Empirical relevance of a variable job destruction rate

- Large consensus built around the empirical findings by Davis and Haltiwanger and others (until the early 2000s)
  - Job destruction rate variable ($\leftrightarrow$ assumed constant $\lambda$)
  - Spikes at cyclical downturns and more variable than job creation
  - Most job destruction due to idiosyncratic shocks
  - Job destruction has persistence
  - Cyclical fluctuations in unemployment are due to variations in job destruction (separation)

- Shimer (2007) finds the opposite result (see 2 Lecture 7)
  - 75% of cyclical variation in unemployment between 1948-2007 due to fluctuations in job creation (job finding rate)
  - During the 1991 and 2001 recessions this number was 95%

- In any case: job destruction is variable over the cycle, spikes at downturns, and contributes to the fluctuation of unemployment
Job destruction vs job creation

- **job creation**: the attaching rate in a market with frictions
  - JC is slow due to frictions
  - cost: the cost of matching (mostly time)
- **job destruction**: the result of an exogenous real productivity shock that hits the firm
  - JD sudden, not suffering from frictions
  - no costs
- **new model**: similar to Lecture 8, but the idiosyncratic shocks have a non-trivial distribution
  → additional firm decision: when to destroy a job depending on the realized productivity shock and the expectations about future shocks
Productivity and timing

\[ V = 0 \rightarrow J(x), W(x) \rightarrow J(x'), W(x')J(x''), W(x'') \lambda \rightarrow px'' \]

- \( p > 0 \) general productivity
- \( x \) idiosyncratic productivity, \( x \sim G(x), x \in [0, 1] \)
- firm can choose \( x \) when creating the job \( \rightarrow x = 1 \) at creation
- after JC at rate \( \lambda \) idiosyncratic shocks hit the job, and the productivity \( x \) changes: \( x \rightarrow x' \)
- firm decision: keep job at new productivity \( x' \) or destroy it
- \( \lambda \in (0, \infty) \rightarrow \) process has persistence, but no memory
Job destruction

- if a prod shock hits the job, the firm and worker have to accept that level of prod
- worker and firm bargain over the new surplus
- value of job depends on its productivity: $J(x)$ intuitively $J'(x) > 0$
- firm destroys the job if $J(x) < 0$
- optimal decision rule for firm: reservation productivity: $J(R_f) = 0$
after productivity shock * if $x < R_f$ destroys
  * if $x \geq R_f$ wants to continue
- is $R_f$ just enough to cover the costs of the worker and the variable costs of the firm?
  NO: jobs have an option value → labor hoarding
Job destruction

- similarly for a worker the value of working depends on productivity: \( W(x) \)
  intuitively \( W'(x) > 0 \)
- keeps working if \( W(x) > U \)
- optimal decision rule: reservation productivity: \( W(R_w) = U \)
  after productivity shock
  * if \( x < R_w \) quits
  * if \( x \geq R_w \) wants to continue

For the match (job) to continue, both firm and worker have to want to continue

\[
R \equiv \max\{R_f, R_w\}
\]

Under the Nash division of the surplus, the worker’s share is

\[
W(x) - U = \beta(J(x) + W(x) - U - V)
\]

therefore \( R_f = R_w \)
Equilibrium and solution method

the new feature of the model requires

- an optimal decision rule for job destruction, $J(R) = 0 \rightarrow$ reservation productivity, $R$ as a function of $\theta$

the equilibrium consists of the value equations for each job productivity (for employer, for worker, for vacancy, for unemployed)

- the job destruction condition, $J(R) = 0$: $R$ as a function of $\theta$ (instead of the wage equation)
- the job creation condition, $V = 0$: $\theta$ as a function of $R$ (instead of $w$)
- the equation for the evolution of the unemployed

$\Rightarrow$ the driver of the economy is the firm, determines $\theta, R$

$w(x)$ is the Nash solution to the wage bargaining, same sharing rule for every $x$
Value equations for continuing matches

Value of a continuing match for the employer for any $x$

$$rJ(x) = px - w(x) + \lambda \left( -J(x) + G(R) \cdot 0 + \int_R^1 J(z)dG(z) \right)$$

$$= px - w(x) + \lambda \int_R^1 [J(z) - J(x)]dG(z) - \lambda G(R)J(x)$$

Value of a continuing match for the worker for any $x$

$$rW(x) = w(x) + \lambda \left( -W(x) + G(R)U + \int_R^1 W(z)dG(z) \right)$$

$$= w(x) + \lambda \int_R^1 [W(z) - W(x)]dG(z) + \lambda G(R)[U - W(x)]$$
Value equations for vacancies and the unemployed

New jobs are created with $x = 1$ the value of a vacancy solves:

$$rV = -pc + q(\theta)(J(1) - V) \quad \text{and} \quad V = 0 \Rightarrow J(1) = \frac{pc}{q(\theta)}$$

Similarly the value of unemployment solves

$$rU = b + \theta q(\theta)(W(1) - U)$$

The initial value of a match for the employer

$$rJ(1) = p - w(1) + \lambda \int_R^1 [W(z) - W(1)]dG(z) + \lambda G(R)[U - W(1)]$$

and for the workers

$$rW(1) = w(1) + \lambda \int_R^1 [J(z) - J(1)]dG(z) - \lambda G(R)J(1)$$
Wage determination

there is costless continuous renegotiation, the Nash bargaining solves:

\[ w(x) = \arg \max (W(x) - U)^{\beta} (J(x) - V)^{1-\beta} \quad \text{where} \quad \beta \in (0, 1) \]

The first order condition is:

\[ \beta (J(x) - V) = (1 - \beta) (W(x) - U) \]

This sharing rule holds for all \( x \in [0, 1] \), hence

\[ \beta (J(1) - V) = (1 - \beta) (W(1) - U) \]

Using this and the expression for \( J(1) \) in the value of the unemployed:

\[ rU = b + \theta q(\theta) (W(1) - U) = b + \theta q(\theta) \frac{\beta}{1 - \beta} (J(1) - V) \]

\[ = b + \theta q(\theta) \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)} = b + \frac{\beta}{1 - \beta} pc\theta \]
Wage determination

Now taking $\beta rJ(x) - (1 - \beta)rW(x)$

$$r(\beta J(x) - (1 - \beta)W(x)) = \beta px - w(x) - \lambda (\beta J(x) - (1 - \beta)W(x))$$

$$+ \lambda \left( G(R)(\beta \cdot 0 - (1 - \beta)U) + \int_1^1 \beta J(z) - (1 - \beta)W(z)dG(z) \right)$$

$$r(\beta V - (1 - \beta)U) = \beta px - w(x) - \lambda (\beta V - (1 - \beta)U)$$

$$+ \lambda (G(R)(\beta V - (1 - \beta)U) + (1 - G(R))(\beta V - (1 - \beta)U))$$

$$w(x) = \beta px + (1 - \beta)rU = \beta px + (1 - \beta) \left( b + \frac{\beta}{1 - \beta} pc\theta \right)$$

$$w(x) = (1 - \beta)b + \beta p(x + c\theta)$$

generalization of previous wage equation for productivity level $x$

* only outside effect is through the market tightness
* equilibrium $\theta$ depends on the distribution of productivities
The job destruction condition

The reservation productivity, $R$, is jointly rational and solves:

$$J(R) - V + W(R) - U = 0 \Rightarrow J(R) = 0 \text{ and } W(R) = U$$

Using the wage equation in the value of a job with productivity $x$

$$(r + \lambda)J(x) = px - w(x) + \lambda \int_{1}^{R} J(z)dG(z)$$

$$= (1 - \beta)(px - b) - \beta pc\theta + \lambda \int_{1}^{R} J(z)dG(z)$$

Evaluate the above for $x = 0$ and subtract it from its value at $x$:

$$(r + \lambda)(J(x) - J(R)) = (1 - \beta)p(x - R) \Rightarrow J(x) = \frac{(1 - \beta)p}{(r + \lambda)}(x - R)$$
The job destruction condition

Substitute the above for \( J(z) \) in the integral in the expression for the value of a continuing match at productivity \( R \):

\[
(r + \lambda)J(R) = (1 - \beta)(pR - b) - \beta pc\theta + \frac{\lambda(1 - \beta)p}{r + \lambda} \int_R^1 (z - R)dG(z)
\]

\[
\frac{b}{p} + \frac{\beta}{1 - \beta} c\theta = R + \frac{\lambda}{r + \lambda} \int_R^1 (z - R)dG(z)
\]

The job destruction condition (instead of the wage equation):

\[
\sqrt{\text{productivity}} \underbrace{\frac{\lambda}{r + \lambda} \int_R^1 (z - R)dG(z)}_{\text{option value}} = \underbrace{\frac{b}{p} + \frac{\beta}{1 - \beta} c\theta}_{\frac{rU}{p}, \text{worker’s outside option}}
\]

The reservation productivity is lower than the reservation wage for the unemployed, as there is a positive option value in occupied jobs. \( \Rightarrow \) labor hoarding
The job creation condition

Again the value of a continuing job:

\[(r + \lambda)J(x) = (1 - \beta)(px - b) - \beta pc\theta + \lambda \int_{R}^{1} J(z)dG(z)\]

Evaluate the above for \(x = 0\) and subtract it from its value at \(x = 1\):

\[(r + \lambda)(J(1) - J(R)) = (1 - \beta)p(1 - R) \iff J(1) = (1 - \beta)p \frac{1 - R}{r + \lambda}\]

The job creation condition as before:

\[J(1) = \frac{pc}{q(\theta)} \quad \text{and} \quad J(1) = (1 - \beta)p \frac{1 - R}{r + \lambda} \Rightarrow (1 - \beta) \frac{1 - R}{r + \lambda} = \frac{c}{q(\theta)}\]

firm’s share of the expected net surplus of a new job = expected recruiting cost
Equilibrium job creation and job destruction

\[
R + \frac{\lambda}{(r + \lambda)} \int_R^1 (z - R) dG(z) = \frac{b}{p} + \frac{\beta}{1 - \beta} c \theta
\]

job creation

\[
(1 - \beta) \frac{1 - R}{r + \lambda} = \frac{c}{q(\theta)}
\]

job destruction
Labor market equilibrium

The job creation rate (vacancies over employment):

\[ JCR = \theta q(\theta) \frac{u}{1 - u} \]

The job destruction rate (job destruction over employment):

\[ JDR = \lambda G(R) \]

The evolution of unemployment:

\[ \dot{u} = \lambda (1 - u) G(R) - m(u, v) = \lambda G(R) - (\lambda G(R) + \theta q(\theta))u \]

The unique steady state gives the Beveridge curve

\[ u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)} \quad \Leftrightarrow \quad \theta q(\theta) \frac{u}{1 - u} = \lambda G(R) \]
Equilibrium vacancies and unemployment

\[
JC, \quad v = \theta^* u
\]

\[
BC, \quad \dot{u} = 0
\]

given \( R^* \) the Beveridge curve is

\[
\frac{v}{u} q \left( \frac{v}{u} \right) \frac{u}{1-u} = \lambda G(R^*)
\]
The effects of general productivity shocks

- the job creation and job destruction conditions hold at all times, \( V = 0 \) and \( J(R) = 0 \) always
- \( \theta \) and \( R \) are jump variables, they instantaneously adjust
- \( u \), the state variable adjusts slowly

Positive productivity shock: \( p \uparrow \)

- employment becomes more profitable: \( \theta \uparrow, R \downarrow \)
  profits and wages rise with \( p \), but profits rise more, as wages are held back by the constant \( b \) ⇒ firm can afford to keep less productive workers
- for a given \( u \) JCR rises and JDR falls
- → \( u \) begins to fall, JDR unaffected, but JCR falls with \( u \)
- equilibrium is reached when JCR falls to the new (lower) level of JDR
The effects of general productivity shocks

negative productivity shock: \( p \downarrow \)

- employment becomes less profitable: \( \theta \downarrow, R \uparrow \)
  profits and wages fall with \( p \), but profits fall more, as wages are kept up by the constant \( b \) \( \Rightarrow \) firm has to lay off more productive workers

- for a given \( u \) JCR falls, and JDR spikes (overshoots)
  there is a mass of jobs between the old and new reservation productivities which are destroyed at the onset of the recession
  once these jobs are destroyed JDR settles at a permanently higher level

- as \( u \) starts to increase JCR starts to increase

- equilibrium is reached when JCR rises to the new (higher) level of JDR
Path of JDR and JCR

- for high frequency shocks the correlation between JCR and JDR is negative
  ~ right after productivity shocks
- for low frequency shocks the correlation between JCR and JDR is positive
  ~ as in the long run
Shimer (2005): the DMP model with exogenous job destruction lacks some amplification mechanism with plausible labor productivity shocks, it cannot generate the observed BC fluctuations in unemployment or labor market tightness

start from the JCR and combine it with the wage equation

\[ (r + \lambda) \frac{c}{q(\theta)} = p - w = (1 - \beta)(p - b) - \beta c \theta \]

let \( m = m + 0u^\eta v^{1-\eta} \), so \( q(\theta) = m_0 \theta^{-\eta} \)

what is the variance of \( \theta \) as \( p \) varies?
Volatility of $\theta$ and $p$

in the data

$$\frac{SD(\log \theta)}{SD(\log p)} = \frac{0.384}{0.021} \approx 18$$

given a stochastic process for $p$ the stochastic process for $\theta$ can be derived from the model

as $\theta$ reacts very quickly, a good approximation to the ratio of the standard errors of the log of $\theta$ to the log of $p$ is the elasticity of $\theta$ with respect to $p$

in the model (with reasonable parameter estimates)

$$\varepsilon_\theta \equiv \frac{\partial \theta}{\partial p} \frac{p}{\theta} = \frac{(1 - \beta)p}{r + \lambda \frac{m_0}{c \eta \theta^n} + \beta c \theta} \ll 18$$
the basic model indeed has a weak amplification mechanism of productivity shocks:

following an increase in productivity

- the higher job-worker match surplus leads firms to post more vacancies
- higher number of posted vacancies reduces the duration of unemployment ⇒ upward pressures on the wage
- in a reasonably calibrated version of the model, the wage absorbs virtually all of the productivity increase
- as a result, a productivity shock barely affects unemployment very large productivity shocks are needed to account for the magnitude of unemployment fluctuations
Possible solutions

1. endogenous job destruction could contribute to unemployment fluctuations
2. wage stickiness
Endogenous job destruction

\[ \Delta u_t = (1 - u_t)s_t - f_t u_t \iff u_t = \frac{s_t + \Delta u_t}{s_t + f_t} \]

the volatility in \( \Delta u_t \) contributes little to the volatility of \( u_t \), so let's simplify

\[ u_t = \frac{s_t}{s_t + f_t} \]

in a model with both exogenous and endogenous job destruction

\[ u = \frac{s + \lambda G(R)}{s + \lambda G(R) + \theta q(\theta)} \]

quantitatively we cannot say much about the importance of endogenous job destruction, as we have no data about the shape of the distribution \( G \)
Endogenous job destruction

in recessions the BC shifts out (↔ with exogenous JD, BC does not shift)

▶ → increases the fluctuations in $u$
▶ → decreases the fluctuations in $v$

⇒ lose the very tight negative relationship between $u$ and $v$

⇒ if we match volatility of $u$, we do even worse on the volatility of $\theta$
The role of wages

the wage equation:
\[ w = (1 - \beta)b + \beta(p + c\theta) \]

▶ continuous Nash bargaining
▶ response of wages to productivity shocks is almost proportional (\( \approx 1 \))

this is at odds with the data, which shows substantial wage stickiness \( \rightarrow \) modification of the model

1. the model can accommodate substantial wage stickiness without violating rationality
2. even small amounts of wage stickiness can amplify the response of job creation to productivity shocks
The role of wages

from

\[(r + \lambda) \frac{c}{q(\theta)} = p - w\]

we get that

\[\varepsilon_\theta = \frac{1}{\eta} \frac{p - \varepsilon_w w}{p - w}\]

▶ if \(\varepsilon_w = 1\), then \(\varepsilon_\theta = 1/\eta = 2\), the Shimer critique

▶ by reducing \(\varepsilon_w\), even a little, we increase \(\varepsilon_\theta\) also through the change of equilibrium \(w\) relative to \(p\)
The role of wages

If in some initial equilibrium the wage equation:

\[ w = (1 - \beta)b + \beta(p + c\theta) \]

then \( W > U \) and \( J > 0 \).

We also know that

\[
W - U = \frac{w - z}{r + \lambda + m_0 \theta^{1 - \eta}}
\]

\[
J = \frac{p - w}{r + \lambda + m_0 \theta^{-\eta}}
\]

so feasibility requires

\[ p \geq w \geq z \]
The role of wages

Hall (2005) argues that

- the feasible range for $w$ is sufficiently large
- given the observed fluctuations in $p$ and $\theta$, $w$ can stay fixed over the cycle
- with fixed $w$ the model predicts the full fluctuation in $\theta$
- continuing vs new matches
  non-responsiveness of wages in continuing matches makes sense → they do not violate rationality → they could create wage norms
  new matches could then simply adhere to the existing wage norms
The role of wages

Pissarides (2008):

- whether wages are sticky is an empirical issue
- individual panel data suggests that wages in new matches are as volatile as productivity
- it is the wage in the new match that is relevant for job creation
- a plausible explanation for the volatility of $\theta$ must imply volatility in wages of new matches
Competitive search and efficiency

» imagine that the labor market is divided into segments
» in each segment there is a market-maker who places workers to firms
» the market-maker posts a pair \((w_i, \theta_i)\)
  \(\rightarrow\) offers matches to firms at rate \(q(\theta_i)\)
  \(\rightarrow\) offers matches to workers at rate \(\theta_i q(\theta_i)\)
  \(\rightarrow\) for a realized match the wage is \(w_i\) until destruction of the job at rate \(\lambda\)
» free entry: anyone can open a segment
» in equilibrium: all surviving markets have to offer the same \(U_i\) to searching workers and the same \(V_i\) to hiring firms
Competitive search and efficiency

The optimal solution for each market-maker can be obtained from the dual

$$\max_{w_i, \theta_i} U_i \quad \text{s.t.} \quad V_i \geq 0$$

or

$$\max_{w_i, \theta_i} V_i \quad \text{s.t.} \quad U_i \geq \bar{U}$$

where $\bar{U} \equiv \max_i U_i$. The value equations are as before:

$$rV_i = -pc + q(\theta_i)(J_i - V_i)$$
$$rJ_i = p - w_i - \lambda J_i$$
$$rU_i = b + \theta_i q(\theta_i)(W_i - U_i)$$
$$rW_i = w_i - \lambda (W_i - U_i)$$
Competitive search and efficiency

Maximization yields:

$$(1 - \eta)(W - U) = \eta(J - V)$$

→ like the Nash sharing rule, but instead of $\beta$ we have $\eta$

▷ with the Nash sharing rule the firm and the worker decide the wage after they meet

▷ → it is unlikely that they internalize the search externalities they do not take into account the effect of their choices on the transition rates of unmatched agents

▷ in equilibrium $\theta$ is a function of $w$, if the matching function is CRS, then there is a unique internalizing rule, which requires $\beta = \eta$, the solution of this wage posting model – Hosios condition

▷ assumption that gives efficiency: workers know both the wage offer and the length of the queue associated with it → relaxation of informational restrictions on workers