \equiv \text{Var}(\tau_{21,t})\), \(S_2 \equiv \text{Var}\left([\tau_{31,t}, \tau_{32,t}]'\right)\) and \(S_3 \equiv \text{Var}\left([\tau_{41,t}, \tau_{42,t}, \tau_{43,t}]'\right)\). This implies that the non-zero and non-unity elements of \(A_t\) which belongs to different rows evolve independently. This assumption drastically simplifies inference, since it allows one to perform Gibbs sampling on the non-zero and non-unity elements of \(A_t\) equation by equation. The choice of the priors and the algorithm procedures are relegated to the Appendix.
3 Reduced-form evidence

The estimation of the TVP-VAR in reduced form yields some interesting results, summarized in Figures 1-6. In what follows, blue solid lines represent the median of a given object among the 10000 draws from the posterior distributions. Red lines represent the 16th and the 84th percentiles, respectively. Figure 1 plots the original data together with the time-varying estimates of the states. The estimation tracks quite well the pattern of the data. The fourth panel shows the huge volatility experienced by the labor market throughout the period under consideration. This also determines the high dispersion of the draws of the states.

Figures 2 and 3 provide evidence of time variation in the VAR coefficients as well as in the volatility of the shocks. In Figure 2, the coefficients displaying the largest time variation and sign changes are those referring to the vacancy equation. This can be interpreted as a first mild evidence that the response of the labor market to the macroeconomy has changed over time. Figure 3 plots $\log \det(R_t)$, where $R_t$ is the variance-covariance matrix of the reduced-form VAR residuals. Following Cogley and Sargent (2005) we interpret this as the total amount of uncertainty hitting the economy at each point in time. Our findings are remarkably similar to figure 4 in Cogley and Sargent (2005) and Figure 2 in Benati and Mumtaz (2007). The variance features a substantial increase from 1965 to 1981; then, it decreases during the Great Moderation period and exhibits two small peaks around 2001 and the recent crisis.

Figures 4-6 disentangle the variance-covariance matrix of the reduced-form residuals into its different components. Figures 4 and 5 show the volatilities of the VAR innovations in absolute terms and relative to GDP, respectively. Innovations to GDP, inflation and real interest rate appear to have been more volatile in the first part of the sample, until 1981. As regards the volatility of innovations to the vacancy rate, however, we can notice two things. First, its magnitude is much larger than the volatility of GDP innovations (up to 9 times). Second, the time pattern is quite different from the other shocks: it spikes in the mid-'70s and at the beginning of the '80s but they are remarkable volatile also during the last decade. Moreover, in the last panel of Figure 5 we can detect a positive trend in the volatility of the residuals in the vacancies equation relative to GDP innovations starting from the '70s.

Finally, Figure 6 plots the correlations between the VAR innovations. The picture shows
substantial time-variation and changes of sign for some of them. The large negative correlation between inflation and real interest rate innovations is consistent with the evidence in Cogley and Sargent (2005). The figure suggests that, given the information set of this econometric model, monetary policy is a reaction to the economic conditions rather than a shock which is able to affect the economy contemporaneously. This seems particularly true during the Great Inflation period: this graph is in line with the conventional wisdom that policy rates tried to catch up with inflation rather than keeping it under control.

The correlation between innovations to vacancies and to output growth are positive, as expected from the theory. Vacancies and inflation innovations are positively correlated for the majority of the sample. The exception are made by period of exceptionally high inflationary pressures. Less clearcut appears the correlation between innovations to vacancies and to the real interest rate: the sign has switched from negative at the beginning of the sample to predominantly positive until 2001 and then negative again in the recent years.

4 Model

The benchmark model combines features of the standard RBC setting and search frictions à la Mortensen-Pissarides in the labor market. Time is discrete. The economy is populated by households, firms and policy authorities. Households consume, invest in the bond market and supply labor. We distinguish between wholesale firms and retailers. Wholesale firms employ labor to produce a homogeneous good sold to retailers in a perfect competitive market. Workers are recruited on a frictional labor market. We consider both a flexible and a sticky wage setting mechanism. In the first case, wages are the outcome of a Nash-bargaining process. In the second case we introduce real wage stickiness à la Hall (2005). Retailers owe a technology which allows them to differentiate the good without any other input. The differentiated good is then sold to households under monopolistic competition.

As for policy, the monetary authority is in charge of setting the nominal interest rate following a standard Taylor rule, while the central government collect lump-sum taxes to finance public expenditure and unemployment benefits. The model is non-stationary for the presence of a unit root in the technological process. In addition, the model is enriched with a large set of transitory shocks. The following sub-sections detail the economic environment and the agent’s decision problem.
4.1 Households

The economy is populated by a continuum of identical households of mass 1. They consume a composite good $C_t$ which incorporates all the varieties produced by the retailers, hold bonds and supply labor. Since in any period workers are either employed or unemployed (i.e. matched or unmatched), a distributional problem may arise. As in Merz (1995), we assume that households pool consumption (they behave like a big family which fully insures each member against unemployment).

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \exp(\varepsilon_{t}^\beta) \beta^t \left( \ln C_t - \psi \exp(\varepsilon_{t}^\psi) \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right)$$

subject to $C_t + \frac{B_t}{R_t P_t} = \frac{B_{t-1}}{P_t} + \frac{w_t}{P_t} N_t + \frac{b_t}{P_t} U_t + \Pi_t - T_t$

where $\sigma_n$ is the inverse of Frisch elasticity$^4$. $\varepsilon_{t}^\beta$ is a shock to the discount rate which we interpret as a demand-non-policy shock. $\varepsilon_{t}^\psi$ accounts for a potential shift in preferences. Households can allocate their income between consumption and nominal bonds, which pay the nominal (gross) interest rate $R_t$. In addition, households supply labor: the labor income is represented by the real wage paid to the household’s members who are employed during the period ($N_t$). Unemployed workers receive benefits $b_t$ from the government. Public expenditure and unemployment benefits are financed by lump-sum taxes $T_t$. Finally, households own firms, whose profits are denoted as $\Pi_t$. $C_t$ is the Dixit-Stiglitz aggregator

$$C_t = \left( \int_0^1 C_{it}^{-\delta} \, di \right)^{-\frac{\delta}{1-\delta}}$$

where $\delta$ is the demand elasticity.

The first order conditions are

$$\frac{1}{C_t} = \lambda_t$$

$^4$Notice that the log specification makes these preferences consistent with balanced growth.
\[ E_t \left[ Q_{t,t+1} \frac{P_t}{P_{t+1}} R_t \right] = 1 \]  

(11)

where \( Q_{t,t+k} = \beta^k \frac{\exp(\varepsilon_{t+k}) \lambda_{t+k}}{\exp(\varepsilon_t) \lambda_t} \) is the stochastic discount factor and \( \lambda_t \) is the marginal value of wealth. Moreover, the demand for variety \( i \) is

\[ C_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} C_t \]

where \( P_t = \left( \int_0^1 P_{it}^{\epsilon-1} di \right)^{\frac{1}{1-\epsilon}} \) is the aggregate retail price index.

Labor supply decisions must take into account the frictions characterizing the labor market, which are relegated in the subsequent sections. Notice that, with perfectly competitive labor markets, the following condition would hold:

\[ \frac{w_t}{P_t} = \psi \exp(\varepsilon_t^\psi) C_t N_t^{\sigma N} = MRS_t \]  

(12)

The above equation states that absent any friction, the wage adjustment mechanism guarantees that there is no voluntary unemployment and households supply labor by equating the wage to the intratemporal marginal rate of substitution.

### 4.2 Labor Market

Labor market clearing is prevented by search and matching frictions à la Mortensen-Pissarides (Mortensen and Pissarides (1994)). Demand and supply conditions (number of vacancies posted and job-seekers, respectively) and labor market characteristics (matching efficiency) jointly determine the employment level.

In order to hire workers, firms must post vacancies on the labor market, incurring the real cost \( k^f_t \). The realized number of matches is the outcome of a Cobb-Douglas technology, which depends on the number of vacancies \( V_t \) and searchers \( U_t^0 \):

\[ M_t \left( V_t, U_t^0 \right) = \]

---

\(^5\)Because of the unit root in technology the vacancy cost is assumed to grow at the same rate of output.
exp(ε^f_t)ξV^0_t (U^0_t)^{1-η}, where ε^f_t is a shock to the efficiency of the matching function. The probability that a firm matches with a worker is \( p^f_t = \frac{M_t(V_t, U^0_t)}{V_t} \). The probability of being hired is then given by \( q^w_t = \frac{M_t(V_t, U^0_t)}{U^0_t q^w} \). The labor market tightness is defined as \( \theta_t = \frac{V_t}{U^0_t} \).

It is easy to show that \( p^f_t \) is a decreasing function of \( \theta_t \) while \( q^w_t \) is an increasing function of it \( (q^w_t = \theta_t p^f_t(\theta_t)) \).

In each period, timing is the following: i) a fraction of productive matches from the previous period exogenously severe and separated workers enter the unemployment pool; ii) unemployed workers and firms search on the labor market and matches are formed; iii) shocks realize; iv) production occurs. Employment dynamic is thus given by:

\[
N_t = (1 - s \exp(ε^s_t)) N_{t-1} + M_t
\]

where \( s \) is the exogenous separation rate which is rendered time-varying by the shock \( ε^s_t \).

The first term in the right hand side of the above equation represents workers matched in the previous period who do not separate (surviving matches); the second term represents new matches realized at the beginning of the period before production occurs. The number of searchers is

\[
U^0_t = \frac{U_t}{1 - q^w_t}
\]

where \( U_t = 1 - N_t \) is current unemployment, which is defined after the matching process has been taking place. Under this timing assumption, the matches become immediately productive.

### 4.3 Wholesale Firms

Wholesale firms employ labor to produce an homogenous good to be sold to retailers at price \( P^w \). Because of the frictions in the labor market, these decisions potentially differ among firms, which we index by \( j \).

In order to hire workers, firms must pay post vacancies on the labor market, by paying the fixed real cost \( k^f_j \). The value of a vacancy for firm \( j \) is
\[ J^V_t (j) = k^f_t + p^f_t J^F_t (j) + \left(1 - p^f_t\right) \mathbb{E}_t \left(Q_{t,t+1} J^V_{t+1} (j)\right) \]

The above equation states that with probability \( p^f_t \) the firm fills the vacancy and gets the value of the match \( J^F_t \) and with a complementary probability the vacancy remains unfilled.

Free entry implies:

\[ J^F_t (j) = \frac{k^f_t}{p^f_t} \forall j \quad (15) \]

The value of a productive match is represented by the following equation:

\[ \frac{k^f_t}{p^f_t} = \frac{1}{\mathcal{M}_t^w} MPN_t - \frac{w_t(j)}{P_t} + \mathbb{E}_t \left(Q_{t,t+1} (1 - s \exp(\varepsilon_{t+1}^s)) \frac{k^f_{t+1}}{p^f_{t+1}}\right) \quad (16) \]

where \( \frac{1}{\mathcal{M}_t^w} = \frac{P^w_t}{P_t} \) is the relative price of the wholesale good in terms of the final good and \( MPN_t = (1 - \alpha) A_t N_t^{-\alpha} \) is the marginal productivity of labor.

The above equation states that firms keep posting vacancies until the real cost they bear (which depends on the fixed cost and the search spell) equates the current productivity gains and the savings on future vacancy costs. Search frictions distort firms’ optimization condition on hiring. For future reference we can define the net hiring costs:

\[ D_t = \frac{k^f_t}{P^f_t} - \mathbb{E}_t \left[Q_{t,t+1} (1 - s \exp(\varepsilon_{t+1}^s)) \frac{k^f_{t+1}}{p^f_{t+1}}\right] \]

As for technology, we assume \( A_t = A^T_t A^P_t \), where \( A^T_t \) denotes the transitory component and \( A^P_t \) is the permanent component.

\[ \ln A^T_t = \rho_\alpha \ln A^T_{t-1} + \sigma_a \varepsilon^a_t \]
\[ \frac{A^P_t}{A^P_{t-1}} = \gamma^a_t \]
\[ \ln \gamma^a_t = \ln \tilde{\gamma}^a + \sigma_a \varepsilon^a_t \]

where \( \varepsilon^a_t \) and \( \varepsilon^a_t \) are standard normals.
4.4 Workers

Workers can be either employed or unemployed. We now characterize their value function in both cases. The value function of worker employed at firm $j$ is:

$$J^w_t(j) = \frac{w_t(j)}{P_t} - MRt + \mathbb{E}_t \left\{ Q_{t,t+1} \left[ (1 - s \exp(\varepsilon_{t+1}^s)) J^w_{t+1}(j) + s \exp(\varepsilon_{t+1}^s) (q_{t+1}^w J^w_{t+1} + (1 - q_{t+1}^w) J^u_{t+1}) \right] \right\}$$

where the second term is the marginal rate of intratemporal substitution, which expresses labor disutility in terms of consumption goods. The terms in brackets is the continuation of the match, which continues with a probability of $1 - s \exp(\varepsilon_{t+1}^s)$. $J^u_t$ is the value of being unemployed, which is given by the following:

$$J^u_t = b_t + \mathbb{E}_t \left\{ Q_{t,t+1} \left[ q_{t+1}^w J^w_{t+1} + (1 - q_{t+1}^w) J^u_{t+1} \right] \right\}$$

where $J^u_t = \int_0^1 \frac{M_t(j)}{M_t} J^w_t(j) \, dj$.

Remember that unemployment is defined after the matches of the current period have taken place. Unemployed agents can find a job in the following period with probability $q_{t+1}^w$ or stay unemployed.

The surplus which accrues to a worker employed at firm $j$ is thus given by:

$$S^w_t(j) = J^w_t(j) - J^u_t = \frac{w_t(j)}{P_t} - (MR_t + b_t) + \mathbb{E}_t \left\{ Q_{t,t+1} (1 - s \exp(\varepsilon_{t+1}^s)) (S^w_{t+1}(j) - q_{t+1}^w S^w_{t+1}) \right\}$$

where $S^w_t = \int_0^1 \frac{M_t(j)}{M_t} S^w_t(j) \, dj$ is the average surplus.
4.5 Nash Bargaining

4.5.1 Flexible Wages

When wages can adjust in every period, they are established through Nash bargaining, thus implying the following relationship:

\[ S_{t}^{w}(j) = \frac{\gamma}{1 - \gamma} S_{t}^{F}(j) \]

where \( S_{t}^{w}(j) \) is defined in above equation, \( \gamma \) is the worker’s bargaining power and \( S_{t}^{F}(j) = J_{t}^{F}(j) \) is the firm \( j \)'s surplus. Eq. (15) implies that in equillibrium all firms offer the same wage. The index \( j \) will be thus omitted in what follows. The following real wage expression can be obtained after some mathematical manipulations:

\[ \omega_{t}^{N} = MRS_{t} + b_{t} + \frac{\gamma}{1 - \gamma} \left[ \frac{k^{f}}{p^{f}} - \mathbb{E}_{t} \left( Q_{t,t+1} \left( 1 - s \exp(\varepsilon_{t+1}) \right) \left( 1 - q_{t+1}^{w} \right) \frac{k^{f}}{p_{t+1}} \right) \right] \]

Where \( \omega_{t}^{N} \) is the Nash-bargained real wage. The equation above shows that workers must be compensated for the disutility of working and for the foregone benefit (as in the competitive framework) but, as long as they have a positive bargaining power, they can also extract part of the firm’s surplus (the term inside the brackets). \(^6\)

4.5.2 Sticky Wages

Following Blanchard and Gali (2010), we introduce wage stickiness by imposing that the real wage prevailing in the economy is a geometric average of the Nash-bargained wage and the wage prevailing in normal times (\( \bar{w} \)):
\[ \omega_t = \bar{\omega}^{\theta_w} \left( \omega_t^N \right)^{1-\theta_w} \]

In what follows we assume that \( \bar{\omega} \) is the Nash bargained wage in steady state. This can be interpreted as a wage norm, in the sense of Hall (2005).

### 4.6 Retailers

The homogeneous wholesale good is sold to retail firms, which differentiate it at no cost and sell it to households. We introduce price stickiness in the form of Calvo prices. Let \( \theta_p \) be the probability of not reoptimizing in a given period. Retailers maximize their profits subject to the demand schedule for each individual good \( i \):

\[
\max_E E_t \sum_{k=0}^{\infty} \theta_p^k Q_{t,t+k} \left( \frac{P_{it}^* - P_{t+k}^w}{P_{t+k}^w} \right) Y_{i,t+k|t} \]

subject to \( Y_{i,t+k|t} = \left( \frac{P_{it}^*}{P_{t+k}^w} \right)^{-\epsilon} Y_{t+k} \)

The price schedule turns out to be

\[
E_t \sum_{k=0}^{\infty} \theta_p^k Q_{t,t+k} \left( \frac{P_{it}^*}{P_{t+k}^w} \right)^{-\epsilon} Y_{t+k} \left[ P_{it}^* \frac{P_{t+k}^w}{P_{t+k}^w} - \frac{\epsilon_t}{\epsilon_t - 1} P_{t-k}^w \right] = 0
\]

where we have omitted the index \( i \) because in equilibrium all firms charge the same price. \( \mathcal{M}_t^p = \frac{\epsilon_t}{\epsilon_t - 1} \) is the mark-up charged by retailers on the marginal cost when prices are perfectly flexible. The desired mark-up changes over time following an AR(1) process, leading to the presence of a cost-push shock in the linearized Philipps curve.

The aggregate price index follows the dynamic given by

\[
P_t = \left[ \theta_p P + (1 - \theta_p)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{17}
\]
4.7 Monetary Authority

The monetary authority sets the nominal interest rate following a standard Taylor rule:

\[ R_t = R_{t-1}^\rho \left( \frac{1}{\beta} \left( \frac{Y_t}{\bar{Y}} \right) \delta_y \left( \frac{P_t}{P_{t-1}} \right) \delta_x \right)^{1-\rho_r} \exp(\varepsilon_t^r) \]  

(18)

where \( \frac{1}{\beta} \) is the steady state value of the interest rate, \( \rho_r \) is the degree of the monetary policy inertia, \( \delta_y \) and \( \delta_x \) express the monetary policy reactions to output gap and inflation, respectively. \( \varepsilon_t^r \) is a contractionary monetary policy shock.

4.8 Fiscal Authority

The government raises lump-sum taxes \( T_t \) to finance public expenditure \( G_t \) and unemployment benefits.

\[ G_t + b_t U_t = T_t \]

Both public expenditure and the unemployment benefits follow a random process:

\[ \ln \tilde{G}_t = (1 - \rho_g) \ln \tilde{G} + \rho_g \ln \tilde{G}_{t-1} + \sigma_g \varepsilon_t^g \]

\[ \ln \tilde{b}_t = (1 - \rho_b) \ln \tilde{b} + \rho_b \ln \tilde{b}_{t-1} + \sigma_b \varepsilon_t^b \]

where \( \varepsilon_t^g \) and \( \varepsilon_t^b \) are standard normals. The tilde denotes stationarized variables (i.e. the original variable divided by the permanent component of technology).

4.9 Closing the model

The resource constraint implies

\[ Y_t = C_t + G_t + k_t^f V_t \]  

(19)
To summarize, the model is driven by one permanent shock to technology and other eight transitory shocks which all follow an AR(1) with own persistences and volatilities. The transitory shocks can be classified as follows: temporary supply, demand-non-policy (shock to the households’ discount factor), monetary policy, cost-push (shock to the elasticity of demand), public expenditure, unemployment benefits, matching efficiency and separation rate.

Because of the presence of unit root in technology, we detrend the non-stationary variables and we then linearized the model around the balanced growth path. The log-linearized model is reported in Appendix A.

4.10 Calibration

We calibrate the model on US quarterly data and we mainly rely on estimates taken from the literature. $\beta$ is 0.99, so that the annual steady state interest rate is around 4 %. a utility function log-linear in leisure (this implying $\sigma_n = 0.5$). We assume a steady state unemployment of 5 %, which corresponds to the average unemployment rate in our sample. We set both the job filling rate ($p^f$) and the job finding rate to 0.7. This implies and exogenous separation rate of 12 %. We impose that the Hosios efficiency condition holds: the elasticity of the matching function ($\eta$) equals the firms’ bargaining power ($1 - \gamma$) at the value of 0.5. The total vacancy expenditure on GDP ($M_p k (1 - \gamma)$) is 0.2 %, which implies that the unit hiring cost is almost 2 % of the nominal wage in steady state. We calibrate $\alpha$ in order to obtain a labor share of 2/3. The price mark up is calibrated at 1.2. The baseline values for price and wage stickiness are both sets at 0.75, so that resets occur once a year on average. We adopt a standard specification of the monetary policy rule, with quite high inertia ($\rho_r = 0.8$), and monetary policy reactions which respect the Taylor principle ($\delta_y = 0.5$ and $\delta_\pi = 1.5$). The variance of the monetary policy shock is calibrated at 0.0025, so that 1 SD contractionary monetary policy shock raises th nominal interest rate by 25 bp. Since we are not interested in the quantitative performance of the model and evidence is scarce, we do not try to find a proper calibration for each shock process. We set all the persistences to 0.9 and standard deviations to 0.01 instead.
5 Structural evidence

5.1 Identification

[TO BE COMPLETED]

We identify four structural shocks. As in Gali and Gambetti (2009), permanent technological shocks are identified by imposing that they are the only ones which affect the level of output in the long run. This is consistent with the model, that has been modified accordingly to take into account the presence of non-stationarity in technology. A time-varying VAR can be written in the following form

\[ Y_t = \mu_{0,t} + C_{t,\infty} u_t \]

where \( C_{t,\infty} = C_0 \tilde{A}_{0,t} \) is the matrix of the cumulative IRFs and takes the following form:

\[
C_{\infty,t} = \begin{bmatrix}
C_{11} & 0 & 0 & 0 \\
. & . & . & . \\
C_{41} & \ldots & C_{44} \\
\end{bmatrix}
\]

\( C_{11} \) is the impact of the first shock on the first variable, in our case the technological shock on GDP growth. We impose zeros on all the elements of the first row except the first one, to say that the technological shock is the only one which potentially have a permanent effect on output. No other long-run restrictions are imposed on the other variables.

The other three structural shocks are identified by sign-restrictions on the impact matrix. These are fully derived from the model described above. For example, following an unexpected monetary policy shock, inflation decreases. Therefore in the impact matrix, the reaction of inflation to an unexpected monetary policy shock is set negative.

However since the structural identification is based on signs obtained from a model, it is necessary to make sure that our choice of time series is enough to identify the underlying

---

\footnote{The MA representation permits us to write a VAR in the following form \( Y_t = \mu_{0,t} + \sum \Psi_k \epsilon_{t-k} \), by using \( \epsilon_t = A_{0,t} u_t \) the VAR can then be written in terms of structural shocks.}
process (i.e. persistence and volatility) of those shocks. For example, suppose that the
time series we include in the TVP-VAR fail to identify cost-push shocks. Then, the signs
derived from the response of the model to the cost-push shock are flawed, since the shock
itself is not identified in the first place. To make sure that shocks are identified using
the time series of our interest we follow Iskrev (2010) and Komunjer and Ng (2011). The
method of Iskrev (2010) is based on calculating the unconditional moments of model and
the method of Komunjer and Ng (2011) is based on the concept of minimality of systems
borrowed from adaptive control theory. Details of the procedure are contained in Appendix
C.

The sign restrictions we impose on the impact matrix are reported in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Monetary Policy</td>
</tr>
<tr>
<td>Inflation</td>
<td>-</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>+</td>
</tr>
<tr>
<td>Vacancies/Labor force</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Short-run sign restrictions on the impact matrix

5.2 Results

Our setup allows us to compute the responses of each variable to each structural shock for
every forecast horizon and every time period. The results for vacancies and for selected
forecast horizons are reported in Figure 12, where the responses have been standardized
for comparability. The upper left panel shows no clear pattern: the effects of permanent
technological shocks are sometimes positive and sometimes negative, with huge variation
during the ’70s. The other panels are more interesting. The Great Inflation period is the
most distinguishable: demand and cost-push shocks have a lower than average impact,
meaning that demand shocks are less expansionary and cost-push shocks are associated to
more severe hiring cuts. Monetary policy, instead, determines a higher than average re-
sponse on impact but more contractionary long-term consequences. Demand and monetary
policy shocks appear to have a larger impact from year 2000 onwards.

Figure 13 plots the impulse responses of vacancies to the different shocks for given quar-
ters. From Figure 12 we know that the responses to non-technological shocks evolve quite smoothly; this is not the case for the responses to permanent technological shocks, which exhibit jumps from one year to the other. In terms of Figure 13, this means that only the IRFs to monetary policy, demand and cost-push shocks can be understood as representative of their respective decade. As already highlighted in the above discussion, during the ’70s cost-push shocks are more recessionary and demand shocks less expansionary. Monetary policy shocks appear to have lower negative impact, but we know that this is only a short-term feature. The reverse is true for 2005: demand and cost-push and monetary policy shocks affect vacancies more severely while cost-push shocks have a smaller impact.

From the IRFs, we can compute the unconditional and conditional correlations, which are plotted in Figure 14. They display the expected sign: vacancies are positively correlated to output and negatively correlated to the nominal interest rate, no matter the shock. The correlation between vacancies and inflation is more diversified. Conditional on cost-push shocks it is negative and fairly constant throughout the sample period. The other conditional and unconditional correlation displays much higher variation over time, instead. They start mildly positive, then touch negative values during the Great Inflation and then start and upward trend which lead them to values above 0.5 in the latest years.

To compute the "median" FEVD, in each period we apply twice the median target (MT) criterion introduced by Fry and Pagan (2005). We make here a methodological point, which we think is sometimes overlooked in practice. As stressed by Fry and Pagan (2011), one should be careful in selecting median estimates for different object of interest which may come from different models. We first consider each shock separately and find a measure of distance of each parameterization (i.e. draw of states from the posterior distribution and impact matrix rotation) to the one that would produce the median fraction of the variance explained by the shock. This is done by considering all forecast horizons jointly. Secondly, we apply again the MT criterion on the previous measures to take into account all shocks at the same time. The output of the this procedure is a "median" set of VAR parameters which corresponds to a unique draw from the posterior distribution of the states and to a unique impact matrix rotation. This is a sufficient condition to ensure orthogonality of the shocks. As a consequence, the variance of the data is fully accounted for and we can safely perform the decomposition.

Figure 15 plots the unconditional variance of the variables included in the our model, as well as the contribution of permanent technological shocks and non-technological shocks
(i.e. the sum of monetary policy, demand and cost-push shocks), respectively. In line with the evidence and the narrative on the Great Moderation, the unconditional variance spikes between the mid '70s and the beginnings of the '80s to remain to much lower levels thereafter. In the case of output and vacancies growth much of the change can be attributed to the lower contribution of non-technological shocks. For inflation and the federal funds rate the two components have roughly followed the same pattern, instead.

The 'median’ FEVD for vacancies is reported in Figure 16. A first look suggests three initial observations: i) the contributions of each shock to the variance of vacancies display considerable time variation; ii) the time patterns are different according to the forecast horizon; iii) technological shocks do not always represent the lion share of the variance of vacancies (at least not for all periods and forecast horizons). At the beginning of our sample period, short term volatility (the upper left panel of Figure 16) is dominated by technological shocks, with a peak of 60% in 1976. However, starting from the beginning of the '80s this contribution has decreased to less than 15% on average. During this latest period 1 third of the variance is explained by demand shocks. A fairly constant fraction of 30% is accounted for by cost-push shocks. Monetary policy shocks have the smallest impact. A different picture emerges at longer horizons. Looking at the bottom right panel of Figure 16, the importance of technological shocks has increased dramatically from the '90s, up to 80% in the middle of the recent financial crisis. This increase has occurred at the expenses of monetary policy and cost-push shocks.

Overall, this evidence draws a more complex picture than the current discussion on the amplification of technological shocks. Especially in recent years short term movements in vacancies are mainly driven by demand and cost-push shocks. At longer horizons, nowadays the unconditional variance in much lower than during the Great Inflation period. This is due to the declining contribution of non-technological shocks (especially monetary policy and cost-push shocks). This implies that more than half of the long-run variance of vacancies is actually explained by permanent supply shocks.

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8 We plot here the 10 years ahead variance of the forecast error.
9 To save space, we report only the results relative to vacancies. The others are available upon request.
6 Conclusions

In this paper we have reinvestigated labor market volatility. We have addressed two issues not generally covered by the existing literature: the contribution of non-technological shocks to the high observed volatility of job creation and the time-varying properties of the labor market in connection to the macroeconomy. We have performed the analysis through a TVP-VAR identified by long-run and sign restrictions derived from a NK-DSGE model with search frictions in the labor market.

We can summarize the main findings regarding job creation as follows: i) the responses to different shocks and the variance display considerable time variation, with the Great Inflation and, to a lesser extent, the recent decade distinguishing from the rest of the sample; ii) the Great Inflation period is characterized by stronger reaction to cost-push and monetary policy shocks (in the long-run) and less expansionary demand shock; the opposite is true in the 2000s; iii) non-technological shocks represented the lion share of the variance until the mid-'90s; since then their contribution to the long-term variance has declined; iv) however, in the short term the contribution of non-technological shocks is still the dominant one (and even more so in the second part of the sample), with cost-push and demand shocks accounting for more than 60% of the variance. No sharp conclusions can be drawn with respect to the response of vacancies to technological shocks identified with long-run restrictions. In line with the mixed results provided by the literature, the response is sometimes positive and sometimes negative, with no clear time pattern.

References


A Log-linearized model

In this appendix, we report the entire log linearized model. The notation is the following:

- $X_t$: original variable as in the main text, potentially exhibiting unit root
- $\tilde{X}_t = \frac{X_t}{A_t^P}$: detrended variable
- $\tilde{X}$: steady state value of the stationary variable $\tilde{X}$
- $\hat{x}_t = \ln \tilde{X}_t - \ln \bar{\tilde{X}}$: deviation of variable $X_t$ from the balanced growth path

This is the log-linearized model:

- Stochastic discount factor: $\mathbb{E}_t(\hat{q}_{t+1}) = \hat{c}_t - \mathbb{E}_t\hat{c}_{t+1} - \epsilon_t^\beta$
- Dynamic IS:
  \[ \hat{c}_t = \mathbb{E}_t(\hat{c}_{t+1}) - [\ln \beta + r_t - \mathbb{E}_t(\pi_{t+1}^p)] + \epsilon_t^\beta \]
- Marginal rate of substitution (eq. 12): $m^s = \hat{c}_t + \sigma_N \hat{n}_t + \epsilon_t^{\psi}$.
- Employment (eq. 13): $\hat{n}_t = (1 - s)\hat{n}_{t-1} + s(\hat{m}_t - \epsilon_t^s)$.
- Searchers (eq. 14): $\hat{u}_0^w = \hat{u}_t + \frac{\tilde{q}_w}{1-q_w} \hat{q}_w^w$.
- Hirings: $\hat{m}_t = \epsilon_t^{\xi} + \eta \hat{n}_t + (1 - \eta)\hat{u}_t^0$.
- Job finding rate: $\hat{q}_w = \hat{m}_t - \hat{u}_t^0$.
- Job filling rate: $\hat{p}_f = \hat{m}_t - \hat{v}_t$.
- Net hiring costs:
  \[ \hat{d}_t = \frac{1}{1 - Q(1 - s)\gamma^A} (-\hat{p}_f^f) + \frac{\tilde{Q}(1 - s)\gamma^A}{1 - Q(1 - s)\gamma^A} \mathbb{E}_t [\hat{p}_{t+1} + \hat{r}_t - \pi_{t+1}^p] \]
- Production function: $\hat{y}_t = a^T_t + (1 - \alpha)\hat{n}_t$.
- Marginal productivity of labor: $m^p = a^T_t - \alpha \hat{n}_t$. 

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• Wages:

\[
\hat{\omega}_t^N = \frac{\tilde{M}RS}{\tilde{\varphi}_N} - \hat{m}_t r_s + \hat{\beta}_t + \frac{\gamma k_f}{\tilde{\varphi}_N} \left[ \left( 1 - \tilde{Q} (1 - s) \tilde{\gamma}^A \right) \tilde{d}_t - q^w \tilde{Q} (1 - s) \tilde{\gamma}^A E_t \left( \hat{p}_{t+1}^f + \hat{\pi}_t - \pi_{t+1}^p - q^w_{t+1} \right) \right]
\]

\[
\hat{\omega}_t = (1 - \theta_w) \hat{\omega}_t^N
\]

\[
\hat{\omega}_t = \hat{\omega}_{t-1} + \pi_t^w - \pi_t^p
\]

• Taylor rule (eq. 18): \( r_t = \rho_r r_{t-1} + (1 - \rho_r) \left[ -\ln \beta + \delta_y \hat{y}_t + \delta_x \pi_t \right] + \epsilon_t^r. \)

• Aggregate resource constraint (eq. 19): \( \hat{y}_t = \hat{C}_t + \hat{\kappa}_f \hat{V}_t + \hat{\kappa} \hat{g}_t. \)

• Job creating condition: \( \hat{\mu}_t^p = m \hat{m}_t - \left[ (1 - \Phi) \hat{\omega}_t + \Phi \hat{d}_t \right], \) where \( \Phi = \frac{\tilde{D}}{\tilde{\varphi} + \tilde{D}}. \)

• Price inflation:

\[
\pi_t^p = \beta E_t \pi_{t+1}^p - \lambda_p \hat{\mu}_t^p + \epsilon_t^u
\]

where \( \lambda_p = \frac{(1 - \theta_p)(1 - \theta_p \beta)}{\theta_p} \) and \( \epsilon_t^u \) is a cost-push shock due to changes in the desired mark-up.

### B Details of Markov-Chain Monte Carlo (MCMC) Procedure

This section describes our choices for the priors and the MCMC algorithm we use to simulate the posterior distribution of the hyperparameters and the states conditional on the data. The choice of the priors largely builds on Benati and Lubik (2012).

#### B.1 Priors

For the sake of simplicity, the prior distribution for the initial values of the states - \( \theta_0 \) and \( h_0 \) - which we postulate all to be normal, are assumed to be independent both from each other and from the distribution of the hyperparameters. In order to calibrate the prior
distributions for $\theta_0$ and $h_0$ we estimate a time-invariant version of (1) based on the first 10 years of data:

$$\theta_0 \sim N \left( \hat{\theta}_{OLS}, 4\hat{V} \left( \hat{\theta}_{OLS} \right) \right)$$

where $\hat{V} \left( \hat{\theta}_{OLS} \right)$ is the estimated asymptotic variance of $\hat{\theta}_{OLS}$. As for $h_0$ we proceed as follows. Let $\Sigma_{OLS}$ be the estimated covariance matrix of $\epsilon_t$ from the time-invariant VAR and let $C$ be its lower-triangular Cholesky factor, i.e. $CC' = \hat{\Sigma}_{OLS}$. We set

$$\ln h_0 \sim N \left( \ln \mu_0, 10 \times I_N \right)$$

where $\mu_0$ is a vector collecting the logarithms of the squared elements on the diagonal of $C$. As stressed by Cogley and Sargent (2005), "a variance of 10 is huge on a natural log scale, making this weakly informative" for $h_0$. Turning to the hyperparameters, we make the following, standard assumptions. The matrix $Q$ is postulated to follow an inverted Wishart distribution

$$Q \sim IW \left( Q^{-1}, T_0 \right)$$

with prior degrees of freedom $T_0$ and scale matrix $T_0Q$. In order to minimize the impact of the prior, thus maximizing the influence of sample information, we set $T_0$ equal to the minimum value allowed, the length of $\theta_t$ plus one. As for $Q$ we calibrate it as $Q = \gamma \Sigma_{OLS}$ setting $\gamma = 1 \times 10^{-4}$ which is slightly more “conservative” (in the sense of allowing for less random-walk drift) than the value of $3.5 \times 10^{-4}$ used by Cogley and Sargent (2005). We assume independent inverse-Wishart distributions also for the blocks of $S$:

$$S_1 \sim IW \left( \tilde{S}_1^{-1}, 2 \right), \quad \tilde{S}_1 = 0.001 \times |\hat{\alpha}_{2,1}|$$
$$S_2 \sim IW \left( \tilde{S}_2^{-1}, 3 \right), \quad \tilde{S}_2 = 0.001 \times \text{diag}[|\hat{\alpha}_{3,1}|, |\hat{\alpha}_{3,2}|]$$
$$S_3 \sim IW \left( \tilde{S}_3^{-1}, 4 \right), \quad \tilde{S}_3 = 0.001 \times \text{diag}[|\hat{\alpha}_{4,1}|, |\hat{\alpha}_{4,2}|, |\hat{\alpha}_{4,3}|]$$

where $\text{diag}(x_1, ..., x_n)$ is a diagonal matrix of order $n$ with elements $x_i$’s on the main diagonal and $|\hat{\alpha}_{i,i}|$ is the $i, i$ element of the correlation matrix of the VAR shocks derived from $\hat{\Sigma}_{OLS}$. As for $\alpha$ we assume: $f(\alpha) = N(\hat{\alpha}, 10 \times |\hat{\alpha}|)$. 

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Finally as for the variance of the stochastic volatility innovations, we follow Cogley and Sargent (2001, 2005) and we postulate an inverse-Gamma distributions for \( \sigma_i^2 \equiv \text{Var}(\nu_{i,t}) \):

\[
\sigma_i^2 \sim IG \left( \frac{10^{-4}}{2}, \frac{1}{2} \right)
\]

### B.2 Simulating the posterior distribution

We simulate the posterior distribution of the hyperparameters and the states conditional on the data via the following MCMC algorithm, which combines procedures found in Cogley and Sargent (2005) and Primiceri 2005. In what follows \( x^t \) denotes the entire history of the vector \( x \) up to time \( t \) i.e. \( x^t = [x'_1, x'_2, ..., x'_t]' \), while \( T \) is the sample length.

a) **Drawing the elements of \( \theta_t \)**

Conditional on \( Y^T, \alpha \) and \( H^T \) the observation equation (1) is linear, with Gaussian innovations and a known covariance matrix. Following Carter and Kohn (2004) the density \( p\left( \theta^T | Y^T, \alpha, H^T \right) \) can be factored as

\[
p\left( \theta^T | Y^T, \alpha, H^T \right) = p\left( \theta_T | Y^T, \alpha, H^T \right) \prod_{t=1}^{T-1} p\left( \theta_t | \theta_{t+1}, Y^T, \alpha, H^T \right)
\]

Conditional on \( \alpha \) and \( H^T \), the standard Kalman filter recursions nail down the first element on the right hand side: \( p\left( \theta_T | Y^T, \alpha, H^T \right) = N (\theta_T, P_T) \), with \( P_T \) being the precision matrix of \( \theta_T \) produced by the Kalman filter. The remaining elements in the factorization can then be computed via the backward recursion algorithm found, e.g. in Kim and Nelson (1999) or Cogley and Sargent (2005). Given the conditional normality of \( \theta_t \) we have

\[
\theta_{t|t+1} = \theta_{t|t} + P_{t|t}P_{t+1|t}^{-1}(\theta_{t+1} - \theta_t)
\]

(B.1)

\[
P_{t|t+1} = P_{t|t} + P_{t|t}P_{t+1|t}^{-1}P_{t|t}
\]

(B.2)

which provides for each \( t \) from \( T-1 \) to 1 the remaining elements in (1), \( p\left( \theta_{t|t+1} | Y^T, \alpha, H^T \right) = \)
\( N\left(\theta_{t\mid t+1}, P_{t\mid t+1}\right) \). Specifically the backward recursion starts with a draw from \( N(\theta_T, P_T) \), call it \( \tilde{\theta}_T \). Conditional on \( \tilde{\theta}_T \), the Kalman formulation above gives us \( \theta_{T-1\mid T} \) and \( P_{T-1\mid T} \) thus allowing us to draw \( \tilde{\theta}_{T-1} \) from \( N\left(\theta_{T-1\mid T}, P_{T-1\mid T}\right) \) and so on till \( t = 1 \).

b) \textbf{Drawing the innovation variance for VAR parameters (Q)}

Conditional on a realization for \( \theta^T \), the VAR parameter innovations (\( \eta \)'s) are observable. Under the linear transition law, \( \eta_t \) is i.i.d. normal. Given an inverse-Wishart prior and a normal likelihood, the posterior is inverse-Wishart.

c) \textbf{Drawing the innovation variances for} \( \alpha_t \) (\( S_1, S_2, S_3 \))

Conditional on the vector of covariance parameters \( \alpha \), the innovations \( \tau_t \)'s are observables and follow a normal distribution. Given the inverse Wishart prior on the innovation variance-covariance matrices \( S_1, S_2, S_3 \) the posterior follows an inverse Wishart distribution as well.

d) \textbf{Drawing the covariance parameters} (\( \alpha_t \))

Conditional on \( Y^T \) and \( \theta^T \), the VAR residuals \( \epsilon_t = Y_t - X_t' \theta_t \) are observable, satisfying \( A\epsilon_t = u_t \), with \( u_t \) being a vector of orthogonalized residuals with known time-varying variance \( H_t \). Following Primiceri (2005), we interpret \( A\epsilon_t = u_t \) as the observation equation and (7) as the unobserved state equation. Then we apply the Carter and Kohn (2004)'s algorithm as in point a) to obtain a draw of \( \alpha \). Given the block-diagonal structure of \( S \), the algorithm can be applied equation by equation.

e) \textbf{Drawing the standard deviation of volatility innovations} (\( \sigma_i \)'s)

Conditional on a specific time path of \( \log(h_t) \), the innovations to the logs of the stochastic volatilities (\( v_{it} \)'s) are directly observable. The \( v_{it} \)'s are i.i.d. normal with mean zero and variance \( \sigma_i^2 \). Assuming an inverse-gamma prior for \( \sigma_i, \ i = 1, ..., 4 \), the posterior is also inverse gamma.

f) \textbf{Drawing the stochastic volatilities} (\( h_{it} \)'s)

Since we assume that the stochastic volatilities evolve independently, we can sample them on a univariate basis by applying the algorithm of Jacquier et al. (1994) element by element.
Summing up, the MCMC algorithm simulates the posterior distribution of the states and the hyperparameters, conditional on the data, by iterating on a)-d). We use a burn-in period of 50,000 iterations to ensure convergence to the ergodic distribution. We then perform other 100,000 iterations sampling every 10th draw in order to reduce the autocorrelation across draws. What we are after are 10,000 draws from the ergodic distributions over which we calculate the statistics reported in the main text.

C Identification in DSGE models (Iskrev (2010))

Iskrev (2010) proposes the following strategy for local identification. A DSGE model can be represented by the following $g$ linear equations, where $\theta$ is the vector of deep parameters of the model and $z_t$ is the vector of variables,

$$
E_t (g (z_t, z_{t-1}, z_{t+1}, u_t) | \theta) = 0
$$

$$
\Gamma_0 (\theta) z_t = \Gamma_1 (\theta) E_t z_{t+1} + \Gamma_2 (\theta) E_t z_{t-1} + \Gamma_3 (\theta) u_t
$$

Assuming a unique solution which exists to the above system of equations, it takes the shape of the following form

$$
z_t = A (\theta) z_{t-1} + B (\theta) u_t
$$

Some of the variables are not observed, the above system is then augmented by the measurement equations in the following form,

$$
x_t = C z_t
$$

The unconditional first and second moment of the model is then

$$
E_t x_t = \mu_x
$$
\[ \mathbb{E}_t x_{t+i} x_t = \Sigma_x(i) \]

where \( \Sigma_x(i) = \begin{cases} C \Sigma_x(0) C' & i = 0 \\ CA^i \Sigma_x(0) C' & i > 0 \end{cases} \) and \( \Sigma_x(0) \) solves the matrix equation

\[ \Sigma_x(0) = A \Sigma_x(0) A' + \Omega \]

where \( \Omega = B(\theta) B'(\theta) \). Define the unconditional second moment for \( T \) observations as

\[ \mathbb{E}_t X_T X_T' = \Sigma_T \]

Then the identification strategy is to check that the following matrix is the same given all parameters, \( \sigma_T = \left[ \begin{array}{cccc} \text{vec}(\Sigma_x(0))' & \text{vec}(\Sigma_x(1))' & \ldots & \text{vec}(\Sigma_x(T-1))' \end{array} \right]' \),

\[ \sigma_T(\theta) = \sigma_T(\theta_0) \iff \theta = \theta_0 \]

The above global identification may never be achieved hence it is well advised to find the conditions resulting into local identification. The theorem states if \( \sigma_T(\theta) \) is continuously differentiable and let \( \theta_0 \) be a regular point of the Jacobian matrix \( J(T) = \frac{\partial \sigma_T}{\partial \theta}' \) then \( \theta_0 \) is locally identifiable if the Jacobian has full column rank at the regular point.

A regular point is a point which there exists an open neighborhood where the rank of the matrix remains constant. This result is due to a classical work by Rothernberg (1971). A necessary condition for identification is that the number of deep parameters does not exceed the number of unique parameters in the utilized moment. If \( \theta_j \) cannot be identified then Jacobian evaluated at that point is zero for all period (i.e. \( T \)). A corollary to the theorem is to look at the rank condition of \( J_2(T) = \frac{\partial \tau}{\partial \theta} \) where \( \tau = \left[ \begin{array}{c} \text{vec}(A)' \quad \text{vec}(C)' \quad \text{vech}(\Omega)' \end{array} \right] \).

We have conducted the exercise using our time series (in our case, GDP growth, vacancy, real interest rate and inflation) and three variables (interest rate, inflation and vacancy). Results show the strong degree of identification. Hence we can make sure the shocks are identified and the implied signs are not flawed.
Figure 1: Original data and median estimated states
Figure 2: Median time-varying coefficients of the reduced form VAR

Figure 3: Time-varying total prediction variance (bps)
Figure 4: Time-varying volatility of the reduced form shocks (bps)
Figure 5: Time-varying volatility of the reduced form shocks relative to GDP

Figure 6: Time-varying correlations of the reduced form shocks
Figure 7: IRFs to a monetary policy shock

Figure 8: IRFs to a demand shock

Figure 9: IRFs to a cost-push shock
Figure 10: IRFs to labor market shocks, similar to demand shocks

Figure 11: IRFs to different shocks, similar to cost-push shocks
Figure 12: Vacancies: standardized time-varying responses at different forecast horizons
Figure 13: Vacancies: time-varying IRFs
Figure 14: Vacancies: time-varying conditional and unconditional correlations
Figure 15: Time-varying conditional and unconditional variance
Figure 16: Vacancies: forecast error variance decomposition (FEVD)
Identification strength with asymptotic Information matrix (log-scale)

relative to param value
relative to prior std

Sensitivity component with asymptotic Information matrix (log-scale)

relative to param value
relative to prior std

Figure 17
Identification strength with asymptotic Information matrix (log–scale)

Sensitivity component with asymptotic Information matrix (log–scale)

Figure 19
Sensitivity bars using derivatives (log-scale)

Moments
Model
LRE model

Figure 20
Figure 21

Figure 22

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