Macroeconomics 2
Lecture 7 - Labor markets: Introduction & the search model

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The neoclassical model of the labor market

central question for macro and labor: what determines the level of employment and unemployment in the economy?

1. theoretically can not deal with unemployment
   ▶ there is supply and demand only
     demand determined by technology or by demand for output
     supply driven by inter-temporal substitution
   ▶ there can be *under-employment* due to wage stickiness, but there is no unemployment in equilibrium
   ▶ or unemployment as leisure
     ⇒ does not fit the statistical definition of unemployment

2. empirically can not explain fluctuations in employment
   ▶ shifting the demand curve according to the business cycle,
     employment and wage movements do not fit the data
Some facts about the labor market

- unemployment is a persistent phenomenon → can wage/price stickiness be the reason?
- large flows of workers between employment, unemployment and non-participation states

\[ \Delta u = \text{inflow} - \text{outflow} \]

- **inflow**: due to job loss or new entry from non-participation
- **outflow**: due to job finding or exit into non-participation (retirement, school, inactivity)

- employed workers often change jobs – with a wage gain or wage reduction
Search theory

- main postulate: there are frictions in the labor market
- source of frictions:
  - heterogeneity of workers and firms
  - imperfect information: invest resources in locating firm/worker
  - mobility costs

\[
\text{neoclassical model} \iff \text{search models}
\]

employment is a state, employment is a state,
a control variable flows are the controls
Why is search theory important?

→ Can we learn more about the macro equilibrium of the labor market by introducing frictions, studying the flows?
→ Is getting information about the stocks through the flows more useful than studying the stocks directly?

▶ Search theory provides a coherent, dynamic theory that can be used to analyze
  ▶ labor supply and demand in a more satisfactory way than in the perfectly competitive models
  ▶ the nature of unemployment, workers and vacancy flows
  ▶ the efficiency / optimality of the level of unemployment
  ▶ the distribution of workers’ wages
  ▶ the dynamic of workers’ wages depending on their labor market history
Why is search theory important?

- provides a strong theoretical background for quantitative questions
  - why was unemployment around 4-5% in the US until the 1970s?
  - why did it increase in the 1970s, 1980s, and then decline again in the late 1990s?
  - why did European unemployment increase in the 1970s, and remain high since then?
  - why is the composition of employment so different across countries?
    - male vs female, young vs old, high vs low wages
- useful tool for policy analysis
How should labor market frictions be modeled?

- incentive problems, efficiency wages
- wage rigidities, bargaining, non-market clearing prices
- search frictions

Search and matching: costly process for workers (firms) to find the right jobs (workers)

→ how do markets function without a Walrasian auctioneer?

→ for empirical analysis we need to develop a tractable and rich model
Facts about job flows

- job creation is mildly pro-cyclical
- job destruction is strongly counter-cyclical
- job destruction leads job creation. It is the driving force of the business cycle – especially in economies with flexible labor markets
- job creation seems to be the main cause of long-run changes in unemployment
Facts about worker flows

- worker turnover about three times as large as job turnover
- worker quits are strongly pro-cyclical
  ⇔ offset by the counter-cyclical job destruction rate
  recession → job destruction increases
    → voluntary quitting decreases
    ⇒ inflow to unemployment increases, but less
- unemployment changes driven mainly by the outflow from unemployment
- in monthly data: employment ↔ non-participation flows ≈ employment ↔ unemployment flows
Key labor market statistics US

Source: Bureau of Labor statistics, monthly, seasonally adjusted data
Figure 6. Unemployment Inflow and Outflow Rates

Outflow Rate, $f$

Inflow Rate, $s$

Outflow rate (left axis)

Inflow rate (right axis)

Source: Bureau of Labor Statistics and authors' calculations. Quarterly averages of monthly data.

Source: Elsby, Hobijn, Sahin (2010), Figure 6.
Inflow and outflow in recessions

1. outflow rate from unemployment
   ▶ markedly pro-cyclical
   ▶ prolonged downswings during recessions

2. inflow into unemployment
   ▶ countercyclical
   ▶ sharp upswings at the onset of recessions, but tend to subside quickly by the end of the recessions

3. in the 1990 and 2001 (relatively mild) recessions, the increase in the inflow rates was relatively muted
**Figure 9. Unemployment Flows by Reason for Unemployment**

Source: Bureau of Labor Statistics and authors' calculations based on Elsby, Michaels, and Solon (2009)

Source: Elsby, Hobijn, Sahin (2010), Figure 9.
Shimer’s exercise

the change in the unemployment rate is

\[ u_{t+1} - u_t = s_t (1 - u_t) - f_t u_t \]

- \( s_t \) – separation rate
- \( f_t \) – job finding rate
- ignore exit from the labor force, and entry from out of labor force

denote average rates by:

\[ \bar{s} = \frac{1}{T} \sum_{t=1}^{T} s_t \quad \text{and} \quad \bar{f} = \frac{1}{T} \sum_{t=1}^{T} f_t \]
Shimer’s exercise

Compare the actual unemployment rate with

1. a hypothetical unemployment rate constructed using the average (a constant) separation rate:

\[ u_{t+1} - u_t = \bar{s}(1 - u_t) - f_t u_t \]

2. a hypothetical unemployment rate constructed using the average (a constant) job finding rate:

\[ u_{t+1} - u_t = s_t(1 - u_t) - \bar{f} u_t \]
The role of the job finding rate

holding the separation rate constant at $\bar{s}$
The role of the separation rate
holding the job finding rate constant at $\bar{f}$

Figure 2: Contribution of Fluctuations in the Job Finding and Employment Exit Rates to Fluctuations in the Unemployment Rate, 1948Q1–2007Q1, quarterly average of monthly data.

The job finding rate $f_t$ is constructed from unemployment and short term unemployment according to equation (4). The employment exit rate $x_t$ is constructed from employment, unemployment, and the job finding rate according to equation (5). The top panel shows the hypothetical unemployment rate if there were only fluctuations in the job finding rate, $\bar{x}/(\bar{x} + f_t)$, and the bottom panel shows the corresponding unemployment rate with only fluctuations in the employment exit rate, $x_t/(x_t + \bar{f})$. Both panels show the actual unemployment rate for comparison. Employment, unemployment, and short term unemployment data are constructed by the BLS from the CPS and seasonally adjusted. Short term unemployment data are adjusted for the 1994 CPS redesign as described in Appendix A.

Source: Shimer (2005)
Lessons from Shimer’s exercise

- separation rate not so important in the evolution of unemployment
- job finding rate is a more important determinant of unemployment
- why?
  separation rates do increase during recessions
  BUT the job finding rate is high in the US, even during recessions
  even if more workers get laid off, they find a job quickly → job separation rate not so important
But countries are different

![Figure 1.—Average Inflow and Outflow rates across countries](image)

Source: Elsby, Hobijn, Sahin (2013), Figure 1.
Shimer’s exercise for other countries

changes in unemployment are due to

- UK: 71% inflow rate, 29% outflow rate
  (Elsby, Smith, Wadsworth (2010))

- Spain: 57% inflow rate, 43% outflow rate
  (Petrongolo and Pissarides (2009))
An overview of search models

- **First generation: one-sided search**
  - focuses on the workers
  - exogenous wage distribution: $F(w)$
  - exogenous job arrival rate: $a$
  - worker’s optimal decision

- **Second generation: two-sided search**
  - endogenous job arrival
    - somebody has to create the job – active job creation by firms
  - matching function $m = m(u, v)$
    - $u$ – stock of unemployed, state variable
    - $v$ number of vacancies, control variable
  - arrival rate: $m(u, v)/u$
  - endogenous wage: reached through bargaining
    - → no wage distribution
  - value of match endogenous
  - exogenous job destruction, $\lambda$
Third generation
- endogenous job destruction
destroy job, if its productivity is not high enough $\rightarrow R$
reservation productivity
- match output from $G(x)$ distribution
  $\Rightarrow$ there is a wage distribution, which depends on $G(x)$

Fourth generation
- endogenous wage distribution
Short, informal introduction to dynamic programming

Consider the following consumption-saving problem:

\[
\sum_{t=0}^{\infty} \beta^t u(c_t)
\]

s.t. \( a_{t+1} = (1 + r)a_t - c_t \) for all \( t \geq 0 \)

\( a_{t+1} \geq 0 \) for all \( t \geq 0 \), for a given \( a_0 > 0 \)

direct approach: set up the Lagrangian, and find the two infinite optimal sequences \( \{a_{t+1}\}_{t=0}^{\infty} \) and \( \{c_t\}_{t=0}^{\infty} \)

dynamic programming approach: find a time-invariant policy function \( h \) mapping wealth at the beginning of period \( t \) into optimal consumption in period \( t \), s.t. the sequence \( \{c_t\}_{t=0}^{\infty} \) generated by iterating

\[
c_t = h(a_t)
\]

\( a_{t+1} = (1 + r)a_t - c_t \)

starting from \( a_0 \) solves the consumption-saving problem
Potential advantages of dynamic programming

while it is unclear that finding a policy function is easier than finding an infinite sequence, but it has three advantages:

1. sometimes we can find a closed-form solution for the policy function $h$

2. sometimes we can characterize theoretical properties of the policy function $h$

3. various numerical methods are available to solve dynamic programs
The value function

we first need to solve for an auxiliary function called the value function, \( V(a) \), which measures the optimal lifetime utility from consumption starting with an initial wealth \( a \):

\[
V(a_0) \equiv \max_{\{c_t,a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

such that for all \( t \geq 0 \)

\[
a_{t+1} = (1 + r)a_t - c_t \\
a_{t+1} \geq 0
\]

we need to determine the value function
To determine the value function:

- we show that the value function is the solution to a particular functional equation called the **Bellman equation**
  - note: not all optimization problems can be represented with a Bellman equation
- the problem must satisfy the **Principle of Optimality**
- which applies only to problems with a **recursive structure**

The value function can be expressed as:

$$V(a_0) = \max\{0 \leq c_t \leq (1+r)a_t\} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

such that for all $t \geq 0$

$$a_{t+1} = (1+r)a_t - c_t$$

let $\Gamma(a) \equiv [0, (1+r)a]$
Heuristic procedure

\[ V(a_0) = \max_{\{a_{t+1} \in \Gamma(a_t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u((1 + r)a_t - a_{t+1}) \]

\[ V(a_0) = \max_{\{a_{t+1} \in \Gamma(a_t)\}_{t=0}^{\infty}} \left[ u((1 + r)a_0 - a_1) + \beta \sum_{t=1}^{\infty} \beta^{t-1} u((1 + r)a_t - a_{t+1}) \right] \]

\[ V(a_0) = \max_{\{a_{t+1} \in \Gamma(a_t)\}_{t=0}^{\infty}} \left[ u((1 + r)a_0 - a_1) + \beta \sum_{t=0}^{\infty} \beta^t u((1 + r)a_{t+1} - a_{t+2}) \right] \]

\[ V(a_0) = \max_{a_1 \in \Gamma(a_0)} \left[ u((1 + r)a_0 - a_1) + \beta \sum_{t=0}^{\infty} \beta^t u((1 + r)a_{t+1} - a_{t+2}) \right] \]

\[ V(a_0) = \max_{a_1 \in \Gamma(a_0)} \left[ u((1 + r)a_0 - a_1) + \beta V(a_1) \right] \]

\[ V(a_0) = \max_{c_0 \in \Gamma(a_0)} \left[ u(c_0) + \beta V((1 + r)a_0 - c_0) \right] \]
The recursive formulation

- this heuristic procedure seems reasonable
- formally it is valid, because the Principle of Optimality applies to the problem
- the Principle of Optimality tells us that the value function defined before can be rewritten as

\[ V(a) = \max_{c \in [0, (1+r)a]} [u(c) + \beta V((1 + r)a - c)] \]

→ recursive formulation of the optimization problem
→ once we determine the value function \( V(a) \), we can solve for the optimal consumption level:

\[ c^* = \arg \max_{c \in [0, (1+r)a]} u(c) + \beta V((1 + r)a - c) \]

→ \( c^* \) is a function of \( a): h(a) \equiv c^* \]
Solving the optimization problem with dynamic programming

This has 5 steps:

1. write down the Bellman equation (the definition of the value function)
2. write down the first order conditions of the optimization problem
3. write down the Benveniste-Scheinkman equation (the envelope condition) to determine the derivative of the value function wrt wealth
4. apply the envelope condition to the next period’s value function and wealth
5. derive the Euler equation, which summarizes the optimal intertemporal behavior of consumption
Step 1: the Bellman equation

\[ V(a) = \max_{c \in [0,(1+r)a]} [u(c) + \beta V((1 + r)a - c)] \]

- a functional equation, the unknown is the function \( V \) itself
  - this is the Bellman equation, let’s assume for now that a solution to this problem exists
- \( a \) is a **state variable**: it contains all the information from the past that is needed to solve the forward-looking optimization problem
- \( c \) is a **control variable**: this is to be chosen in the current period
  - it determines the state variable’s next period value, \( a' \), according to the **transition equation**

\[ a' = (1 + r)a - c \]
Step 2: the FOC

first rewrite the Bellman equation → we choose $a'$ rather than $c$

$$V(a) = \max_{a' \in [0, (1+r)a]} [u((1 + r)a - a') + \beta V(a')]$$

→ this is useful for the envelope condition
→ let’s assume $V$ exists and is differentiable

The FOC wrt to $a'$:

$$\frac{dV}{da'}(a) = -\frac{\partial u}{\partial c}((1 + r)a - a') + \beta \frac{\partial V}{\partial a}(a') = 0$$

$$\frac{\partial u}{\partial c}(c) = \beta \frac{\partial V}{\partial a}(a')$$
Step 3 & 4: the Envelope Condition & one step forward

we need to determine $\frac{\partial V}{\partial a}$ for the first order condition

$$V(a) = \max_{a' \in [0, (1+r)a]} [u((1+r)a - a') + \beta V(a')]$$

$$\frac{\partial V}{\partial a}(a) = \frac{\partial u}{\partial c}(c) \left( (1+r) - \frac{da'}{da} \right) + \beta \frac{\partial V}{\partial a}(a') \frac{da'}{da}$$

$$= \frac{\partial u}{\partial c}(c)(1+r) + \frac{da'}{da} \left( -\frac{\partial u}{\partial c}(c) + \beta \frac{\partial V}{\partial a}(a') \right)$$

$$= 0 \text{ due to the FOC}$$

$$\frac{\partial V}{\partial a}(a) = \frac{\partial u}{\partial c}(c)(1+r)$$

this holds in all periods, so forwarding by one period:

$$\frac{\partial V}{\partial a}(a') = \frac{\partial u}{\partial c}(c')(1+r)$$
Step 5: the Euler equation

we plug

$$\frac{\partial V}{\partial a}(a') = \frac{\partial u}{\partial c}(c')(1 + r)$$

into

$$\frac{\partial u}{\partial c}(c) = \beta \frac{\partial V}{\partial a}(a')$$

to get

$$\frac{\partial u}{\partial c}(c) = \beta \frac{\partial u}{\partial c}(c')(1 + r)$$

$$u'(c) = \beta(1 + r)u'(c')$$

the optimization problem can thus be reduced to finding $h(a)$ for all $a$ such that:

$$\frac{\partial u}{\partial c}(h(a)) = \beta(1 + r)\frac{\partial u}{\partial c}(h((1 + r)a - h(a)))$$
First generation models
The search model

based on McCall partial equilibrium search model (1968)
- the simplest model of search frictions
- people get draws from a given wage distribution
- decision: which jobs to accept and when to start work
- jobs are sampled sequentially
Environment

- people are risk neutral $\rightarrow$ utility function?
- continuous time (very similar in discrete time)
- a worker’s objective is to maximize expected discounted lifetime utility
- every worker starts as unemployed
- all jobs are identical, except for their wages
  - wages are drawn (iid) from an exogenous stationary distribution $F(w)$
  - with bounded support $w \in [0, A]$
The search model

- an unemployed worker
  - receives income $b$ per unit of time
    - the value of leisure and home production activities
    - unemployment benefit
    - net of search costs
  - actively searches for a job
    → jobs arrive according to a Poisson process with rate $a > 0$
  - if a job offer arrives, the worker has to decide whether to accept it or not

let $U_t$ denote the expected present discounted value of a unit of unemployed labor at time $t$
- the value of an unemployed worker

let $W_t(w)$ be the value of an employed worker at wage $w$
- employment is an absorbing state: once accepts a job with wage $w$, remains employed at that wage forever
- alternative: allow for on-the-job search
Arrival rate – Poisson process

- in each period of length $\delta t$ a worker can potentially receive $n = 0, 1, 2, \ldots$ offers, with probability $a(n, \delta t)$
- let $N(t)$ denote the number of offers received by time $t \geq 0$, such that $N(0) = 0$
- $a(n, \delta t)$ follows a Poisson distribution:

$$a(n, \delta t) = \frac{e^{a\delta t}(a\delta t)^n}{n!}$$

- the number of offers received is independent across periods
- $\Rightarrow \{N(t), t \geq 0\}$ is a Poisson process with rate $a > 0$

$$P(N(\delta t) = 1) = a\delta t + O(\delta t)$$
$$P(N(\delta t) \geq 2) = O(\delta t)$$

A function $f$ is $O(\delta t)$ if $\lim_{\delta t \to 0} \frac{f(\delta t)}{\delta t} = 0$
The value of an unemployed worker

assuming

- infinite horizon
- constant discount rate, \( r \)
- the length of a period is \( \delta t \geq 0 \), but small

\( U_t \) satisfies the Bellman equation:

\[
U_t = \frac{b \delta t}{1 + r \delta t} + a \delta t \frac{\max(W_{t+\delta t}, U_{t+\delta t})}{1 + r \delta t} + (1 - a \delta t) \frac{U_{t+\delta t}}{1 + r \delta t}
\]

- very similar to the evaluation of an asset
- dividend yield
- option arrives \( \rightarrow \) expectation of the capital gain
- option does not arrive \( \rightarrow \) continuation value
The value of an unemployed worker

rearranging terms and dividing by $\delta t$

$$rU_t = b + a \left( \max(W_{t+\delta t}, U_{t+\delta t}) - U_{t+\delta t} \right) + \frac{U_{t+\delta t} - U_t}{\delta t}$$

taking the limit as $\delta t \to 0$ and omitting subscripts for convenience yields:

$$rU = b + a \left( \max(W, U) - U \right) + \dot{U}$$

this is an arbitrage equation for the valuation of human capital:

- investing the value at a safe return
- if you leave the asset in the labor market
  - flow return
  - expected capital gain from change of state
  - capital gains from changes in evaluation – in the steady state, $b, a, r$ are all constant + infinite horizon $\Rightarrow$ stationary solutions to the valuation equations, i.e. $\dot{U} = 0$
Closing the model

assume that

- employment is an *absorbing state*
- a worker earns forever the wage, \( w \), that he/she was hired at

\[
W = \frac{w}{r}
\]

and the stationary solution to \( U \) satisfies

\[
rU = b + a \left( \max \left( \frac{w}{r}, U \right) - U \right)
\]
When does the unemployed take a job?

\[ W \geq U \]
\[ \frac{w}{r} \geq U \]
\[ w \geq rU \]

⇒ optimal stopping rule: reservation wage, the minimum acceptable wage

\[ \xi \equiv rU \]

if you stay unemployed, you can never be worse off than \( rU \), so unless someone pays at least this, you won’t start working
For a known wage offer distribution

assume that offers arrive from a known wage distribution, $F(w)$

$$rU = b + a \left( \int_0^A \max \left( \frac{w}{r}, U \right) dF(w) - U \right)$$

wages under the reservation wage, $\xi = rU$ are rejected:

$$rU = b + a \int_{\xi}^A \left( \frac{w}{r} - U \right) dF(w)$$

collecting terms and solving for the reservation wage

$$\xi = \frac{r}{r + a(1 - F(\xi))} b + \frac{a}{r + a(1 - F(\xi))} \int_{\xi}^A wdF(w)$$

$a(1 - F(\xi))$ is the transition from unemployment to employment
On-the-job search

this is critical for understanding the difference between worker and job flows

assume that

- employed workers get job offers at rate $a_e$
- unemployed workers get job offers at rate $a_u$

new reservation rules for an offer of wage $w$

- unemployed, accept if $W(w) \geq U \rightarrow \xi$
- employed with wage $w_1$, accept if $W(w) \geq W(w_1) \rightarrow \xi(w_1)$
Value of the employed and the reservation wage

value of an employed worker with wage $w_1$

$$rW(w_1) = w_1 + a_e \int_{\xi(w_1)}^{A} (W(w) - W(w_1)) dF(w)$$

- $W'(w) > 0$
- for $w = w_1$, $W(W) = W(w_1)$

$\Rightarrow$ the reservation wage for on-the-job search is the current wage

$$\xi(w_1) = w_1$$

note the absence of job changing costs
let $\xi$ be the reservation wage if unemployed

$$rU = b + a_u \int_{\xi}^{A} (W(w) - U) \, dF(w)$$

the value of a job with wage $w = \xi$ is

$$rW(\xi) = \xi + a_e \int_{\xi}^{A} (W(w) - W(\xi)) \, dF(w)$$

note that $W(\xi) = U$, hence

$$\xi = b + (a_u - a_e) \int_{\xi}^{A} (W(w) - U) \, dF(w)$$
The reservation wage of the unemployed

\[ \xi = b + (a_u - a_e) \int_{\xi}^{A} (W(w) - U) \, dF(w) \]

the reservation wage for the unemployed depends on the arrival rates:

- if \( a_u = a_e \), then \( \xi = b \)
- if \( a_u > a_e \), then \( \xi > b \)
- if \( a_u < a_e \), then \( \xi < b \)

note that with on-the-job search there is a much bigger flow of workers than jobs

workers quit for a better job, and leave behind a vacancy, which is filled by someone further down the chain

\( \Rightarrow \) vacancy chains, with the unemployed at the bottom