The informational role of supermajorities

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Abstract

We study a collective decision making environment where an agenda setter makes strategic proposals to privately informed voters who vote strategically. We show that, consistent with empirical evidence, it can be optimal for the agenda setter to propose supermajorities. Due to an informational role that we unveil, optimal supermajorities can be less costly than minimum winning coalitions, even though more voters are awarded a positive share. We also examine consequences in terms of quality of decision making. We show that the introduction of a strategic agenda setter can lead to socially suboptimal decisions.

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1. Introduction

Group decision making environments often share two important characteristics. First, the proposals submitted to a vote can be shaped strategically by an agenda setter. Second, voters often hold private information about the uncertain benefits of the proposals. Consider for instance a legislature deciding on the construction of major infrastructures such as a network of highways. The agenda setter proposes the precise location of the roads, in other words how the benefits are to be shared between legislators. The legislators vote based on private information they obtained, either from their constituents or from experts they consulted, on the benefits and costs of the project. Casual and empirical evidence suggests that in such legislative bargaining environments the proposed coalitions often involve more legislators than the simple majority needed for approval. In the present paper we show that the two characteristics previously identified, the existence of a strategic agenda setter and privately informed voters, can explain the emergence of these larger coalitions. The agenda setter exploits the informational role of supermajorities.

We unveil this mechanism in a theoretical model of legislative bargaining with privately informed voters. The environment is one where an agenda setter submits a proposal that specifies how to share the benefits of a project with uncertain returns. The legislators, who vote simultaneously, first obtain private signals about the project’s returns. Legislators vote strategically and thus consider only the case where their vote is pivotal.

Our main result shows that, under conditions that we derive, the agenda setter proposes supermajorities. We define a supermajority proposal as one that attributes a strictly positive share of the benefits to strictly more legislators than the number of votes in favor required for approval. This result rests on the role of supermajorities as tools to aggregate information. The intuition...
is that optimal supermajorities include some legislators voting informatively. All legislators, can thus obtain some information about other legislators’ signals from considering the case where their vote is pivotal. If this information makes them confident that the returns of the project are high, they need a smaller share of the benefits to vote in favor of the proposal.2 Even though more legislators are included in the coalition, the overall cost can thus be decreased. However, such proposals also decrease the probability of acceptance. We derive a condition such that the first effect dominates and thus provide an explanation for the emergence of supermajorities, based on strategic agenda setting and information aggregation.

Our second result relates to the quality of decision making. We find that the introduction of a strategic agenda setter will sometimes lead to socially suboptimal decision. We show that supermajorities can lead to mistaken rejections whereas minimum winning coalitions, where no information is aggregated, can lead to mistaken acceptances. The conclusions of the literature on strategic voting, that predict that the socially optimal decision will be taken with probability one in sufficiently large legislatures, are therefore altered by the introduction of strategic agenda setting.

In the last section we examine how a debate phase prior to the vote affects the proposals made by the agenda setter and changes the final outcome. A natural conjecture is that information will be revealed during the debate by the different legislators. We first show that proposals granting different strictly positive shares to different groups of legislators cannot lead to full information revelation during the debate. More specifically, one group at the most will be able to truthfully reveal their information. Second, we show that it can be optimal for the agenda setter to make such proposals, creating conflicts of interest, so as to limit the amount of information shared.

We present the informational role of supermajorities in the context of a legislature. There are numerous other situations where a proposer can strategically distribute benefits to privately informed voters. A potential buyer of a firm can design tender offers that specify different prices for different shares. Shareholders often have private information about the potential benefits of the takeover. A firm in bankruptcy negotiations can grant different amounts to shareholders and creditors. One of the groups can thus be induced to vote informatively. Offers made by a firm to a union can treat different members differently. In all these applications it can be beneficial for the proposer to strategically use information aggregation, as emphasized in our model.

The contribution of this paper can be viewed in different ways. The first is to introduce private information in a legislative bargaining context and in vote-buying models.3 This leads us to provide an informational justification for the emergence of supermajorities. The second contribution is to introduce strategic agenda setting in the literature on information aggregation in elections. We find that strategic agenda setting can lead to socially suboptimal decisions even when information is aggregated.4

We discuss the related literature, and in particular alternative explanations for the emergence of supermajorities (Groseclose and Snyder, 1996; Baron, 2006) and a related paper (Bond and Eraslan, 2007), later in the paper after having described our main results. Our paper is organized as follows. In Section 2 we describe the model and institutional environment. In Section 3 we present an example that introduces the main mechanisms. In Section 4 we expose our main result on supermajorities. In Section 5 we examine the introduction of a debate prior to the vote before discussing the related literature in the last section. Finally in Section 7 we discuss how these results could be tested empirically.

2. The model

We study a legislature composed of one agenda setter and N legislators.5 The agenda setter submits a proposal which specifies how to split the benefits from a project between the different legislators. Specifically, the proposal is of the form \( (b_1, \ldots, b_N) \), where \( b_i \) is the share of benefits allocated to legislator \( i \). The agenda setter will keep the remaining share \( 1 - \sum_{i=1}^{N} b_i \).

The legislators cast their vote simultaneously and we suppose, without loss of generality, that the agenda setter does not vote. If more than \( M \) legislators vote in favor of the proposal, the project is undertaken and the benefits are distributed as specified by the proposal. If strictly less than \( M \) legislators vote in favor, the project is abandoned and all players obtain zero utility. Note that \( M \), the majority needed for approval, is a fixed characteristic of the legislature. We assume \( M < N \), as the case of unanimity rule is of no interest in the present study where we want to explain the emergence of supermajorities.

The project has uncertain returns that can be either high \( s_H \) or low \( s_L \) (with \( s_L < s_H \)). The agenda setter and the legislators initially share the same prior belief \( \pi \) that the state is high. Before voting, each legislator \( i \in [1, \ldots, N] \) receives a private signal \( \sigma_i \in \{0, 1\} \). The signals are conditionally independent and satisfy \( P(\sigma_i = 1|s_H) = P(\sigma_i = 0|s_L) = p \), where \( p > \frac{1}{2} \) measures the quality of the information.

The project also has a total cost \( c \) that will be incurred only if the project is undertaken. We suppose without loss of generality that the cost is equally divided between all legislators. The individual cost is therefore \( \frac{c}{N} \).

The agenda setter and the legislators are risk neutral. The expected utility of legislator \( i \) faced with the proposal \( (b_1, \ldots, b_N) \) is given by: \( b_i \pi P(\text{pass} \cap s_H|s_H) + P(\text{pass} \cap s_L|s_L) - \frac{c}{N} \), where \( \pi \) passes refers to the event where the proposal is accepted. Legislators vote strategically and, when voting, consider only the case when their vote is pivotal. Specifically they will compare the expected utility of voting in favor if their vote is pivotal (event noted \( \text{piv} \) ) to zero (utility from voting against the proposal when your vote is pivotal). The agenda setter’s utility is given by \( P(\text{pass} \cap s_H|s_H) + P(\text{pass} \cap s_L|s_L) - \sum_{i=1}^{N} b_i \). She will maximize her expected utility by choosing the shares \( (b_1, \ldots, b_N) \) appropriately, taking into account the voting behavior of the legislators.

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2 We will show how the proportion of legislators voting informatively needs to be optimally adjusted to make legislators confident that the state is high.

3 We show in Section 6 that our model can be reexpressed as a vote buying model.

4 This is in contrast with the literature on information aggregation in elections (Feddersen and Pesendorfer, 1996, 1997) that we discuss later on in the paper.

5 We refer throughout the main text to the group of decision makers as a legislature, even though the analysis can be applied to other groups such as shareholders.
To summarize, the timing of the game is the following: (1) the agenda setter first makes a proposal (2) the legislators receive private signals (3) the legislators vote simultaneously on the proposal (4) the outcome of the vote is determined and if the proposal is accepted, the project is undertaken and the benefits are split accordingly.

3. A simple example

The following simple example is useful to introduce some of the ideas of this paper. Suppose that three neighbors are deciding whether to rehabilitate a wasteland situated between their respective properties. The rules of the local community appoint an outside member as the agenda setter who proposes how to split this common land if the rehabilitation project is adopted and keeps all the remaining share for herself. The rules also require two votes in favor to accept the proposal. The value of this wasteland after rehabilitation is unknown: one in the good state and zero otherwise, i.e. $s_t = 1$ and $s_t = 0$. All neighbors share a prior belief of one half that the state is high ($\pi = \frac{1}{2}$) and receive a private signal of precision $p = \frac{3}{4}$. All three neighbors incur a cost of 0.1 if the project is undertaken. We suppose for simplicity that the agenda setter knows for sure that the state is high before making the proposal.

The agenda setter can potentially make a large number of offers but we concentrate on two of them to convey the main insight. Offer 1 proposes a minimum winning coalition: one neighbor receives a share of zero and the other two receive the minimal share such that they vote in favor regardless of their signal. Offer 2 proposes a supermajority: all three neighbors are offered the minimal share such that they vote in favor regardless of their signal. We examine these offers separately.

Offer 1 is accepted for sure: the positive shares offered are such that the two legislators who receive them vote in favor regardless of their message. We now determine the cost of the proposal. The shares offered are such that the neighbor is indifferent between voting in favor or against if he gets a signal of zero. When voting he considers only the case where his vote is pivotal, which for offer 1 does not provide any additional information since the other neighbors vote a certain way regardless of their signals. The neighbor will thus only consider his own signal when determining his voting behavior. After he receives a bad signal, his posterior probability that the state is high is $\frac{1}{2}$. To be indifferent between voting in favor or against after a bad signal, he needs to be offered a share $b_i$ such that $b_i \frac{3}{4} = \frac{1}{2}$ (i.e. $b_i = 0.4$). The total cost of the proposal is then 0.8 and the utility of the agenda setter is $1 - 0.8 = 0.2$.

On the contrary, offer 2 might be rejected. The neighbors in this case obtain some information from considering the pivotal case: one neighbor received a signal of one and voted in favor and the other received a signal of zero and voted against. After they themselves receive a signal of one, their posterior probability that the state is high is therefore $\frac{1}{2}$. To be indifferent between voting in favor or against after a good signal, they need to be offered a share $b_i$ such that $b_i \frac{3}{4} = \frac{1}{2}$ (i.e. $b_i = 0.4$). The total cost of the proposal is 0.4, which is smaller than in the case of offer 1 even though more neighbors are offered positive shares. However, the probability with which the proposal is accepted also decreases (it is now $1 - (1 - p)^3 = 0.98$). The overall utility of the agenda setter is 0.59, which is greater than in the case of offer 1.

We learned from this simple example that offering positive shares to more neighbors might decrease the overall cost of the proposal. Indeed, some neighbors can then be induced to vote informatively (all three neighbors in the case of offer 2). When voting, they can thus derive information about the other neighbor signals when they consider the case where their vote is pivotal and if this information raises their posterior belief that the state is high, they require a smaller share of benefits to vote in favor. Overall this reduces the cost of the proposal. However, given that some neighbors vote informatively, this proposal also has a lower probability of acceptance. In this simple example, the first positive effect on costs dominates. We now examine, in the general model, under what conditions proposing supermajorities (1) decreases the overall cost of the proposal (2) increases the expected utility of the agenda setter.

4. Aggregation of information and supermajorities

4.1. Proposals

We start by describing the set of proposals the agenda setter makes in equilibrium. In general, she is not restricted to the two offers we compared in the previous example. However, the following lemma indicates that individual shares offered will only be of three types, precisely those that we described in the example.

Lemma 1. In equilibrium, a legislator will be offered either:

1) A zero share.
2) A share $b_i$ such that the legislator is indifferent between voting in favor or against when he receives a signal one.
3) A share $b_k (b_k < b_i)$ such that the legislator is indifferent between voting in favor or against when he receives a signal zero.

Lemma 1 indicates that there are at most three types of legislators in equilibrium: those who are offered a zero share and always vote against the proposal, those who are offered $b_i$ and always vote in favor (denoted by $j$) and those who are offered $b_k$ and vote

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6 In other words the share such that the neighbor is indifferent between voting in favor or against if he receives a signal of zero. We suppose that they break the indifference in favor of the proposal.
7 In other words the share such that they are indifferent between voting in favor or against if they receive a signal of one.
8 We use the fact that the agenda setter knows for sure that the state is high.
9 This assumes as we did at the beginning of this section that the agenda setter knows that the state is high.
10 In other words, when the positive effect on the cost of the proposal will dominate the negative effect on the probability of acceptance.
informatively (denoted by $k$). Voting informatively means that they vote in favor if and only if they get a signal of one.\footnote{We suppose that if they are indifferent, they vote in favor of the proposal.} The intuition of this result rests on the fact that any other offer leads to the same voting behavior as one of those three offers, but is more costly for the agenda setter. Consider for instance, a legislator who is offered a share $b \in (b_k, b_j)$. The share $b$ is such that he votes in favor if he gets a signal of one but is not large enough to induce him to vote in favor if his signal is zero. His voting behavior is identical to that of type $b_k$ but he obtains a higher share of the profits if the proposal is accepted. The agenda setter never in cludes such a legislator in her coalition.

A consequence of Lemma 1 is that, to maximize her expected benefits, the agenda setter can restrict herself to choosing the number of legislators who vote non-informatively (denoted by $J$) and the number of legislators who vote informatively (denoted by $K$). The following lemma establishes how the values of $K$ and $J$ affect the individual shares offered in equilibrium.

Lemma 2. Given a choice of $K$ and $J$, the proposal made in equilibrium is characterized by the shares:

Share offered to one of the $J$ legislators voting non-informatively:

$$b_j = \frac{c}{N s_H p_j + s_L \left(1 - p_j\right)} \text{ where } p_j = \frac{1}{\frac{1 - \pi}{\pi} \left(\frac{p}{1 - p}\right)^{K+2J-2M+1}}.$$ 

Share offered to one of the $K$ legislators voting informatively:

$$b_k = \frac{c}{N s_H p_k + s_L \left(1 - p_k\right)} \text{ where } p_k = \frac{1}{\frac{1 - \pi}{\pi} \left(\frac{p}{1 - p}\right)^{K+2J-2M}}.$$ 

Proof. See Appendix A. \hfill \square

The share $b_j$, by definition, is such that the legislator is indifferent between voting in favor or against when he receives a signal of zero. The probability $p_j$ is his posterior belief that the state is high when he receives a signal of zero and considers the case where his vote is pivotal. Therefore, the share $b_j$ is such that the expected benefits from the project given beliefs $p_j$ exactly equal the costs $c$. The intuition for the share $b_k$ is identical.

It is important to notice that, if $K+2J$ is significantly different from $2M-1$, the share the agenda setter needs to offer to a legislator voting informatively ($b_k$) is very close to the one she needs to offer a legislator who votes in favor regardless of his message ($b_j$). To understand why the comparison between $K+2J$ and $2M-1$ is relevant, consider the situation where $K+2J=2M-1$ in the special case where all voters in the coalition vote informatively (i.e., $J=0$).\footnote{In other words the case where $J=0$ and $K=2M-1$.} A legislator who considers the event where his vote is pivotal, knows that $M-1$ legislators received a signal of one and voted in favor and the $K-M$ remaining legislators who were offered a positive share, received a signal of zero and voted against. If $K=2M-1$, the number of signals in favor and against are equal, and the fact that a legislator’s vote is pivotal does not affect his posterior belief. In this case, the only information he uses is his own signal and the difference between shares $b_j$ and $b_k$ is then large.\footnote{A similar argument can be made when $J=0$: the votes in favor are then $M-1$, the votes against $K-M+J$ and if $K+2J=2M-1$, these two numbers are equal.} On the contrary, when $K+2J$ is significantly different from $2M-1$, most of the information legislators take into account does not come from their own signal but from the information aggregated and the shares $b_j$ and $b_k$ are very close. The fact that in general the shares offered to legislators voting informatively and the shares offered to those voting non-informatively are close will be an important element to explain why supermajorities can be the cheapest option for the agenda setter.

4.2. Aggregation of information in large legislatures

We present in this section the main result of our paper which shows that, under a condition that we derive, the agenda setter proposes a supermajority. This result is obtained in large legislatures, in other words when the size of the legislature $N$ tends to infinity. In this context it is useful to reexpress the main variables as a proportion of the legislature size $N$: $k = \frac{K}{N}$, $J = \frac{J}{N}$ and $m = \frac{M}{N}$.\footnote{Where $m$ is not constrained to be equal to $1/2$ but is different from 1 by assumption.}

We also introduce the notation $R = \frac{1 - \pi}{\left(\frac{s_H \pi + s_L \left(1 - \pi\right)}{s_H + s_L \left(1 - \frac{1}{\pi} - \frac{1}{\pi}\right)}\right)} - \pi$. This parameter $R$, which is strictly smaller than one, will be useful in the main result that follows.

Proposition 1. In large legislatures, if the total cost of the project is relatively high compared to the benefits in the low state, $mc \in [RS_L, S_H]$, the agenda setter proposes supermajorities in equilibrium.

Proof. See Appendix A. \hfill \square
majority). We concentrate on the result relative to the lower bound: \( mc \geq Rs_L \). The intuition follows closely that of the example of Section 3. Well designed supermajorities decrease the total cost of the proposal but will also decrease the probability of acceptance. The trade-off between these two effects leads to the condition \( mc \geq Rs_L \) given in Proposition 1. We now elaborate on the details of this intuition.

Consider a supermajority proposal such that \( pk + j = m \). In a large legislature, the proportion of legislators voting in favor of the proposal among those voting informatively (type \( k \)) is \( pk \) if the state is high and \( (1 - p)k \) if the state is low. Given that \( p > \frac{1}{2} \), we have \( m > (1 - p)k + j \). The conditions \( pk + j = m > (1 - p)k + j \) imply that the proposal is accepted if and only if the state is high. When a legislator determines his voting behavior and considers the case when his vote is pivotal, he will aggregate information and know for sure that the state is high and require a smaller share of the benefits to vote in favor of the proposal. Furthermore, following the intuition of Lemma 2, legislators voting informatively and those voting non-informatively will demand very similar shares.\(^{15} \) The overall cost of the proposal will be low.

If a minimum winning coalition is proposed, on the one hand it is accepted regardless of the state, but on the other hand, no information is aggregated and each legislator needs to be at least compensated for his expected cost after obtaining a signal zero. Both the cost of the proposal and the probability of acceptance are higher. The balance between these two effects is reflected in the condition \( mc \geq Rs_L \). If the total cost of buying a minimum coalition (which is approximately \( mc \)) is high, decreasing the cost of the proposal becomes a priority and supermajorities become attractive. On the contrary, if the cost \( mc \) is low, guaranteeing acceptance, becomes more important.

The type of proposals made in equilibrium will also have consequences in terms of the quality of the decision making. The following proposition addresses this question.

**Proposition 2.** In large legislatures, if the total cost of the project is relatively high compared to the benefits in the low state (\( mc \in [Rs_L, s_H] \)), the proposal made in equilibrium is accepted if and only if the state is high. Furthermore if \( s_L > c \), the equilibrium proposal leads to mistaken rejections.

**Proof.** See Appendix A. \( \blacksquare \)

The result of Proposition 2 follows closely that of Proposition 1. Under condition \( mc \in [Rs_L, s_H] \), supermajorities are proposed in equilibrium and the project is undertaken if and only if the state is high. The second part of the proposal highlights the fact that this will not always lead to socially optimal decisions. Consider the case where \( Rs_L : mc \) (a supermajority is proposed) and \( s_L < c \). If the state is low, the proposal is rejected when it should have been optimally accepted. Therefore, one important conclusion of this work is that, contrary to the conclusions of the literature on strategic voting, the socially optimal decision will not be taken with probability one when the agenda is set strategically. If the optimal choice of the agenda setter is a supermajority, we might observe mistaken rejections. Notice also that if the optimal choice is a minimal winning coalition, we might witness mistaken acceptance.

In the following result, we examine the effect of the quality of the information \( p \) on proposals made by the agenda setter.

**Corollary 1.** As the quality of the information held by the legislator increases:

(a) the likelihood that the agenda setter proposes a supermajority increases.

(b) if a supermajority is proposed, the size of the coalition \( j + k \) decreases.

**Proof.** See Appendix A. \( \blacksquare \)

The intuition for result (a) is the following. When the agenda setter proposes a minimum winning coalition, she has to offer legislators a share such that they are indifferent between voting for or against the proposal when they get signal zero. Therefore, if the quality of the signal is high (\( p \) high), the legislators demand a higher share to vote in favor of the proposal. On the contrary, the cost of optimal supermajorities decreases when the quality \( p \) increases.

However, result (b) states that in cases where the agenda setter does include more legislators than a simple minimum winning coalition, the size of the supermajority decreases with the quality of the signals. The intuition is that the agenda setter just needs enough legislators voting informatively so that others are confident that the state is high in the pivotal case. If the quality of the individual signals is high, fewer legislators are needed to aggregate the same amount of information. Therefore, as the quality of the information held by the legislators increases, fewer of them are included in the proposed coalition. If one of the concerns of the designer is to have benefits equally distributed among the different legislators and through them, their constituents, Corollary 1 indicates that information of better quality is not necessarily beneficial.

4.3. Small legislature

All the results of the previous sections were derived for large legislatures. For small legislatures, we cannot obtain a closed form solution to describe the optimal proposal.\(^{17} \) However, we report the results of some simulations. These simulations are performed in the special case where \( s_L = 0, s_H = 1 \) and \( m = 0.5 \). According to Proposition 1, given that \( s_L = 0 \), in large legislatures supermajorities will always be proposed. Table 1 in Appendix A shows that this is not necessarily the case in small legislatures for low values of \( p \).

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\(^{15} \) If \( pk + j = m \), given that \( p > 0.5 \), this means that \( K + 2j \) is significantly different from \( 2m - 1 \) and that \( b \) and \( h \) will be very similar.

\(^{16} \) The exact value of the total cost in the case of a minimum coalition is larger than \( mc \):

\[ mc < \left( \frac{2m}{2m - 1} \right) \]

\(^{17} \) It is not possible to find a closed form solution for the proposal that maximizes the utility of the agenda setter. In large legislatures we are able to use uniform convergence to first take the limit and then determine the maximum which greatly simplifies our problem. Such a technique is not feasible in finite legislatures.
(Table 1 in Appendix A, p=0.6). For that value of p, when the legislature is small, minimum winning coalitions are preferable. However, when the legislature size N becomes larger, consistent with the results of Proposition 1, we observe that supermajorities are always preferred. The second table shows that when p is larger (Table 2 in Appendix A, p=0.85), supermajorities are the optimal choice regardless of the size of the legislature. The comparison between Tables 1 and 2 in Appendix A also confirms Corollary 1: once a supermajority is proposed, if the quality p increases, the size of the coalition decreases.18

5. How debates affect proposals

The results of the previous sections rely on the fact that the agenda setter strategically uses information aggregation. It is natural to ask whether communication prior to the vote will affect the agenda setter’s incentives and thus proposals made in equilibrium.

In this section, we add a phase to the game previously considered. Prior to the vote, legislators simultaneously exchange non-verifiable public messages m∈{0,1}. To be more precise, we consider the following timing: (1) the agenda setter makes a proposal (2) legislators receive private signals (3) legislators simultaneously send public messages (4) legislators vote simultaneously on the proposal (5) the outcome of the vote is determined and the benefits are distributed.

The first point we want to make is that there will always exist an equilibrium where no information is revealed truthfully. In that case the results of the previous sections are unaffected. We will study in this section the most informative equilibrium.

5.1. Back to the example

Let us return to the example of Section 3 to illustrate these questions. We concentrate on proposals where one neighbor is excluded from the coalition (he is offered a zero share). With the added debate phase more individual equilibrium offers can be made. In an equilibrium where some information is truthfully revealed, the offers can be conditioned on the number of positive signals revealed. In equilibrium, a neighbor included in the coalition could be offered shares such that:

- He is indifferent between voting for or against at (0, 0)
- He is indifferent between voting for or against at (0, 1)
- He is indifferent between voting for or against at (1, 0)
- He is indifferent between voting for or against at (1, 1).

The notation (u,v) used is such that: u is the neighbor’s own signal, v is the message sent by the other neighbor during the debate. We denote b_{u,v} the share that leaves the neighbor indifferent between voting in favor or against the proposal when the information is (u,v).

Consider the case where one neighbor is offered b_{0,1} and the other b_{1,1}. We find that there cannot exist an equilibrium where both report their signals truthfully. Suppose type b_{1,1} is reporting truthfully in the debate and consider the decision type b_{0,1} faces. He only has to consider the case where his message is pivotal, in other words if he sends message one, the proposal passes and if he sends message zero, it is defeated. In such a case, type b_{0,1} knows that the other neighbor (type b_{1,1}) must have obtained signal one (this type only votes in favor if both his signal and the message received are one). This is sufficient information for type b_{0,1} to want the proposal to pass regardless of his message. He will have an incentive to always send a message of one. The same logic applies if b_{0,1} is reporting truthfully and type b_{1,1} obtains signal one. If they are offered different amounts, truthful reporting by both neighbors cannot occur in equilibrium.

5.2. Do all legislators reveal their information in debates?

Can we generalize the result of this simple example? We first need to determine the individual offers that can be made in equilibrium. The introduction of a debate increases the number of potential offers compared to those described in Lemma 1. In an equilibrium where information is truthfully revealed, the offers will depend on the number of signals revealed during the debate and will be different for legislators revealing truthfully and for the others. We denote T as the number of legislators revealing truthfully in equilibrium. The individual equilibrium offers are given by the following lemma.

**Lemma 3.** In equilibrium, a legislator will be offered either:

1) A zero share.
2) If he reveals truthfully, a share b_k such that he is indifferent between voting in favor or against if k signals of one were revealed during the debate, including potentially his own (k∈(0,..., T)).
3) If he does not reveal truthfully, a share b_k such that he is indifferent between voting for or against if k signals of one were revealed truthfully and he obtained a signal of zero or if k − 1 signals of one were revealed truthfully and he obtained a signal of one (k∈(0,..., T+1)).

As in the proof of Lemma 1, all other individual offer will lead to the same outcome of the vote but will be more costly and therefore suboptimal for the agenda setter. We now study the communication strategy during the debate. The first thing to note is that legislators who are offered no share of the benefits cannot credibly reveal any information during the debate as they always

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18 For instance for N=200, the coalition is of size 181 for p=0.6 and size 109 for p=0.85.
want proposals to be defeated independently of their signal. It is also trivially true that when all legislators in the proposed coalition are offered the same share, an equilibrium where they truthfully reveal their information during the debate phase will always exist. However, the following result confirms the simple example: in equilibrium, truthful revelation will not always occur.

**Proposition 3.** In the most informative equilibrium, if different groups of legislators are offered different amounts, only one group will truthfully reveal their signal in equilibrium.

**Proof.** See Appendix A.

Proposition 3 indicates that two groups of legislators who are offered different amounts in equilibrium cannot both reveal their information truthfully. The intuition for this result has two parts. First, in equilibrium according to Lemma 3, if two groups are offered different amounts and truthfully reveal their information, legislators in one group will need at least one extra positive signal revealed during the debate than the other group to vote in favor of the proposal. Second, the legislators when they decide whether to report truthfully or not, consider only the case where their message is pivotal for the final vote. If the group that needs more positive signals is truthfully revealing, this means for a legislator of the other group that, even without his message, the information is good enough for him to want to pass the proposal. He therefore has an incentive to always report a signal of one.

Proposition 3 therefore indicates that at most one group of legislators will truthfully reveal their information during the debate. The following proposition characterizes this group. We call the legislator who receives the $M$th highest individual share, the pivotal legislator, where $M$ is the number of votes in favor required to pass the proposal.

**Lemma 4.** In equilibrium, if $b_k^i$ is the offer made to legislators who reveal their signal truthfully, the pivotal legislator can only receive one of three offers: $b_k^i$, $b_{k-1}^i$ or $b_{k+1}^i$ (with $b_{k+1}^i < b_k^i < b_k^i$).

**Proof.** See Appendix A.

Lemma 4 states that the incentives of the pivotal legislator and those of a legislator revealing truthfully his signal need to be closely aligned in equilibrium. Consider the decision of a legislator to report or not his signal. Suppose that the pivotal legislator needs many more positive signals than he does to vote in favor of the proposal. The legislator, to determine his communication strategy, examines the case where his message is pivotal, in other words changes the vote of the pivotal legislator. Because this pivotal legislator needs many more positive signals to vote in favor, the information obtained from the pivotal case is sufficient to want the proposal to pass irrespective of his signal. Such a legislator will not report truthfully. The intuition is identical if the pivotal legislator needs less positive evidence. The only legislators who can report truthfully are therefore those who need a similar number of positive signals as the pivotal legislator to vote in favor of the proposal.

### 5.3. Preventing information sharing in debates

We have studied in the previous section the behavior of legislators in the deliberation phase. In this section we use these results to examine the agenda setter’s optimal behavior and determine what proposals she will make in equilibrium? We show that it can be optimal for her to limit the amount of information shared during the debate by designing proposals where different groups have diverging interests. We first describe the different individual equilibrium proposals in the following lemma.

**Lemma 5.** When a debate occurs prior to the vote, in equilibrium the agenda setter makes only two types of proposals:

1. **Type I:** all legislators in the coalition receive the same share $b_i$ such that they are indifferent between voting for or against if $l$ positive signals were revealed during the debate. Furthermore, they all reveal their information truthfully in the debate.
2. **Type II:** involves two group of legislators receiving different strictly positive amounts in equilibrium. Type $b_i^m$ reveals truthfully. The other type is either $b_{k-1}^i$ or $b_{k+1}^i$.

**Proof.** See Appendix A.

The previous lemma indicates that the agenda setter will optimally choose among a limited set of proposals. The first type is one where all the information held by members of the coalition is revealed during the debate. The second type of proposal involves two groups: one group of legislators who truthfully reveal their information during the debate and the other group who cannot communicate information. The proposals in that category are designed to create a conflict of interest between legislators and thus prevent full information sharing. However, Lemma 5 leaves the question open: can it be optimal for the agenda setter to make a proposal of type II creating a conflict of interest between the different legislators? The following proposition addresses this question in large legislatures.

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19 Indeed they have exactly aligned preferences and therefore in the case where their message is pivotal will always want to reveal it to avoid deceiving the other identical legislators.

20 Suppose the legislators who reveal their signal truthfully need $k$ signals of one to vote in favor. The pivotal legislator will either be the legislator who reveals truthfully or a legislator who does not reveal and needs either $k$ or $k+1$ signals of one including potentially his own to vote in favor.

21 That is all legislators receiving a strictly positive share.
Proposition 4. In large legislatures, it can be optimal for the decision maker to choose a proposal of type II where only a finite amount of information is shared during the debate.

Proof. See Appendix A. □

We prove this result by presenting an example where, for a large legislature, a proposal of type II provides a higher expected utility than any proposal of type I. In the case of large legislatures, all legislators included in the coalition reveal truthfully their information and the state is perfectly known after the debate.22 There are therefore only two proposals of type I. The first proposes to M legislators a share such that they are indifferent between voting for or against if the state is known to be high: therefore such a proposal has a low cost,23 but it also has a low probability of acceptance. We name this proposal high for sure. The other offer of type I, that we name low for sure, compensates legislators when they know the state is low for sure, is always accepted but at a much higher cost.24 Proposals of type II in effect constitute intermediate solutions: they are accepted less often than the low for sure proposal but need to abandon a smaller share of the benefits to the legislators. They also are more costly than the high for sure proposal but are accepted more often.

The proof of Proposition 4 is based on one of several specific examples where a proposal of type II, where only a few signals are revealed during the debate, dominates all proposals of type I. It is difficult to provide a more general statement as there is no systematic way to compare the discrete utilities of the infinite number of proposals of type II.25 However, this initial result already shows that it can be optimal for the agenda setter to prevent full information sharing during the debate. She can achieve this by making proposals that create conflicts of interests.

6. Related literature

Our paper is linked to two main strands of the literature and makes contributions to both. First, it can be seen as introducing private information in the vote-buying literature. Reviewing this literature allows us to discuss two alternative explanations for the emergence of supermajorities based on competition between two lobbyists. Second, it can be seen as introducing strategic agenda setting in the literature on information aggregation. We show that, contrary to the results in that literature, strategic agenda setting will lead to mistaken decisions, even when information is aggregated. As Piketty (1999) indicates in his survey paper, before the “information aggregation approach” was introduced (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996, 1997) most of the public choice literature viewed politics as “a game where selfish rational actors seek to divide the pie produced by economics”. We can view our paper as combining these two approaches.

6.1. Vote-buying literature

The main contribution of our paper is to give an explanation for the observed occurrence of supermajorities, based on strategic agenda setting and information aggregation. Two main alternative explanations exist in the literature on vote buying (Groseclose and Snyder, 1996 and Baron, 2006) and are based on competition between several lobbyists. Notice that our model can be relabeled as a vote-buying game with no agenda setter and a single lobbyist. The same type of arguments as in our setup imply that it can be optimal for the single lobbyist to buy a supermajority to decrease the amount spent on vote buying.26

Groseclose and Snyder (1996) present an interesting mechanism in a model with two competing vote buyers, who have opposed interests and make sequential offers to legislators. They show that buying a minimum winning coalition might not be the cheapest solution for the first lobbyist. Instead, if she makes such an offer, she will need to spend large amounts to defend every single vote in her coalition, under the threat that the competing lobbyist only needs to buy back one vote to guarantee a favorable outcome.27

Several authors (Baron, 2006; Dekel et al., 2004) have pointed out that the existence of a final offer in the model of Groseclose and Snyder (1996), is crucial for the result on supermajorities as it grants a substantial advantage to the second lobbyist. Dekel et al. (2004), in a model with two competing lobbyists, who make sequential offers with a minimum increment and no final round, find that supermajorities will not be the observed outcome, as long as the winning lobbyist did not have a supermajority in his favor before the game started.

Baron (2006) not only relaxes the assumption of a final offer but also introduces several roles for the agenda setter: bargaining with the lobbyists, setting the agenda and voting on the proposal. In the model, at each round one legislator is randomly selected to set the agenda. Lobbyists not only buy votes but also spend resources to influence the agenda setting activity. If the stakes of the

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22 Due to the law of large numbers.
23 The legislators know the state is high for sure and therefore need a smaller share or vote in favor.
24 Note that for the legislator it is much more costly than without a debate because legislators will share information and will therefore know that the state is low for sure.
25 Type II proposals, even in large legislatures can contain a finite number of signals truthfully revealed in equilibrium. The agenda’s setter expected utility from making a Type II proposal will contain complex discrete probabilities. There does not seem to be a systematic way to obtain a closed form solution for the maximum.
26 The vote buyer can pay a high amount for a vote in favor to buy the vote for sure or pay an intermediate amount to induce the legislator to vote informatively as the agenda setter did in our model.
27 The intuition of their model is the following: suppose that a first vote buyer has bought a minimum winning coalition, then the other vote buyer needs to win over only one member of that coalition. It therefore becomes very expensive for the first lobbyist to defend every single vote. If, on the other hand, she increases her coalition by one member, she might, even if she spends less per legislator, make it more expensive for the second mover to win over a majority that would now require him to buy two votes.
lobbyists are high or if legislators are impatient, supermajorities will be bought as they increase the probability that the randomly selected agenda setter will submit a favorable proposal to a vote.

We provide an alternative explanation based on a completely different mechanism involving private information and strategic agenda setting. In terms of broad differences the informational role of supermajorities can explain their emergence in the absence of competition between different vote buyers. Furthermore the mechanism is general and does not rely on the particular institutional details of legislative bargaining. We describe in Section 7 how these alternative theories could be empirically tested.

6.2. Literature on strategic voting

The second perspective on the paper is that it introduces private information in a legislative bargaining context, similar to the model in the seminal work by Baron and Ferejohn (1989). The introduction of private information in elections naturally ties our paper to the literature on information aggregation (Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996, 1997), Duggan and Martinelli (2001) and others). In these papers the agenda is set exogenously. We effectively introduce strategic agenda setting in these environments.

We show that the introduction of strategic agenda setting has important consequences in terms of the quality of decision making. One of the messages of the literature on information aggregation is that in large legislatures, information will be efficiently aggregated and the socially optimal decision taken with probability one. For instance, Feddersen and Pesendorfer (1997) show that even if the fraction of informed voters is arbitrarily small, the socially preferred policy wins with probability one with a large enough electorate. On the contrary, we show in Proposition 2, that the existence of a strategic agenda setter can lead to mistaken rejections if supermajorities are proposed or mistaken acceptances if minimum winning coalitions are the optimal choice. Strategic agenda setting leads to potential mistakes.

As was pointed out in the introduction, an independent work by Bond and Eraslan (2007), considers a related question. They also examine situations where the agenda is set endogenously by an agenda setter and voters hold private information. However, they focus on proposals where all legislators are treated equally (they are not interested in legislative bargaining situations) and consider a different set of questions. They reexamine the classical comparison between unanimity and majority rules.

Finally, we want to point that our work is also linked to a paper on vote trading by Piketty (1994). When voters do not hold information, vote trading enhances social welfare as voters with stronger preferences can voice them in the market for votes. However, Piketty shows that this result can be overturned when information is introduced. If there exists a proportion of uncertain voters who hold private information on the state of the world and if their signal is sufficiently imprecise, there exists an equilibrium where all uncertain voters sell their vote. In such circumstances voting does not aggregate any information. As is the case in our paper, the author combines two strands of the literature (vote-buying and information aggregation approaches) and uncover as we do in Proposition 2 deviations from the socially optimal outcome.

6.3. Literature on debates

Finally we point out the links with the literature on debates, relevant to our analysis in Section 5. There is a relatively large literature on debates (for instance Spector, 2000). A few recent papers have examined more specifically the impact of debates on the outcome of a vote in a game-theoretic framework. An important result was obtained by Gerardi and Yariv (2003): under general conditions, the set of sequential equilibria outcomes achievable by augmenting a voting game by a deliberation phase is constant in the voting rule. The central observation leading to that result is that all individuals voting unanimously can be part of a sequential voting equilibrium. Therefore, individuals behave as if the final voting phase did not matter. In contrast, Austen-Smith and Feddersen (2002), examine equilibria where agents always condition their vote on being pivotal. They show that majority rule induces more information sharing than unanimity.

Two other papers are particularly relevant to our study. Meirowitz (2007) studies a case where voters have potentially opposed preferences. In a model where the proportion of the different types of voters is not known with certainty, he finds that truthful equilibria exist only if the agents are confident enough that the other participants have preferences similar to theirs. In a related paper, Meirowitz (2006) determines a necessary condition for an equilibrium with full revelation: either all participants believe they have the majority opinion or no individual signal can change the outcome. The result emphasizes the fact that participants who have minority interests generally have incentives to misrepresent their information. The result we obtain in Lemma 4 follows a similar logic: only the legislators who have closely aligned preferences to those of the pivotal voter can reveal their information truthfully.

7. Empirical validation

The theory we develop in this paper makes precise testable predictions regarding the size of coalitions and the emergence of supermajorities. We therefore believe it is important to outline how our results could be tested and to relate our work to the existing empirical literature.

28 The model can involve either a strategic agenda setter and no lobbyist or a single vote buyer.
29 Contrary to previous results, that found that unanimity just led to less information aggregation and therefore worse decisions, they find that with an endogenous agenda, it can become a Pareto dominant voting rule. The agenda setter is forced to make more attractive proposals as under unanimity, low offers will lead to mistaken rejections.
30 Note that our model is a special case of a market for votes where votes can only be traded with the agenda setter who is essentially setting the price through the proposal she makes.
7.1. Empirical literature on legislative bargaining

There exists a relatively large empirical literature linking the variation in public spending across states to the distribution of votes in a legislature (including Atlas et al., 1995; Rodden, 2002). These papers consistently find that higher benefits per capita are obtained by states that have a larger share of votes per capita (usually the smaller states). These papers do not concentrate on voting patterns but on aggregate outcomes associated to different procedural rules. Another strand of the literature examines coalitional bargaining to form governments. For instance, Diermeier and Merlo (2004) examine government formation across different European countries and determine empirically the rule that most often governs the proposal power.

Two papers are particularly related to our work. The first by Knight (2005), already mentioned in the introduction, examines the allocation of transportation projects in the US Congress. Although he concentrates on the effect of proposal power he also reports that the proposed coalitions often included more legislators than the required majority.31 Mattila and Lane (2001) examine the voting patterns in the Council of the European Union. They report that even in sectors where a qualified majority was required,32 the proposals were often approved unanimously. They point out that several competing theories could explain such over-sized coalitions. We describe in the next section how the predictions of this model differ from those of the alternative explanations.

7.2. Testing alternative theories

We described in Section 6.1 the alternative explanations proposed in the literature for the existence of supermajorities. A certain number of predictions differentiate our work from these theories and could provide the basis of an empirical test.

First and most importantly, information plays a key role in our setup. In particular we show in Corollary 1 that as the quality of the information held by legislators increases, the size of supermajorities decreases provided that a supermajority is indeed proposed. All else equal, we should therefore observe larger supermajorities in sectors where the effects of legislation are more uncertain. Furthermore the model could be extended to the case where legislators hold information of varying quality to test whether better informed legislators have a higher probability of being included in proposed coalitions. Note that in the competing models, uncertainty has no effect on the proposed coalitions.

Second, our theory, as opposed to the models of Groseclose and Snyder or Baron does not rely on lobbying. Our model predicts the emergence of supermajorities even in the absence of competing lobbyists. According to our results variations in the degree of lobbying activity should have no effect on the size of supermajorities.

Third, our model predicts that inside a proposed coalition different groups will receive different amounts. In particular, legislators who vote informatively will receive a smaller share than those who always vote in favor. On the contrary, in Groseclose and Snyder’s model, assuming that preferences are initially identical across legislators, every vote in the coalition needs to be defended against the competing lobbyist and therefore all legislators in the coalition receive the same amount. Of course we acknowledge that differences in preferences could also explain differential amounts inside a coalition. Nevertheless, all else equal, we expect that the distribution of shares among coalition members to be more dispersed if our model is an accurate representation of reality.

Finally, we want to point out that in our model the required number of votes needed for approval affects the relative size of coalitions. It also does in the alternative models but the effects are presumably different.33 We describe in the next section how the differences we uncovered could be exploited and what type of data would be appropriate.

7.3. Data requirements

The previous section indicates that a valuable empirical exercise would be to test how well variations in coalition sizes are explained by differentials in uncertainty, information held by different legislators, size of the required majorities and lobbying activity. Furthermore it also suggests that the dispersion of shares allocated to legislators in the coalition could also be explained by similar variables. In this section we attempt to determine what type of data would be most appropriate to conduct such an analysis.

Knight (2005) highlights the fact that the data that he used was well adapted to test predictions of legislative bargaining models for a number of reasons: the payoffs associated to each legislators were easily measurable,34 the stakes were high and bargaining was clearly important. For our purposes however this data has some limitations. First the author concentrated on transportation projects whose effects were easily predictable. There is presumably little variation in uncertainty across the different bills submitted to a vote. Second, given that all proposals he uses are made in the same policy sector (transportation), they were presumably characterized by similar degrees of lobbying activity.

Accordingly, we believe that an empirical analysis aimed at testing these alternative theories on the size of coalitions would require data varying across several policy sectors. It would be difficult to attach a numerical value to the degree of uncertainty

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31 He reports that in 1998 the coalitions included on average 82% of all districts while 50% of districts was sufficient for approval.
32 The ratio for approval is approximately five votes in favor for every seven votes.
33 We have not determined the differences but this could be an interesting extension.
34 Each transportation project was evaluated as part of the proposal.
across sectors but a discrete classification could probably be established. For instance in the European Union, the effects of legislation associated with agricultural policy are more predictable (i.e., less uncertain) than the effects of common commercial policy (that refers mostly to trade policy). In that respect it is interesting to note that Mattila and Lane report that 67% of bills were accepted unanimously for agricultural policy while 99% were for commercial policy. This result is broadly consistent with our predictions, although more systematic analysis of this data is clearly needed. A measure of lobbying activity could also be obtained. Liebert (1995) indicates that data on the number of groups lobbying the European Parliament exists. These groups could probably be associated with different policy sectors to obtain a measure of lobbying activity across policy sectors.

Based on these findings, it seems that the data used in Mattila and Lane on votes in the European Council of Ministers is well adapted for an empirical test of the competing theories. We already pointed out that discrete measures of uncertainty as well as measures of lobbying activity could be included. Furthermore, as pointed out by Mattila and Lane, party discipline is not an issue for this assembly, while it usually is in legislative contexts. However, one limiting factor is that communication inside this forum is relatively easy and this affects our predictions on coalition sizes as indicated in Section 5. We note that data on the European Parliament or on the German legislature could also be well adapted (measures of lobbying activity can also be obtained in these legislatures according to Liebert, 1995).

8. Conclusion

We have examined situations where an agenda setter submits proposals to privately informed voters who vote strategically. We show that it can be optimal for the agenda setter to propose supermajorities. The use of these larger coalitions can decrease the overall cost due to their informational role. In terms of quality of decision making, we show that the introduction of a strategic agenda setter in an environment with private information, can lead to socially suboptimal decisions. Finally when a debate precedes the vote, we have shown that it can be optimal for the agenda setter to make proposals that create conflicts of interest preventing full information sharing.

The paper was motivated by the application to legislative bargaining but we mentioned in the introduction numerous other applications of this model. One interesting avenue for future research might be to examine the application of the model to the design of takeover bids and the choice between “any-and-all” offers, conditional offers or “multi-tiered offers”. Shareholders tend to have private information on the potential benefits of the takeover and the strategic use of information aggregation could influence the design of the bid. In particular “multi-tiered offers” could include shareholders who decide informatively and others that are given a sufficient price to always want to tender.

Another direction for future work could be to examine several rounds of bargaining. This is a difficult problem: indeed at each round legislators would not only vote on the proposal but also possibly signal their information (see Piketty, 2000 for a similar problem without an agenda setter). It could also be interesting to modify the rules of the debate and allow for sequential communication. A recent paper by Caillaud and Tirole (2007) examines the optimal communication strategy by a sponsor of a proposal to a group of voters and shows that it can be optimal for the sponsor to provide information to key members to trigger persuasion cascades. It would be natural to introduce endogenous agenda setting in these environments. Finally, as we pointed out in Section 7, an empirical test of this theory seems both possible and important.

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Appendix A

Lemma 2.

Legislators vote strategically and consider only the situation where their vote is pivotal. The information aggregated in the pivotal case differs for the J legislators voting non-informatively and for the K voting informatively.

For one of the J legislators, being pivotal implies M−1 votes in favor and J−1 of those regardless of the signal obtained. Therefore M−J signals of value one were obtained and the K−M+J other legislators who were voting informatively got signals zero.

Overall, if the legislator’s own signal is zero, the information aggregated is (M−J) signals of value one and (K+J−M+1) of value zero. The share bj is such that the legislator is indifferent between voting in favor or against if his signal is zero. Given the information aggregated, the legislator will be exactly compensated for his cost: \( \frac{J}{K+J-M+1} = bj [sJpJ + sJ(1−pJ)] \) where \( pJ \) where \( pJ = \frac{1}{(M-J)} \) is a share of the vote. Applying Baye’s rule, this leads to the result stated in Lemma 2.

For one of the K legislators voting informatively, being pivotal implies a slightly different information: (M−J−1) signals of value one and (K−M+J) of value zero.

The share bj is such that the legislator votes in favor only if he gets a signal of one. Aggregating all this information he knows that (M−J) signals of value one and (K−M+J) of value zero were obtained. Applying Baye’s rule leads to the result stated in Lemma 2.

\[ \frac{J}{K+J-M+1} = bj [sJpJ + sJ(1−pJ)] \]

35 It should be checked whether lobbying of the European Parliament is similar to lobbying of the European Council.

36 “Any-and-all” offers propose a positive price for any share; conditional offers are paid only if a certain number of shares are tendered and multi-tiered offers specify a different price for different groups of shares.
Proposition 1.

Using Lemma 2, we can determine the expected utility of the agenda setter making a proposal involving a proportion \( j \) of legislators voting non-informatively and \( k \) voting informatively:

\[
U_N(k, j) = \left| s_{1\pi}P[\text{pass}|s_{1\pi}] + s_L(1 - \pi)P[\text{pass}|s_L]\right| \left[ 1 - jC_j - kC_k \right]
\]

where: \( P[\text{pass}|s_j] \) is the probability that at least \((M-j)\) signals of value one are obtained among \( K \) signals (i.e. the proposal passes) given that the state is high and \( p_j \) and \( p_k \) are the posterior probabilities specified in Lemma 2.

We want to determine \( \lim_{N \to \infty} \max_k U_N(k, j) \). It is not possible to find a closed form formula for the maximum and therefore difficult to calculate directly the limit. We will however show that on a certain interval we have uniform convergence and thus \( \lim_{N \to \infty} \max_k U_N(k, j) = \max_k \lim_{N \to \infty} U_N(k, j) \).

**Step 1:** Uniform convergence of \( U_N(k, j) \)

Let \( \varepsilon > 0 \). We study the domain \( A_k \) such that \( k + 2j \leq 2m - \varepsilon \) and show we have uniform convergence of \( U_N(k, j) \) on this interval. It is important to note that \( A_k \) is different from the domain of \( j \) and \( k \). In particular it does not include the case \( j = m \) that is minimum winning coalitions. This will be important in what follows.

From Lemma 2 we have \( p_j = 1/\left[ 1 + \frac{1}{n} \left( \frac{p}{p - \pi} \right)^{-N(k+2j-2m)+1} \right] \) and \( p_k = 1/\left[ 1 + \frac{1}{n} \left( \frac{p}{p - \pi} \right)^{-N(k+2j-2m)} \right] \). On the domain \( A_k \), both \( p_j \) and \( p_k \) converge uniformly to 1.

Indeed, \( \forall \varepsilon > 0. \exists N_0 \) such that \( \forall N > N_0 \), \( 1/\left[ 1 + \frac{1}{n} \left( \frac{p}{p - \pi} \right)^{-N(k+2j-2m)+1} \right] > 1 - \eta \). Furthermore, on \( A_k \), \( k + 2j - 2m \leq \varepsilon \), so for all \( (k, j) \in A_k \), if \( N > N_0 \), \( \varepsilon > p_j \geq 1 - \eta \). We have shown uniform convergence of \( p_j \) to 1 and the proof for \( p_k \) is identical.

We now examine the uniform convergence of \( P[\text{pass}|\theta = 1] \) and \( P[\text{pass}|\theta = 0] \).

\[
P[\text{pass}|\theta = 1] = \sum_{(k \geq 1j \geq M-j)} c_k p_j(1 - p_k)^{k-j}
\]

We can use the approximation of the binomial by a normal variable:

\[
\int_{N(\mu, \delta)} \frac{1}{\sqrt{2\pi} \delta} e^{-\frac{x^2}{2\delta^2}} dx
\]

Using the change of variable \( x = Np \), we can rewrite the integral:

\[
\int_{m-j}^{\infty} \frac{N}{\sqrt{2\pi} \delta} e^{-\frac{(Np - \mu)^2}{2\delta^2}} dy
\]

This can be reexpressed as \( P[X \geq m-j] \) with \( X \sim N \left( \mu \sqrt{(k+1-\pi)} \right) \). So, \( P[\text{pass}|\theta = 1] \) uniformly converges to \( 1_{m-j<0} \) (the proof of uniform convergence follows the same lines as for the uniform convergence of \( p_j \). The uniform bound on \( N \) can be found for the case where \( k = 2m - \varepsilon \).

Following the same logic, \( P[\text{pass}|\theta = 0] \) converges uniformly to \( 1_{m-j \geq 0} \). However, on the interval \( A_k \), \( k + 2j \leq 2m \), in other words \( m-j \geq 0 \), \( p_j \) and \( p_k \) converge uniformly to 1 on this interval.

Overall we have uniform convergence of \( U_N(k, j) \) to \( \left[ s_{1\pi}P[\text{pass}|s_{1\pi}] + s_L(1 - \pi)P[\text{pass}|s_L]\right] \) on this interval.

On the interval \( A_k \) we have proved uniform convergence and we can therefore invert limit a maximization:

\[
\lim_{N \to \infty} \max_k U_N(k, j) = \max \left( \left[ s_{1\pi} P[\text{pass}|s_{1\pi}] + s_L(1 - \pi) P[\text{pass}|s_L]\right] \right) \]

The maximum is attained for a supermajority proposal where \( j + pk > m \) and \( k > 0 \) (in other words \( j + k > m \) which characterizes a supermajority).

**Step 2:** Case of minimum coalitions

Minimum coalitions are defined by the fact that \( j + k = m \).

The first case is \( j = m \) (in the strict sense a minimum winning coalition). In this case, for all values of \( N \), no information is aggregated, the proposal always passes and the legislators have to be compensated for their prior updated after a bad signal. We find:

\[
U_N(k, j) = \left[ s_{1\pi} + s_L(1 - \pi) \right] \left[ 1 - mc \right]
\]

The second case is \( j + k = m \) with \( k > 0 \). In this case we can find an upper bound on \( U_N(k, j) \):

\[
U_N(k, j) \leq \left[ s_{1\pi} + s_L(1 - \pi) \right] \left[ 1 - mc \right]
\]

**Step 3:** Comparison of maximum values

The final step is to compare the maximum value attained in the case of supermajorities to the case of minimum coalitions. The proof of uniform convergence is valid for any value of \( \varepsilon \). We therefore want to compare \( \left[ s_{1\pi} P[\text{pass}|s_{1\pi}] + s_L(1 - \pi) P[\text{pass}|s_L]\right] / \left[ 1 - mc \right] \) to the values obtained in Step 2.
We first notice that the case \(j + k = m\) with \(k > 0\) will lead to a lower expected utility for the agenda setter at the limit. The important comparison is therefore with the case \(j = m\).

A supermajority will be proposed by the agenda setter if:

\[
[s_H \pi] \left[ 1 - \text{mc} \frac{1}{s_H} \right] \geq [s_H \pi + s_L (1 - \pi)] \left[ 1 - \text{mc} \left[ \frac{1}{s_H} \left( 1 + \frac{1}{\pi} \frac{1}{1 - p^R} \right) + s_L \left( 1 - \frac{1}{1 + \frac{1}{\pi} \frac{1}{1 - p^R}} \right) \right] \right].
\]

Simplifying, we find that the condition is equivalent to the condition given in the main text: \(mc > R_{s_L}\) with \(R = [1 - \pi] / ([s_H \pi + s_L (1 - \pi)] / (s_H \left[ 1 + \frac{1}{\pi} \frac{1}{1 - p^R} \right] + s_L \left( 1 - \frac{1}{1 + \frac{1}{\pi} \frac{1}{1 - p^R}} \right)) - \pi] \).

The result relative to the upper bound \(mc \leq \pi s_H\) is obvious. If \(mc > R_{s_L}\), the agenda setter cannot make an offer that is accepted and leaves her with positive utility. Indeed, even in the best case where the state is known to be high, the total cost is still too high for a simple majority of \(M\) legislators.

**Proposition 2.**

If \(mc \in [R_{s_L}, s_H]\), we know from Proposition 1 that the agenda setter will propose a supermajority. We also saw in Step 1 of the proof of Proposition 1 that in that case \(P[\text{pass} \mid s_H] \) converges uniformly to 1 and \(P[\text{pass} \mid s_L] \) converges uniformly to 0.

Furthermore if \(s_L \sim c\), the socially optimal choice is to conduct the project even if the state is low. However, if \(mc > R_{s_L}\), a supermajority is proposed and rejected if the state is low. A socially suboptimal decision is reached.

**Corollary 1.**

(a). When \(p\) increases, the cost of a minimum winning coalition increases; indeed each legislator in the coalition needs a higher compensation as he needs to be made indifferent between voting for or against after obtaining a signal of zero.

In the case of a supermajority, the compensation given to each legislator is given by Lemma 1. Therefore as \(p\) increases the amounts given to each legislator in the coalition weakly decrease (an optimal proposal is necessarily such that \(k + 2j < 2m\)). Therefore the cost of proposing a supermajority weakly decreases.

These two results imply that as \(p\) increases, the likelihood that the agenda setter will propose a supermajority increases.

(b). If it is optimal to propose a supermajority, we have seen in Step 1 of the proof of Proposition 1 that the optimal proposal is such that: \(m - j \leq pk\). In this area a proposal such that \(m - j < pk\) is never accepted. Indeed, with such a proposal, the agenda setter keeps a larger share of the benefits. The optimal proposal will be such that \(m - j = pk\) and as \(p\) increases, the size of the coalition \(j + k\) decreases.

**Proposition 3.**

Suppose there exists an equilibrium proposal involving at least two groups revealing truthfully and offered different equilibrium amounts \(b_k^L\) and \(b_k^R\) with \(k' > k\). According to Lemma 3, these are the only possible equilibria for legislators revealing truthfully.

Legislators when they decide whether to reveal truthfully their signal, consider only the case where their message is pivotal: if they reveal one, the proposal passes, if they reveal zero it is defeated. Both type \(k\) and \(k'\) obtain the same information when they consider the case where their signal is pivotal. The fact that type \(k'\) reveals truthfully his signal means that the information obtained in the pivotal case combined with an extra signal of one is sufficient to make him want to pass the proposal. However, type \(k\) needs at least one signal of one less to pass the proposal. Therefore, for type \(k\), the information obtained in the pivotal case is sufficient to want the proposal to pass independently of his signal. Type \(k\) will therefore not reveal truthfully.

**Lemma 4.**

The notation in this lemma is that \(b_k^L\) is the offer made to the type that reveals truthfully in equilibrium: he needs at least \(k\) signals of one to be revealed in the debate to vote in favor of the proposal.

Suppose type \(b_k^L\) with \(k' > k + 1\) is the pivotal legislator.

When type \(b_k^L\) determines his communication strategy he considers the case where his signal is pivotal: if he reveals one the pivotal legislator votes in favor, if he reveals zero he votes against. He can conclude that, given the offer made to the pivotal legislator \(b_k^R\) with \(k' > k + 1\), abstracting from his own signal, at least \(k\) signals of one have already been revealed. This is a sufficient information to want the proposal to pass. He has an incentive to always report a signal of one. Both groups cannot report their signal truthfully.

The proof is identical if the pivotal voter is of type \(b_k^R\) with \(k' < k\).

**Lemma 5.**

We need to prove that the proposals of type II can be reduced to the set described in Lemma 5. From Lemma 4, we know that only three types can be the pivotal legislators: \(b_k^L, b_k^R, b_k^L\). We will examine each case in turn.

**Case 1:** \(b_k^L\) is the pivotal legislator.

First, no type \(b_k^L\) with \(k' < k\) will be included: they will not change the outcome of the vote as \(k\) signals of one need to be revealed for the proposal to pass \(b_k^L\) is the pivotal legislator), but are more expensive than type \(b_k^L\). Furthermore they don’t reveal their signal: \(b_k^L\) is the type truthfully revealing.

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Note that there could exist other groups in equilibrium, revealing or not their signal. This will not impact our proof.
Case 3: they vote in favor if no positive signal is revealed and they get a signal of one or if a signal of one is revealed and they get a signal of utility of the agenda setter in a large legislature\(^{38}\)

This specifies the utility of the agenda setter in a large legislature.

Second, no type \(b^k_h\) with \(k^*>k\) will be included: they will not change the outcome of the vote given that if they vote in favor it means that at least \(k\) signals have already been revealed and the proposal would be accepted even without their vote (i.e. \(b^k_h\) is the pivotal legislator). Furthermore they don’t reveal their signal.

Therefore in this case the proposal can contain only two types receiving positive shares: type \(b^k_h\) and \(b^0_0\)

Case 2: \(b^0_0\) is the pivotal legislator.

Following exactly the same arguments as in case 1 only two types of positive individual offers can be made in this case: type \(b^k_h\) and \(b^0_0\). Type \(b^k_h\) might be included to avoid granting the same amount \(b^0_0\) to all legislators and thus create a conflict of interest.

Case 3: \(b^0_0\) is the pivotal legislator.

Following exactly the same arguments as in case 1 only two types of positive individual offers can be made in this case: type \(b^0_0\) and \(b^0_0\). Note that even if type \(b^0_0\) does not change the outcome of the vote we cannot exclude the individual offer in equilibrium as legislators in this group serve a purpose in revealing their signal.

We have shown that all proposals of type II will contain only two groups of legislators receiving strictly positive offers. □

Proposition 4.

The goal of this proof is to provide a specific example of a proposal of type II that will be preferred by the agenda setter to all proposals of type I in large legislatures.

In large legislatures, with proposals of type I, an infinite amount of information is revealed and legislators will have perfect information about the state. Therefore there are only 2 possible proposals of type I:

(a) One such that the legislator is indifferent between voting for or against when the state is low: he gets \(x_i\) such that \(x_iS_L = \frac{s}{n}\).

Therefore the expected utility of the agenda setter in this case is

\[
U = \frac{nS_H + (1 - \pi)S_L}{nM_s + (1 - \pi)S_L} \left(1 - \frac{s}{n}\right).
\]

(b) The same when the state is high: the legislator obtains \(x_i\) such that \(x_iS_H = \frac{s}{n}\). The proposal passes only if the state is high and therefore:

\[
U_H = \frac{nS_H}{nM_s - \pi}\left(1 - \frac{s}{n}\right).
\]

We now propose a specific proposal of type II. Consider the equilibrium where \(M=6\) legislators are offered a share \(b^0_i\) such that they vote in favor if no positive signal is revealed and they get a signal of one or if a signal of one is revealed and they get a signal of zero and where the pivotal group, composed of six members, vote in favor if one positive signal is revealed, i.e. get offer \(b^0_i\), and truthfully reveal their signal.

We need first of all to calculate \(b^0_i\).

The legislator is indifferent between voting for or against in the case where he receives signal one and all six legislators reveal during the debate signals zero. Therefore \(b^0_i\) is such that:

\[
\left[P_{posS_H} + (1 - P_{pos})S_L\right]|b^0_i = \frac{s}{n}\text{ where }P_{pos}\text{ is the probability that the state is high given six signals of zero and one signal of one.}
\]

Furthermore, the proposal will only pass if at least one positive signal is obtained. Therefore, we can determine the expected utility of the agenda setter in a large legislature\(^{38}\)

\[
U_{alt} = \left[\frac{nS_H}{nM_s - \pi} \left(1 - (1 - P_{pos})^6\right) + (1 - \pi)S_L (1 - P_{pos})^6\right] * \left[1 - Mb^0_i\right].
\]

We take specific values: \(p = 0.51, \pi = 0.2, s_L = 0.5, s_H = 2\) and \(m = 0.4\). We find that \(U_L = 0.16, U_H = 0.32\) and \(U_{alt} = 0.37\). Therefore in this specific example, the optimal choice for the agenda setter will be a proposal of type II. □

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\(^{38}\) For a large legislature we can approximate the share lost by the agenda setter by \(mc\times b_i\) the share given to the six pivotal voters is negligible.
Table 2
Quality of information; \( p = 0.85, m = 0.5 \)

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(1) Size of the legislature.
(2) Size of the “excess coalition”.
(3) Number of legislators voting non-informatively.
(4) Number of legislators voting informatively.
(5) Probability of making the correct decision.

References