ABSTRACT. This paper presents a model in which bank competition plays a central role for the propagation of adverse shocks and the credit channel of monetary policy. The competition between banks amplifies shocks through two adverse feedback loops on the liability side through deposits and the asset side through loans. These adverse feedback loops can lead to a collapse in economic activity if the shocks are large enough. We define an operational measure of economic stability – the distance-to-crash – as the smallest shock leading to a crash. In the absence of adverse shocks, we characterize the first-best (conventional) interest-rate and (unconventional) balance-sheet monetary policies. We characterize situations of stress in which the central bank should depart from the first-best monetary policy to increase the distance-to-crash by lowering interest rates or using its balance-sheet to subsidize the operations of banks or non-financial firms.

* Rady School of Management, University of California, San Diego and Imperial College Business School, London. Email: sylvain@ucsd.edu.

In a speech at the London School of Economics on January 13, 2009, Ben Bernanke declared that “In taking these actions [ie. bringing down its target for the federal funds rate by a cumulative 325 basis points], we aimed both to cushion the direct effects of the financial turbulence on the economy and to reduce the virulence of the so-called adverse feedback loop, in which economic weakness and financial stress become mutually reinforcing.” This statement raises several questions. First, how should we think of the initial shock, the “financial turbulence”? What are the propagation mechanisms and “feedback loops” that undermine the stability of the economy? Second, what measures can be used to assess the fragility (or resilience) of the economy? What operational measures should be used? Third, how are the effects of monetary policy transmitted to the economy and how can they prevent crashes? In times of stress, what objectives justify a decisive use of monetary policy?

In this paper we present a model in which banks propagate and amplify shocks but also transmit the effects of monetary policy. A crash is a situation in which the profitability of banks and non-financial firms becomes negative and economic activity collapses. First we show how adverse shocks, whether productivity shocks or idiosyncratic liquidity shocks affecting non-financial firms or banks, can lead to crashes. Second we define the distance-to-crash (relative to productivity or liquidity shocks) as the size of the smallest shock that leads to a crash. We show that the distance-to-crash increases with competition. Third, we study the role of money – the riskfree claim backed by a stock of goods held by the policymaker (“central bank”).\footnote{Money is riskfree in the sense that its return is known when the investor makes his portfolio decision.} The model has no nominal frictions in order to focus on the bank lending channel of monetary policy. In the absence of adverse shocks, we characterize the first-best interest-rate and balance-sheet monetary policies. In particular, we consider two “unconventional” balance-sheet policies that complement the “conventional” interest-rate policy: an interest-rate neutral subsidy to bank entry and a money-supply neutral subsidy to bank entry. When the probability of large adverse shocks increases, the central bank should depart from the first-best policies, and choose to lower interest rate or use the balance-sheet of the central bank to subsidize bank operations. These operations increase bank and firm profit, increase the distance-to-crash and prevent the propagation of shocks.
**Figure 1.**

**A model of the allocation of capital.** A risk-averse representative investor can invest in a riskfree security for the return $r_M$ or in banks for the risky return $r$. Banks in turn provide loans to firms for a return $r(1 + \xi)$ where $\xi$ is the intermediation markup and $\frac{A - (1 + \xi)r}{A}$ is the investment markup. Markups are positive in equilibrium and free entry drives profit to zero.

![Diagram of the allocation of capital](image)

**Figure 2.**

**Liability-side and asset-side liquidity spirals.**

![Diagram of liquidity spirals](image)
In the model, the credit market equilibrium is based on two main ingredients: investors make a portfolio choice between a riskfree security (“money”) and risky deposits; a population of financial intermediaries (“banks”) compete to raise these deposits and to lend the funds to a population of entrepreneurs. Figure 1 illustrates the setup. Both entrepreneurs and bank managers set distortionary markups to cover their costs of operation, and free entry drives their profits to zero and pins down in equilibrium the endogenous numbers of banks and non-financial firms, the competition among banks, the size of the industrial sector and the risk-diversification of the representative investor. In contrast to the “principal-agent view of credit markets” developed in Bernanke and Gertler (1989, 1990); Holmstrom and Tirole (1997) in which the investment wedge (between the project return and the loan rate) and the intermediation wedge (between the loan rate and the deposit rate) are due to an agency problem, we present an “imperfect-competition view of credit markets” in which the wedges are determined by the endogenous monopolistic competition between firms and between banks. Two types of feedback loops propagate shocks to the economy (see Figure 2). In liability-side liquidity spirals, a decrease in bank deposits lowers bank profit, and in equilibrium, forces some banks to exit. Bank competition is relaxed and banks charge higher markups and offer lower deposit rates, leading to a further decrease in bank deposits. In asset-side liquidity spirals, a decrease in the demand for loans from non-financial firms lowers bank profit and forces some banks to exit. Bank competition is also relaxed and banks charge higher markups with higher lending rates, which lowers firm profit, leads to the exit of more firms and a further decrease in the demand for bank loans. When the shocks are large enough, these vicious dynamics can lead to crashes in which neither firms nor banks have incentives to operate and the unique equilibrium is one with no bank or firm entry.  

2Bank competition is modeled as spatial competition (Besanko and Thakor, 1992; Sussman, 1993; Chiaipori, Perez-Castrillo, and Verdier, 1995). The complementarity between production and financing can be interpreted either as geographical complementarity (Petersen and Rajan, 1995; Degryse and Ongena, 2005) or informational complementarity (Berger, Miller, Petersen, Rajan, and Stein, 2005).

3In models of imperfect competition and strategic entry complementarities (Pagano, 1990; Cooper, 1994; Ciccone and Matsuyama, 1996), there are multiple equilibria when the complementarities are strong. We assume that agents (investors, entrepreneurs, bank managers) coordinate in the Pareto dominating equilibrium.
are similarly defined in Varian (1979); Gennaioli and Leland (1990); den Hann, Ramey, and Watson (2003); Brunnermeier and Pedersen (2009); Challe and Ragot (2010).

To assess the resilience of the economy to adverse shocks, we introduce the operational measure of “distance-to-crash” defined as the size of the smallest shock that leads to a crash. The distance-to-crash answers questions such as: how much would aggregate productivity have to decrease to lead a crash? how many banks (resp. firms) would have to fail to trigger further bankruptcies and lead to a systemic failure? This framework formalizes the assessments of the ability of the financial and industrial sectors to overcome adverse shocks performed routinely by central banks and international institutions using stress testing methods (see Bank of England, 2008; European Central Bank, 2008; International Monetary Fund, 2008; Board of Governors of the Federal Reserve System, 2009).

In this model, we call “money” the riskfree claim on the stock of goods held by the policymaker (“central bank”). More precisely, the central bank issues money for three reasons: to “store” goods for repayment in the final period; to directly subsidize the operations of banks and firms; and to produce a public good. We refer to the first task as the conventional interest rate policy of the central bank which controls the return on money (money is more desirable to hold when more goods are stored). This induces a classic bank lending channel (Bernanke and Blinder, 1989; Disyatat, 2010): given a lower riskfree rate, investors rebalance their portfolio towards risky deposits, thereby stimulating the entry of banks and firms. The second task is part of “unconventional” balance-sheet policies (or credit policies) as the central bank uses its own balance-sheet to directly stimulate the economy (Borio and Disyatat, 2009; Gertler and Kiyotaki, 2010, forthcoming). The third task is the provision of public goods. It creates an opportunity cost for the use of resources of the central bank. The utility of investors provides a welfare measure that can be used to evaluate the effects of monetary policy. We characterize the first-best policies (in the absence of adverse shocks and stability concern). In particular, we study two balance-sheet policies: an interest-rate neutral

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4 If the price level is the inverse of the value of money, a lower riskfree rate implies higher inflation.

5 Keister (2010) use a similar setup to discuss fiscal policy in times of stress.
subsidy to bank entry and a money neutral-neutral subsidy to bank entry. Faced with adverse shocks, the central bank can mitigate the vulnerability of the economy by lowering the riskfree rate or by subsidizing the entry of firms. Curdia and Woodford (2010) characterize two main assumptions under which the balance-sheet of the central bank does not matter. While the model of Section I satisfies the first criteria put forward by Curdia and Woodford (“the asset in question are valued only for their pecuniary returns,” page 5), the setup with imperfect bank and firm competition relaxes the second criteria (“all investors can purchase arbitrary quantities of the same assets at the same (market) prices”, page 5). In this case, the balance-sheet of the central bank matters to stimulate the economy. By fostering bank competition, it can increase aggregate supply and modify the equilibrium allocation of capital and the prices. Since Diamond and Dybvig (1983), the focus is on understanding bank runs and the role of the lender of last resort to prevent a given institution from collapsing. While policy targeted at a particular institution is important, we follow Farhi and Tirole (2009) in focusing on non-targeted policy and its impact on the decisions of a population of firms. In cases in which it is impossible or undesirable to prevent the collapse of an individual institution, we describe general equilibrium linkages through which the failure spills over to the economy and we study ways in which policy makers can “manage the systemic risk of bank failures” (Richardson and Roubini, 2009). This paper is complementary to studies in which the interbank market can help absorb adverse liquidity shocks (Allen and Gale, 2000b; Freixas, Martin, and Skeie, 2009; Heider, Hoerova, and Holthausen, 2010).

I. Model

There are five dates, \( t = 0, 1, 2, 3, 4 \). At date 0, policy decisions are made. At date 1, the shocks (to be defined below) are realized. At date 2, the entry decisions of entrepreneurs and bank managers are made. At date 3, bank managers set the lending and deposit rates, entrepreneurs make the investment decision taking the lending rates as given and the risk-averse investor makes the portfolio decision taking the deposit rates as given. Output is realized at date 4 and repayments take place.

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A crash is an ex-ante (date 2) situation in which no entrepreneur and no bank manager has an incentive to enter. In this case, the revenues of the bank managers and entrepreneurs are not large enough to cover the cost of operation. The stability of the economy is characterized by the size of smallest aggregate or idiosyncratic shock that leads to a crash (distance-to-crash). Bank competition is measured by the size of the intermediation markup (the difference between the lending and deposit rates) and non-financial firm competition, by the size of the investment markup (the difference between the technological productivity and the lending rate paid to the banks).

I.1. Uncertainty and the supply of capital. Identical investors with aggregate capital $Y$ have log-preferences and choose between immediate consumption $c$ and investment for future consumption. They can invest $k_M$ in a riskfree security yielding the return $r_M$. They can also invest in a set of risky projects.

We make specific assumptions on the correlation structure of risky projects (to be discussed below). There is an infinite countable set of industrial sectors, each indexed by $\varphi$ in the unit interval and there is an endogenous number $N_e$ of risky projects per sector ($N_e > 1$). Investing in a firm is equivalent to buying a security that pays in two states of nature, either with all the firms of its sector or alone. More precisely, for each risky project indexed by $(\varphi, n) \in [0, 1] \times [1, N_e]$ promising a deposit rate $r(\varphi, n)$ if it generates revenue (and by limited liability, zero otherwise), the objective function of investors is

$$U = \log(c) + \beta q \log(r_M k_M) + \beta p_0 \int_0^1 \log \left[ r_M k_M + \sum_{n=1}^{N_e} r(\varphi, n) k(\varphi, n) \right] d\varphi + ...$$

$$... + \beta p_1 \sum_{n=1}^{N_e} \int_0^1 \log \left[ r_M k_M + r(\varphi, n) k(\varphi, n) \right] d\varphi. \quad (1.1)$$

where $\beta$ is the discount rate, $q = 1 - p_0 - N_e p_1 > 0$ is the probability that no project pays off and $\langle k_M, k(\varphi, n) \rangle$ are the investments in the riskfree and risky securities. The first integral in Equation (1.1) represents the expected utility when all firms in a sector pay out and the second integral, when only one firm in a sector pays out. This formulation is an extension of Acemoglu and Zilibotti (1997); Martin and Rey (2004); Champonnois (2008) which has three main advantages: different projects are imperfectly correlated so that there is safety
in variety; depending on $p_0$ and $p_1$, any correlation between projects (positive or negative) is possible; and it is a parsimonious representation with ex-ante identical projects.\footnote{The setup in Acemoglu and Zilibotti (1997); Martin and Rey (2004) with pure Arrow securities ($p_0 = 0$) imposes a negative correlation between projects, which has been criticized by Okawa and van Wincoop (2009).} The budget constraint is

$$c + k_M + \sum_{n=1}^{N_e} \int_0^1 k(\varphi, n) d\varphi = Y.$$ 

Because of the logarithmic preferences, the first period consumption is $c = Y/(1 + \beta)$. Denote $y = \beta Y/(1 + \beta)$ as the disposable income in the later period. For a given sector $\varphi$, we study the supply of capital for a given project $n$ (all other projects in the industry are indexed by $j$).

**Lemma 1.** Under the condition that $(p_0 + p_1)r_n \geq r_M$, the supply of capital $k$ is given by

$$\frac{p_0}{r_M k_M + r_n k_n + \sum_{j \neq n} r_j k_j} + \frac{p_1}{r_M k_M + r_n k_n} = \frac{1}{r_n y}. \quad (1.2)$$

Equation (1.2) in Lemma 1 defines implicitly the supply of capital $k = K(r)$ for investment in a project $(\varphi, n) \in [0, 1] \times [1, N_e]$ which is an increasing concave function in the return $r$. The elasticity of the supply of capital is increasing in the number of firms $N_e$ and in the riskfree money investment $k_M$.\footnote{This setup is related to international trade models with heterogeneous firms. In particular, the fact that the elasticity of the supply of capital is non-constant is related to Melitz and Ottaviano (2008).}

**I.2. Demand for capital.** Given the supply of capital from the portfolio decision of the representative investor, the demand for capital depends on the investment decisions of entrepreneurs and the intermediation decision of bank managers.

**Investment decision.** The projects (“firms”) are run by risk-neutral entrepreneurs. The production technology has constant returns $A$ and they set the return $r$ to maximize

$$(p_0 + p_1)AK(r) - r(1 + \xi)K(r) - f_e,$$

where $\xi > 0$ is an intermediation markup fixed by the bank and $f_e$ is the operation/entry cost.
Lemma 2. The first-order condition of entrepreneurs implies

\[
a - (1 + \xi)r_n = \nu_n k_n y \left[ \frac{p_0}{(r_M k_M + r_n k_n + \sum_{j} r_j k_j)^2} + \frac{p_1}{(r_M k_M + r_n k_n)^2} \right]
\]

The left-hand side of equation (1.3) in Lemma 2 is the markup charged by entrepreneurs. This markup is strictly positive as each project is important for the diversification of the investor and the entrepreneur running it has some marginal monopoly power in setting the markup. As in monopolistic competition models, the free entry of entrepreneurs drives profit to zero:

\[
(p_0 + p_1)(r_n k_n)^2 a y \left[ \frac{p_0}{(r_M k_M + r_n k_n + \sum_{j} r_j k_j)^2} + \frac{p_1}{(r_M k_M + r_n k_n)^2} \right] - f_e = 0. \tag{1.4}
\]

In contrast to model in which the investment decision is based on a moral hazard problem (Bernanke and Gertler, 1989, 1990; Holmstrom and Tirole, 1997), the markup in (1.3) is endogenous and depends on the competition between firms. Note that Equations (1.2) and (1.3) imply

\[
\frac{1 + \xi}{\nu a y} = \frac{p_0}{r_M k_M + r_n k_n + \sum_{j} r_j k_j} + \frac{p_1}{r_M k_M + r_n k_n} - \cdots - \frac{p_0 r_n k_n}{(r_M k_M + r_n k_n + \sum_{j} r_j k_j)^2} - \frac{p_1 r_n k_n}{(r_M k_M + r_n k_n)^2}. \tag{1.5}
\]

The markup \(\xi\) only depends on prices/returns \(r_n\) or on quantities \(k_n\) only through “dividends” (the terms \(r_n k_n\) or \(r_j k_j\)). This property will be useful to solve for the equilibrium.

Intermediation decision. There is a finite number of financial intermediaries (“banks”) uniformly distributed on \([0, 1]\).\(^9\) They are separated by the endogenous distance \(2z\) so that there are \(N_b = \frac{1}{2z}\) banks in equilibrium. Each bank is specialized in providing financial services close to its location. If a bank is located at a distance \(\varphi\) from a firm, it incurs the cost \(f_\varphi \varphi\) when lending to it. A lower intermediation cost \(f_\varphi\) makes bank competition tougher while a larger \(f_\varphi\) increases “segmentation” and softens bank competition. Subject to the contestability by other banks, a bank manager maximizes its expected revenue \((p_0 N_e + p_1)\xi r(\xi)k(\xi)\) for each industry \(\varphi\). We assume that banks are constrained to offer only one intermediation

\(^9\)We ignore “border effects”: the segment \([0, 1]\) is a circle. In this paper, we focus on symmetric banks. Vogel (2008) studies a model of spatial competition in which asymmetric agents face location decisions.
Figure 3.

Contestability and intermediation pricing. This figure illustrates the result of Lemma 3. If bank 1 sets the intermediation markup too low, then some firms are not served (the distance cost is above the revenues). If bank 1 sets the intermediation markup too high, then some firms in $[0, z]$ are served by bank 2 if it sets its intermediation markup marginally below.

price $\xi$ to all firms (non-discriminatory pricing) and a bank can contest the loans offered by other banks (subject to the distance cost). Banks also have the option not to lend to a particular firm or industry. By symmetry, a bank serves all firms with a distance of at most $z$ in equilibrium. We focus here on the equilibrium in which all industries are served and Appendix VI.3 discusses other equilibria.

Lemma 3 (Intermediation decision). When all loan terms are contested by the closest bank, we have $f_\varphi z = (p_0 N_e + p_1) \xi r k$ or

$$\frac{f_\varphi}{2N_b} = (p_0 N_e + p_1) r_n k_n \left[ \frac{p_{0ay}}{r_M k_M + r_n k_n + \sum_{-i} r_j k_j} + \frac{p_{1ay}}{r_M k_M + r_n k_n} - \ldots \right]
\ldots - r_n k_n \left( \frac{p_{0ay}}{(r_M k_M + r_n k_n + \sum_{-i} r_j k_j)^2} + \frac{p_{1ay}}{(r_M k_M + r_n k_n)^2} \right) - 1. \quad (1.6)$$

Figure 3 illustrates the spatial setup. For a given distance between banks $2z$, if a bank sets the markup $\xi$ such that the revenue $(p_0 N_e + p_1) \xi r k$ is strictly above $f_\varphi z$, then for $\epsilon$ small, the industries located in $z - \epsilon$ is not served. The bank can therefore increase its profit by raising its revenue $(p_0 N_e + p_1) \xi r k$. If a bank sets the markup $\xi$ such that the revenue $(p_0 N_e + p_1) \xi r k$ is strictly above $f_\varphi z$, then the two closest banks can set a marginally lower intermediation markups such that their revenues are strictly between $f_\varphi z$ and $(p_0 N_e + p_1) \xi r k$ and serve industries at a distance larger than $z$ from them. In both case, these strategies
of bank 1 are not compatible with the equilibrium and it can increase its profit by setting $\xi$ such that $f_\phi z = (p_0 N_e + p_1) \xi r k$

**Lemma 4** (Bank free entry). The bank free entry condition imposes $\frac{f_\phi}{2 N_b^2} - \frac{f_\phi}{4 N_b^2} - f_b = 0$ or $N_b = \frac{1}{2} \sqrt{\frac{f_\phi}{f_b}}$. (1.7)

The assumption of the allocation of banks on the circle (Sussman, 1993) is a convenient way to model the imperfect competition between banks. Banks have a local monopoly power which allows them to be imperfect substitute to each other and to charge a markup which is strictly positive on average. Setups with islands (Cooper and Corbae, 2002; Gertler and Kiyotaki, 2010, forthcoming) have generally the same property of imperfect substitutability between banks.

I.3. **Aggregate budget constraint and equilibrium.** Given the ex-ante symmetry in firms and banks, the aggregate budget constraint is written as

$$1 = \frac{k_M}{y} + N_e k. \quad (1.8)$$

We assume that if no banks enter, firms receive no capital and have no revenues. Similarly when no firms enters, banks have no use for their deposits and therefore no revenues. The no entry case is an equilibrium. We define now the equilibrium for the entry of firms and banks, taking policy decisions on $r_M$ as given

**Definition 1.** An equilibrium with entry is a set of variables $\langle N_e, N_b, r, k, \xi \rangle$ satisfying

1. the pricing conditions (1.6)
2. the entry conditions for entrepreneurs and bank managers (1.4) and (1.7)
3. the budget constraint of the representative investor (1.8),
4. Pareto dominance: no other allocation $\langle N_e, N_b, r, k \rangle$ satisfying the items 1. to 3. delivers a higher utility to the investor.

As noted before, this model has many equilibria and we focus on the equilibrium that delivers the highest utility to the investor which is an equilibrium where all industries are served and Equation (1.6) is the valid pricing condition for bank markups.
I.4. **Renormalization.** Even though we have a production economy, following the intuition in Equation (1.5), it is easier to solve the equilibrium in the space of dividends paid to the investors (rather than prices/returns or investments/portfolio shares). Introduce $\lambda = \frac{p_1}{p_0}$ and
d
$$
d = \frac{rk}{p_0 ay}; \quad d_M = \frac{rMk_M}{p_0 ay};$$

$$
\phi_e = \sqrt{\frac{f_e}{p_0(p_0 + p_1)ay} - \lambda} \quad \text{and} \quad \phi_b = \sqrt{\frac{f_b(p_0 + p_1)}{p_0^2 ay[f_e - p_1(p_0 + p_1)ay]}}.
$$

We will call $d_M$ the “money dividend” as the liquidation value of money. The system of equation is then

$$
\phi_e^2 = \frac{d^2}{(d_M + N_e d)^2} - \frac{2\lambda d_M(d_M + 2d)}{(d_M + d)^2},
$$

$$
\phi_b \phi_e = (N_e + \lambda) d \left( \frac{d_M + (N_e - 1)d}{(d_M + N_e d)^2} + \frac{\lambda d_M}{(d_M + d)^2} - 1 \right).
$$

(1.9)

(1.10)

The first equation is the condition of free entry for firms while the second equation is the intermediation pricing condition for banks. Although this system is highly nonlinear, it is possible to show that the solutions of this system can be reduced to one polynomial equation in one variable using basic techniques from Elimination Theory (see Appendix VI.1). However, to exploit closed form solutions, we focus in the following sections on particular assumptions in the correlation structure.

I.5. **Equilibrium.** In rest of the paper, we focus on the special case in which idiosyncratic risk converges to 0 ($\lambda = 0$) and projects within industries are perfectly correlated. For intuition on the role of idiosyncratic risk, we derive the results on stability of Section II.1 under different assumptions with $\lambda > 0$ in Appendix VI.4. When $\lambda = 0$, the system in the variables $(d, N_e)$ is

$$
\phi_e = \frac{d}{d_M + N_e d}; \quad \phi_b \phi_e = N_e (\phi_e - \phi_e^2 - d),
$$

or with $d = \frac{\phi_e d_M}{(1 - \phi_e N_e)}$,

$$
0 = \phi_b - N_e (1 - d_M - \phi_e + \phi_b \phi_e) + N_e^2 \phi_e (1 - \phi_e).
$$

(1.11)

Equation (1.11) has in general two roots and following the previous discussion on equilibrium selection, we focus on the root that implies the largest number of firms. There are two
constraints: the positive discriminant of Equation (1.11) and the condition that at least one firm per industry operates $N_e \geq 1$.

**Lemma 5** (Existence of equilibrium). If

$$\phi_b \leq \begin{cases} 
1 - \phi_e - \frac{d_M}{1-\phi_e} & \text{if } \phi_e \leq 1 - \sqrt{d_M} \\
\frac{1-\phi_e - \sqrt{d_M}}{\phi_e} & \text{if } \phi_e \geq 1 - \sqrt{d_M}
\end{cases},$$

(1.12)

an equilibrium exists and the equilibrium number of firms $N_e \geq 1$ is decreasing in the entry costs $f_b$ and $f_e$ and the money dividend $d_M$:

$$N_e = \frac{(1 - d_M - \phi_e + \phi_b \phi_e) + \sqrt{(1 - d_M - \phi_e + \phi_b \phi_e)^2 - 4\phi_b \phi_e (1 - \phi_e)}}{2\phi_e (1 - \phi_e)}.$$

When the cost of firm entry $\phi_e$ is high, then the “$N_e = 1$”-constraint is the most binding one, while when $\phi_e$ is lower, then the discriminant constraint is the most binding one. When the cost $f_e$ and $f_b$ are high, the profit of entrepreneurs is low and few firms enter. Similarly, when the money dividend $d_M$ is high, money is a high share of the portfolio of the investor and the supply of capital in Equation (1.2) is very elastic, the markups that bank managers and entrepreneurs charge are low and the profits are low too, leading few firms to enter. Similarly, when there is perfect competition between banks ($f_b f_\phi = 0$), the discriminant condition is never binding. While when $f_b \geq f_e$, the “$N_e = 1$-constraint is never binding.

The following Lemma shows that the main measures of competition (the markups charged by banks and firms) are increasing in the entry costs and the money dividend.

**Corollary 1.** The markups (bank markup $\xi$, firm markup $1 - \frac{(1+\xi)r}{a}$ and total markup $1 - \frac{r}{a}$) are increasing in $\phi_e$, $\phi_b$ and $d_M$.

$$\xi = \frac{(1 - \phi_e \phi_b - d_M - \phi_e) - \sqrt{(1 - \phi_e \phi_b + d_M - \phi_e)^2 - 4d_M (1 - \phi_e)}}{2d_M},$$

$$1 - \frac{(1+\xi)r}{a} = \phi_e,$$

$$1 - \frac{r}{a} = \frac{(1 + \phi_e \phi_b - d_M + \phi_e) - \sqrt{(1 - \phi_e \phi_b + d_M - \phi_e)^2 - 4d_M (1 - \phi_e)}}{2}.$$
II. Economic stability

In this section, we study the economy stability of the equilibrium with entry of banks and firms. Following Allen and Gale (2000a), we perturb the equilibrium and study its existence and the entry of firms and banks.

II.1. Distance-to-crash. We study the critical values in productivity, number of firms or banks at which a crash could take place. The distances to these critical values represent measures of economic stability and they allow to evaluate the size and shape of the basin of attraction of the equilibrium with entry of firms and banks. By analogy to the credit risk literature which uses the distance-to-default to evaluate the probability of default, we define the distance-to-crash as the size of the smallest shock leading to a crash. In this section, we show how these measures of economic stability are decreasing in the entry costs $f_e$, $f_b$ and in the money dividend $d_M$.

Assumption 1. Aggregate shock on productivity: the productivity $a$ decreases to $a(1 - \omega_a)$. 

**Figure 4.**

The frontier for the existence of an equilibrium. This figure shows the Equation (1.12) in $\phi_b$ and $\phi_e$ with $d_M = .1$. For low value of $\phi_e$, the discriminant is the binding condition while for high value of $\phi_e$, the condition $N_e = 1$ is the binding one.
Lemma 6. The distance-to-crash relative to aggregate shocks $\omega_a$ is decreasing in the entry costs $f_e, f_b$ and the money dividend $d_M$:

$$
\frac{1}{\sqrt{1 - \omega_a}} = \begin{cases} 
X_a(\theta, \phi_e, d_M) & \text{if } \theta \geq \sqrt{d_M} \\
\sqrt{\frac{1 + \theta + \sqrt{d_M}}{\theta \phi_e} - \frac{1}{2} \sqrt{d_M} - \frac{1}{2}} & \text{if } \theta \leq \sqrt{d_M}
\end{cases}
$$

where $\theta = \frac{\phi_b}{\phi_e} = \sqrt{\frac{f_e f_b}{f_e}}$ and $x = X_a(\theta, \phi_e, d_M)$ is the unique solution in $\left[0, \frac{1}{\phi_e \sqrt{1 + \theta}}\right]$ of the equation $[1 - (1 + \theta)\phi_e^2 x^2](1 - \phi_e x) = d_M$.

Assumption 2. Idiosyncratic liquidity shock on banks: a number $\omega_b N_b$ of banks go bankrupt for exogenous reasons and are not immediately replaced (sticky entry).

Lemma 7. The distance-to-crash relative to idiosyncratic bank shocks is decreasing in $f_e, f_b$ and the money dividend $d_M$

$$
\frac{1}{1 - \omega_b} = \begin{cases} 
\frac{1 - \phi_e - d_M}{\phi_b} & \text{if } \phi_e \leq 1 - \frac{\sqrt{d_M}}{\phi_b} \\
\frac{2 \phi_e (1 - \phi_e)}{(\sqrt{1 - \phi_e} - \sqrt{d_M})^2} & \text{if } \phi_e \geq 1 - \frac{\sqrt{d_M}}{\phi_b}
\end{cases}
$$

Assumption 3. Idiosyncratic liquidity shock on firms: a number $\omega_e N_e$ of firms go bankrupt for exogenous reasons and are not immediately replaced (sticky entry).

Lemma 8. The distance-to-crash relative to idiosyncratic firm shocks is decreasing in $f_e, f_b$ and the money dividend $d_M$

$$
\frac{1}{1 - \omega_e} = \begin{cases} 
\frac{(1 - d_M - \phi_e + \phi_b \phi_e) + \sqrt{(1 - d_M - \phi_e + \phi_b \phi_e)^2 - 4 \phi_b \phi_e (1 - \phi_e)}}{2 \phi_e (1 - \phi_e)} & \text{if } 4d_M^2 - 4d_M(1 - 2\phi_e \phi_b) - (\phi_e \phi_b)^2(1 - 2\phi_e \phi_b) < 0 \\
\frac{1 - X_e(\theta, \phi_e, d_M)^2}{2 \phi_e (1 - \phi_e) X_e(\theta, \phi_e, d_M)} & \text{if } 4d_M^2 - 4d_M(1 - 2\phi_e \phi_b) - (\phi_e \phi_b)^2(1 - 2\phi_e \phi_b) \geq 0
\end{cases}
$$

where $x = X_e(\theta, \phi_e, d_M)$ is the solution of

$$x \left(1 - \frac{(1 - 2x)d_M}{(1 - x)^2}\right) = 2\phi_e \phi_b. \quad (2.1)$$
II.2. **Systemic risk and too-big-to-fail banks and firms.** The reaction of the economy to adverse shocks is characterized by two adverse feedback loops (see Figure 2). In the case of the liability-side liquidity spiral, when the economy is hit by an adverse productivity shock, the deposits of banks decrease, and the direct effect is to force the exit of some banks and the relaxation of bank competition, leading to an increase in intermediation markups and a decrease in the deposit rate. The indirect effect from the decrease in the deposit rate is a decrease in the supply of deposits. If the indirect effect is stronger than the direct effect, the vicious circle leads to a crash. The key friction is that bank managers and entrepreneurs have to charge distortionary markups to cover the fixed operation costs $f_e$ and $f_b$. When the revenues are insufficient, firms and banks cannot operate and economic activity breaks down.

In the case of the asset-side liquidity spiral, when the economy is hit by an idiosyncratic shock, for instance the exit of some banks for exogenous reasons, bank competition is relaxed and lending rates increase and the direct effect is to force the exit of firms due to lower profit. But indirectly, the demand for bank loans decreases which might also force the exit of some banks. Again, when the indirect effect is stronger than the direct effect, the vicious circle leads to a crash. When the failure of a single bank leads to a crash ($\omega_b N_b = 1$), all banks are “too-big-to-fail” and the economy is exposed to systemic risk.

II.3. **Numerical illustration.** We set the normalized entry parameters

- the normalized firm entry cost is $\phi_e = \sqrt{\frac{f_e}{p_0 \alpha y}} = .145$,
- the normalized bank entry and distance costs are $\sqrt{\frac{f_b}{p_0 \alpha y}} = .187$ and $\sqrt{f_e} = 2.246$.

In this case, $\theta = \sqrt{\frac{f_b f_e}{\phi_e}} > 1$ and the only constraint for the stability of the economy in Equation (1.12) is the discriminant. The money dividend is $d_M = .05$. In equilibrium, the number of firms and banks are $N_e = N_b = 6$ and the dividend and intermediation markup are $d = .053$ and $\xi = 1.32$. The distances-to-crash are

$$1 - \omega_e = 50.5\%; \quad 1 - \omega_b = 85.6\%; \quad 1 - \omega_a = 87\%.$$

In Figure 5, we illustrate the discriminant as well as the distances-to-crash relative bank and firm liquidity shocks for different values $(\phi_b, \phi_e)$ (left panel) and $(d_M, \phi_e)$ (right panel).
III. Monetary Policy

In this section we study how the instruments of monetary policy affect the efficiency and the stability of the allocation of capital. Since the number of firms $N_e$ depends on monetary policy through the money dividend $d_M$ in Equation (1.11), the equilibrium demand for money from the aggregate budget constraint of the representative investor and the return $r_M$ are also endogenous

$$k_M \frac{y}{y} = 1 - p_0 \phi_e N_e; \quad r_M = \frac{d_M}{1 - p_0 \phi_e N_e}.$$ 

A preliminary remark is that for a constant value of $\theta = \frac{\phi_b}{\phi_e}$, a higher value of the money dividend $d_M$ reduces the probability that the equilibrium exists.

Corollary 2. The condition (1.12) for the existence of the equilibrium can rewritten as

$$d_M \leq \begin{cases} 
[1 - \phi_e(1 + \theta)](1 - \phi_e) & \text{if } \phi_e \leq 1 - \theta \\
(\sqrt{1 - \phi_e} - \sqrt{\phi_e \phi_b})^2 & \text{if } \phi_e \in \left[1 - \theta, \frac{\sqrt{1 + \theta - \frac{1}{\theta}}}{\theta}\right]. 
\end{cases}$$

III.1. Policy objectives and instruments. An advantage of proceeding from explicit microfoundations is that the welfare of private agents (here the utility of investors since bank managers and entrepreneurs are risk-neutral and make zero profit) provides a natural
objective in terms of which alternative policies can be evaluated. The first-best welfare measure without crash is the utility of the representative investor

$$U_{\text{entry}} = \alpha \log(G) + (1 - p_0) \log(r_M k_M) + \int p_0 \log(r_M k_M + N_e r_k) d\varphi$$

$$= \alpha \log(G) + \log(p_0^2 ay) + \log(d_M) - p_0 \log(1 - p_0 \phi_e N_e)$$

(3.1)

$$= U_{\text{no entry}} - p_0 \log(1 - p_0 \phi_e N_e).$$

(3.2)

where $G$ is some public goods financed by the issuance of money and $U_{\text{no entry}} = \alpha \log(G) + \log(p_0^2 ay) + \log(d_M)$ is the utility of the representative investor if there is no entry of firms and banks (crash). Taking into account the likelihood of adverse shocks and crashes, the expected utility of the representative investor is

$$U = U_{\text{entry}} \times (1 - P_{\text{crash}}) + U_{\text{no entry}} P_{\text{crash}}$$

$$= U_{\text{no entry}} - p_0 \log(1 - p_0 \phi_e N_e) \times (1 - P_{\text{crash}}).$$

where the probability of a crash $P_{\text{crash}}$ is a function of the distances-to-crash. The balance-sheet of central bank is

$$k_M = k_0 + T + G.$$

where $k_0$ is stored at rate $r_0$, $T$ is the subsidy for the operations of banks and firms and $G$ is a public good. We study the following non-targeted objectives for the policymaker (central bank):

- Interest rate policy: the central bank faces an exogenous riskfree “storage” rate $r_0$ and chooses the value of $d_M$ to maximize the utility of the investor taking into account the provision of public goods $G$.
- Balance-sheet policy with public goods: the balance-sheet of the central bank is directly used to subsidize the entry of banks and firms.

---

10 This is analogous to New Keynesian models in which the utility of consumers is used as a welfare measure to evaluate different inflation paths (Woodford, 2003).

11 See Farhi and Tirole (2009) for a discussion of targeted versus non-targeted policies.
The first instrument is often described as conventional monetary policy while the second objective is as unconventional monetary policy (Borio and Disyatat, 2009). In particular, the balance-sheet policy is related to the Capital Purchase Program of the US Treasury in 2008 (including the Trouble Asset Relief Program, TARP). In 2008, the Federal Reserve has started to provide liquidity facilities through provided non-targeted short-term financing at a cheaper rate that could be found on the financial markets (see Gertler and Kiyotaki, 2010, forthcoming, for a detailed description of the role of the Federal Reserve during the 2007-2009 financial crisis).

III.2. Conventional monetary policy: interest rate policies. In this section, we set the subsidy to zero $T = 0$ and focus on interest rate policies.

Reduced-from interest rate policy. We first look at a “reduced-form” interest rate policy in which the central bank directly controls the riskfree rate.

Lemma 9. The first-best policy $(d_{fb}^{fb}, r_{fb}^{fb})$ to maximize the utility of investors upon entry with no public goods is

\[
\begin{align*}
    d_{fb}^{fb} &= \frac{\theta \phi_c^2 p_0^2 + 2 \theta \phi_c^2 (1 - p_0) + 2 (1 - p_0)(1 - \phi_c) - \phi_c (2 - p_0) \sqrt{\theta (\theta \phi_c^2 p_0^2 + 4 (1 - p_0)(1 - \phi_c))}}{2 (1 - p_0)}; \\
    r_{fb}^{fb} &= \frac{2 (1 - p_0 \theta \phi_c^2 - \phi_c)^2 + \theta^2 \phi_c^4 p_0 (1 - p_0) - \phi_c [2 (1 - \phi_c) - p_0 \theta \phi_c^2] \sqrt{\theta (\theta \phi_c^2 p_0^2 + 4 (1 - p_0)(1 - \phi_c))} - \theta \phi_c}{2 (1 - \phi_c)}.
\end{align*}
\]

The optimal $d_M$ is an interior solution with respect to the existence constraint in Equation (1.12) (positive discriminant and $N_e = 1$) so that the policymaker would never choose under this objective any distance-to-crash equal to zero. Nevertheless under some scenarios, the distance-to-crash is insufficient and the policy maker would decrease the riskfree rate to increase the distance-to-crash.

Corollary 3. If there is a high risk of exit of banks or firms, it may be optimal to choose $d_M < d_{M}^{fb}$ by choosing a lower interest rate $r_M$.

\footnote{We distinguish here targeted emergency lending (such as the support to Northern Rock in the UK, Bear Stearns, AIG, Citigroup in the US, UBS in Switzerland) from non-targeted fiscal policy.}
When $r_M$ decreases, investors rebalance their portfolio towards the risky deposits and firms and banks can charge higher markups which allows them to absorb the adverse shocks. Lower interest rates (below the optimal $r^{fb}_M$) stimulate the economy and lead to more firm entry, more production and more expected output, as well as more stability. The fact that monetary policy can mitigate the effect of adverse shocks is consistent with the fact that the Fed cut interest rates sharply after the October 1987 stock market crash, the Russian default in 1998 and the market turmoil of September 2008 (failure of Lehman Brothers and Washington Mutual, bailout of AIG, and Fereral takeover of Fannie Mae and Freddie Mac). However, the lower interest rates come at the expense of investors who are less compensated on their investment portfolio.

**Interest rate policy with the provision of public goods.** In this section, we take into account the balance-sheet of the policy maker (central bank). The public goods $G$ it provides are financed by the issuance of money and we have $G = 1 - p_0 \phi_e N_e - \frac{p_{ad} M}{r_0}$ where the amount to be delivered against money at date 4 is stored for the return $r_0$.

**Lemma 10.** The first-best $d^{fb}_M = F(\phi_e|p_0, \alpha, \theta, r_0)$ with public goods is the solution of a 4th-order polynomial equation.

To stimulate the economy, the central bank lowers the riskfree rate by decreasing the storage allocation $k_0$ and increasing the provision of public goods $G$. With a lower riskfree rate, the investor rebalances its portfolio away from money and towards the risky deposits which stimulates the economy and in particular increases the number of firms $N_e$. This also increases the distance-to-crash as entrepreneurs and bank managers compete with a lower riskfree rate $r_M$. The tradeoff for the central bank is that the utility of the investor is increased through a higher provision of public goods and more economy stability but the investor is less well diversified given that the allocation to the riskfree security (money) has decreased.

**III.3. Unconventional monetary policy: balance-sheet policy.** The balance-sheet of the central bank is $k_M = k_0 + T + G$ and we now consider and non-zero subsidy $T$ which is
**Figure 6.**

First-best conventional monetary policy (interest rate policy) and distances-to-crash. This figure shows the optimal monetary policy (riskfree policy and provision of public goods) in the absence of stability considerations and the bank distance-to-crash (when 17% of the banks go bankrupt for idiosyncratic reasons) and the firm distance-to-crash (when 50% of the firms go bankrupt). Calibration: $\theta = \sqrt{\frac{f_b}{f_e}} = 20$, utility parameter $\alpha = .236$.

in general positive. If $T < 0$, this is in fact a tax on entry. Moreover, we assume that $T$ is only used to influence the entry of banks and a similar logic applies to the entry of firms.

**Constant interest rate.** We impose that the intervention of the central bank keeps a constant interest rate $r_M$. The number of firms is given by the free entry condition $\frac{f_b}{4N_b^2} = \hat{f}_b - \frac{T}{N_b}$.

We then have

$$\phi_b \phi_e = \sqrt{f_b \left( \frac{f_b - T}{N_b^2} \right)} = \sqrt{f_b f_e + T^2 - T} \leftrightarrow \frac{T}{p_0^2} = \frac{f_b f_e - (\phi_b \phi_e)^2}{2 \phi_b \phi_e}.$$  

and it is equivalent for the central bank to choose $T$ or $\phi_b$ and in what follows we focus on the banking entry cost as the choice variable. A constant $r_M$ leads to $d_M = \left( \frac{r_M}{p_0} \right) (1 - p_0 \phi_e N_e)$ and the provision of the public good is

$$G = 1 - p_0 \phi_e N_e \frac{p_0 a d_M}{r_0} - T = (1 - p_0 \phi_e N_e) \left( \frac{r_0 - r_M}{r_0} \right) - \frac{T}{y},$$  

where $N_e$ depends on $\phi_b$ through Equation (1.11).

**Lemma 11.** The first-best cost $\phi_b^{fb} = H(\phi_e | p_0, \alpha, \theta, r_0)$ is the solution of a 7th-order polynomial equation.
To stimulate the economy, the central bank increases the subsidy $T$ to bank entry. With more bank competition, the deposit rates are higher and the investor rebalances its portfolio towards risky deposits so that its demand for money $k_M$ decreases and the central bank decreases the stored amount of goods $k_0$. Although the riskfree rate $r_M$ remained constant, part of the channel for the transmission of monetary policy is through a rebalancing of the portfolio of the investor. In the next section, we consider an intervention that keeps the portfolio of the investor unchanged.

*Constant money supply.* We assume now that the money supply $k_M$ is constant. This intervention has similarities to sterilized interventions in the foreign exchange markets that keep the domestic supply of money constant.

**Lemma 12.** The first-best entry cost $\phi_b = H(\phi_e|p_0, \alpha, \theta, r_0)$ is the solution of a 3rd-order polynomial equation.

When the central bank fosters bank competition with higher bank subsidies $T$, the risky deposit rate increases and to keep a constant money supply, the central bank needs to also increase the riskfree rate $r_M$ by increasing the allocation of stored goods $k_0$ and the provision of public good $G$ decreases.

**IV. Extensions of the results**

**IV.1. Financial fragility and definition of a crash.** There are many papers studying financial fragility and the propagation and amplification of shocks (most notably Bernanke and Gertler, 1989, 1990; Kiyotaki and Moore, 1997; Brunnermeier and Pedersen, 2009). Allen and Gale (2007) “use the phrase ‘financial fragility’, to describe situations in which small shocks have a significant impact on the financial system.”¹³ For a formalization of the propagation of shocks, introduce an economy where the equilibrium (steady state) is

$$J(x|y, \theta) = 0,$$  

(4.1)

¹³The definition of “fragility” in Brunnermeier and Pedersen (2009) is slightly different and includes the fact that the the endogenous variables “cannot be chosen to be continuous in the exogenous shocks.” In this paper, we refer to situations in which such discontinuities arise as “crashes.”
Figure 7.

First-best unconventional monetary policy (balance-sheet policy) and distances-to-crash. This figure shows the optimal monetary policy (riskfree policy and provision of public goods) in the absence of stability considerations and the bank distance-to-crash (when 17% of the banks go bankrupt for idiosyncratic reasons) and the firm distance-to-crash (when 50% of the firms go bankrupt). Calibration: \( \theta = \sqrt{\frac{\phi f}{\bar{f}}} = 20 \) and utility parameter \( \alpha = 0.236 \).

where \( x \) is a vector of endogenous variables, \( y \) is an exogenous stochastic variable distributed with mean \( y^* \) and standard deviation \( \sigma^* \) and \( \theta \) is the policy instrument. Equation (4.1) defines possibly multiple equilibria and we select the Pareto dominating equilibrium \( x^*(y, \theta) \) as a function of the exogenous parameters \((y, \theta)\). For instance, \( x \) is a vector that summarizes the numbers of firms and banks in the economy, \( y \) is the level of aggregate liquidity, and \( J(x|y, \theta) \) represents the profit of firms and banks which is zero in equilibrium because of free entry. The question is by how much the numbers of firms and banks decrease when the aggregate liquidity \( y \) dries up. Papers on financial fragility and the amplification of shocks study the elasticity of endogenous variables to exogenous shocks \( \frac{\partial x}{\partial y} \) where\(^{14} \)

\[
\frac{\partial x}{\partial y} = -[\nabla J(x|y, \theta)]^{-1} \left( \frac{\partial J}{\partial y} \right),
\]

\(^{14}\text{An example is in Brunnermeier and Pedersen (2009). } x \text{ is the price of some risky security, } y \text{ is the wealth of speculators and in Proposition 5, liquidity spirals are characterized by the price sensitivity to speculator wealth shocks.}\)
where $\nabla J(x|y,\theta)$ is the Jacobian of $J$. In this paper, we define a crash as the critical value $y = \bar{y}(\theta)$ for which the system is singular:

$$\det[\nabla J(x|\bar{y}(\theta), \theta)] = 0; \quad J(x|\bar{y}(\theta), \theta) = 0.$$  

When the Jacobian $\nabla J(x|y,\theta)$ becomes closer to a singular matrix, small shocks lead to larger movements in the endogenous variables. Financial crises take place when there are very large variations, and at the limit, discontinuities, in the endogenous variables. This is related to Bifurcation theory and Catastrophe theory in Mathematics Arnol’d (1992); Demazure (1991). A bifurcation takes place at the value in which the system is singular (the “critical value” or “tipping point”). A “catastrophe” takes place when the bifurcation leads to a jump in the endogenous variables. It should be noted that crashes are essentially nonlinear phenomena that disappear if the system is linearized (see the remarks in Mishkin, 2009, forthcoming; Brunnermeier and Sannikov, 2009). For a given equilibrium $x^*(y,\theta)$, we are interested in the size and shape of the basin of attraction. The distance-to-crash is the distance $y - \bar{y}(\theta)$. The economy can withstand any adverse shock smaller than the distance to crash. When the probability of large adverse shocks increases, the policymaker can adjust the policy instrument $\theta$, which will increase or decrease the distance-to-crash.

V. Conclusion

Bernanke in a speech on August 21, 2009 claimed that “[t]his strong and unprecedented international policy response proved broadly effective. Critically, it averted the imminent collapse of the global financial system, an outcome that seemed all too possible to the finance ministers and central bankers that gathered in Washington on October 10.” The creation of macroprudential regulation in Europe (the European Systemic Risk Board) and discussions in the US on the creation of a similar agency, either as a stand-alone entity or within the Federal Reserve, have put the management of macroeconomic risk into the limelight. Recently Bernanke also addressed the need to prepare for an “exit strategy” for ending the support to financial institutions and the period of very low interest rates.

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15See Varian (1979) for an early discussion of Catastrophe theory in the context of Economics.
In this paper, we propose a framework to guide macroprudential policies aimed at averting crashes. We present a general equilibrium model with two main ingredients: imperfect bank competition and a bank-lending channel for the transmission of monetary policy. In assessing the stability of the economy, we take a “stress-test approach” in which we focus on particular aggregate and idiosyncratic shocks and study their impacts on the equilibrium. Liability-side and asset-side liquidity spirals illustrate how shocks propagate through the economy. In contrast to models in which there is a unique equilibrium and adverse shocks lead the economy to temporary move away from this equilibrium, this paper presents a multiple-equilibria setup in which adverse shocks can lead to durable consequences. A crash is a situation in which the Pareto-dominant equilibrium disappears and only Pareto-dominated equilibria remain. We introduce the distance-to-crash as a metric to evaluate the likelihood that shocks will cause a collapse in financial intermediation and economic activity. When the economy is more vulnerable to adverse shocks with a smaller distance-to-crash, a decrease in the riskfree rate improves the stability by reducing the competition between banks and firms.

The model in this paper can be extended in several ways. First the model is static and a dynamic model would allow to study in more detail the propagation mechanisms of adverse shocks. Second the model has no nominal frictions. Introducing a nominal price level would allow to connect with traditional models of monetary policy and inflation (Woodford, 2003). Third, the paper studies the systemwide ability of the economy to absorb aggregate and idiosyncratic shocks. Individual bank capital as well as interbank lending provide other avenues to absorb shocks. Extending the model to introduce risk-averse bank managers would create a motive to hold riskfree capital as well as participate on the interbank market (Allen and Gale, 2000b; Freixas, Martin, and Skeie, 2009). Fourth, bank or firm heterogeneity would allow to study the effect of adverse shocks on different population of firms and banks and the use of targeted policy responses. Fifth, following cite Adrian, Adrian2, introducing risk-shifting by bank managers and entrepreneurs would allow to study how the incentives of agents over the business cycle influenced by bank competition.
VI. Appendix

VI.1. Elimination theory. Elimination theory deals with the problem of finding conditions on parameters of a polynomial system so that these equations have a common solution (Emiris and Mourrain, 1999). For the system

\[ P(x) = p_0 + p_1 x + \ldots + p_r x^r = 0 \]
\[ Q(x) = q_0 + q_1 x + \ldots + q_s x^s = 0, \]

the Sylvester matrix is

\[
A[P, Q] = \begin{bmatrix}
p_0 & p_1 & \ldots & p_r & 0 & 0 & 0 \\
0 & p_0 & p_1 & \ldots & p_r & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & p_0 & p_1 & \ldots & p_r \\
q_0 & q_1 & \ldots & q_s & 0 & 0 & 0 \\
0 & q_0 & q_1 & \ldots & q_s & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & q_0 & q_1 & \ldots & q_s \\
\end{bmatrix}.
\]

The condition \( \det A[P, Q] = 0 \) on the parameters \( \{p_i, q_j\} \) is imposed if \( P \) and \( Q \) have a common solution. The discriminant of a polynomial equation \( P \) is the determinant of the Sylvester matrix of \( P \) and its first derivative \( P' \). The discriminant characterizes the condition on the parameters for which some solution disappear and plays a crucial role in Bifurcation theory (Arnol’d, 1992; Demazure, 1991).

VI.2. Proofs.

VI.2.1. Proof of Lemma 1. The condition \( (p_0 + p_1)r_n \geq r_M \) is a sufficient condition when there are \( N_e \) ex-post identical firms. In this case the function determining \( k \) is a second degree polynomial equation

\[
k^2 N_e (r - r_M)^2 + k(r - r_M) [r_M (1 + N_e - p_1 - N_e p_0) + N_e r_M - (p_0 + N_e p_1) r] - r_M (1 - p_0 + p_1) [(p_0 + p_1) r - r_M]
\]

This equation has a positive (and a negative) root when \( (p_0 + p_1) r_n \geq r_M \).

VI.2.2. Proof of Lemma 2. The first-order condition imposes

\[
\frac{a - r(1 + \xi)}{r} \left( \frac{\partial \log k}{\partial \log r} \right) = \frac{a - r(1 + \xi)}{r} \left( \frac{\partial \log(r k)}{\partial \log r} - 1 \right) = 1 + \xi.
\]
VI.2.3. Proof of Lemma 4. From Equation (1.6), the revenue per sector is $\frac{f_e}{2N_e}$, a bank serves 2z sectors so that its profit is
\[
0 = \frac{f_e \times 2z}{2N_b} - f_e \int_{-z}^{z} \varphi d\varphi - f_b
\]
where $z = \frac{1}{2N_b}$.

VI.2.4. Proof of Lemma 5. We have $\phi_b^2 = N_e \left( \frac{d}{dM + N_e d} - \frac{d^2}{(dM + N_e d)^2} - 1 \right) = N_e \left( \phi_c - \phi_c^2 - d \right)$ or
\[
\phi_b = N_e \left( 1 - \phi_c - \frac{dM}{(1 - \phi_c N_e)} \right)
\]
The conditions are
\[
\Delta = (1 - d_M - \phi_e + \phi_b \phi_e)^2 - 4\phi_b \phi_e (1 - \phi_e) \geq 0
\]
\[
N_e = \frac{(1 - d_M - \phi_e + \phi_b \phi_e) + \sqrt{(1 - d_M - \phi_e + \phi_b \phi_e)^2 - 4\phi_b \phi_e (1 - \phi_e)}}{2\phi_e (1 - \phi_e)} \geq 1
\]
Note that if $N \geq 1$ is a solution, then $N \leq \frac{1}{\phi_e}$ and the condition $d \geq 0$ is automatically satisfied. The intersection of the $N = 1$ constraint and the discriminant is
\[
\phi_e^* = 1 - \sqrt[3]{d_M}; \quad \phi_e^* = \sqrt[3]{d_M} \left( 1 - \sqrt[3]{d_M} \right)
\]
The number of firms $N_e$ is decreasing in $d_M$ and $\phi_b \phi_e$ and $\phi_e$

- Equation (1.11) is increasing in $d_M$ and $\phi_b$ (since $1 - \phi_e N_e 0$) which decreases the largest root.
- if $\phi_e < \frac{1}{2}$ and $\phi_b < 1$, then the Equation (1.11) is always increasing in $\phi_e$.
- if $\phi_e > \frac{1}{2}$ and $\phi_b < 1$, then the derivative of Equation (1.11) in $\phi_e$ is zero when $N = \frac{1 - \phi_b}{2\phi_e - 1}$ and for this value of $N$, Equation (1.11) is
\[
(1 - \phi_e - \phi_b \phi_e)^2 + d_M (2\phi_e - 1)(1 - \phi_b) > 0
\]
The largest root is then smaller than $\frac{1 - \phi_b}{2\phi_e - 1}$ and therefore Equation (1.11) is increasing in $\phi_e$.

VI.2.5. Proof of Corollary 1. We have $\xi = \frac{(1 - \phi_e)(1 - \phi_e N_e)}{dM} - 1$, $1 - \frac{(1 + \xi)\alpha}{a} = \phi_e$, and $1 - \frac{\xi}{a} = 1 - \frac{dM}{1 - \phi_e N_e}$. Introduce $h = \frac{dM}{1 - \phi_e N_e}$ and $z = (1 - \phi_e)(1 - \phi_e N_e)$. Then $h$ and $z$ are the two roots of the equation ($h$ is the largest root, $z$ the smallest):
\[
d_M (1 - \phi_e) - h(1 - \phi_e \phi_b + d_M - \phi_e) + h^2 = 0
\]
We study the effect of $d_M$ and $\phi_e$ on $h$

- since $d_M (1 - \phi_e) - (1 - \phi_e)(1 - \phi_e \phi_b + d_M - f_e) + (1 - \phi_e)^2 = \phi_e \phi_b (1 - \phi_e) > 0$, then $h < (1 - \phi_e)$ and $\frac{\partial h}{\partial d_M} < 0$. A higher $d_M$ allows for fewer firms and bank markups $\xi$ and higher total markups $1 - \frac{\xi}{a}$.
- since $d_M (1 - \phi_e) - d_M (1 - \phi_e \phi_b + d_M - f_e) + d_M^2 = \phi_e \phi_b d_M > 0$, then $h < d_M$ and $\frac{\partial h}{\partial \phi_e} < 0$.
- similarly $z$ is increasing in $\phi_e$. 
VI.2.6. Proof of Lemma 6. The two constraints can rewritten as

\[ 1 - (1 + \theta)\phi_e^2(1 - \phi_e) = d_M; \quad \sqrt{1 - \phi_e} - \sqrt{\theta \phi_e} = \sqrt{d_M} \]

Note that \((1 + \theta)\phi_e^2 = \phi_e^2 + \phi_e \phi_b = \frac{f_e + \sqrt{f_b f_e}}{\phi_b} \).

VI.2.7. Proof of Lemma 7. A decrease in the number of banks \(N_b\) translates immediately in an increase in \(\phi_b\) since \(N_b = \frac{1}{2} \sqrt{f_b}\) and the left-hand side of bank pricing equation is \(\frac{f_e \phi_b}{N_b^2}\).

VI.2.8. Proof of Lemma 8. The critical number of firms for the entry of banks is defined when the derivative of the bank competition is zero, which implies

\[
d = \frac{d}{d_M + N d} - (\tilde{N} + 2) \left( \frac{d}{d_M + N d} \right)^2 + 2\tilde{N} \left( \frac{d}{d_M + N_e d} \right)^3 \quad (6.1)
\]

The bank competition condition is

\[
0 = \left( \frac{d}{d_M + N d} \right)^2 - \frac{d}{d_M + N d} + d + \frac{\phi_b \phi_e}{N}
\]

The critical number of firms for the entry of banks is defined when the derivative is zero, which implies

\[
d = \frac{d}{d_M + N d} - (\tilde{N} + 2) \left( \frac{d}{d_M + N d} \right)^2 + 2\tilde{N} \left( \frac{d}{d_M + N_e d} \right)^3 \quad (6.2)
\]

Introduce \(x = \frac{\tilde{N} d}{d_M + N d}\). \(x\) can be interpreted as a (normalized) measure of aggregate investment at the critical number of firms for some bank entry. Then using Equation (6.2), we

\[
\tilde{N} = \frac{2x}{1 - \frac{d_M}{(1-x)^2}}; \quad d = \frac{d_M x}{(1 - x)\tilde{N}}
\]

Plugging these \(\tilde{N}\) and \(d\) and rearranging, we find Equation (2.1). Since the left-hand side of Equation (2.1) is increasing in \(x\), then the equilibrium \(x\) is increasing in \(\phi_e \phi_b\) and \(d_M\). Moreover the critical \(\tilde{N}\) is increasing in \(x\) and \(d_M\), therefore increasing in \(d_M\) and \(\phi_b \phi_e\). On the other hand the number of firms \(N_e\) is decreasing in \(d_M\), \(\phi_b \phi_e\) and \(\phi_e\) so that the distance-to-crash defined as the ratio \(\frac{N_e}{\tilde{N}}\) is decreasing in the entry costs \(f_e\) and \(f_b\) and the money dividend \(d_M\).

The two constraint (critical \(\tilde{N}\) and \(N = 1\)) intersect when \(\tilde{N} = 1\). In this case, there are two constraints in \(x\)

\[
\frac{2x}{1 - \frac{d_M}{(1-x)^2}} = 1; \quad x \left( 1 - \frac{(1 - 2x)d_M}{(1 - x)^2} \right) = 2\phi_e \phi_b
\]

Eliminating \(x\) by using the determinant of the Sylvester matrix of these two equations, we find

\[
4d_M^2 - 4d_M (1 - 2\phi_e \phi_b) - (\phi_e \phi_b)^2 (1 - 2\phi_e \phi_b) = 0
\]

Note that if \(\phi_e \phi_b \geq \frac{1}{2}\), this equation has not solution.
VI.2.9. Proof of Corollary 2. The critical value is at \( \phi_e = 1 - \sqrt[3]{d_M} \) which is equivalent to \( \phi_b = \phi_e(1 - \phi_e) \) or \( \phi_e = 1 - \frac{\phi_b}{\phi_e} \).

VI.2.10. Proof of Lemma 9. From the equilibrium equation on \( N_e \) in Equation (1.11) we have

\[
\frac{\partial N_e}{\partial d_M} = - \frac{N_e}{2N_e \phi_e(1 - \phi_e) + d_M + \phi_e(1 - \theta) - 1}
\]

The first-order condition on maximizing \( \log(p_0ay) + \log(d_M) - p_0 \log(1 - \phi_e N_e) \) yields

\[
0 = \frac{1}{d_M} + \frac{\partial N_e}{\partial d_M} \frac{p_0 \phi_e}{1 - \phi_e N_e}
\]

The equation in \( r_M \) is

\[
r_M^2 (1 - \phi_e) + r_M (-\phi_e^2 p_0^2 \theta^2 - 4 \phi_e^3 p_0 \theta + 2 \phi_e^2 \theta + 4 \phi_e^2 p_0 \theta - 2 \phi_e^2 \theta - 2 \phi_e^2 + 4 \phi_e - 2) + (1 - \phi_e)(1 - \theta \phi_e^2 - \phi_e)^2
\]

VI.2.11. Proof of Lemma 3. When \( d_M \leq d_M^* \), then \( \frac{\partial r_M}{\partial d_M} > 0 \).

VI.2.12. Proof of Lemma 10. The first-order condition is now

\[
0 = \frac{1}{d_M} - \frac{\alpha p_0 a}{1 - p_0 \phi_e N_e - \frac{p_0 d_M}{\alpha}} + \frac{\partial N_e}{\partial d_M} \left[ \frac{p_0 \phi_e}{1 - \phi_e N_e} - \frac{\alpha p_0 \phi_e}{1 - p_0 \phi_e N_e - \frac{p_0 d_M}{\alpha}} \right]
\]

This equation is equivalent to a 3rd-degree polynomial function in \( N_e \). Using Equation (1.11), we can write the Sylvester matrix of the two polynomial functions which yields a 4th-degree polynomial in \( d_M \).

With the condition \( G = 0 \), the first-order condition becomes \( \frac{\partial N_e}{\partial d_M} p_0 \phi_e + \frac{p_0 a}{r_0} = 0 \). Using Equation (1.11) and \( G = 0 \) allows to eliminate \( d_M \) and \( N_e \) and to find the condition

\[
\left( \frac{r_0}{p_0 a} \right)^2 - 2 \left( \frac{r_0}{p_0 a} \right)(1 - \phi_e + \phi_e^2 \theta - 2 \phi_e p_0 \theta) + (1 - \theta \phi_e^2 - \phi_e)^2 \leq 0
\]

VI.2.13. Proof of Lemma 7?. Introduce \( F = \phi_b \phi_e - (\phi_e N_e)(1 - d_M + \phi_c \phi_b - \phi_e) + (\phi_e N_e)^2(1 - \phi_e) \). We have

\[
\phi_e \phi_b = \sqrt{f_e \left( f_b - \frac{f_e}{N_e} \right)}; \quad \phi_e = \sqrt{f_c - \frac{f_e}{N_e}}
\]

\[
\frac{\partial (\phi_e \phi_b)}{\partial T_b} = -\frac{1}{2N_b p_0 a y} \left( \frac{f_e}{f_b - \frac{f_e}{N_e}} \right); \quad \frac{\partial \phi_e}{\partial T_b} = \frac{1}{2p_0 a y \phi_e N_e} \frac{1}{\phi_e N_e} = 1 - \phi_e N_e; \quad \frac{\partial F}{\partial T_b} = \phi_e N_e (1 - \phi_e N_e)
\]

Since \( \frac{\partial (\phi_e N_e)}{\partial T_b} = \frac{\partial (\phi_e N_e)}{\partial T_e} \frac{\partial T_e}{\partial T_b} = \frac{1 - \phi_e N_e}{\sigma(\phi_e, N_e)} \frac{\partial T_e}{\partial T_b} = \frac{\partial (\phi_e N_e)}{\partial T_e} \frac{\partial T_e}{\partial T_b} = \frac{1 - \phi_e N_e}{\sigma(\phi_e, N_e)} \frac{\partial T_e}{\partial T_b} \), we have

\[
\frac{\partial (\phi_e N_e)}{\partial T_b} > \frac{\partial (\phi_e N_e)}{\partial T_e}
\]
VI.2.14. Proof of Lemma 11. We have \( \frac{\sqrt{f}}{2N_e} = \sqrt{f_0 + \frac{p_0}{\ell}} - \frac{T}{\sqrt{f_0}} \). Differentiating the endogenous variables, we have

\[
\frac{\partial \left( \frac{f}{\sqrt{f_0}} \right)}{\partial (\phi_e \phi_b)} = -\frac{1}{2} \left[ 1 + \frac{f f_0}{\sqrt{f_0}} \right] (\phi_e \phi_b)^2
\]

The equation determining \( N_e \) is now

\[
0 = \phi_e \phi_b - (\phi_e N_e) \left( 1 - \frac{r_M}{p_0 a} - \phi_e + \phi_b \phi_e \right) + (\phi_e N_e)^2 \left( 1 - \phi_e - p_0 \frac{r_M}{p_0 a} \right) \tag{6.3}
\]

We then have

\[
\frac{\partial (\phi_e N_e)}{\partial (\phi_b \phi_e)} \bigg|_{r_M} = -\frac{1 - \phi_e N_e}{2 \phi_e N_e} \left( 1 - \phi_e - p_0 \frac{r_M}{p_0 a} \right) - \left( 1 - \frac{r_M}{p_0 a} - \phi_e + \phi_b \phi_e \right)
\]

The utility is

\[
U_{\text{entry}} = \log(1 - p_0 \phi_e N_e) - p_0 \log(1 - \phi_e N_e) + \alpha \log \left( 1 - p_0 \phi_e N_e \right) \left( \frac{r_0 - r_M}{r_0} \right) - \frac{T}{y} + \log \left( \frac{r_M}{p_0 a} \right)
\]

The first-order condition is

\[
\left( -\frac{1}{1 - p_0 \phi_e N_e} + \frac{1}{1 - \phi_e N_e} - \frac{\alpha \frac{r_0 - r_M}{r_0}}{1 - p_0 \phi_e N_e} \frac{\partial (\phi_e N_e)}{\partial (\phi_b \phi_e)} \right) \bigg|_{r_M} = \frac{\alpha p_0 a}{(1 - p_0 \phi_e N_e) \frac{r_0 - r_M}{r_0} - \frac{T}{y}} \frac{\partial (\frac{f}{\sqrt{f_0}})}{\partial (\phi_e \phi_b)}
\]

The discriminant for Equation (6.3)

\[
\phi_b \phi_e = \frac{(\sqrt{r_M(1 - p_0)} - \sqrt{p_0 a (1 - \phi_e)} - p_0 r_M)^2}{p_0 a}
\]

The number of firms when this discriminant is equal to zero is \( N_e = 1 - \sqrt{\frac{r_M(1 - p_0)}{p_0 a (1 - \phi_e) - p_0 r_M}} \).

VI.3. No competition equilibrium. There are two regimes illustrated on Figure 8. In the competition regime, the distance between banks is small and banks are contesting each other’s loans. In the no-competition regime, the distance between banks is larger, some industries are not served and banks operate on “isolated islands” without feeling the direct influence of other banks.

In the no-competition regime where loan terms are not contested, banks set the intermediation markup \( \xi \) to maximize their revenues which imply the equation

\[
1 = \frac{p_0 ay}{r_M k_M + r_n k_n + \sum_{-i} r_j k_j} + \frac{p_1 ay}{r_M k_M + r_n k_n} - \ldots
\]

\[
\ldots - 3r_n k_n \left( \frac{p_0 ay}{(r_M k_M + r_n k_n + \sum_{-i} r_j k_j)^2} + \frac{p_1 ay}{(r_M k_M + r_n k_n)^2} \right) + \ldots
\]

\[
\ldots + (r_n k_n)^2 \left( \frac{p_0 ay}{(r_M k_M + r_n k_n + \sum_{-i} r_j k_j)^3} + \frac{p_1 ay}{(r_M k_M + r_n k_n)^3} \right) \tag{6.4}
\]
**Figure 8.**

**Competitive regimes.** The left panel illustrates the intermediation decisions of banks when the distance $2z$ between banks is small. In this case, the intermediation cost of bank 2 is below its monopoly revenues for some industries close to bank 1. Contestability brings the bank revenues down (competitive revenues line). The right panel illustrates the intermediation decisions of banks when the distance $2z$ between banks is small. In this case, the intermediation cost of bank 2 is above the monopoly revenues for all industries between bank 1 and $\tilde{z}$ it can not contest loans. Industries between $\hat{z}$ and $\tilde{z}$ are not served because the intermediation costs for banks 1 and 2 are above the monopoly revenue.

Introduce $z_m$, the maximum “distance” to industries which are served: $f_\varphi z_m$ is the the monopoly revenues from serving a given industry and an industry is served if the revenue it generates is above the cost $f_\varphi z_m \geq f_\varphi z$. In this case, the free entry condition on the profit of banks is

$$f_\varphi z_m(2z_m) - f_b - f_\varphi z_m^2 = 0 \iff f_\varphi z_m = \sqrt{f_b f_\varphi}$$

Since the distance between banks is $z = \frac{1}{2N_e} \geq z_m$, the expected profit of entrepreneurs (set to zero because of free entry) is

$$(p_0 + p_1)2z_m N_b (r_n k_n) ay \left[ \frac{p_0}{(r_M k_M + r_n k_n + \sum_j r_j k_j)^2} + \frac{p_1}{(r_M k_M + r_n k_n)^2} \right] - f_e = 0$$

The system in the variables $(d, N_e, N_b, z_m)$ is

$$1 = \frac{1}{d_M + N_e d} + \frac{\lambda}{d_M + d} - \frac{3d}{(d_M + N_e d)^2} - \frac{3d\lambda}{(d_M + d)^2} + \frac{2d}{(d_M + N_e d)^3} + \frac{2d^2\lambda}{(d_M + d)^3}$$

$$\phi_e \phi_b = N_e \rho \left[ (d_M + (N_e - 1) d)^2 + \frac{\lambda d_M}{(d_M + d)^2} - 1 \right]$$

$$\phi_e^2 = \frac{N_b}{2} \frac{\lambda d}{f_b} \left[ \frac{1}{(d_M + N_e d)^2} + \frac{\lambda}{(d_M + d)^2} \right]$$
When $\lambda = 0$, the system simplifies to

\[
1 = \frac{1}{d_M + N_e d} - \frac{3d}{(d_M + N_e d)^2} + \frac{2d^2}{(d_M + N_e d)^3}
\]

\[
\phi_e \phi_b = N_e d \left( \frac{1}{d_M + N_e d} - \frac{d}{(d_M + N_e d)^2} - 1 \right)
\]

\[
N_b = \frac{d}{\phi_e(d_M + N_e d)}^2
\]

Introduce $x = \frac{d}{d_M + N_e d}$. Then $d = \frac{d_M}{1 - N_e x}$ and from the first equation, $N_e x = 1 - \frac{d_M}{(1 - x)(1 - 2x)}$. Plugging this into the second equation we find

\[
\phi_e \phi_b = 2x(1 - x) \left( 1 - \frac{d_M}{(1 - x)(1 - 2x)} \right) \text{ or }
\]

\[-\phi_e^2 \theta - 2x(-\theta \phi_e^2 + d_M - 1) - 6x^2 + 4x^3 = 0\]

There are potentially three constraints: the last equation has a positive solution; there are at least one firm per sector $N_e \geq 1$; and there is no contestability in loan pricing: $N_b \leq \frac{1}{2} \sqrt{\frac{L}{f_b}}$ or $x \leq \phi_e$. It turns out the first constraint is never binding when at least one of the other two is. And the other two are stricter constraints than the condition of existence in the full competition case (Equation (1.12)). One can show the property

- if $\theta < 1$, only the constraint $N_b \leq \frac{1}{2} \sqrt{\frac{L}{f_b}}$ is binding: the entry cost of banking $\sqrt{f_b f_e}$ is relatively cheap and the constraint that no all sector are served by a bank is the binding one.

- if $\theta > 2$, only the constraint $N_e > 1$ is binding: now the entry cost of bank is high, which requires and high intermediation markup and hurts the entry of firms. The constraint $N_e = 1$ is then more binding.

- when $\theta \in [1, 2]$, there is an intersection between the two constraints at $\phi_e = \frac{2 - \theta}{2}$ and $d_M = \frac{(\theta - 1)\theta^2}{4}$. When $\phi_e < \frac{2 - \theta}{2}$, the constraint $N_b \leq \frac{1}{2} \sqrt{\frac{L}{f_b}}$ is the binding one; and when $\phi_e \in \left[\frac{2 - \theta}{2}, \frac{1}{2\sqrt{\theta}}\right]$, the constraint $N_e > 1$ is the binding one.

VI.4. Allocation with no money: $r_M = 0$. We set $r_M \to 0$ and look at the system of equation with idiosyncratic risk $\lambda > 0$. While this assumption allows to study the role of idiosyncratic risk, it obviously prevents to talk address the role of policy and it also prevents from making any welfare statements (since the utility of the investor is not defined). The system of equation in full competition regime is

\[
\phi_e = \frac{1}{N_e}; \quad \phi_b \phi_e = (N_e + \lambda) \left( \frac{N_e - 1}{N_e} - d \right)
\]

or equivalently

\[
N_e = \frac{1}{\phi_e}; \quad d = \phi_e \left( 1 - \phi_e - \frac{\phi_b \phi_e^2}{1 + \lambda \phi_e} \right)
\]

The condition for the existence of the equilibrium is $d \geq 0$ or

\[(1 - \phi_e)(1 + \lambda \phi_e) \geq \phi_b \phi_e^2\]
Competition is characterized by the markups

\[ 1 + \xi = (1 + \lambda) \left( \frac{1}{1 - \frac{\phi_b \phi_e^2}{(1 + \lambda \phi_e)(1 - \phi_e)}} \right) \]

\[ 1 - \frac{r}{a} = 1 - \frac{\phi_e^2}{1 + \lambda \phi_e} \left( 1 - \phi_e - \frac{\phi_b \phi_e^2}{1 + \lambda \phi_e} \right) \]

\[ 1 - \frac{(1 + \xi)r}{a} = \frac{1 + \phi_e^2 \lambda}{\phi_e (1 + \phi_e \lambda)} \]

Economic stability is characterized by the distances-to-crash

\[ \frac{1}{1 - \omega_a} = \frac{\phi_e^2 + \lambda}{X_{a,1} (\lambda, \sqrt{f_b f_e})^2 + \lambda} \]

where \( x = X_{a,1} (\lambda, \sqrt{f_b f_e}) \) is the solution of

\[ (1 - x)(1 + \lambda x) = \sqrt{f_b f_e} f_e (x^2 + \lambda) x \]

and

\[ \frac{1}{1 - \omega_b} = \frac{(1 - \phi_e)(1 + \lambda \phi_e)}{\phi_b \phi_e} \]

\[ \frac{1}{1 - \omega_e} = \frac{\sqrt{(1 - \lambda - \phi_e \phi_b)^2 + 4 \lambda} - (1 - \lambda - \phi_e \phi_b)}{2 \lambda \phi_e} \]

**Lemma 13.** Competition is decreasing in \( \lambda \). Distances to crash are increasing in \( \lambda \).

**References**


