A brief history of money in macro 1.

1. Hume: money has a wealth effect
   more money $\Rightarrow$ increase in aggregate demand $\Rightarrow$ $Y \uparrow$

2. Friedman - Schwartz: decline in the money supply is the source of the great depression

   money demand is the key to transmission:
   $M^s \uparrow \rightarrow r \downarrow$ for $M^d \uparrow \rightarrow Y \uparrow$, $C \uparrow$

4. econometrics
   empirical development: money leads output
   very consistent with IS-LM
   big consensus: IS-LM + data fit $\Rightarrow$ Tobin: macro is dead
A brief history of money in macro 2.

5. Barro: the theory that says money ⇒ expansion is observationally equivalent to FED competence, i.e. the economy will do well, so FED starts pushing money into the economy
⇒ data is not conclusive
+ theory does not make sense: it assumes price rigidity, 
\[ M^s \uparrow \nRightarrow \frac{M^s}{P} \uparrow \]

6. Friedman + Phelps + Lucas: \( M^s \uparrow \nRightarrow \frac{M^s}{P} \uparrow \), because \( P \uparrow \) so \( \frac{M^s}{P} \) does not change necessarily
skepticism of price rigidity (without is IS-LM collapses)

7. Lucas: island model with information asymmetries
people don’t know what a price change means: is it a relative price change or inflation?
as a producer sees the price of their product go up, they start to produce more, but if it went up due to inflation, then everybody produces more ⇒ expansion
8. Taylor, Fischer, Calvo
   micro-founded models of price and wage rigidity – restoration of old views

9. Romer and Romer: sometimes money supply change is truly exogenous, not responding to expectations, but a new policy looking at these episodes the strong response it still there (↔ Barro)

10. Sims and VAR: separation of endogenous and exogenous

11. put money into DSGE models

**Meanwhile:** tool of mon pol is short term interest rate (not $M^s$)
   empirical models, VAR, theory is focusing on FFR
Introducing money raises a lot of questions:

- why money? what kind of money?
- can there be competing currencies?
- is money a unit of account, a medium of exchange or a store of value?
- fiat money or commodity money?

most of the time: money is fiat money, it is used in transactions, and it is the numeraire and the medium of exchange at the same time
Taking the above as given many other questions arise:

- how different is the economy with money?
- what determines the demand for money, the price level and the nominal interest rate?
- does the presence of money affect the consumption/saving choice?
- how do changes in the growth rate of money affect real activity and inflation?
Output and money in the data

- long-held belief that money has big effects on output → money in macro
- in the data (Stock and Watson): inflation lags output, strong correlation
- money leads output, strong, positive correlation
- but what does this prove? Fed competence? ...
- Christiano, Eichenbaum and Evans: structural VAR estimate 1 % increase in FFR: long lasting effects on output, employment, unemployment, on price level only after 6 quarters
  what are exogenous monetary policy shocks?
The cash-in-advance model

benchmark model, abstract from labor/leisure choice and ignore uncertainty

consumers solve the following problem:

\[ \sum_{i=0}^{\infty} \beta^i U(C_{t+i}) \]

subject to

\[ P_t C_t + M_{t+1} + B_{t+1} + P_t K_{t+1} = W_t + \Pi_t + M_t + (1 + i_t)B_t + (1 + r_t)P_t K_t + X_t \quad \text{(BC)} \]

and

\[ P_t C_t \leq M_t + X_t \quad \text{(CIA)} \]
\[ P_tC_t + M_{t+1} + B_{t+1} + P_tK_{t+1} = \\
= W_t + \Pi_t + M_t + (1 + i_t)B_t + (1 + r_t)P_tK_t + X_t \]

- **\( P_t \)** the price level, i.e. the price of goods in terms of the numeraire
- **\( M_t, B_t, K_t \)** are holdings of money, bonds and capital at the start of period \( t \)
- **\( W_t, \Pi_t \)** are the nominal wage and the nominal profit of each household
- **\( i_t \)** is the nominal interest rate paid on bonds
- **\( r_t \)** is the gross real rental rate paid on capital (in terms of goods)
- note: money pays no interest → cost of holding money
- **\( X_t \)** the value of a nominal transfer paid by the government → this will be the method of increasing money supply, the extra printed money will go to the households – as a ”helicopter drop” of money
So we saw the cost of holding money. But what is the benefit of holding money?

- consumers only care about consumption
- i.e. they would not hold money unless they needed to, as money does not improve their utility, and as opposed to bonds and capital, which have a positive return, money does not
- but people must enter each period with enough nominal money supply to pay for consumption - **cash in advance** constraint

\[ P_t C_t \leq M_t + X_t \]

- why? – liquidity services of money
  * worker - consumer household
  * money balances reduce the costs of buying consumption goods
Solving the consumer’s problem

The Lagrangian can be written as:

\[ \mathcal{L} = \sum_{i=0}^{\infty} \beta^i U(C_{t+i}) \]

\[ - \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \left[ P_{t+i} C_{t+i} + M_{t+i+1} + B_{t+i+1} + P_{t+i} K_{t+i+1} \right. \]

\[ \left. - (W_{t+i} + \Pi_{t+i} + M_{t+i} + (1 + i_{t+i}) B_{t+i} + (1 + r_{t+i}) P_{t+i} K_{t+i} + X_{t+i}) \right] \]

\[ - \sum_{i=0}^{\infty} \beta^i \mu_{t+i} \left( P_{t+i} C_{t+i} - M_{t+i} - X_{t+i} \right) \]

the consumer chooses \( C_{t+i}, M_{t+i+1}, K_{t+i+1}, B_{t+i+1} \) for every \( i \geq 0 \)
The FOCs for the consumer in period $t$ (for $i = 0$) are:

\[ C_t : \quad U'(C_t) = (\lambda_t + \mu_t)P_t \]

\[ M_{t+1} : \quad \lambda_t = \beta(\lambda_{t+1} + \mu_{t+1}) \]

\[ B_{t+1} : \quad \lambda_t = \beta\lambda_{t+1}(1 + i_{t+1}) \]

\[ K_{t+1} : \quad \lambda_t P_t = \beta\lambda_{t+1}(1 + r_{t+1})P_{t+1} \]
Nominal and real interest rates

Combine the FOC of $B_{t+1}$ and $K_{t+1}$:

$$(1 + i_{t+1}) = (1 + r_{t+1}) \frac{P_{t+1}}{P_t}$$

$$= (1 + r_{t+1})(1 + \pi_{t+1})$$

where we defined the inflation rate as $1 + \pi_{t+1} = \frac{P_{t+1}}{P_t}$

as an approximation:

$$r_{t+1} \approx i_{t+1} - \pi_{t+1}$$
The Euler equation

Combine the FOC of $C_t$ and $M_{t+1}$:

$$\lambda_t = \beta \frac{U'(C_{t+1})}{P_{t+1}} \quad \lambda_{t+1} = \beta \frac{U'(C_{t+2})}{P_{t+2}}$$

Use these in the FOC of $B_{t+1}$:

$$\frac{U'(C_{t+1})}{P_{t+1}} = \beta (1 + i_{t+1}) \frac{U'(C_{t+2})}{P_{t+2}}$$

divide by $(1 + i_{t+1})$ and multiply by $P_{t+1}$:

$$\frac{U'(C_{t+1})}{1 + i_{t+1}} = \beta \frac{U'(C_{t+2})}{P_{t+2}} = \beta (1 + r_{t+2}) \frac{U'(C_{t+2})}{1 + i_{t+2}}$$
\[
\frac{U'(C_{t+1})}{1 + i_{t+1}} = \beta(1 + r_{t+2}) \frac{U'(C_{t+2})}{1 + i_{t+2}}
\]

- since people have to hold money one period in advance, the effective price of consumption is \((1 + i)\) rather than 1
- after adjusting for this price effect, it boils down to the same as before: marginal utility today has to equal marginal utility tomorrow, times the real interest rate and discounted
- both the value or real and nominal interest rates matter – latter mainly through the effective price
- if the nominal interest rate is constant \(\Rightarrow\) the equation reduces to the original Euler equation (except one period ahead - why?)
Money demand

from the FOC of $M_{t+1}$ and $B_{t+1}$:

$$\mu_{t+1} = i_{t+1} \lambda_{t+1}$$

if the interest rate, $i_{t+1} > 0$, then $i_{t+1} \lambda_{t+1} > 0$ holds as well $\Rightarrow$ $\mu_{t+1} > 0$ as well $\Rightarrow$ the CIA constraint is binding

$$\frac{M_t + X_t}{P_t} = C_t$$

pure quantity theory of money: the only reason to hold money is for transactions

note: no elasticity wrt the interest rate
Equilibrium

- firms:
  \[ P_t F_L(K_t, L_t) = W_t \quad F_K(K_t, L_t) = R_t \]
  where \( r_t = R_t - \delta \)
- as before, CRS and perfect competition \( \Rightarrow \Pi_t = 0 \)
- no labor/leisure choice \( \Rightarrow L_t = 1 \)
- \( X_t \) is the transfer to households from the government, the extra money printed:
  \[ M_{t+1} - M_t = X_t \]
- bonds are issued by households, closed economy, identical agents
  \[ B_{t+1} = B_t = 0 \]
The budget constraint of the household was:

\[ P_t C_t + M_{t+1} + B_{t+1} + P_t K_{t+1} = M_t + X_t = 0 \]

\[ = W_t + \prod_t + M_t + (1 + i_t) B_t + (1 + r_t) P_t K_t + X_t = 0 \]

which simplifies to

\[ P_t C_t + P_t K_{t+1} = W_t + (1 + r_t) P_t K_t \]

\[ = P_t F_L(K_t, 1) + (1 + F_K(K_t, 1) - \delta) P_t K_t \]

\[ = P_t F(K_t, 1) + (1 - \delta) P_t K_t \]

Divide by \( P_t \) and reorganize to get the usual capital accumulation equation:

\[ K_{t+1} = F(K_t, 1) + (1 - \delta) K_t - C_t \]
The equations that characterize the equilibrium:

\[(1 + i_{t+1}) = (1 + r_{t+1})(1 + \pi_{t+1})\]

\[\frac{U'(C_{t+1})}{1 + i_{t+1}} = \beta(1 + r_{t+2}) \frac{U'(C_{t+2})}{1 + i_{t+2}}\]

\[\frac{M_t + X_t}{P_t} = C_t\]

\[1 + r_t = 1 + F_K(K_t, 1) - \delta\]

\[K_{t+1} = F(K_t, 1) + (1 - \delta)K_t - C_t\]
The steady state of the model

- assume that the growth rate of money is constant at $\gamma$, then $X_t/P_t = \gamma M_t/P_t$
- in the steady state all of the **real variables** are constant
- real money supply has to be constant:
  
  $$\frac{M_{t+1}}{P_{t+1}} = \frac{M_t}{P_t} \Rightarrow \frac{M_t(1 + \gamma)}{P_t(1 + \pi_{t+1})} = \frac{M_t}{P_t}$$

- the inflation rate equals the growth rate of money:
  $\pi_t = \pi = \gamma$
- the nominal interest rate:
  $$i = r + \pi = r + \gamma$$
then from the Euler equation of the consumer and the rental rate of the firm we get:

\[ 1 + r = 1 + F_K(K, 1) - \delta = \frac{1}{\beta} \]

this is the same as in the economy without money
→ modified golden rule

from the resource constraint consumption is given by:

\[ C = F(K, 1) - \delta K \]

**superneutrality of money** any change in the growth rate of money has no real effects
Putting together

\[ i = r + \pi = r + \gamma \]

and

\[ 1 + r = 1 + F_K(K, 1) - \delta = \frac{1}{\beta} \]

we get the **Fischer effect**:  

- the real interest rate is pinned down by preferences, and is independent of any monetary measures  
- any change in the growth rate of money is absorbed by a one-to-one increase in the inflation rate

Welfare costs of inflation?  

*here* none (in the steady state)  
*if* labor-leisure choice or money in the utility function there will be welfare costs
Dynamics

- much harder to characterize effects come from the consumption side: changes in the effective price of goods: $\gamma \rightarrow \pi \rightarrow i$
- for some simple cases easy to guess the solution
- an unexpected permanent increase in the money supply (in levels) leads to a proportional increase in the price level and none of the real variables are affected
- an unexpected permanent increase in the growth rate of money supply leads to a proportional increase in the current price level, and an increase in the inflation rate, which affects the nominal interest rate, and none of the real variables are affected
- only anticipated future shocks can have real effects by changing the inter-temporal consumption/saving choice
Summary so far

- introduction of money as a medium of exchange in general equilibrium models
- does not look too promising so far: economy with money does not look very different
  - consumption/saving choice a little bit modified
  - no real effects in the steady state - neutrality & superneutrality
  - some dynamic effects, but quite limited
- very simple CIA model, more sophisticated: Baumol-Tobin households decide how often to go to the bank, the higher the interest rate, the more often they go
  Changes in money have distributional effects, which has (limited) real effects
- have to look further
Money in the utility function

Sidrauski’s model (1967)

Consumers maximize:

$$\sum_{i=0}^{\infty} \beta^i U(C_{t+i}, \frac{M_{t+1}}{P_{t+1}})$$

subject to

$$P_t C_t + M_{t+1} + B_{t+1} + P_t K_{t+1} = W_t + \Pi_t + M_t + (1 + i_t) B_t + (1 + r_t) P_t K_t + X_t$$

the utility function is a **reduced form** (a short cut) of a more complicated problem, where holding money makes households more efficient in shopping, and increases leisure time

what properties do you think $U(\cdot)$ has?

$U_m > 0, U_{mc} \geq 0$
The FOCs from the consumer’s problem

- still ignoring uncertainty
- and using the usual $\beta^i\lambda_{t+i}$

\[ C_t : \quad U_C(C_t, \frac{M_t}{P_t}) = \lambda_t P_t \]

\[ M_{t+1} : \quad \lambda_t = \beta \left( \lambda_{t+1} + \frac{1}{P_{t+1}} U_m(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}) \right) \]

\[ B_{t+1} : \quad \lambda_t = \beta \lambda_{t+1} (1 + i_{t+1}) \]

\[ K_{t+1} : \quad \lambda_t P_t = \beta \lambda_{t+1} (1 + r_{t+1}) P_{t+1} \]
We can rearrange these as:

the **inter-temporal condition**:

\[ UC \left( C_t, \frac{M_t}{P_t} \right) = \beta(1 + r_{t+1}) UC \left( C_{t+1}, \frac{M_{t+1}}{P_{t+1}} \right) \]

modified Euler equation the **intra-temporal condition**:

\[
\frac{U_m \left( C_t, \frac{M_t}{P_t} \right)}{UC \left( C_t, \frac{M_t}{P_t} \right)} = i_t
\]

the MRS between consumption and real holdings of money has to equal the opportunity cost of holding money, which is \( i \), the nominal interest rate
Example of log utility

\[ U \left( C, \frac{M}{P} \right) = \log(C) + \phi \log \frac{M}{P} \]

the first order conditions become:

\[ \frac{1}{C_t} = \beta (1 + r_{t+1}) \frac{1}{C_{t+1}} \]
\[ \frac{M_t}{P_t} = \phi \frac{C_t}{i_t} \]

we can interpret these as an IS and an LM relation:

- IS: a higher interest rate implies a lower consumption today given future expected consumption
- LM: the money demand depends positively on consumption (transactions) and negatively on the opportunity cost of holding money, \( i \)

note: under separability same Euler equation as before
Steady state

Nothing has changed on the firm side:

\[ 1 + r = 1 + F_K(K, 1) - \delta = \frac{1}{\beta} \]

\[ C = F(K, 1) - \delta K \]

\[ \frac{U_m(C, \frac{M}{P})}{U_c(C, \frac{M}{P})} = i = \gamma + r \]

- same real allocation as before
- superneutrality of money still holds
- \( \frac{M}{P} \) inversely related to \( \pi = \gamma \)
Optimal growth rate of money supply

- money is costless to produce, utility increasing in real money holdings
- $\gamma$ has to be such that the marginal utility of real money is zero $\iff i = \gamma + r$
- this implies $\gamma = -r$, negative money growth and inflation, equal to the negative of the marginal product of capital
- put it another way: private opportunity cost of holding money: $i$, social marginal cost of producing money is zero, optimal growth rate should drive this wedge to zero
- known as: **Optimum quantity of money**
  
  see Bailey (1956), Friedman (1969), Lucas (2000)
Welfare cost from deviating from this rule

- Bailey – welfare cost is the area under the money demand curve (as a function of $i$), measures consumer surplus lost due to $i > 0$

- Lucas – estimates the welfare costs in the US, it is between 0.85 and 3% of real GNP per percentage rise in the nominal interest rate above zero; loss equivalent to 88-310 billion 2002$ per year

Cost is larger than the cost of fluctuations
Dynamics

- in general there are some real effects, but they are quite limited
  (some exceptions when no real effects: for example log utility)
- but it doesn’t look like the real effects of money in the real world
- overshooting of inflation, decrease in employment and output following a rise in employment
Summary

- CIA, MIU model
- money often superneutral
- effect of interest rates: \( i \) affects money demand, \( r \) affects consumption/saving choice
- quantitative effects of monetary shocks: mostly through price adjustment, real effects die out quickly
- qualitatively at odds with the observed fluctuations
- money as a medium of exchange, without nominal rigidities provides a way of thinking about the price level, the nominal interest rate, but does not tell us much about fluctuations
- welfare costs of reducing inflation could be very important (potential overestimation due to superneutrality)