Recap of last lecture

Last lecture we:

- went over the business cycle facts
- considered the neoclassical growth model, and different ways of solving it
- looked at the effects of uncertainty
- replicated fairly well the co-movements in output, consumption and investment
- preview of the methods of solving such models

This lecture:

- to really assess the model, add labor/leisure choice → the RBC model, initially due to Prescott
- choose a special case → which we can solve in closed form
Baseline RBC model

Neoclassical growth model with three modifications:

- it’s in discrete time
- it’s stochastic
- it features a labor-leisure choice

competitive markets and no money

Production function

\[ Y_t = K_t^\alpha (A_t L_t)^{1-\alpha} \quad 0 < \alpha < 1 \]

goal: match the data

in the data the capital share is constant \( \Rightarrow \) Cobb-Douglas
Technological progress

\[ A_t = A_t^* \tilde{A}_t \]

- deterministic component

\[ A_t^* = G^t \bar{A} \]

long-run non-stochastic log-linear trend, \( G > 1 \)

- shock process

\[ \ln(\tilde{A}_t) = \rho \ln(\tilde{A}_{t-1}) + \varepsilon_{A,t} \]

\( E(\varepsilon_{A,t}) = 0 \) and is iid
Capital accumulation

\[ K_{t+1} = (1 - \delta)K_t + Y_t - C_t \]

Household objective function

\[ U = E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, 1 - L_{t+i}) \]

\( C_t \) is consumption and \( L_t \) is the fraction of time spent working
\( U_1, U_2 > 0 \) and \( U_{11}, U_{22} < 0 \)
Imagine the following:

- the economy is initially on its non-stochastic BGP
- there is a sudden realization of $\varepsilon_{A,t} \neq 0$
- this moves the economy away from its non-stochastic BGP
- over time the economy moves back to its BGP
- interpret the economy’s deviation from the BGP as a business cycle
- does it look like what we see in the data?
Household behavior I.

inter-temporal FOC

\[ U_1(C_t, 1 - L_t) = \beta E_t (R_{t+1} U_1(C_{t+1}, 1 - L_{t+1})) \] (1)

interpretation (should be familiar)

the household should not prefer any perturbation to the solution, i.e. should be indifferent:

- decrease consumption by \( \varepsilon \), so decrease utility by \( U_1(C_t, 1 - L_t)\varepsilon \)
- save and get \( R_{t+1}\varepsilon \) next period, so an increase in expected utility of \( E_t (R_{t+1} U_1(C_{t+1}, 1 - L_{t+1})) \varepsilon \)
Household behavior II.

intra-temporal FOC

\[ U_1(C_t, 1 - L_t)W_t = U_2(C_t, 1 - L_t) \]  \hspace{1cm} (2)

interpretation

the household should not prefer any perturbation to the solution, i.e. should be indifferent:

- increase work by \( \varepsilon \), so decrease in utility by \( U_2(C_t, 1 - L_t)\varepsilon \)
- extra wage: \( W_t\varepsilon \rightarrow \) increase consumption \( \rightarrow \) increase in utility by \( U_1(C_t, 1 - L_t)W_t\varepsilon \)
\[ U_1(C_t, 1 - L_t)W_t = U_2(C_t, 1 - L_t) \]

if \( W_t \) was constant, then if \((1 - L_t) \downarrow \Rightarrow U_2(C_t, 1 - L_t) \uparrow\)
\[ \Rightarrow \text{for intra-temporal FOC to hold we need:} \]
\[ U_1(C_t, 1 - L_t)W_t \uparrow \Rightarrow C_t \downarrow \]
\[ \iff \text{work more} \Rightarrow \text{consume less from the intra-temporal FOC} \]

Barro and King (1984) insight: to have both consumption and labor pro-cyclical (as in the data) need:
* pro-cyclical wage and/or
* high substitutability between leisure and consumption

quantitative question: is the observed pro-cyclicality of the wage enough given the large pro-cyclicality of labor?
Balanced growth restrictions

To progress further we need to specify the utility function of the household.

What can the utility function look like?

Imposing balanced growth path restrictions might help. What do they mean? And are they reasonable?
in the steady state:

- $L$, labor is constant $\Rightarrow$ production side: we need labor augmenting technological progress
- $C$ and $W$ are growing at rate $G$, which is the rate of technological progress

can identify a set of utility functions that allow a BGP when technology is labor augmenting (King, Plosser, Rebelo (1988, JME))

two cases that are often used (where utility is separable in leisure and consumption):

1. $U(C, 1 - L) = \ln C + b \ln (1 - L)$
   $\rightarrow$ use this today

2. $U(C, 1 - L) = \ln C + \theta \frac{(1- L)^{1-\gamma}}{1-\gamma}$
   $\rightarrow$ most used New-Keynesian specification, look at it later on
Are balanced growth restrictions reasonable?

Greenwood and Vandebroucke: Hours worked: Long run trends, Figure 1, NBER working paper 11629
Are balanced growth restrictions reasonable?
Using the specification $U(C, 1 - L) = \ln(C) + \nu(1 - L)$ we get the following:

- the intra-temporal foc becomes:

$$\frac{W_t}{C_t} = \nu'(1 - L_t)$$

equalize the marginal utility of leisure to the wage times the marginal value of capital

- while the inter-temporal foc becomes:

$$1 = E_t (\beta R_{t+1} \frac{C_t}{C_{t+1}})$$

this is the usual condition for consumption
What are the effects of a positive technological shock? It increases current and future $R$ and $W$.

- **Consumption**
  - Income effect: people feel richer $\Rightarrow$ consumption up
  - Substitution effect: saving is worth more $\Rightarrow$ consumption down
  - Net effect is probably consumption up

- **Leisure**
  - Income effect: people feel richer $\rightarrow$ they want to enjoy more leisure $\rightarrow$ leisure up
  - Substitution effect: higher wage $\Rightarrow$ leisure down
  - Net effect depends on the relative strength of the two forces

* Transitory shock $\rightarrow$ smaller wealth effect and stronger substitution effect
* Permanent shock $\rightarrow$ it is possible that consumption goes up and employment goes down
Employment effects another way

combine inter- and intra-temporal conditions and assume that \( v(1 - L) = b \ln(1 - L) \)

- the intra-temporal condition is:
  \[
  \frac{W_t}{C_t} = \frac{b}{1 - L_t}
  \]

- using this in the inter-temporal condition we get:
  \[
  1 = E_t \left( \beta R_{t+1} \frac{W_t}{W_{t+1}} \frac{1 - L_t}{1 - L_{t+1}} \right)
  \]

* transitory shock \( \rightarrow W_t \uparrow \) but not \( W_{t+1} \Rightarrow (1 - L_t)/(1 - L_{t+1}) \downarrow \)
  \( \rightarrow \) employment increases today

* permanent shock \( \rightarrow W_t/W_{t+1} \) pretty much constant \( \Rightarrow (1 - L_t)/(1 - L_{t+1}) \) constant as well \( \rightarrow \) employment does not change
Back to our very special case

Two additional assumptions:

- $\delta = 1$, full depreciation $\Rightarrow$ almost like a two-period model
- separable log-utility:

\[ U(C_t, 1 - L_t) = \ln(C_t) + b \ln(1 - L_t) \]

The two FOCs become:

\[ \frac{1}{C_t} = \beta E_t \left( \frac{R_{t+1}}{C_{t+1}} \right) \quad (3) \]

and

\[ \frac{W_t}{C_t} = \frac{b}{1 - L_t} \quad (4) \]
As in the homework, using that $C_t = (1 - s_t) Y_t$, that $R_t = \alpha \left( \frac{A_t L_t}{K_t} \right)^{1-\alpha} + (1 - 1)$, and that $K_{t+1} = s_t Y_t$, we can manipulate (3) to get:

$$\frac{s_t}{1 - s_t} = \alpha \beta E_t \left( \frac{1}{1 - s_{t+1}} \right)$$

the optimal saving rate has to be constant \( \Rightarrow s_t = \alpha \beta \Rightarrow \)

$$\frac{C_t}{Y_t} = 1 - \alpha \beta$$
Using that $\frac{C_t}{Y_t} = 1 - \alpha \beta$ and that $W_t = (1 - \alpha) \frac{Y_t}{L_t}$ in (4) gives

$$\frac{(1 - \alpha) Y_t}{(1 - \alpha \beta) Y_t} = \frac{(1 - \alpha) L_t}{(1 - \alpha \beta) L_t} = \frac{b}{1 - L_t}$$

rearranging for $L_t$ we get:

$$L_t = \frac{1 - \alpha}{b(1 - \alpha \beta) + 1 - \alpha}$$

1. $\Rightarrow$ constant working hours
2. very pro-cyclical wage

not very good news - far from where we want to be
Intuition:

- re-write (4) as: $L_t = 1 - \frac{bC_t}{W_t}$
- can think of $C_t$ as capturing the income effects (remember: $1/C_t$ is the marginal value of wealth)
  $C_t$ moves around a lot in cycles $\Rightarrow$ the income effect is very large $\Rightarrow$ people feel much richer $\Rightarrow$ they want to consume more and enjoy more leisure
- and of $W_t$ as capturing the substitution effects
  higher $W_t$ $\Rightarrow$ people want to work harder to take advantage of the higher wages
- here they exactly cancel each other out

In models where $C_t$ is less variable, maybe substitution effect can dominate $\Rightarrow$ labor up in booms
Summary so far:

- consumption is too pro-cyclical
too pro-cyclical, moves one-for-one with $Y_t$

- investment $I_t = s_t Y_t = \alpha \beta Y_t$ also pro-cyclical
  but not enough
investment in the data is super pro-cyclical, with constant
saving rate it is only pro-cyclical
we need a pro-cyclical saving rate to match the data:

$$Var(\ln I_t) = Var(\ln s_t) + Var(\ln Y_t) + 2 Cov(\ln s_t, \ln Y_t)$$

for this a pro-cyclical interest rate is needed

effects of $R$ on saving?
$r \uparrow \Rightarrow$ inter-temporal substitution effect, $C_t \downarrow$
  $\Rightarrow$ income effect, $C_t \uparrow$
(in this model they cancel out)

- $W_t$ is too pro-cyclical
Output dynamics

- constant saving rate and labor supply ⇒ like the Solow model in discrete time: $y_t^* = \frac{Y_t^*}{A_t^*L_t^*} = \left(\frac{\alpha\beta}{G}\right)^{\frac{\alpha}{1-\alpha}}$, rearranged:

$$Y_t^* = \left(\frac{\alpha\beta}{G}\right)^{\frac{\alpha}{1-\alpha}} A_t^*L_t^*$$

- What we are interested in is $\ln \tilde{Y}_t = \ln Y_t - \ln Y_t^*$. Assume that initially we are in the steady state, $\ln \tilde{Y}_0 = 0$. Then we can express

$$\ln \tilde{Y}_t = (1-\alpha) \left(\alpha^t \ln \tilde{A}_0 + \alpha^{t-1} \ln \tilde{A}_1 + ... + \alpha \ln \tilde{A}_{t-1} + \ln \tilde{A}_t\right)$$
One way to show this (normalize $L^* = 1$ for simplicity):

\[
Y_1 = K_1^\alpha A_1^{1-\alpha} = (sY_0)^\alpha A_1^{1-\alpha} = (sY_0^*)^\alpha A_1^{1-\alpha} \\
= \left(s \left(\frac{s}{G}\right)^{\frac{\alpha}{1-\alpha}} A_0^*\right)^\alpha A_1^{1-\alpha} \\
= \left(\frac{s}{G}\right)^\alpha G^\alpha \left(\frac{s}{G}\right)^{\frac{\alpha^2}{1-\alpha}} (A_0^*)^\alpha A_1^{1-\alpha} \\
= \left(\frac{s}{G}\right)^{\frac{\alpha(1-\alpha)}{1-\alpha} + \frac{\alpha^2}{1-\alpha}} G^\alpha (A_0^*)^\alpha A_1^{1-\alpha} \\
= \left(\frac{s}{G}\right)^{\frac{\alpha}{1-\alpha}} (A_1^*)^\alpha A_1^{1-\alpha} = \left(\frac{s}{G}\right)^{\frac{\alpha}{1-\alpha}} (A_1^*)^\alpha (A_1^* \tilde{A}_1)^{1-\alpha} \\
= \left(\frac{s}{G}\right)^{\frac{\alpha}{1-\alpha}} A_1^* \tilde{A}_1^{1-\alpha} = Y_1^* \tilde{A}_1^{1-\alpha}
\]

Now move to $Y_2$ and keep going.
The economy’s reaction to a shock

experiment: there is one shock at time 0 and then never again

\[ \ln \tilde{A}_t = \rho^t \varepsilon_{A,0} \]

what is the economy’s log deviation from trend? what is the economy’s **impulse response function**? use formula from before:

\[
\ln \tilde{Y}_t = (1 - \alpha) \left( \alpha^t \ln \tilde{A}_0 + \alpha^{t-1} \ln \tilde{A}_1 + \ldots + \alpha \ln \tilde{A}_{t-1} + \ln \tilde{A}_t \right) \\
= (1 - \alpha) \left( \alpha^t + \alpha^{t-1} \rho + \ldots + \alpha \rho^{t-1} + \rho^t \right) \varepsilon_{A,0}
\]

in the long run output goes back to the BGP level:

\[ \lim_{t \to \infty} \ln \tilde{Y}_t = 0 \]

plot: \( \varepsilon \) (green), technology (red), output (blue)
Higher persistence, higher $\rho$

\[ \rho = 0.3, \alpha = 0.33333 \]
Higher persistence, higher $\rho$

$\rho = 0.5, \alpha = 0.33333$
Higher persistence, higher $\rho$

$\rho=0.7, \alpha=0.33333$
Higher persistence, higher $\rho$
Higher capital share, higher $\alpha$
Higher capital share, higher $\alpha$
Higher capital share, higher $\alpha$
Higher capital share, higher $\alpha$
Higher capital share, higher $\alpha$
The graphs show

- hump shape in the impulse response of $Y_t$
  - this is due to the increase in total investment, since the saving rate is constant, and output increases
  - so there is **amplification**
  - this comes in part from $\alpha$, but not only
  - output today is only higher because $A$ is higher, output tomorrow is higher because $K$ is higher (due to $\alpha$) and $A$ is higher (due to $\rho$)

- output stays above trend longer than the shock (which is just one period) → so there is **persistence**

  two sources:

1. exogenous persistence, $\rho > 0$, this is the intrinsic persistence of the technology shock
   - this determines mostly the persistence and it is chosen by us
2. endogenous persistence, $\alpha > 0$, which works through investment
Conclusions so far

special case does not look good

- not enough persistence: very quick return to the BGP unless very high $\rho$
- not enough amplification
- need a more general model
- and possibly need to look for other sources of shocks

need to:

- get consumption to be less pro-cyclical
  and investment to be more pro-cyclical
- get labor effort to respond
- get more endogenous persistence