Graduate Macroeconomics 2
Lecture 1 - Introduction to Real Business Cycles

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Sciences Po

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About the course I.

- 2-hour lecture every week, Tuesdays from 10:15-12:15
- 2 big topics covered:
  1. economic fluctuations
  2. unemployment
- office hour by appointment, in office E.404
- email: zsofia.barany@sciencespo.fr
- lecture notes will become available weekly before the lecture
About the course II.

- besides the lectures, there will be classes every Monday afternoon
- tutorials in 3 groups
- for most of the classes you will have to solve problem sets
- you will have to hand in 2 problem sets, these will constitute 30 percent of your final grade
- you will receive the solution to the problem sets
Let’s start!
Why study business cycles?

- last term you studied long-run growth
  → clearly a very important determinant of welfare
- now we will look at short-term fluctuations
- why do we care about these?
  - fluctuations are unpleasant
  - especially increases in unemployment and inflation
- ⇒ governments tend to want to dampen these cycles
- we need to understand the driving forces behind the fluctuations of the economy
A very-very brief history 1.
the Great Depression and Keynes

- Keynes: short-run rigidity of wages (and prices) $\Rightarrow$ stable equilibrium with unemployment possible
  aggregate demand is the key $\Rightarrow$ government intervention
- others formalized his ideas:
  - $\rightarrow$ the IS (investment and saving) – LM (money demand and supply) model
  - later addition: Phillips curve - tradeoff between inflation and unemployment
- $\rightarrow$ similar much bigger models with several hundred equations
- these models are based on empirically observed relationships between: output and consumption, money demand, inflation, unemployment, etc
- the aim of these models was to predict the effects of policies
- they were pretty successful at it until the 1960s
A very-very brief history 2.

the Neo-Classicals

- monetarism - Friedman
  - permanent income hypothesis $\rightarrow$ small fiscal policy multipliers
  - bad monetary policy causes economic fluctuations (not "animal spirits") $\rightarrow$ prescription: steady growth in monetary aggregates
  - Phillips curve relationship does not hold in the long run $\Leftrightarrow$
    money is neutral
    it appears in the short run due to unanticipated inflation and money illusion

- the rational expectations revolution - Lucas
  - Keynesian empirical relationships estimated given a policy for an alternative policy the relationship would be different due to changes in expectations
    $\rightarrow$ Keynesian models not suitable for policy evaluation
  - monetary policy only matters if it surprises people
  - $\rightarrow$ systematic monetary policy aimed at stabilizing the economy will not work
A very-very brief history 3.

the Neo Classical Real Business Cycle Theory

- prices adjust instantaneously to clear markets
- economic actors optimize, and are forward looking
- rational expectations
- cause of fluctuations: random shocks to technology
- key mechanism: intertemporal substitution in consumption and leisure as a response to these shocks

today:

- methodological contribution is most emphasized
- important to assess how close reality is to a perfectly working, frictionless environment
  → to understand the importance of market imperfections
A very-very brief history 4.

the New Keynesians

- general disequilibrium
  - tools of general equilibrium to analyze the allocations of resources when markets do not clear
  - how does the not clearing of one market influence supply and demand in another market
  - fixed prices and wages
  - → different regimes can arise, i.e. ”Keynesian regime” excess supply in goods and labor

- rational expectations without market clearing
  - systematic mon policy can stabilize the economy

- why don’t wages and prices clear the markets?
  - menu costs
  - efficiency wages
  - wage and price setters aren’t perfectly rational
  - market power → wedge between privately and socially optimal price adjustment
Dynamic Stochastic General Equilibrium (DSGE) models

D some things don’t make sense in static models (for example: investment) relatively short-term analysis

S shocks hit the economy, and force it off the balanced growth path (BGP) ⇒
fluctuations do not mean dis-equilibrium, this is the reaction of the economy to an outside shock

GE this is macro
  ▶ the models are based on
    ▶ perfectly/monopolistically competitive markets
    ▶ optimizing agents
  ⇒ the economy is in EQUILIBRIUM
  ▶ micro-founded models

There are different schools of thought...
<table>
<thead>
<tr>
<th>METHODOLOGICAL DIFFERENCES</th>
<th>SUBSTANTIVE DIFFERENCES</th>
<th></th>
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<tbody>
<tr>
<td>simple models/qualitative insights</td>
<td>supply shocks</td>
<td>demand shocks rigidity</td>
</tr>
<tr>
<td>$A \cdot L^\alpha K^{1-\alpha}$ growth model</td>
<td>SALT WATER contributed what is important RIGIDITY</td>
<td></td>
</tr>
<tr>
<td>complex models quantitative matching</td>
<td>FRESH WATER contributed the METHODOLOGY</td>
<td>now there is more harmony in macro</td>
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→ aim: see the mechanisms very clearly

→ transparency is the cost

↓
we are going to start here, because it is easier
Path to get there:

1. real business cycle (RBC) models
2. introduce market power - New Keynesian (NK) model 1.
3. introduce money
4. introduce nominal rigidities - NK model 2.

The predictions in the end are not very different from the IS-LM.

However, better sense

- of the role of distortions
- of optimal policy
Business Cycle Facts I.

properties of quarterly detrended macro time series

\[ x_t = \log(X_t) - \log(X_t^*) \]

is the percentage deviation of variable \( X \) from its trend, \( X^* \)

how is the trend defined? (you have seen this in class yesterday)
- first linear
- now more sophisticated bandpass filter
→ growth theory serves as a guidance
Business Cycle Facts II.

Look at the highest correlation with GDP

$$\rho(x_t, y_{t+k}) \quad k = -6, -5, ..., 0, ..., 5, 6$$

- if $\rho > 0$, then $x$ is pro-cyclical
- if $\rho < 0$, then $x$ is counter-cyclical
- if $k < 0$, then $x$ lags behind output
- if $k > 0$, then $x$ leads output
### Business Cycle Facts III.

<table>
<thead>
<tr>
<th>Series</th>
<th>Standard deviation</th>
<th>Correlation with GDP</th>
<th>Lag</th>
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<tbody>
<tr>
<td>y</td>
<td>1.66</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>1.26</td>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td>i</td>
<td>4.97</td>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td>g</td>
<td>2.49</td>
<td>0.15</td>
<td>-6</td>
</tr>
<tr>
<td>hours</td>
<td>1.61</td>
<td>0.9</td>
<td>-1</td>
</tr>
<tr>
<td>n</td>
<td>1.39</td>
<td>0.8</td>
<td>-1</td>
</tr>
<tr>
<td>tfp</td>
<td>2.29</td>
<td>0.9</td>
<td>-1</td>
</tr>
<tr>
<td>(\frac{w}{P})</td>
<td>0.64</td>
<td>0.16</td>
<td>0</td>
</tr>
</tbody>
</table>

- everything quite pro-cyclical
- except: government spending → does not seem to support the statement that the cause if BC is gov spending
- real wage is mildly pro-cyclical → big problem in many models aggregation bias: average wage evolves differently than the wage of a continuously employed worker
Business Cycle Facts IV.

Standard deviations (unit of measure is percentage deviation from trend) ⇒ they are comparable

▶ GDP is more volatile than consumption
▶ investment is much more volatile than GDP
▶ government spending is pretty volatile
▶ working hours is almost exactly as volatile as GDP
▶ vast majority of the volatility of working hours is explained by the employment volatility
 → weird, because it should be cheaper to adjust the working hours of employees than to hire/fire people
▶ TFP is very volatile
 → school of thoughts disagree whether this is a cause or consequence of business cycles
Basic model

Start with the most basic model, has to contain

- uncertainty – in the form of productivity shocks
- consumption/saving choice

⇒ Ramsey model with two modifications:

- discrete time
- stochastic, due to technological shocks

Many limitations: infinite horizon, no heterogeneity, no money, no labor/leisure choice

Good starting point: analyze the effect of shocks, propagation mechanisms, consumption smoothing, show equivalence of centralized/ decentralized economy
The centralized problem

$$\max_{\{C_{t+i}, S_{t+i}, K_{t+i+1}\}_{i=0}^{\infty}} E_t \left( \sum_{i=0}^{\infty} \beta^i U(C_{t+i}) \right)$$

subject to

$$C_{t+i} + S_{t+i} \leq Z_{t+i} F(K_{t+i}, 1)$$

$$K_{t+i+1} = (1 - \delta) K_{t+i} + S_{t+i}$$

$$K_{t+i+1} \geq 0; \ K_0 > 0 \text{ given}$$

- CRS production: $F(\lambda K, \lambda L) = \lambda F(K, L)$ for every $\lambda > 0$
- usual conditions: $F(0, L) = 0, F_1 > 0, F_2 > 0$, $\lim_{K \to 0} F_1(K, L) = \infty, \lim_{K \to \infty} F_1(K, L) = 0$
- utility: $U' > 0, U'' < 0, \lim_{c \to 0} U'(c) = \infty$
- $Z_t$ random variable with mean $Z$

Goal: understand the dynamic effect of $Z$ on $Y, C, S$
The first order conditions

Combining the two constraints we get:

\[ K_{t+i+1} \leq (1 - \delta)K_{t+i} + Z_{t+i}F(K_{t+i}, 1) - C_{t+i} \]

There are two options now:
1. Solve the problem as it is with the Lagrangian

The Lagrangian can be written as:

\[
\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}) - \\
E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i}(K_{t+i+1} - (1 - \delta)K_{t+i} - Z_{t+i}F(K_{t+i}, 1) + C_{t+i})
\]

note:
(1) Lagrange multiplier \( \beta^i \lambda_{t+i} \) rather than just \( \lambda_{t+i} \)
\( \rightarrow \) purely for convenience
(2) the non-negativity constraints on \( K_t \) are obviously not binding (why?), hence they are omitted from the Lagrangian
Expand the Lagrangian by writing out the $t + i$ and $t + i + 1$ terms:

$$\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}) -$$

$$E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} (K_{t+i+1} - (1 - \delta)K_{t+i} - Z_{t+i}F(K_{t+i}, 1) + C_{t+i}) =$$

$$E_t \left( U(C_t) + \beta U(C_{t+1}) + \ldots \beta^i U(C_{t+i}) + \ldots \right) -$$

$$E_t \lambda_t (K_{t+1} - (1 - \delta)K_t - Z_tF(K_t, 1) + C_t) -$$

$$E_t \beta \lambda_{t+1} (K_{t+2} - (1 - \delta)K_{t+1} - Z_{t+1}F(K_{t+1}, 1) + C_{t+1}) -$$

$$\ldots$$

$$E_t \beta^i \lambda_{t+i} (K_{t+i+1} - (1 - \delta)K_{t+i} - Z_{t+i}F(K_{t+i}, 1) + C_{t+i}) -$$

$$E_t \beta^{i+1} \lambda_{t+i+1} (K_{t+i+2} - (1 - \delta)K_{t+i+1} - Z_{t+i+1}F(K_{t+i+1}, 1) + C_{t+i+1}) -$$

$$\ldots$$
Take the derivative with respect to $C_{t+i}$:

$$\frac{\partial \mathcal{L}}{\partial C_{t+i}} = E_t(\beta^i U'(C_{t+i}) - E_t(\beta^i \lambda_{t+i}))$$

$$= E_t(U'(C_{t+i})) - E_t(\lambda_{t+i}) = 0$$

there is one optimum condition on $C$ for each time period, i.e. $i = 0, 1, 2, ...$

Take the derivative with respect to $K_{t+i+1}$:

$$\frac{\partial \mathcal{L}}{\partial K_{t+i+1}} = E_t(\beta^i \lambda_{t+i} - \beta^{i+1} \lambda_{t+i+1} ((1 - \delta) + Z_{t+i+1} F_1(K_{t+i+1}, 1)))$$

$$= E_t(\lambda_{t+i} - \beta \lambda_{t+i+1} ((1 - \delta) + Z_{t+i+1} F_1(K_{t+i+1}, 1))) = 0$$
The first order conditions for every time period (every \( i \geq 0 \)) are:

\[
\frac{\partial \mathcal{L}}{\partial C_{t+i}} = E_t(U'(C_{t+i})) - E_t(\lambda_{t+i}) = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial K_{t+i+1}} = E_t(\lambda_{t+i} - \beta \lambda_{t+i+1} ((1 - \delta) + Z_{t+i+1} F_1(K_{t+i+1}, 1))) = 0
\]

By defining \( R_{t+i+1} \equiv (1 - \delta) + Z_{t+i+1} F_1(K_{t+i+1}, 1) \) as the return on capital, we get the following two conditions for \( i = 0 \):

\[
U'(C_t) = \lambda_t
\]

\[
\lambda_t = E_t(\beta \lambda_{t+1} R_{t+1})
\]
\[ U'(C_t) = \lambda_t \]
\[ \lambda_t = E_t(\beta \lambda_{t+1} R_{t+1}) \]

intuition:

- the marginal utility of consumption must equal the marginal value of capital

- the marginal value of capital must equal the expected value of the marginal value of capital tomorrow times the gross return on capital, times the subjective discount factor

Putting the two together we get the **Euler equation**:

\[ U'(C_t) = E_t(\beta R_{t+1} U'(C_{t+1})) \]
2. Simplify the model further

Remember that the constraint was:

\[ K_{t+i+1} \leq (1 - \delta)K_{t+i} + Z_{t+i}F(K_{t+i}, 1) - C_{t+i} \]

Define \( f(K_{t+i}) \equiv (1 - \delta)K_{t+i} + Z_{t+i}F(K_{t+i}, 1) \), the problem can be written as (why?):

\[
\max_{\{K_{t+i+1}\}_{i=0}^{\infty}} E_t \left( \sum_{i=0}^{\infty} \beta^i U(f(K_{t+i}) - K_{t+i+1}) \right)
\]

subject to

\[ 0 \leq K_{t+i+1} \leq f(K_t); \quad K_0 > 0 \text{ given} \]

These constraints are obviously not binding. The FOC for \( i = 0 \) is:

\[ U'(f(K_t) - K_{t+1}) = E_t(\beta f'(K_{t+1})U'(f(K_{t+1}) - K_{t+2})) \]

This is another version of the Euler equation.
A variational argument for the Euler equation

\[ U'(C_t) = E_t(\beta R_{t+1} U'(C_{t+1})) \]

- on the optimal path a consumer should be indifferent between reducing his consumption by $\varepsilon$ today and using the savings for consumption in the next period
- saving $\varepsilon$ today reduces utility by $U'(C_t)\varepsilon$ today
- the $\varepsilon$ saving today is worth $E_t(R_{t+1})\varepsilon$ tomorrow
- which increases the consumers utility by $E_t(R_{t+1} U'(C_{t+1}))\varepsilon$ tomorrow
- the present value of this future increase in utility is $E_t(\beta R_{t+1} U'(C_{t+1}))\varepsilon$
We saw that the Euler equations are **necessary** conditions for optimality. But are they **sufficient** as well?

The Euler equations check for **unilateral** variations on the optimal plan. They require that for $\forall t \geq 0$

$$U(f(K_t^*) - K_{t+1}^*) + \beta U(f(K_{t+1}^*) - K_{t+2}^*) \geq U(f(K_t^*) - K_{t+1}) + \beta U(f(K_{t+1}) - K_{t+2})$$

for $\forall K_{t+1}$ such that $0 \leq K_{t+1} \leq f(K_t^*)$ and $0 \leq K_{t+2} \leq f(K_{t+1})$

However, the problem is infinite, and the Euler equation does not check for a deviation in all periods.
The transversality condition

FOC only check for one period deviations $\Rightarrow$ need to check in addition the **transversality condition**:
if an interior sequence $\{K_{t+1}^*\}_{t=0}^{\infty}$ satisfies the Euler equations (given $K_0$) and

$$\lim_{T \to \infty} \beta^T U'(f(K_T^*) - K_{T+1}^*)f'(K_T^*)K_T^* \leq 0$$

then it is a globally optimal sequence under some assumptions (functions $U$ and $f$ bounded, increasing, continuous, concave, and differentiable)

Intuition: FOCs are sufficient for global finite period optimality. What about infinite deviations? Since $\beta^T U'f'$ is the $t = 0$ price of capital $K_T$, if

$$\lim_{T \to \infty} \beta^T U'(f(K_T^*) - K_{T+1}^*)f'(K_T^*)K_T^* > 0$$

then the agent is holding valuable capital, i.e. the value of capital has not been exhausted and perhaps his utility can be increased
The optimal solution to the centralized problem

Is fully characterized by a sequence of $K_{t+1}^*$ for $\forall t \geq 0$ which satisfies

- the Euler equation:
  \[ U'(f(K_t^*) - K_{t+1}^*) = E_t(\beta f'(K_{t+1}^*) U'(f(K_{t+1}^*) - K_{t+2}^*)) \]

- the transversality condition:
  \[ \lim_{T \to \infty} \beta^T U'(f(K_T^*) - K_{T+1}^*) f'(K_T^*) K_T^* \leq 0 \]

- feasibility given $K_0 > 0$
  \[ 0 \leq K_{t+i+1}^* \leq f(K_t^*) \]
To formulate the decentralized problem we have to decide

- whether firms rent or buy the capital
- how firms are financed (through retained earnings, bonds, equity)

Here I assume that the capital is owned by the consumers, who rent out the capital to firms.

Main assumption: the goods, labor and capital markets are perfectly competitive
Firms

- firms’ technology as before: \( Y_t = Z_t F(K_t, L_t) \), where \( F(\cdot, \cdot) \) is CRS:
  \[
  F(\lambda K_t, \lambda L_t) = \lambda F(K_t, L_t)
  \]
- firms rent labor and capital; their profit is therefore given by
  \[
  \pi_t = Y_t - W_t L_t - \tilde{R}_t K_t
  \]
  where they take the rental rate, \( \tilde{R}_t \), and the wage rate, \( W_t \) as given
- firms maximize their profit each period, which implies:
  \[
  \frac{\partial \pi_t}{\partial L_t} = Z_t F_2(K_t, L_t) - W_t = 0 \iff MPL = W_t
  \]
  \[
  \frac{\partial \pi_t}{\partial K_t} = Z_t F_1(K_t, L_t) - \tilde{R}_t = 0 \iff MPK = \tilde{R}_t
  \]
Consumers

- Consumers maximize the PV of lifetime utility from consumption

\[ E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}) \]

- Supply one unit of labor inelastically in a competitive labor market, for wage \( W_t \) in period \( t \)

- Saving \( (S_t) \), is only by accumulating capital, which is rented out to firms every period in a competitive market for rental services, at rental rate \( \tilde{R}_t \)

- The consumers own the firms (in equal shares), and receive all profits

- The total amount of labor supplied is equal to the measure of consumers, we can normalize this: \( L_t = 1 \)
To derive the budget constraint of consumers, we need to calculate the profit of the firms. We will show that if there is perfect competition & CRS production function ⇒ firms’ profits = 0

The profit of the firm is:

$$\pi = Z \cdot F(K, L) - \tilde{R}K - WL$$

1. for a CRS function the following holds:

$$\frac{\partial f(\lambda a, \lambda b)}{\partial \lambda} = \frac{\partial \lambda f(a, b)}{\partial \lambda} = f(a, b)$$

2. now find $$\frac{\partial f(\lambda a, \lambda b)}{\partial \lambda}$$, denote $$\lambda a \equiv a$$ and $$\lambda b \equiv b$$:

$$\frac{\partial f(a,b)}{\partial \lambda} = \frac{\partial f(a,b)}{\partial a} \frac{\partial a}{\partial \lambda} + \frac{\partial f(a,b)}{\partial b} \frac{\partial b}{\partial \lambda}$$

$$= \frac{\partial f(a,b)}{\partial a} a + \frac{\partial f(a,b)}{\partial b} b$$
3. this is true for $\lambda = 1$ as well, which implies $a = a$ and $b = b$:

$$\frac{\partial f (a, b)}{\partial \lambda} = \frac{\partial f (a, b)}{\partial a} a + \frac{\partial f (a, b)}{\partial b} b$$

4. according to (1) this also implies:

$$f(a, b) = \frac{\partial f (a, b)}{\partial a} a + \frac{\partial f (a, b)}{\partial b} b$$

Applying this to our production function: $Z \cdot F(K, L)$:

$$Z \cdot F(K, L) = \frac{\partial Z \cdot F(K, L)}{\partial K} K + \frac{\partial Z \cdot F(K, L)}{\partial L} L$$

$$= Z \cdot F_1(K, L) K + Z \cdot F_2(K, L) L = \tilde{R} K + WL$$

Plugging this into the profit function, we see that it is zero.
the period $t$ budget constraint of the consumer is therefore given by:

$$S_t + C_t \leq \tilde{R}_t K_t + W_t + \pi_t$$

the evolution of capital is given by

$$K_{t+1} = (1 - \delta) K_t + S_t$$

the consumer’s problem can be summarized as:

$$\max \left\{ \sum_{i=0}^{\infty} E_t \beta_i U(C_{t+i}) \right\}_{i=0}^{\infty} \{C_{t+i}, S_{t+i}, K_{t+i+1}\}_{i=0}^{\infty}$$

subject to

$$S_{t+i} + C_{t+i} = \tilde{R}_{t+i} K_{t+i} + W_{t+i}$$
$$K_{t+i+1} = (1 - \delta) K_{t+i} + S_{t+i}$$
$$K_{t+i} \geq 0; \quad K_0 > 0 \quad \text{given}$$
denote by $R_t \equiv 1 + \tilde{R}_t - \delta$ the return on capital; merge the budget constraint (with equality (why?)) and the law of motion for capital to get:

$$C_t = R_t K_t + W_t - K_{t+1}$$

the objective function of the consumer can then be reformulated to get:

$$\max_{\{K_{t+i+1}\}_{i=0}^\infty} \mathbb{E}_t \sum_{i=0}^\infty \beta^i U(R_{t+i} K_{t+i} + W_{t+i} - K_{t+i+1})$$

where $K_0 > 0$ is given and $0 \leq K_{t+i+1} \leq R_{t+i} K_{t+i} + W_{t+i}$ (which again is not binding)

the first order condition for $i = 0$ is then:

$$U'(C_t) = \mathbb{E}_t (R_{t+1} \beta U'(C_{t+1}))$$
Equivalence

Using the fact that $\tilde{R}_t = Z_tF_1(K_t, 1)$ and that $R_t = 1 - \delta + \tilde{R}_t$ we get:

$$U'(C_t) = E_t(\beta(1 - \delta + Z_{t+1}F_1(K_{t+1}, 1))U'(C_{t+1}))$$

which is the same as the previous Euler equation. Using the budget constraint of the consumer:

$$K_{t+1} = (1 - \delta)K_t + \tilde{R}_tK_t + W_t - C_t$$
$$= (1 - \delta)K_t + Z_tF_1(K_t, 1)K_t + Z_tF_2(K_t, 1) - C_t$$
$$= Z_tF(K_t, 1) - C_t$$

which using perfect competition and the CRS property of $F$ is the same as under the planner’s problem.
The non-stochastic steady state

in the non-stochastic steady state $C^*$ and $K^*$ are constant (as there is no growth): $C_t = C_{t+1} = C^*$

also $R^* = 1 - \delta + ZF_1(K^*, 1)$

from the Euler equation: $U'(C^*) = \beta R^* U'(C^*) \Rightarrow \beta R^* = 1 \Rightarrow 1 - \delta + ZF_1(K^*, 1) = \frac{1}{\beta}$

This gives us the optimal level of capital, $K^*$. The condition can be re-written as:

$$ZF_1(K^*, 1) - \delta = \frac{1 - \beta}{\beta}$$

and the optimal $C^*$ is given by:

$$ZF(K^*, 1) - \delta K^* = C^*$$
The effects of an unexpected increase in $Z$?

An increase in $Z$ has two opposing effects on $C$:

**Income effect**: higher $C$ due to the increase in production, as $ZF(K, 1)$ is higher with the same $K$.

**Inter-temporal substitution effect**: lower $C$ due to consumption shifting towards the future.

EE:

$$U'(C_t) = E_t(\beta(1 - \delta + Z_{t+1}F_K(K_{t+1}, 1))U'(C_{t+1}))$$

as $Z$ is higher $\Rightarrow \beta R_{t+1}$ is higher $\Rightarrow$ incentive to save.
- net effect on $C$ ambiguous, depends on the utility function
- $S, I$ unambiguous increase
- $Y$ increase, and further increase over time
- if increase in $Z$ is transitory $\Rightarrow C$ up less, $I$ up more for less time
- good news: positive co-movements
The effects of shocks

Solving the model is very hard, there are several approaches one can take:

1. very special cases, which we can solve explicitly
   → results may be misleading
2. linearize or log linearize, get an explicit (numerical or analytical) solution
   → lose non-linearities
3. formulate a stochastic dynamic programming problem, and solve numerically
   → might not work
   not covered in this course
4. ignore uncertainty, use continuous time and solve with phase-diagram
   → lose interesting effects of uncertainty
   not covered in this course
1. A very special case

\[ U(C_t) = \log C_t \]

\[ Z_t F(K_t, 1) = Z_t K_t^\alpha \]

\[ \delta = 1 \]

HW: Find the solution to this problem, and give an intuition for it for next week’s class.
2. Log-linearization

The first order conditions (FOCs) are a non-linear system of equations. The main idea is that log-linearizing the model around its steady state gives a system of linear equations, which then can be solved in many ways.

Have to follow these steps:

1. write down the FOCs that govern the model
2. solve for the non-stochastic BGP
3. rewrite the model in terms of log deviations from the non-stochastic BGP
4. study this alternative model, which is log-linear, and an approximation of the original model around the non-stochastic BGP

We will go over this in detail two weeks from now.