The Dynamics of Marriage and Divorce*

Gustaf Bruze\textsuperscript{a}, Michael Svarer\textsuperscript{a} & Yoram Weiss\textsuperscript{a,b}

\textsuperscript{a}Department of Economics and Business, Aarhus University

\textsuperscript{b}Eitan Bergalas School of Economics, Tel Aviv University

February 19, 2012

Abstract

We formulate and estimate a dynamic model of marriage, divorce, and remarriage using 27 years of panel data for the entire Danish cohort born in 1960. The marital surplus is identified from the probability of divorce, and the surplus shares of husbands and wives from their willingness to enter marriage. Education and marriage order are complements in generating gains from marriage. Educated men and women receive a larger share of the marital gains but this effect is mitigated when their proportion rises. Education stabilizes marriage and second marriages are less stable. As the cohort ages, uneducated men are the most likely to be single.

\footnote{Acknowledgement: We thank the Danish Social Science Research Foundation, The Danish Council for Independent Research in Social Sciences, and the Cycles, Adjustment, and Policy Research Unit (CAP) at Aarhus University for financial support. We also thank Rune Vammen Lesner and Jonas Maibom Pedersen for excellent research assistance. Finally, we thank Jerome Adda, B J Christensen, John Knowles, Dale Mortensen, Victor Rios-Rull, and Aloysius Siow for useful comments on previous drafts.}
1 Introduction

Modern marriage markets are characterized by high turnover; men and women divorce more but also remarry more than in the past. However, different individuals have different marital histories; they marry, divorce, and remarry at different rates. To explain this variation it is crucial to understand the two-sided aspect of marriage markets. Marriages form and dissolve based on preferences and expectations of two different individuals who operate in a "marriage market" with many competing agents.

In this paper, we examine the marriage patterns within a large cohort consisting of all men and women in Denmark who were born in 1960, which we observe from age 20 to age 46. We have information not only on their own attributes and marital history but also on that of their various spouses. The spouses can belong to different cohorts and thus may be younger or older than the respondents. We consider two alternative definitions of marriage: legal marriages and partnerships that include legal marriages and cohabitation. Most initial partnerships involve cohabitation but upon approaching age 46 most partnerships are marriages. The hazards of entry into and exit out of partnerships are initially much larger than the hazards of entry and exit into regular marriage. However long term partnerships have similar exit and entry rates to legal marriages. We mainly analyze legal marriages. We then show that, in the long term, the matching patterns are similar under the two definitions.

We focus on marriage patterns by completed education at age 46, when men and women are classified into three education groups. We then ask which types of individuals are more likely to marry, to stay married, and to remarry and which types of marriage (based on the education of both partners) are more stable. We analyze the complex dynamic interactions between marriage, divorce and assortative matching as they evolve over time. Assuming that agents are forward-looking such analysis requires a structural model that specifies the marital preferences of men and women for different types, their expectations of future marital options and the marital choices that they make based on these expectations. We also need to specify how the marriage market operates in terms of the meeting technology and the ability of spouses to transfer economic resources between each other in order to attract partners into marriage and maintain the stability of the marriage.

We formulate and estimate such a model and show that it fits the data well. The general approach is matching without frictions. The basic simplifying assumption is that
individuals can be classified into few categories. Every man (woman) is indifferent between all women (men) in the same category but still has idiosyncratic preferences over categories (meeting one person of each category is sufficient). Given a large number of men and women in each category, the share that each agent receives of the systematic gains from marriage is the same with all potential spouses of a given category. These common share components satisfy stability conditions and respond to the number of single men and women of each type in a given year.

A major challenge for any economic analysis of marriage is that in most cases the "prices" or the division of marital gains are not observed. The methodological innovation of this paper lies in the ability to infer such transfers from data in which one observes both marriage and divorce. Information on the dissolution of marriages of different types provides information on the joint value of the marriage to both partners. The willingness of each single man and woman to enter into a particular type of marriage gives us information on the gains from marriage that each one of them expects separately. The paper provides a dynamic analysis of the gains from marriage, its division and the sorting by education in a given cohort over time. In this regard we extend and complement previous work by Choo and Siow (2006) and Chiappori, Salanie and Weiss (2011). The panel data that we use allows us to observe both the inflows and the outflows into different types of marriage, which provides more information on the gains from marriage and its division than what can be obtained from cross-section data.

Similar to previous studies, we find strong assortative matching by education. A novel aspect of our data is to show that such sorting operates gradually via the selection into first and second marriages and also by the selection out of these marriages, based on the education level of both partners. In particular we find that men with low education are increasingly sorted out of marriage and remain or end up single. This happens because marriages in which both partners or the men have low education dissolve at a higher rate. Our model captures these basic features by showing that the marital output flow and the expected marital surplus is supermodular in the education of the husband and the wife which implies that the total aggregated marital output flow over all possible assignments, is maximized when agents marry a spouse with the same education. Our results also indicate that education raises the marital output flow, and that more educated men and women receive a larger share of the marital surplus. In addition, women receive a lower share of the marital surplus in second marriage.

A closely related paper is Brien, Lillard and Stern (2006). They estimate a dynamic
model of marital transitions for women in the US. They include learning about match quality, children as an exogenous event, and a three-way choice between singlehood, cohabitation, and marriage. Our approach complements their work by emphasizing the two-sided aspects of the marriage problem and the role of competition. A recent paper by Gemici and Laufer (2011) extends that work to include the different division of labor of cohabiting and married couples. Another related paper is Aiyagari et al. (2000) that construct a dynamic model of marriage and divorce and use calibration to simulate the impact of policy changes such as child support. Although our data includes information about children, we do not use that information in this work. Instead, our focus is on assortative matching by schooling as it evolves over time and the associated shares in the gains of marriage that partners with different schooling obtain.

We proceed by first describing our unique Danish data in terms of the assignment patterns by education within a cohort as it ages. We provide details on the transitions between marriage, divorce and remarriage, separating first and second marriages. We then describe the model and highlight the main assumptions. Finally, we present and discuss the estimated parameters, the derived marital output flow, the estimated expected marital surplus, and the husband’s and wife’s surplus and marital shares.

2 The sample

The data we use is register-based and is collected by Statistics Denmark. All information is derived from public registers which are combined according to a social security number (all Danish citizens have a social security number). The data set follows the 1960-cohort annually from 1980-2006 (i.e. from ages 20 to 46). In order to be included in a given year, individuals must have an address in Denmark. This implies that individuals who are not living in Denmark in a given year are not sampled in that year and do not have a complete marital history. We restrict the sample to individuals with records in all years which implied that we lost around 24% of the sample (50% of these were immigrants who were born in 1960 but entered Denmark after 1980). In addition, we eliminated those who were married at age 20. This additional restriction caused us to lose about 4 percent of our sample, mainly women. This leaves us with 61780 individuals.
2.1 Education groups

We distinguish between three education groups, based on level of completed education at age 46: high school or less, vocational education, and college (defined as some college or more).

Table 1 below shows the distribution by level of education for men and women. There is about an equal number of men and women in the sample. However, the distribution by education differs between men and women. We see that the proportion with high school or less is slightly higher for men, the proportion with vocational education is also higher for men but the proportion with some college or more is higher for women.\(^1\)

TABLE 1

Sample distribution of male and female completed education

<table>
<thead>
<tr>
<th>Completed Education (at 46)</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>High school or less</td>
<td>0.33</td>
<td>0.30</td>
</tr>
<tr>
<td>Vocational education</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>Some college or more</td>
<td>0.26</td>
<td>0.33</td>
</tr>
</tbody>
</table>

No. of observations 31835 29945

2.2 Individual Marriage Patterns

2.2.1 Cohabitation and marriage

Cohabitation is a common phenomenon in Denmark (see e.g. Svarer 2004). More than 60% of all young couples are cohabiting and it is somewhat more common among men and women with a high level of completed education. To be classified as cohabiting by Statistics Denmark, an individual must share an address with an opposite sex person who is not family-related, and who is no more than 15 years older or younger than the individual. This measure is more noisy than legal marriage, since e.g. two students of opposite sex who share a flat but are not in a relationship will be registered as cohabiting.

\(^1\)As mentioned, we exclude individuals for which we do not have complete marriage market histories. The sample attrition tends to sort out the more highly educated from the final sample.
In this paper we do not try to model the cohabitation decision explicitly but consider two alternative definitions: legal marriages and partnerships that include legal marriages and cohabitation. A three-state model with cohabitation as a separate choice is left for future work.\footnote{There are different legal implications of cohabitation vis-à-vis formal marriage. In some respects, couples who cohabited for more than two years least two years are considered as married. For instance, the law stipulates that if a couple has cohabited for more than two years the partner has the right to keep the apartment if the other partner (who originally rented the apartment) dies.}

As one can see in the Figures 1, 2 and 3, most initial partnerships involve cohabitation but upon approaching age 46 most partnerships are marriages. Also, the hazards of entry into and exit out of partnerships are initially much larger than the hazards of entry and exit into regular marriage.

2.2.2 Marriage patterns by education and gender

Figure 4 shows the proportions of men and women in marriage and partnership, respectively, by education. At early ages, women with medium education enter marriage and partnership at the highest rate. Highly educated men and women delay their entry, but by age 46 they are the most likely to be married. Men with low education are increasingly left behind and by age 46 about 55% of them are unmarried and 41% are without a partner. These time patterns are very similar for marriages and partnerships.

2.2.3 Marriage and divorce hazards

The time patterns of the proportions married can be traced back to differences in the hazard to enter marriage, exit from marriage and to remarry in our sample. Figures 5 and 6 show the hazards of entry and exit into and out of first marriages. As one would expect, the hazard rates into first marriage are higher for women at early ages and highly educated men and women delay their marriage. Divorce hazards are highest in marriages involving low education agents and lowest when the two partners are highly educated. Hazards of entry into second marriage are higher than into first marriage.

2.2.4 Who marries whom

Conditioned on his/her education, each man (woman) can marry three types of spouses distinguished by their level of education. The actual choices made depend on gender and
the agent’s own level of education as shown in Figures 9 and 10. We see that highly educated men marry mainly highly educated women. This proportion rises sharply as the cohort advances in age and reaches 60% at age 46. The probability for a highly educated man to have a wife with high school or less declines sharply, reaching 13% at age 46. About half of the wives of men with vocational education have vocational education themselves and this proportion remains stable as the cohort ages. Men with low education marry mainly women with low education. With the passage of time, the proportion of marriages of low education men with women of higher education rises, reflecting the higher stability of these marriages.

The marital choices of women are similar to those of the men with some noticeable differences: The proportion of highly educated women that have a highly educated husband at age 46 is 53%, which is lower than the proportion of highly educated men that have a highly educated wife, which is 60%. The proportion of highly educated women that have a husband with low education at age 46 is 16%, which is higher then the proportion of highly educated men that have a wife with low education which is 13%. These differences reflect the fact that women in our sample are more educated than men and, therefore, forced to marry a husband with less education.

The choice probabilities are similar when we replace marriages by partnerships (i.e. legal marriage or cohabitation). For instance, conditioned on being in a partnership, the probability that a highly educated man will be in a partnership with a highly educated woman is 61%. The corresponding probability for educated women is 49%.

2.2.5 Second marriages

Individuals who divorce remarry quickly and the hazards of entry into second marriage exceed the hazards of entry into first marriage (see Figure 7). This indicates that individuals who divorce wish and are able to remarry rather quickly (see Browning, Chiappori, and Weiss 2011, chapter 1). Men enter second marriages faster than women, especially educated men. The time patterns of the marital choices of different types of agents, conditioned on first and second marriages, are also quite close to each other. This is true for both men and women. Thus, from the point of view of the assignment, it does not matter much if one is in a second or first marriage when one gets beyond age 30 or so.\(^3\) However, second marriages are substantially less stable than first marriages, especially if

\(^3\)One exception is that the proportion of marriages in which both partners have medium education is higher in first marriages.
both spouses have low education. This is consistent with the findings of Svarer (2004) using Danish data, and the findings of Parisi (2008) using British data.

### 2.2.6 Sorting in the marriage market

As members of the cohort advance in age and make their marriage and divorce decisions each year, the composition of married couples by the education of the spouses and the composition of singles by their own education both change. Changes in the degree of sorting over time occurs mainly through two mechanisms: First, marriages in which both partners are highly educated are the most stable and, therefore, their proportion among all marriage rises. Second, men and women with low education are more likely to remain single following marital dissolution and, therefore, marriages involving individuals with low education become increasingly rare. This trend is noticeably stronger among men. By age 46, 54% of the unmarried men have low education compared with the 31% who have high education. The corresponding percentages among women are 46 % and 36%, respectively. The role of education in stabilizing marriages and the difficulties that less uneducated men face in marriage are not special to Denmark. Similar results for the US are reported by Weiss and Willis (1997), Stevenson and Wolfers (2007), and Chiappori, Salanie, and Weiss (2011).

### 3 A dynamic model of marriage and divorce

#### 3.1 The general approach

We now propose a dynamic model that captures and interprets the main features observed in our data. The general approach is matching with transferable utility and without frictions. Within this framework, the basic simplifying assumption is that individuals can be classified into few types. Every man (woman) is indifferent between all women (men) of the same type but still has idiosyncratic preferences over types. Given a large number of men and women in each type, the share that each agent receives of the systematic gains from marriage is the same for all members of a given type. These common shares are determined by stability conditions and generally respond to market conditions. When making choices, individuals anticipate the aggregate effects that are transmitted through the shares of the marital surplus that husbands and wives receive each year during their lifetime. In this framework, we analyze individual choices to marry, divorce, and remarry.
We also analyze aggregate outcomes such as the number of men and women that remain single, the types of marriage that are formed and survive, and the educational composition of different marital groups over time.

### 3.2 Types of Men and Women

Time is discrete and the economy is populated by men and women who can choose to be single or marry a member of the opposite sex. All men and women live for $T$ periods, die, and then receive no further utility. Utility flows over these $T$ time periods are discounted geometrically with the discount factor $R$. We denote the time period by $t$.

Men and women are characterized by their educational attainment $e$ which can be either low, medium, or high. We denote the set of these three educational types by $E = \{l, m, h\}$. Men and women are also characterized by their marital histories $p$. We distinguish between men and women who have never previously been married ($p = f$) and men and women who have been married previously at least once ($p = s$). The set of these two divorce types is denoted by $P = \{f, s\}$. Finally, men and women are characterized by their preferences for being single. These preferences are observed by all agents in the model but not by the econometrician. These preference types are intended to capture the permanent unobserved differences in the willingness to marry. We let $U = \{1, 2\}$ denote the set of the two different preference types.

In total there are thus 12 types of men which we index by the letter $i \in E \times P \times U$, 12 types of women which we index by the letter $j \in E \times P \times U$, and a total of 144 different types of marriages as given by all possible combinations of husband and wife types $i, j$.

### 3.3 Marriage

If married, a man of type $i$ and a woman of type $j$ generate together a marital output flow that they can divide between them. We assume that the systematic component of the marital output flow, denoted by $\zeta^{i,j}$, depends on the education and marital histories of both partners, but not on their preference type or age (although age has an indirect effect on utility via the marital histories of the partners). We allow for full interactions in the education of the husband and the wife so that we can test for the complementarity of male and female education. In addition, the order of marriage enters in a separable additive manner which allows us to rank marriages by their order. Specifically, the systematic part
of the marital output flow is given by

$$\zeta^{i,j} = \sum_{k=1}^{9} \alpha_k \cdot E^{i,j}_k + \sum_{l=1}^{4} \beta_l \cdot H^{i,j}_l,$$

(1)

where $E^{i,j}_k$ is a dummy variable that takes on a different value for each of the nine possible combinations of husband and wife education, and $H^{i,j}_l$ is a dummy variable that takes on a different value for each of the four combinations of husband and wife marital histories. By assumption, the marital output flow is constant over time in each type of marriage. We assume that upon dividing the marital output, utility is transferred between the husband and the wife at a one-to-one exchange rate.

In addition to the systematic part $\zeta^{i,j}$, partners receive a flow utility from the quality of their match $\theta_t$. The quality of match is an iid match-specific random variable drawn from a standard normal distribution, which is revealed to the partners only at the end of each period. In this regard, marriage is an "experience good". In particular, single agents who marry at time $t$ do not know the quality of their match $\theta_t$ and expect it to equal the mean which is set to zero.

Following the realization of their match quality, the partners decide whether to continue the marriage or not. Since utility is transferable in marriage, divorce depends only on the sums of utilities upon separation and upon remaining single. Hence, husbands and wives "agree" on when to divorce. The total utility flow of the partners who stay married is then $\zeta^{i,j} + \theta_t$. By divorcing, the partners can avoid a bad realization of their match-specific quality. However, divorce also entails a fixed cost of separation $s_t$ which depends on the duration of the marriage $d_t$, through the function $s_t = s(d_t)$. This divorce cost plays an important role in the model since it reduces "experimental short marriages" and limits turnover.

### 3.4 Singlehood

Single agents receive a flow utility of $\varphi_t$ each period which depends on their gender, age, education, and preference type. More specifically, the flow utility of single agents is a sum

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4. This specification is borrowed from Browning et al. (2011, Chapter 6). Partners may have different evaluations of the marriage, $\theta^a$ and $\theta^b$. We denote the sum of these evaluations by $\theta$. Only the sum matters as long as partners can transfer resources to compensate each other for differences in $\theta$. This is always feasible if the evaluations do not differ much. To simplify, we assume here that this is the case.

5. For a more general treatment of learning on match quality see Brien, Lillard and Stern (2006).
of two terms: \( \mu_t \) which depends on the gender, age, and education of the single agent, and a constant term \( u_p \) that represents the fixed preference of the agent to be single and takes two values \((p = 1 \text{ or } p = 2)\). Thus, the flow utility of being single for a man of type \( i \) in year \( t \) is given by

\[
\varphi^i_t = \mu^i_t + u^i_1 \quad \text{if preference Type 1} \tag{2}
\]

\[
\varphi^i_t = \mu^i_t + u^i_2 \quad \text{if preference Type 2}
\]

and the flow utility of being single for a woman of type \( j \) in year \( t \) is given by

\[
\varphi^j_t = \mu^j_t + u^j_1 \quad \text{if preference Type 1} \tag{3}
\]

\[
\varphi^j_t = \mu^j_t + u^j_2 \quad \text{if preference Type 2}.
\]

Because we deal with a model of discrete choice, some normalizations are required. In a static model it is customary to normalize the flow value from being single to zero. However, in a multiperiod model, the value of marriage relative to singlehood can vary over time. At ages above 30, we set the utility component \( \mu_t \) equal to zero for all single agents but at ages of 30 or below, we allow the utility component \( \mu_t \) to depend on the gender, age, and the education type of the single agents. We also normalize the utility flow \( u_2 \) of Type 2 agents to zero in all periods.

A divorced person must remain single for a whole period. At the end of every time period \( t \), single agents receive iid shocks to their preferences over remaining single or entering a marriage in period \( t + 1 \) with a husband of type \( i \) or a wife of type \( j \), where \( i \in E \times P \times U \), and \( j \in E \times P \times U \). We denote these shocks by \( \varepsilon^0_{t+1}, \varepsilon^i_{t+1}, \varepsilon^j_{t+1} \), respectively, and assume that they are drawn from a standard extreme value distribution. These random variables represent unobserved transitory taste considerations (or optimization and classification errors). These taste shocks allow observationally identical individuals to make different choices with regard to remaining single or marrying a particular type of spouse at time \( t \). We assume that preference shocks revealed at the end of the time period \( t \), prior to the decision of single agents to remain single or marry, have no impact on the utility flows from marriage or singlehood in subsequent periods beyond \( t + 1 \).

Since the shocks to the preferences of single agents are related to the types of spouse whom they marry and not to individual agents, single men and women are indifferent between marrying different agents of the same type. As a consequence, in equilibrium, these agents must receive the same share of the expected marital surplus in a given type.
of marriage. We formalize this property by letting $\gamma_{ij}^t$ be the share of the expected marital surplus that is obtained by a man of type $i$ who marries a woman of type $j$ in time period $t$ (the woman of type $j$ receives the share $1 - \gamma_{ij}^t$). We emphasize that these parameters reflect the share from the expected surplus of the marriage, as expected at the time of marriage.

3.5 Bellman Equations

With this characterization of the economic environment, we can introduce the Bellman equation for the value of marriage. For that purpose, let $W_{ij}^t (d_t)$ be the total expected value of an ongoing marriage between a man of type $i$ and a woman of type $j$ at time $t$, which has lasted for $d_t$ years, prior to the realization of the match quality $\theta_t$. The value of the marriage after the shock is revealed is $W_{ij}^t (d_t) + \theta_t$ and it satisfies the Bellman equation

$$W_{ij}^t (d_t) + \theta_t = \zeta_{ij}^t + \theta_t + R \cdot E_t \left[ \max \{ W_{ij}^{t+1} (d_{t+1}) + \theta_{t+1}, V_{t+1}^i + V_{t+1}^j - s (d_{t+1}) \} \right]. \tag{4}$$

Equation (4) states that the total value of marriage plus the realized match quality equals the sum of the systematic marital output flow $\zeta_{ij}^t$ and the match quality $\theta_t$ in the current period $t$ plus the discounted expected value of the marriage in the next period. The expectation is taken over the maximum of the two options that are available to the couple in the next period: remain married or divorce. If the couple remains married, they receive a utility flow equal to the total value of the marriage in the next period $W_{ij}^{t+1} (d_{t+1})$ plus the subsequent realization of the match quality $\theta_{t+1}$. If the couple decides to divorce, each partner receives his/her expected utility as single in the next period net of his/her share of the divorce costs $s (d_{t+1})$. Because utility is transferable both within marriage and after divorce, divorce does not depend on the surplus shares that the partners receive within marriage nor does it depend on the division of the cost of separation, $s (d_{t+1})$ (see Chiappori, Iyigun, and Weiss, 2009).

The Bellman equation for marriage prior to the realization of the match quality is

$$W_{ij}^t (d_t) = \zeta_{ij}^t + RE_t \left[ \max \{ W_{ij}^{t+1} (d_{t+1}) + \theta_{t+1}, V_{t+1}^i + V_{t+1}^j - s (d_{t+1}) \} \right]. \tag{5}$$

Note that (5) is obtained from (4) by eliminating the match quality shock $\theta_t$ which appears both on the left and the right hand sides of the equality sign in (4).

In a similar fashion, we can specify the Bellman equation for the value of singlehood. For that purpose, let $V_t^i$ be the expected value of being single for a man of type $i$ at time
period $t$, prior to the realization of the preference shocks $\varepsilon_{t+1}$ which are observed at the end of period $t$ but only affect the utility of the agent in the next time period $t+1$. The value $V_t^i$ then satisfies the Bellman equation

$$V_t^i = \varphi_t^i + RE_t \left[ V_{t+1}^i + \max_{j \in E \times P \times U} \{ \varepsilon_{t+1}^0, \gamma_{t+1}^{ij} [W_{t+1}^{ij} (1) - V_{t+1}^i - V_{t+1}^j] + \varepsilon_{t+1}^j \} \right].$$

Equation (6) states that the value of being single in time period $t$ is the flow value of being single in the current period $\varphi_t^i$ plus the discounted expected value of being single in the next period. The discounted expected value of being single in period $t + 1$ is the expectation of the maximum over the options available to a single man: remain single or marry a woman of a given type $j \in E \times P \times U$. If the man chooses to remain single, he receives a utility equal to the value of being single in the next time period plus the realization of the preference shock. If the man decides to marry, he receives a utility equal to the value of being single, plus his share of the marital surplus in the particular type of marriage, plus the realization of the preference shock. Finally, the value $V_t^j$ of being single for a woman of type $j$ at time $t$ satisfies the Bellman equation

$$V_t^j = \varphi_t^j + RE_t \left[ V_{t+1}^j + \max_{i \in E \times P \times U} \{ \varepsilon_{t+1}^0, (1 - \gamma_{t+1}^{ij}) [W_{t+1}^{ij} (1) - V_{t+1}^i - V_{t+1}^j] + \varepsilon_{t+1}^i \} \right].$$

As is clear from the equations above, we view marriage as a risky investment that has an asset value that can be divided between partners. The expected marital surplus that depends only on the type of marriage is $W_{t+1}^{ij} (1) - V_{t+1}^i - V_{t+1}^j$. Individuals with a high draw of $\varepsilon$ may enter a marriage even if the expected marital surplus is negative, provided that $\varepsilon_{t+1}$ is sufficiently high.

As we noted above, the shares of the expected marital surplus of the wife and husband in marriage $i, j$, $(1 - \gamma_{t+1}^{ij}) [W_{t+1}^{ij} (1) - V_{t+1}^i - V_{t+1}^j]$ and $\gamma_{t+1}^{ij} [W_{t+1}^{ij} (1) - V_{t+1}^i - V_{t+1}^j]$, respectively, do not depend on the $\varepsilon$’s because a single agent with a given idiosyncratic preference for a given type of spouse is indifferent among all spouses of this type and will not pay his/ her prospective spouse more than the "going price". These prices are determined competitively by market conditions. Hence there is no scope for bargaining at the time of marriage as one would have in a model with frictions. However, the separation costs generate ex post rents and bargaining. We assume that partners commit at the time of marriage on the shares within marriage with an option to renegotiate if match quality or market conditions change. Given this flexibility, divorce is efficient under transferable utility. As researchers, we cannot observe revisions in the shares but we assume that the
agents fully anticipate these revisions at the time of marriage when \( \gamma \) is formed. Indeed, we assume perfect foresight of all agents regarding all future variables that can affect their current choices.

4 Estimation

4.1 Econometric Specification

In the econometric specification of the shares of the marital surplus that husbands and wives expect in equilibrium, one could in principle maintain a different share for each type of marriage in each time period. But such a specification would involve too many parameters to be estimated. Instead, we model the surplus shares parametrically as a function of husband and wife education, husband and wife marital histories, and time (age). Specifically, we assume a quadratic function of time, \( \rho^{ij} + \kappa^{ij}t + \lambda^{ij}t^2 \), which is then embedded in the ratio of two exponential functions to ensure that each share takes on a value between zero and one:

\[
\gamma^{ij}_t = \frac{\exp\{\rho^{ij} + \kappa^{ij}t + \lambda^{ij}t^2\}}{1 + \exp\{\rho^{ij} + \kappa^{ij}t + \lambda^{ij}t^2\}}.
\]

(8)

As noted before, the share \( \gamma^{ij}_t \) is the share of the expected surplus of the marriage, as expected at the time of marriage.

To parameterize the fixed cost of divorce as a function of the duration of marriage, we introduce ten different dummy parameters corresponding to the costs of divorce after 1, 2, 3, ... , 10 or more years of marital duration \( d_t \).

4.2 Likelihood function

The structural model is estimated by maximum likelihood, using the full set of marital transitions from 1980 to 2006 for each of the 61780 individuals in the data set. We take the initial state of all men and women at age 20 as given, and maximize a conditional likelihood function. To construct the theoretical transition probabilities, it is assumed that all men and women live to the age of 72 and receive no further utility after death. The value of being single and the total value of marriage is then computed recursively back to the initial observation year. The discount factor is set to \( R = \frac{1}{1.03} \).
In a given year, an agent of a particular gender and education can find himself or herself in a total of 242 states. A first state is defined as being single with no previous marriage, and a second state is defined as being single with one or more previous marriages. The additional 240 states are marriages characterized by the marital history of the agent, the marital history of the spouse, the education and type of the spouse, and the duration of the marriage.

To describe the conditional likelihood function, we index male individuals by $m = 1, 2, ..., N^m$, female individuals by $f = 1, 2, ..., N^f$, and time periods by $t = 1, 2, ..., T$. Let $O_{mt}$ be the observed outcome of male $m$ at time $t$, and let $O_{ft}$ be the observed outcome of female $f$ at time $t$. Assuming independence between individuals, the conditional likelihood $L$ of observing our sample given the initial states of all men $S_{m0}$ and women $S_{f0}$, is

$$L = \prod_{m} \Pr (O_{1m}, ..., O_{Tm} | S_{m0}) \prod_{f} \Pr (O_{1f}, ..., O_{Tf} | S_{f0}) = \prod_{m} [q^m \Pr (O_{1m}, ..., O_{Tm} | S_{m0}, p = 1) + (1 - q^m) \Pr (O_{1m}, ..., O_{Tm} | S_{m0}, p = 2)] \\
\prod_{f} [q^f \Pr (O_{1f}, ..., O_{Tf} | S_{f0}, p = 1) + (1 - q^f) \Pr (O_{1f}, ..., O_{Tf} | S_{f0}, p = 2)],$$

where $q^m$ is the fraction of Type 1 agents among men, $q^f$ is the fraction of Type 1 agents among women, and $p \in \{1, 2\}$ is an index for the two unobserved preference types.

In calculating the probabilities, we take into account the selection on unobservables. In particular, we update the type probabilities of married partners based on the duration of marriage. To maximize the conditional log-likelihood function, we use a simplex optimization algorithm. The standard errors of the parameter estimates are then computed using the outer product of the scores of the conditional log-likelihood function.

The model that we estimate assumes that all divorced men and women have to be single for at least one year before they can remarry. In the data, we observe a few cases (2.3%) for which men and women move directly from one marriage to another without being recorded as single in between. For these few cases, we assume that the first year of the new marriage is singlehood.

4.3 Identification

The parameters of the structural model are identified from the observed divorce and marriage probabilities of different types of agents. First of all, the assumption of transferable
utility in marriage implies that divorce probabilities are only a function of the marital surplus in each type of marriage and the costs of divorce. The observed divorce probabilities of different types of marriage at different durations of marriage therefore identify the size of the marital surplus and the costs of divorce (as the divorce cost depends on the duration of the marriage but not on the marriage type). Specifically, it is seen from Bellman equation (5) that

\[ V_i^t + V_j^t - s(d_t) - W_{ij}^t = \Phi^{-1}(\text{prob of divorce in year } t), \]  

where \( \Phi \) is the standard cumulative normal distribution function. Secondly, observations on the inflows to different types of marriages by different types of agents identify the relative shares of the total marital surplus that these types of agents expect to receive. This can be seen by noting that equations (6) and (7) together with the assumed multinomial structure for the probability of entry into marriage, imply that

\[ \frac{\gamma_{ij}^t}{1 - \gamma_{ij}^t} = \frac{W_i^t - V_i^t}{W_j^t - V_j^t} = \]  

\[ = \ln \left( \frac{\text{prob( single man } i \text{ selects wife } j \text{ in year } t)}{\text{prob( man } i \text{ remains single in year } t)} \right) / \ln \left( \frac{\text{prob( single woman } j \text{ selects husband } i \text{ in year } t)}{\text{prob( woman } j \text{ remains single in year } t)} \right) \]

where, \( W_i^t \) and \( W_j^t \) are the absolute shares of the husband and wife, respectively, in the expected marital output at the time of marriage, \( W_{ij}^{t+1} \). Thus, the parameter \( \gamma_{ij}^t \) which governs the shares of the marital surplus obtained by the husband and the wife is identified from the willingness of single men of type \( i \) relative to the willingness of single women of type \( j \) to enter in to a \( i,j \)-marriage at time \( t \).

Given the expected surplus and the surplus shares, the utility flows in marriage and in singlehood can be backed up from the Bellman equations (5), (6), and (7) using the assumptions that life ends at age 72. Finally, the parameters of the utility level and the share in the sample of the two unobserved preference types are identified from the slopes of the hazards of entering marriage.

A basic assumption of the dynamic model is that agents are forward looking in making marital decisions. Given this assumption, parameters of interest such as the flow utility in marriage or costs of divorce, are identified from both the marriage and divorce choices
of the agents. A simple illustration of potential forward looking behavior is indicated by the data presented in Table 2. We see that agent types who are more likely to divorce at ages 31-46 (averaged over durations) are less likely to marry at age 30. For instance, low education males and females have relatively unstable marriages regardless of whom they marry. We also see that men and women with low education are less likely to enter marriage at age 30.

<table>
<thead>
<tr>
<th>Wife Education</th>
<th>Male Inflow</th>
<th>Wife Education</th>
<th>Male Inflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>.0487</td>
<td>.0458</td>
<td>.0598</td>
</tr>
<tr>
<td>M</td>
<td>.0507</td>
<td>.0311</td>
<td>.0485</td>
</tr>
<tr>
<td>H</td>
<td>.0567</td>
<td>.0454</td>
<td>.0275</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Husband Education</th>
<th>Male Inflow</th>
<th>Wife Education</th>
<th>Male Inflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>.0507</td>
<td>.0311</td>
<td>.0485</td>
</tr>
<tr>
<td>H</td>
<td>.0567</td>
<td>.0454</td>
<td>.0275</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Female Inflow</th>
<th>Male Inflow</th>
<th>Male Inflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>.0709</td>
<td>.0810</td>
</tr>
<tr>
<td>35</td>
<td>.0434</td>
<td>.0571</td>
</tr>
</tbody>
</table>

5 Results

We now present our estimation results. We first examine the overall fit of the model and then interpret the coefficients. We also discuss the time patterns of the estimated expected utilities associated with marriage and divorce states at different ages. Finally, we present and discuss the estimated shares from the surplus and the gains from marriage. The detailed estimated coefficients and their standard error are presented in the Appendix.

5.1 Quality of fit

The model fits well the proportion married and the hazards of entry into first and second marriages (see Figures 11, 12, and 14). One exception is the sharp peaks in the hazards of entry into first marriage which are observed at ages 30 and 40 (see Figure 12). The model

6These peaks also appear for the cohort born in 1961. Notice, however, that when marriage and cohabitation are combined, their are no such peaks (see Figure 2). A possible explanation for this
also fits reasonably well the divorce hazards for first and second marriages (see Figures 13 and 15), although it overpredicts divorce rates in first marriages and underpredicts divorces in second marriages (especially for couples with low education).

5.2 The marital output flow

Based on the estimated coefficients of equation (1) we can calculate the marital output flows by the education of the spouses in different marriages. In Table 3, we present the values of the marital output when both partners are in their first marriage.

<table>
<thead>
<tr>
<th>Wife education</th>
<th>H</th>
<th>M</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Husband education</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>-1.007</td>
<td>-0.919</td>
<td>-1.001</td>
</tr>
<tr>
<td>M</td>
<td>-1.036</td>
<td>-0.890</td>
<td>-0.968</td>
</tr>
<tr>
<td>H</td>
<td>-1.056</td>
<td>-0.906</td>
<td>-0.812</td>
</tr>
</tbody>
</table>

Before analyzing the details of this matrix, we note that the estimated marital output flow is negative in our model. This occurs since we have normalized the utility flow for singles after age 30 to be zero. Under this normalization, the values of being single at earlier ages are positive (see Appendix table). If all the options in the multinomial logit model (the 12 marriage types and singlehood) had a utility value of zero, then they would have an equal probability of $\frac{1}{13}$. Given that at least 30% of agents in the sample remain single, a typical marriage has a probability less than $\frac{1}{13}$ which requires a negative surplus. This in turn causes the (constant) marital output flow to be negative for all marriages.

We now note two important features of the marital output flow matrix:

**Monotonicity:** Moving along the main diagonal in Table 3, we see that the utility flow is lowest if both partners have a low education, then rises if both have medium education, and then reaches its highest point if both have high education. This monotonicity result is significant at the 1% level.

behavior is that individuals who are cohabiting want to celebrate their 30th or 40th birthdays together with their marriage and time their marriage accordingly.
Complementarity: Under the additivity assumption, the complementarity in education is independent of the marriage order. We can therefore test for the supermodularity of husband and wife education based on the estimated coefficients for education in the marital output flow for first marriages which are presented in the Table 3. Denoting marriages by the education of the husband and the wife, this amounts to jointly testing if

\begin{align*}
(L, L) + (M, M) &> (L, M) + (M, L) \\
(M, M) + (H, H) &> (M, H) + (H, M) \\
(L, M) + (M, H) &> (M, M) + (L, H) \\
(M, L) + (H, M) &> (H, L) + (M, M)
\end{align*}

Replacing these utility expressions with the estimated marriage output flows in first marriage, we find that all of these four inequalities are satisfied as

\begin{align*}
-1.007 - 0.890 &> -0.919 - 1.036 \\
-0.890 - 0.812 &> -0.968 - 0.906 \\
-0.919 - 0.968 &> -0.890 - 1.001 \\
-1.036 - 0.906 &> -1.056 - 0.890
\end{align*}

This supermodularity result is significant at the 1% level.

The marriage order has a large and asymmetrical impact on marital output. Surprisingly, a second marriage for both partners provides a higher flow than a first marriage for both, given any level of education. This finding is consistent with the higher hazard of entry rate into second marriages compared to first marriages as displayed in figure 6 and 7. We see that the matrix in Table 4 is also super modular, which means that the marriage order of the two partners complement each other. However, we observe asymmetry by gender. A match of a divorced man with a woman who is married for the first time generates a higher marital output flow than a match between a divorced woman and a man married for the first time.\footnote{Both of these differences in marriage order coefficients mentioned above are significant at the 1% level.} A possible reason for this asymmetry is the presence of children from previous marriages that a divorced wife brings into the new marriage (see Beaujouan 2010). Recall that we do not control explicitly for having children.
TABLE 4

Effects of marriage order on the marital output flow

<table>
<thead>
<tr>
<th></th>
<th>Wife first marriage</th>
<th>Wife second marriage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Husband first marriage</td>
<td>-0.0379</td>
<td>-0.157</td>
</tr>
<tr>
<td>Husband second marriage</td>
<td>-0.0792</td>
<td>0.0918</td>
</tr>
</tbody>
</table>

5.3 Costs of divorce

The estimated costs of divorce are quite high and constitute about half of the sum of the values of being single of the two partners. Since we do not control directly for children and because the costs of separation are likely to rise over time as the average number of children rises, we would expect the costs of separation to rise with the duration of marriage. However we do not find such an increasing trend. Rather, the first year of marriage has a relatively low cost of divorce and from the second year on we observe a mild U-shape pattern with respect to duration.\(^8\)

TABLE 5

Costs of divorce by duration of marriage

<table>
<thead>
<tr>
<th>Marital duration</th>
<th>Cost of divorce</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.7</td>
</tr>
<tr>
<td>2</td>
<td>17.2</td>
</tr>
<tr>
<td>3</td>
<td>15.4</td>
</tr>
<tr>
<td>4</td>
<td>14.1</td>
</tr>
<tr>
<td>5</td>
<td>14.9</td>
</tr>
<tr>
<td>6</td>
<td>15.4</td>
</tr>
<tr>
<td>7</td>
<td>14.0</td>
</tr>
<tr>
<td>8</td>
<td>15.2</td>
</tr>
<tr>
<td>9</td>
<td>16.0</td>
</tr>
<tr>
<td>10+</td>
<td>17.9</td>
</tr>
</tbody>
</table>

\(^8\)The model restricts the costs of divorce to be the same in first and second marriages. We have tested this by allowing different divorce costs and cannot reject equality.
5.4 Unobserved types

The estimated parameters of the unobserved types are presented in Table 6. As seen in the table, a majority of the sample have a negative unobserved flow value for being single and, therefore, enter marriage quicker than one would expect based on observables such as education and age. A minority of about six percent of the sample are Type 2 individuals with an unobserved utility flow of zero. This type marries at a lower rate and divorces more. These results are similar for men and women, with a slightly higher tendency towards marriage among women.

Following Eckstein and Wolpin (1999) we estimate the probability that each individual in our sample is a Type 1 agent (with an estimated low utility of being single) using Bayes’ rule. We then calculate the fraction of Type 1 agents in subgroups of our sample by taking the average of these estimated Type 1 probabilities. In the bottom of Table 6 we see that the subset of men and women who enter into marriage from singlehood have a higher fraction of Type 1 agents. We also see that the subset of married men and women who divorce have a lower fraction of Type 1 agents. The fraction of Type 1 agents (with an estimated low utility of being single) is lowest among those who never marry and second lowest among those who divorce twice or more.

TABLE 6.

Estimated utilities of Type 1 agents and distribution of preference types in the population and selected groups.

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility $u_1$</td>
<td>-0.372</td>
<td>-0.345</td>
</tr>
<tr>
<td>Fraction Type 1, entire population</td>
<td>0.942</td>
<td>0.936</td>
</tr>
<tr>
<td>Fraction Type 1, never married</td>
<td>0.834</td>
<td>0.782</td>
</tr>
<tr>
<td>Fraction Type 1, married at least one</td>
<td>0.976</td>
<td>0.960</td>
</tr>
<tr>
<td>Fraction Type 1, divorced at least once</td>
<td>0.932</td>
<td>0.890</td>
</tr>
<tr>
<td>Fraction Type 1, married at least twice</td>
<td>0.953</td>
<td>0.918</td>
</tr>
<tr>
<td>Fraction Type 1, divorced at least twice</td>
<td>0.897</td>
<td>0.828</td>
</tr>
</tbody>
</table>
We then proceed by correlating the estimated probability that an agent is of Type 1 with a number of observed exogenous characteristics that we did not include in our model. In Table 7 below, we show the results of this exercise.

**TABLE 7**

*Correlation estimated probability agent is Type 1 and individual characteristics*

<table>
<thead>
<tr>
<th>Trait</th>
<th>Correlation</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parents living together in 1980</td>
<td>0.0463</td>
<td>56262</td>
</tr>
<tr>
<td>Father’s completed education in 1980</td>
<td>-0.0243</td>
<td>47895</td>
</tr>
<tr>
<td>Mother’s completed education in 1980</td>
<td>-0.0262</td>
<td>56856</td>
</tr>
<tr>
<td>Father’s wealth in 1980</td>
<td>0.0411</td>
<td>53095</td>
</tr>
</tbody>
</table>

We first construct a dummy variable equal to one if the parents of the agent are living together in 1980. The Pearson correlation coefficient between this dummy and the estimated probability that the agent is of Type 1 is positive, indicating that agents whose parents are living together have a stronger preference for marriage. We then assign the numerical values of 1, 2, and 3 to the parents of the agents depending on whether they have low, medium, or high education. We then compute the Spearman rank correlation coefficient between this index and the estimated probability that the agent is of Type 1. As can be seen in Table 7, the correlation is negative for both father’s and mother’s education. This negative correlation could reflect the fact that more educated agents marry later. Finally, we compute the Pearson correlation coefficient for father’s wealth in 1980 and the estimated probability that an agent is of Type 1. This correlation is positive, implying that agents whose fathers are wealthier, are more likely to be married. All of the correlations listed in Table 7 are significantly different from zero at the 1% level.

### 5.5 Values

The values that the model generates for marriage and singlehood summarize the impact of the estimated parameters on the discounted life-time utility that individuals and couples expect at different ages. The values of being single and being married are interrelated as a high value for being single entails a high share of the marital output and, at the same time, high expected values for prospective marriages entail a high value for being single.
Both values depend on the unobserved preference to be single and in our presentation we report values that are averaged over types.

### 5.5.1 Values of singlehood

The estimated values of being single \( V \) are generally positive and decline with age (see Figure 16). Among the never married, men have a higher value than women of remaining single at every level of education, reflecting the fact that men marry later (see Figure 5). Among all groups, men with low education have the highest value of being single.

### 5.5.2 Surplus

The expected marital surplus is defined here as the expected joint value of a marriage at the time of marriage minus the sum of the expected values in singlehood of the two partners at that time, \( W_{ij}^t (1) - V_i^t - V_j^t \). As such, it reflects properties of the utility flows in both marriage and following a divorce. As already mentioned, the utility flows are generally nonnegative in singlehood and negative in marriage. Therefore, the expected surplus is negative. In such a case, marriage is driven by the taste shocks, \( \varepsilon \), of the partners. The general time pattern of the expected marital surplus at the time of marriage is a rise during the early years (ages), as individuals delay their marriage, followed by stability for ages beyond 30 (see Figures 17a to 17d). Similar to the marital output flow, the marital surplus is generally supermodular in the education of the husband and the wife. The surplus is also monotonically increasing in the education of the husband and the wife when comparing marriages in which both spouses have the same education. In this regard, the expected surplus inherits the properties of the systematic part of the marital output flow. This is the case, for instance, for the surplus generated by first marriages at age 33 in Table 8a below and the surplus generated in second marriages at age 33 in Table 8b below.

We see that by age 33, the expected surplus is very similar in first and second marriages. This holds despite the larger utility flow in second marriages that we observed in Table 4. The reason is that the higher marital output in second marriage is offset by differences by the higher utility from being single for agents in second marriage. This happens despite the fact that, beyond age 30, the time dependent part of utility flow from being single \( \mu \) is set to zero for all agents. The important element here is the unobserved fixed utility flow \( u_p \) of being single which on average takes on a higher value among men.
and women in second marriage, and thus lowers the marital surplus for this group relative to those who are married for the first time (see Table 6).

TABLE 8a
Marital surplus of couples who first married at age 33, by education of husband and wife

<table>
<thead>
<tr>
<th></th>
<th>Wife education</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>-12.78</td>
</tr>
<tr>
<td>Husband education</td>
<td>M</td>
</tr>
<tr>
<td>H</td>
<td>-12.82</td>
</tr>
</tbody>
</table>

TABLE 8b
Marital surplus of couples who entered second marriage at age 33, by education of husband and wife

<table>
<thead>
<tr>
<th></th>
<th>Wife education</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>-12.70</td>
</tr>
<tr>
<td>Husband education</td>
<td>M</td>
</tr>
<tr>
<td>H</td>
<td>-12.79</td>
</tr>
</tbody>
</table>

5.6 Surplus shares and shares from the marital gains

We have estimated 72 coefficients corresponding to the specification (8). The implied average values of the surplus share \( \gamma \) are displayed in tables 9 to 12. We recall that by equation (10), if \( \gamma > .5 \) the husband is more keen than the wife to enter the marriage and the opposite holds if if \( \gamma < .5 \). We see that in first and second marriages for both, \( \gamma \) roughly equals .5. However, in marriages when only one of the partners is divorced, the partner who is marrying for the first time is more willing to enter the marriage. Specifically, if the marriage is the first for the husband and the second for the wife \( \gamma > .5 \). while if the husband is married for the second time and the wife is married for the first time then \( \gamma < .5 \).

Some care must be taken in interpreting the results in tables 9 to 12 because the \( \gamma \)'s represent shares from expected surplus values that are negative. For instance, we see in
Table 9 that low educated men married to highly educated women get a higher $\gamma$ than a highly educated men married to highly educated women. This does not mean that educated men are worse off in such marriages, because they receive a higher share from a negative expected surplus, which may differ in different marriages. All we can say is that in a particular marriage, say a first marriage of highly educated man and woman, the wife is more keen to enter the marriage (see Table 9).

### TABLE 9

*Estimated average surplus share $\gamma$ in first marriage by education of husband and wife*

<table>
<thead>
<tr>
<th>Wife education</th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>.515</td>
<td>.510</td>
<td>.515</td>
</tr>
<tr>
<td>Husband education</td>
<td>M</td>
<td>.524</td>
<td>.507</td>
</tr>
<tr>
<td>H</td>
<td>.495</td>
<td>.488</td>
<td>.481</td>
</tr>
</tbody>
</table>

### TABLE 10

*Estimated average surplus share $\gamma$ in first marriage for husband and second marriage for wife by education of husband and wife*

<table>
<thead>
<tr>
<th>Wife education</th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>.593</td>
<td>.617</td>
<td>.620</td>
</tr>
<tr>
<td>Husband education</td>
<td>M</td>
<td>.606</td>
<td>.612</td>
</tr>
<tr>
<td>H</td>
<td>.542</td>
<td>.570</td>
<td>.618</td>
</tr>
</tbody>
</table>

### TABLE 11

*Estimated average surplus share $\gamma$ in second marriage for husband and first marriage for wife by education of husband and wife*

<table>
<thead>
<tr>
<th>Wife education</th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>.401</td>
<td>.410</td>
<td>.417</td>
</tr>
<tr>
<td>Husband education</td>
<td>M</td>
<td>.436</td>
<td>.399</td>
</tr>
<tr>
<td>H</td>
<td>.419</td>
<td>.396</td>
<td>.384</td>
</tr>
</tbody>
</table>
The surplus shares $\gamma$ are just one component in the gains from marriage that each partner receives. A broader concept is to look at what single men and women of different types expect to receive if they marry and make the best choice based on their drawn preferences, $\varepsilon$, the systematic gains from the marriage, and the surplus shares that they receive. Recalling that at any period $t$, a single man of type $i$ draws a set of taste preferences for being single or marrying a woman of type $j$, the expected utility from his best choice as viewed by the researcher is

$$U^i_t = V^i_t - \ln(p^i_{t0})$$

where $p^i_{t0}$ is the probability that a single man of type $i$ will choose to remain single at time $t$. Similarly the expected utility for a woman of type $j$ is

$$U^j_t = V^j_t - \ln(p^j_{t0})$$

where $p^j_{t0}$ is the probability that a single woman of type $j$ will choose to remain single at time $t$. The expected gains from marriage are then $U^i_t - V^i_t = -\ln(p^i_{t0})$ and $U^j_t - V^j_t = -\ln(p^j_{t0})$ for men of type $i$ and woman of type $j$, respectively. We can now define the share of the expected gains of the husband in a marriage type $i,j$ as

$$\Gamma^{i,j}_t = \frac{U^i_t - V^i_t}{U^i_t - V^i_t + U^j_t - V^j_t} = \frac{-\ln(p^i_{t0})}{-\ln(p^i_{t0}) - \ln(p^j_{t0})}$$

\[(11)\]

(See McFadden, 1984 and Chiappori, Salanie, and Weiss, 2011). In contrast to the surplus shares $\gamma_{ij}$ that represent the relative willingness of partners to enter marriage $i,j$, (if faced with that choice), the average shares $\Gamma^{i,j}$ represents the relative willingness of

<table>
<thead>
<tr>
<th>Wife education</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>.481</td>
<td>.481</td>
<td>.522</td>
</tr>
<tr>
<td>M</td>
<td>.507</td>
<td>.502</td>
<td>.547</td>
</tr>
<tr>
<td>H</td>
<td>.476</td>
<td>.433</td>
<td>.515</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Husband education</th>
<th>M</th>
<th>.507</th>
<th>.502</th>
<th>.547</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>.481</td>
<td>.481</td>
<td>.522</td>
<td></td>
</tr>
</tbody>
</table>
these agents to marry (if allowed to choose whom to marry). The average values of these alternative shares at different types of marriages are displayed in tables 13-16.

**TABLE 13**  
*Estimated average surplus share $\Gamma$ in first marriage for husband and wife by education of husband and wife*

<table>
<thead>
<tr>
<th>Wife education</th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0.404</td>
<td>x</td>
<td>0.411</td>
</tr>
<tr>
<td>Husband education</td>
<td>M</td>
<td>0.440</td>
<td>0.445</td>
</tr>
<tr>
<td>H</td>
<td>0.500</td>
<td>x</td>
<td>0.506</td>
</tr>
</tbody>
</table>

**TABLE 14**  
*Estimated average surplus share $\Gamma$ in first marriage for husband and second marriage for wife by education of husband and wife*

<table>
<thead>
<tr>
<th>Wife education</th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0.245</td>
<td>x</td>
<td>0.186</td>
</tr>
<tr>
<td>Husband education</td>
<td>M</td>
<td>0.275</td>
<td>0.234</td>
</tr>
<tr>
<td>H</td>
<td>0.334</td>
<td>x</td>
<td>0.264</td>
</tr>
</tbody>
</table>

**TABLE 15**  
*Estimated average surplus share $\Gamma$ in second marriage for husband and first marriage for wife by education of husband and wife*

<table>
<thead>
<tr>
<th>Wife education</th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0.716</td>
<td>x</td>
<td>0.722</td>
</tr>
<tr>
<td>Husband education</td>
<td>M</td>
<td>0.704</td>
<td>0.708</td>
</tr>
<tr>
<td>H</td>
<td>0.752</td>
<td>x</td>
<td>0.759</td>
</tr>
</tbody>
</table>
TABLE 16

*Estimated average surplus share* $\Gamma$ *in second marriage for husband and wife by education of husband and wife*

<table>
<thead>
<tr>
<th>Wife education</th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Husband education</td>
<td>L</td>
<td>0.515</td>
<td>0.455</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>0.501</td>
<td>0.441</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>0.559</td>
<td>0.500</td>
</tr>
</tbody>
</table>

One can see that in first marriage and in second for both partners, the husbands’ average share is about half. However, in marriages in which partners have different marital history, the spouse for whom this is the second marriage receives a substantially higher share. This large effect can be traced back to the fact that entry hazards into second marriages are substantially higher for second marriages than for first marriages. We also see that, holding fixed the education of the spouse, the shares tend to increase with the own level of schooling. This is especially true for changes in the education of the husband while holding fixed the education of the wife.

Over all, the surplus shares, $\gamma$, and the shares from the gains from marriage, $\Gamma$, tell a very similar story. This becomes apparent when we recall that $\gamma$ is a share from a negative number and $\Gamma$ is a share from a positive number. The estimated surplus shares, $\gamma$, and the shares from the gains from marriage, $\Gamma$, also resemble the surplus shares estimated in the collective household literature which are often increasing in the education and lifetime income of the two spouses (see for example Browning et al. 1994). These patterns are often interpreted as "power" relationships within couples.

5.7 Changing the definition of marriage

In this subsection we discuss briefly an application of our model to partnerships that may be either legal marriage or cohabitation, using the same sample (see Gemici and Laufer 2011 for a comparison of the two types of relationships). As one would expect the estimated costs of divorce for partnerships are lower than for legal marriages at short durations of the partnership (see also Brien-Lillard and stern, 2006 and Gemici and Laufer 2011). However, these costs tend to increase with duration because long cohabitations
are increasingly similar to marriages (see Appendix Tables 2 and 3). As in marriage, the marital flow in partnerships is monotone in education. It is also the case that the marital flow is supermodular with respect to husband and wife education. These findings reflect a feature of the data noted above, namely that the assignment patterns in partnerships and marriage are similar. The qualitative effects of marriage order on the marital output flow are also the same. Overall, the basic model is flexible enough to encompass both partnerships and marriages and most important features of our results are robust to the definition of marriage that we use.

The quality of fit in the combined sample is also very good, except for the entry into first partnerships where the model overpredicts entry at later ages. Again, unobserved heterogeneity plays an important role. We find that the utility of Type 1 agents from being single is substantially smaller for partnerships than for marriages. Thus, compared to marriages, more heterogeneity is required to explain the dynamics of partnerships.

5.8 Copenhagen sample

We recognize that the marital surplus shares may differ across marriage markets with different distributions of types even if individual preferences over types remain constant. To examine the impact of exogenous variation in the distribution of educational types, we therefore selected a sample of men and women who resided in the Copenhagen area at least during the ages 25-30. The distribution of agents in this sample by gender and education is shown in Table 17 below. Comparing with Table 1 for the Denmark sample, we see that there are substantially more highly educated women in Copenhagen than in Denmark as a whole. We also note that whereas there are more men than women in the Denmark sample, the sex ratio is reversed in the Copenhagen sample where there are more women than men.
TABLE 17
Sample distribution of male and female completed education in Copenhagen sample

<table>
<thead>
<tr>
<th>Completed Education (at 46)</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>High school or less</td>
<td>34.0</td>
<td>30.4</td>
</tr>
<tr>
<td>Vocational education</td>
<td>33.3</td>
<td>32.0</td>
</tr>
<tr>
<td>Some college or more</td>
<td>32.7</td>
<td>38.6</td>
</tr>
</tbody>
</table>

No. of observations 5852 6000

We then reestimate the model for the Copenhagen sample under the restriction that all parameters are the same as those estimated from the Denmark sample, except for the distribution of the unobserved utility of being single and the surplus shares in marriage. In Table 18 below, we show the estimated average surplus shares $\Gamma$ in first marriage for both the husband and the wife by the education of both spouses. Comparing these expected shares with the estimates in Table 13 for the Denmark sample, we notice two main differences. First of all, women with high education receive a lower share of the expected marital surplus in the Copenhagen sample. This is consistent with the large fraction of highly educated women in Copenhagen. Secondly, men receive a larger share of the expected marital surplus in the Copenhagen sample (this is true for seven out of the nine marriage types displayed in Table 18). This larger share for men in Copenhagen is consistent with the difference in the overall sex ratios between Denmark as a whole and Copenhagen that we mentioned above.

TABLE 18
Estimated average surplus share $\Gamma$ in first marriage for husband and wife by education of husband and wife in Copenhagen sample

<table>
<thead>
<tr>
<th>Wife education</th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Husband education</td>
<td>L</td>
<td>0.421</td>
<td>0.415</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>0.483</td>
<td>0.476</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>0.508</td>
<td>0.502</td>
</tr>
</tbody>
</table>
6 Conclusions

In this paper, we study the complex dynamics of marriage and divorce within a large Danish cohort which we follow from age 20 to age 46. We focused on the assortative matching by education and the implications for the marital gains from marriage of different agents. We observe that matching patterns according to education change sharply as the cohort ages, because agents with different education are sorted gradually and because matches with two highly educated partners are more stable. As the cohort ages, men with low education are the most likely to be single as they enter marriage at a lower rate and divorce at a higher rate.

We found that the observed assortative matching by education can be rationalized by the complementarity in traits within couples, as predicted by theory. The estimated shares in marriage also respond to the sex ratio and the distributions of female and male traits in the manner predicted by theory. We found a strong impact of marital history on the estimated flow of benefits from marriage and on the relative expected gains from marriage of husbands and wives. Surprisingly, second marriages are associated with higher marital output and a higher share of the marital output. At the same time, second marriages are less stable and associated with a higher divorce rate. We resolve this apparent contradiction by allowing agents to differ in preferences for singlehood in a way that is independent of their education. We then show that agents that marry twice have a larger than average proportion of the type with a higher fixed preference to be single.
### 7 Appendix: Estimated Coefficients

<table>
<thead>
<tr>
<th>Marital output flow</th>
<th>Legal marriages</th>
<th>Partnerships</th>
</tr>
</thead>
<tbody>
<tr>
<td>Husband low education</td>
<td>-0.620 (3.39E-04)</td>
<td>-0.709 (2.76E-04)</td>
</tr>
<tr>
<td>Husband medium education</td>
<td>-0.587 (3.35E-04)</td>
<td>-0.527 (2.65E-04)</td>
</tr>
<tr>
<td>Husband high education</td>
<td>-0.607 (3.32E-04)</td>
<td>-0.512 (2.71E-04)</td>
</tr>
<tr>
<td>Wife low education</td>
<td>-0.411 (4.91E-04)</td>
<td>-0.585 (2.94E-04)</td>
</tr>
<tr>
<td>Wife medium education</td>
<td>-0.261 (4.79E-04)</td>
<td>-0.484 (2.86E-04)</td>
</tr>
<tr>
<td>Wife high education</td>
<td>-0.343 (4.90E-04)</td>
<td>-0.492 (2.88E-04)</td>
</tr>
<tr>
<td>Both spouses low education</td>
<td>0.0621 (1.27E-05)</td>
<td>0.0720 (1.22E-05)</td>
</tr>
<tr>
<td>Both spouses medium education</td>
<td>-4.06E-03 (1.18E-05)</td>
<td>-0.0122 (1.12E-05)</td>
</tr>
<tr>
<td>Both spouses high education</td>
<td>0.176 (1.67E-05)</td>
<td>0.222 (1.23E-05)</td>
</tr>
<tr>
<td>1st marriage for both spouses</td>
<td>-0.0379 (2.82E-04)</td>
<td>0.0605 (2.32E-04)</td>
</tr>
<tr>
<td>Husband 1st, wife 2nd marriage</td>
<td>-0.157 (2.85E-04)</td>
<td>-0.101 (2.31E-04)</td>
</tr>
<tr>
<td>Husband 2nd, wife 1st marriage</td>
<td>-0.0792 (2.82E-04)</td>
<td>0.0976 (2.29E-04)</td>
</tr>
<tr>
<td>2nd marriage for both spouses</td>
<td>0.0918 (2.78E-04)</td>
<td>0.160 (2.19E-04)</td>
</tr>
</tbody>
</table>

| Cost of divorce | |
|-----------------|-----------------|-----------------|
| 1 year of marriage | 12.72 (1.61E-04) | 9.38 (6.59E-05) |
| 2 years of marriage | 17.15 (1.21E-03) | 10.31 (6.86E-04) |
| 3 years of marriage | 15.42 (1.20E-03) | 11.08 (7.44E-04) |
| 4 years of marriage | 14.08 (1.15E-03) | 11.99 (8.82E-04) |
| 5 years of marriage | 14.88 (1.16E-03) | 14.20 (1.19E-03) |
| 6 years of marriage | 15.40 (1.14E-03) | 15.52 (1.35E-03) |
| 7 years of marriage | 14.02 (1.08E-03) | 17.29 (1.43E-03) |
| 8 years of marriage | 15.22 (1.21E-03) | 18.42 (1.56E-03) |
| 9 years of marriage | 16.02 (1.24E-03) | 20.03 (1.66E-03) |
| 10+ years of marriage | 17.87 (5.94E-04) | 24.40 (6.00E-04) |

Estimated coefficients and standard errors in parentheses.
<table>
<thead>
<tr>
<th></th>
<th>Legal marriages</th>
<th></th>
<th>Partnerships</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utility Singles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Man, 20-22, low education</td>
<td>0.0516 (8.65E-02)</td>
<td>0.0354 (2.25E-02)</td>
<td></td>
</tr>
<tr>
<td>Man, 23-25, low education</td>
<td>0.172 (6.23E-02)</td>
<td>-0.193 (8.87E-03)</td>
<td></td>
</tr>
<tr>
<td>Man, 26-28, low education</td>
<td>0.244 (9.35E-03)</td>
<td>0.110 (5.45E-03)</td>
<td></td>
</tr>
<tr>
<td>Man, 29-30, low education</td>
<td>0.0633 (1.72E-02)</td>
<td>-0.172 (2.00E-03)</td>
<td></td>
</tr>
<tr>
<td>Man, 20-22, medium education</td>
<td>0.272 (8.65E-02)</td>
<td>0.0738 (2.25E-02)</td>
<td></td>
</tr>
<tr>
<td>Man, 23-25, medium education</td>
<td>0.262 (6.23E-02)</td>
<td>-0.125 (8.86E-03)</td>
<td></td>
</tr>
<tr>
<td>Man, 26-28, medium education</td>
<td>0.245 (9.35E-03)</td>
<td>0.0770 (5.46E-03)</td>
<td></td>
</tr>
<tr>
<td>Man, 29-30, medium education</td>
<td>0.0395 (1.72E-02)</td>
<td>-0.191 (1.99E-03)</td>
<td></td>
</tr>
<tr>
<td>Man, 20-22, high education</td>
<td>0.523 (8.65E-02)</td>
<td>0.572 (2.25E-02)</td>
<td></td>
</tr>
<tr>
<td>Man, 23-25, high education</td>
<td>0.507 (6.23E-02)</td>
<td>0.154 (8.87E-03)</td>
<td></td>
</tr>
<tr>
<td>Man, 26-28, high education</td>
<td>0.474 (9.35E-03)</td>
<td>0.276 (5.44E-03)</td>
<td></td>
</tr>
<tr>
<td>Man, 29-30, high education</td>
<td>0.133 (1.72E-02)</td>
<td>-0.171 (2.00E-03)</td>
<td></td>
</tr>
<tr>
<td>Woman, 20-22, low education</td>
<td>0.395 (8.65E-02)</td>
<td>-0.0718 (2.25E-02)</td>
<td></td>
</tr>
<tr>
<td>Woman, 23-25, low education</td>
<td>0.301 (6.23E-02)</td>
<td>0.165 (8.87E-03)</td>
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</tr>
<tr>
<td>Woman, 26-28, low education</td>
<td>-0.0214 (9.34E-03)</td>
<td>-0.0784 (5.46E-03)</td>
<td></td>
</tr>
<tr>
<td>Woman, 29-30, low education</td>
<td>0.141 (1.72E-02)</td>
<td>0.273 (1.99E-03)</td>
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</tr>
<tr>
<td>Woman, 20-22, medium education</td>
<td>0.707 (8.65E-02)</td>
<td>0.0754 (2.25E-02)</td>
<td></td>
</tr>
<tr>
<td>Woman, 23-25, medium education</td>
<td>0.520 (6.23E-02)</td>
<td>0.200 (8.88E-03)</td>
<td></td>
</tr>
<tr>
<td>Woman, 26-28, medium education</td>
<td>-0.0159 (9.34E-03)</td>
<td>-0.103 (5.45E-03)</td>
<td></td>
</tr>
<tr>
<td>Woman, 29-30, medium education</td>
<td>0.0622 (1.72E-02)</td>
<td>0.170 (2.00E-03)</td>
<td></td>
</tr>
<tr>
<td>Woman, 20-22, high education</td>
<td>1.150 (8.65E-02)</td>
<td>0.159 (2.25E-02)</td>
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</tr>
<tr>
<td>Woman, 23-25, high education</td>
<td>0.615 (6.23E-02)</td>
<td>0.387 (8.86E-03)</td>
<td></td>
</tr>
<tr>
<td>Woman, 26-28, high education</td>
<td>0.215 (9.35E-03)</td>
<td>0.0214 (5.45E-03)</td>
<td></td>
</tr>
<tr>
<td>Woman, 29-30, high education</td>
<td>0.147 (1.72E-02)</td>
<td>0.221 (1.99E-03)</td>
<td></td>
</tr>
<tr>
<td>Utility $u_1$ men</td>
<td>-0.372 (2.77E-05)</td>
<td>-0.500 (2.05E-05)</td>
<td></td>
</tr>
<tr>
<td>Fraction Type 1 men</td>
<td>0.942 (3.18E-05)</td>
<td>0.913 (1.69E-05)</td>
<td></td>
</tr>
<tr>
<td>Utility $u_1$ women</td>
<td>-0.345 (2.48E-05)</td>
<td>-0.483 (2.04E-05)</td>
<td></td>
</tr>
<tr>
<td>Fraction Type 1 women</td>
<td>0.936 (3.21E-05)</td>
<td>0.909 (1.81E-05)</td>
<td></td>
</tr>
</tbody>
</table>

Estimated coefficients and standard errors in parentheses.
References


8 Figures

FIGURE 1.
Fraction married or in partnerships (marriage plus cohabitation) by age.
FIGURE 2.
Hazard into first marriage or partnership (marriage plus cohabitation).

FIGURE 3.
Divorce hazard for first marriage or partnership (marriage plus cohabitation).
FIGURE 4.
Fraction men and women married (top) or in partnerships (bottom) by age and education.
FIGURE 5.
Hazard rate into first marriage for men and women by age and education.

FIGURE 6.
Divorce hazards for first marriages by education of the husband and wife.
Hazard rate into second marriage for men and women by education.

Divorce hazards when at least one spouse is in second marriage, by education of the husband and wife.
FIGURE 9.
Distribution of marriages for men with low (top), medium (middle), or high (bottom) education.
FIGURE 10.
Distribution of marriages for women with low (top), medium (middle), or high (bottom) education.
FIGURE 11.
Fraction married in data and in model by gender.
FIGURE 12.
Hazard into first marriage in data and in model by gender.

FIGURE 13.
Divorce hazard for first marriage in data and in model by education and gender.
FIGURE 14.
Hazard for entry into second or higher marriages in the data and in the model by gender.

FIGURE 15
Divorce hazard when at least one spouse is in second marriage, in data and in model by education and gender.
FIGURE 16.
Value of being single for never married by education.
FIGURE 17a.
Expected marital surplus at marriage, both spouses never previously married.

FIGURE 17b.
Expected marital surplus at marriage, man never previously married, woman divorced.
FIGURE 17c.
Expected marital surplus at marriage, man divorced, woman never previously married.

FIGURE 17d.
Expected marital surplus at second marriage, both spouses divorced.
FIGURE 18a.
Husband’s share $\Gamma$ of expected marital gain when none of the spouses have divorced.

FIGURE 18b.
Husband’s share $\Gamma$ of expected marital gain, first marriage for husband, second marriage for wife.
FIGURE 18c.
Husband’s share $\Gamma$ of expected marital gain, second marriage for husband, first marriage for wife.

FIGURE 18d.
Husband’s share $\Gamma$ of expected marital gain when both spouses have divorced.