Macroeconomics 2
Lecture 8 - Labor markets: The search and matching model

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Last week

- an overview of some labor market facts
- a brief reminder of dynamic programming
- we covered the search model
  - unemployed worker gets job offers sequentially
  - the offers arrive at exogenous rate $a$
  - from exogenous, stationary wage distribution $F(w)$
  - the unemployed has to decide whether to take a job offer or not
  - we concluded that the optimal decision is a reservation wage rule, such that $W(\xi) = U$
- $\xi$ solves

$$
\xi = \frac{r}{r + a(1 - F(\xi))}b + \frac{a}{r + a(1 - F(\xi))} \int_{\xi}^{A} wdF(w)
$$

→ depends on $r$, $a$, $b$ and the distribution $F(w)$
The one-sided search model is a partial equilibrium model

- where does the arrival rate, \(a\), come from?
  - the arrival rate of job offers should depend on the number of vacancies firms create
  - this should be the result of a profit maximizing decision

- where does the wage distribution, \(F(w)\), come from?
  wages are endogenous, different views of what determines them:
  
  - wage posting: firms post wages and commit to it
    - choose wages to maximize profits
    - competitive search models
    - efficiency wages
  
  - wage bargaining: wage determined by the meeting worker-firm pair after contact was made
The Rothschild and Diamond paradoxes

in the one-sided search model of last week – exogenous, non-degenerate wage distribution, $F(w)$

→ where does this come from?

One option: this is the result of profit maximizing wage posting by firms

Paradox: a non-degenerate wage distribution cannot be the result of profit maximizing behavior by firms
The Rothschild paradox

- somebody must be offering each wage in the support of the distribution
- critique: it is difficult to rationalize the distribution $F(w)$ as a result from profit maximizing choices of firms
- workers search sequentially
- they discover the firm’s wage offer when they meet
- worker has to accept/reject before he can sample another firm
- worker’s optimal decision: accept any wage s.t.
  $$W(w) \geq U \iff w \geq \xi$$
- firm’s objective: hire workers at the lowest wage possible
- profit maximization then leads to a common wage offer $w = \xi$
The Diamond paradox

- Diamond goes even further (chronologically he was first)
- suppose that all firms offer jobs at wage, $w$
- now suppose that a single firm deviates, and offers $w - \varepsilon$
- the unemployed who gets the offer $w - \varepsilon$ will accept it
- the reservation wage in this case is

$$\xi = \frac{r}{r + a(1 - F(\xi))} b + \frac{a}{r + a(1 - F(\xi))} \int_{\xi}^{A} wdF(w)$$

$$= \frac{rb + aw}{r + a} \leq w \quad \text{with} \quad = \text{if} \ w = b$$

- the firm has the power to set wages
  $\Rightarrow$ extracts all the surplus from job creation: $w = b$
- if there are out-of-pocket search costs, the market breaks down
Resolving the Diamond paradox

in several situation the Diamond paradox disappears

- competitive search models
  - let worker have more than one offer
  - give the worker more ex ante information
- efficiency wage models
  workers have incentives to work harder when wages are harder
- models with wage distribution
  - workers search on the job → the worker can have at one time two wage offers (current wage and new offer)
  - assume that workers have different search costs
  - allow for ex post wage heterogeneity
Second generation models
Two-sided matching

to close the one-sided search model, to derive a proper equilibrium of the economy, we need to

- make the arrival rate, $a$, endogenous
  - specify decision of firms whether or not to offer jobs

- ensure that employment is not an absorbing state
  - exogenous job destruction, at rate $\lambda$
  
interpretation: negative shocks arrive to existing matches, that destroy the match
  - the worker becomes unemployed, the job is destroyed

- the wage is the result of bargaining between the matched worker and firm
  
assume for now: single wage offer, which satisfies $w \geq \xi$
  - all job offers are accepted, outflow rate from unemployment is $a$
Value of employed and unemployed

- The value of an unemployed worker is (as before)
  \[ r_U = b + a(W - U) \]

- Using that all jobs pay the same wage rate, \( w \)
  \[ \rightarrow \] nobody has an incentive to quit or to search for another job while employed

- The value of an employed worker is
  \[ r_W = w - \lambda(W - U) \]

Combining the above two and rearranging:

\[ W - U = \frac{w - b}{r + a + \lambda} \]
The matching function

key assumption: the aggregate flow satisfies and aggregate matching function

- black box: gives the outcome of the matching process as a function of the inputs into the search process
- \( m = m(u, v) \) – matches
- assumptions on \( m(\cdot, \cdot) \)
  - continuous, differentiable
  - positive first partial derivatives
  - negative second partial derivatives
  - CRS
- uncoordinated random search implies the following matching function
  \[ m = v \left( 1 - e^{-\frac{ku}{v}} \right), \text{ where } k > 0 \]
- the empirical literature found that a Cobb-Douglas function matches the data well
  \[ m = Au^\eta v^{1-\eta} \]
The matching function

job matching is pairwise ⇒

\[ m = au = qv \]

- arrival rate of workers to vacant jobs

\[ q = \frac{m(u, v)}{v} = m \left( \frac{u}{v}, 1 \right) \equiv m(\theta^{-1}, 1) \equiv q(\theta) \]

- \( q \) is a decreasing function of \( \theta \): \( q'(\theta) < 0 \)
- the elasticity of \( q \) wrt \( \theta \) is \( \frac{\partial q}{\partial \theta} \frac{\theta}{q} \equiv -\eta \in (-1, 0) \)

- arrival rate of jobs to workers

\[ a = \frac{m(u, v)}{u} = m \left( 1, \frac{v}{u} \right) = a(\theta) = \theta q(\theta) \]

- \( a \) is an increasing function of \( \theta \): \( a'(\theta) > 0 \)
- the elasticity of \( a \) wrt \( \theta \) is \( \frac{\partial a}{\partial \theta} \frac{\theta}{a} \in (0, 1) \)
Market tightness

- $\theta = \frac{\nu}{u}$ is a measure of **market tightness**
- $u$ is a state
  - $\nu$ is the firm’s control $\rightarrow$ this drives unemployment
- however, probably both firms and workers ignore their effect on $\theta$ when they make their search choices $\rightarrow$ **search externalities**
Equilibrium

Things you need to know

- derivation of the aggregate flows from individual transitions
- derivation of individual transitions from optimizing decisions under rational expectations about the constraints and the future path of variables
- monopoly rents and wage determination
- derivation of the dynamics of the stock of employment and unemployment from the aggregate flows
- the equilibrium rate of unemployment (also called the natural rate)
Timing of decisions

1. Firm posts vacancy
   ▶ the firms post vacancies until there are rents to be made
   ▶ the value of a vacancy in equilibrium has to be zero
   ⇔ $V = 0$
   ▶ new jobs produce with the best technology

2. Worker arrives
   ▶ wage bargaining takes place
   ▶ acceptable to both parties
   ⇔ $W \geq U$ and $J \geq 0$
   ▶ job creation takes place if there is an agreement
   ▶ production begins

3. Idiosyncratic productivity shock arrives
   ▶ investment is irreversible
   if the shock reduces the net value of the job below zero
   ⇔ $J + W < U$
   ▶ job destruction
Timing of decisions

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   - the firms post vacancies until there are rents to be made
   - the value of a vacancy in equilibrium has to be zero $\iff V = 0$
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3. **idiosyncratic productivity shock arrives**
   - investment is irreversible
     - if the shock reduces the net value of the job below zero \( \iff J + W < U \)
   - **job destruction**
Constraints

- the distribution of productivities
- the processes that
  - bring together firms and workers – the matching function
    → affects unemployed people and vacant jobs
  - change the productivity of a job – the idiosyncratic shocks
    → affects a pair of employed person and a firm/job
Flow of people and jobs

\[ v, \text{ vacancies control value: } V \]
\[ u, \text{ unemployment state value: } U \]
\[ q(\theta) \]
\[ m(u, v) \]
\[ 1 - u, \text{ employment productivity } p \]
\[ \text{value for worker: } W \]
\[ \text{value for firm: } J \]
\[ \lambda \]
\[ \text{workers} \]
\[ \text{job destroyed } p = 0 \]
The value of vacancies and jobs

- \( V \) – value of a vacant job
- \( J \) – value of an occupied job
- A job is an asset owned by the firm, and its value is determined by arbitrage equations

\[
\begin{align*}
    rV &= -pc + q(\theta)(J - V) \\
    rJ &= p - w - \lambda J
\end{align*}
\]

- \( r \) – discount rate
- \( p \) – value of product, productivity \( \rightarrow \) higher value for better workers
- \( pc \) – cost of maintaining vacancy \( \rightarrow \) it is more costly to find people to fill skilled vacancies
- \( w \) – wage rate
Job creation

A firm creates a job vacancy when there are gains from entering the market and there is free entry.

⇒ zero profits, $V = 0$

$$rV = -pc + q(\theta)(J - V)$$

$$V = 0 \iff J = \frac{pc}{q(\theta)}$$

- $J$ – the value of having a worker = the PV(expected profits)
- $\frac{1}{q(\theta)}$ – the expected duration of a vacancy
- $\frac{pc}{q(\theta)}$ – the expected total cost of finding a worker
Job creation

the value of a filled job

\[ rJ = p - w - \lambda J \]

\[ J = \frac{p - w}{r + \lambda} \]

note: for the firm to accept a wage \( w \) \( \Leftrightarrow p \geq w \)

combining the two equations on the value of filled jobs:

\[ J = \frac{pc}{q(\theta)} \quad & \quad J = \frac{p - w}{r + \lambda} \]

\[ p - w - \left( \frac{(r + \lambda)pc}{q(\theta)} \right) = 0 \]

profit flow expected cost of finding a worker

**job creation condition**: generalization of the labor demand condition, downward sloping in the \( \theta - w \) space
The value of the unemployed and the employed

Human capital as an asset valued by arbitrage conditions

\[ rU = z + \theta q(\theta)(W - U) \]
\[ rW = w + \lambda(U - W) \]

- \( z \) – income during unemployment (previously \( b \))
- \( \theta q(\theta) = a(\theta) \) – arrival rate of jobs
- \( w \) – wage rate
Wage determination

How does the firm and employee divide the surplus?

▶ the worker does not get his marginal product because there are sunk costs (the search is costly, long) → matches enjoy some local monopoly rents due to the frictions

▶ wages are determined by a bargain usually by Nash bargaining

▶ net rent: $J + W - (U + V) \geq 0$ → is the room for bargaining → given this range, where do they fix wages?

▶ worker rent: $W - U$

▶ firm rent: $J - V$

▶ contract: initial wage and continuation wage both depend on job productivity and on outside conditions
Wage bargain

- $J$ – firm’s reward from agreement
- $V$ – firm’s payoffs when there is no agreement
- $W$ – worker’s rewards from agreement
- $U$ – worker’s payoff when there is no agreement

Assumption: wage rate maximizes the Nash product

$$(W - U)^{\beta}(J - V)^{1-\beta} \text{ where } \beta \in (0, 1)$$

Solution:

$$W - U = \beta(J + W - V - U)$$

$\beta$ share of the net (monopoly) rents go to the worker, which implies

$$w = (1 - \beta)z + \beta p(1 + c\theta)$$
this is a weighted average of the income (flow value) inside and outside the firm:

\[ w = (1 - \beta)z + \beta p(1 + c\theta) \]
Wage bargain

this is a weighted average of the income (flow value) inside and outside the firm:

\[ w = (1 - \beta)z + \beta p + \beta pc\theta \]

\[ = (1 - \beta) \left( z + \frac{\beta}{1 - \beta} pc\theta \right) + \beta p \]
Wage bargain

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\[ = (1 - \beta)\left( z + \frac{\beta}{1 - \beta} \frac{pc}{q(\theta)}q(\theta)\theta \right) + \beta p \]
Wage bargain

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\[ = (1 - \beta) \left( z + \frac{\beta}{1 - \beta} (J - V) q(\theta)\theta \right) + \beta p \]
Wage bargain

this is a weighted average of the income (flow value) inside and outside the firm:

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\[ = (1 - \beta) \left( z + \frac{\beta}{1 - \beta} (J - V)q(\theta)\theta \right) + \beta p \]
\[ = (1 - \beta) (z + (W - U)q(\theta)\theta) + \beta p \]
Wage bargain

this is a weighted average of the income (flow value) inside and outside the firm:

\[ w = (1 - \beta)z + \beta(p + c\theta) \]

\[ = (1 - \beta) \left( z + \frac{\beta}{1 - \beta} p c \theta \right) + \beta p \]

\[ = (1 - \beta) \left( z + \frac{\beta}{1 - \beta} \frac{p c}{q(\theta)} q(\theta) \theta \right) + \beta p \]

\[ = (1 - \beta) \left( z + \frac{\beta}{1 - \beta} (J - V) q(\theta) \theta \right) + \beta p \]

\[ = (1 - \beta)(z + (W - U)q(\theta)\theta) + \beta p \]

\[ = (1 - \beta)rU + \beta p \]
Equilibrium wages and market tightness

**wage curve** – bargained wage as a function of $\theta$

$$w = (1 - \beta)z + \beta p(1 + c\theta)$$

**job creation curve** – zero profit from vacancies, optimal $\theta$ as a function of $w$

$$p - w - \frac{(r + \lambda)pc}{q(\theta)} = 0$$
Equilibrium

in this economy the equilibrium is a path for

the controls:
- wages – from the wage equation: \( J \geq 0, W \geq U \)
- job vacancies (or tightness) – from job creation \( V = 0 \)

the state:
- employment – condition for the evolution of unemployment

\[
\dot{u} = \lambda(1 - u) - m(u, v) = \lambda - (\lambda + \theta q(\theta))u
\]

given \( \theta^* \), this has a unique stable solution, the natural rate of unemployment

\[
u = \frac{\lambda}{\lambda + \theta^* q(\theta^*)}
\]

the above is the **Beveridge curve**
Equilibrium tightness and unemployment

- $\theta$ is a control variable, its equilibrium is independent of $u$
- $u$ is a state variable, and it is stable
- the $\dot{\theta} = 0$ line is the saddle path
- $\theta$ jumps to its equilibrium value, and the economy moves along the saddle path to the steady state
Equilibrium vacancies and unemployment

\[ v = \theta^* u \]

- \( v \) is a control variable
- \( u \) is a state variable, and it is stable
- the \( v = \theta^* u \) line is the saddle path
- \( v \) jumps on the saddle path, and the economy moves along it to the steady state
The Beveridge curve in the US

the Beveridge curve shifted out in the great recession in the US

Source:
https://sites.google.com/site/robertshimer/
What caused the rightward shift of the BC?

- data: fewer hirings per job openings compared to historical data
  → decline in the matching efficiency?
- composition of labor demand changed:
  away from high labor turnover industries (construction) towards low labor turnover industries (engineering, medical care)
  → might seem like a decline in matching efficiency
- skill mismatch
- extended unemployment insurance benefits