A Dynamic Equilibrium Model of Imperfectly Integrated Financial Markets*

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Abstract

This paper analyzes the determination of global equity portfolios and stock returns in the context of imperfectly integrated stock markets. We consider a continuous-time, two-country endowment economy, where the level of financial integration is captured by a proportional tax on foreign dividends. Despite the investor heterogeneity induced by this tax, we obtain approximate closed-form expressions for asset prices, and characterize equity holdings and the joint process followed by country-level stock returns in equilibrium. Our model is consistent with a broad range of empirical findings on international financial integration. When the (endogenous) cross-country return correlation is high, small frictions in equity markets can generate a substantial home bias in portfolios. In the baseline version of our model, the cross-country return correlation is driven by the fundamental correlation and portfolio rebalancing. In a two-good extension of the model, the adjustment of relative good prices can generate a high stock return correlation even for a low level of fundamental correlation, magnifying the impact of the financial friction on portfolios. We assess the quantitative performance of the model in a calibration exercise using data from G7 countries.

Keywords: Two Trees, Asset Pricing with Heterogeneous Investors, Home Bias in Portfolios, International Stock Return Correlations, Financial Integration.

JEL Codes: G15, F30, G11, G12

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1 Introduction

Over the last decades most equity markets around the world have been liberalized and cross-border equity holdings have surged.[1] However a number of frictions remain in international equity markets: transaction costs, withholding taxes, as well as informational and agency problems still act as impediments to cross-border investment. In a sense, the mere existence of home bias in equity portfolios, initially documented by French and Poterba (1991), indicates that some frictions are still at play.[2] In 2008, U.S. investors held 77.2% of their stock portfolios in domestic stocks and a portfolio home bias is observed in all developed countries (Sercu and Vanpee (2007) and Coeurdacier and Rey (2011)). Broadly speaking, it is probably fair to describe international equity markets today as neither perfectly integrated nor totally segmented.

In this paper, we analyze the workings of international financial markets in between the polar cases of perfect financial integration and complete segmentation. We consider a two-country endowment economy with one Lucas tree in each country and equity claims on national output. The friction which induces equity markets to be partially segmented takes the form of a proportional cost that shareholders have to pay on dividends earned abroad. Naturally, the size of the home bias in portfolios depends on the size of the friction in equity markets, but it also depends on the international correlation of returns which determines the benefits of diversification. At the same time, this correlation is affected by cross-border equity holdings, since portfolio rebalancing effects can generate comovement in asset prices. Our main achievement is to determine both the joint distribution of asset returns and portfolio holdings in equilibrium for various levels of financial integration. We believe our setting is appropriate to make sense of (i) the extent of international portfolio diversification, (ii) the joint behavior of country stock markets, and (iii) how they are affected by the process of financial integration.

Modeling *imperfectly integrated* financial markets is appealing for the sake of realism, but it is technically challenging. Any form of financial segmentation implies some heterogeneity in investment opportunity sets across agents, a feature which makes the pricing of assets more complicated, because asset prices and the endogenous cross-sectional wealth distribution must be jointly determined. We keep the problem tractable by capturing in a simple way the partial segmentation of international financial markets. The friction we consider essentially acts as a withholding tax on foreign dividends.[3] Withholding taxes are relevant in practice (e.g., pension funds have to bear such taxes on their foreign equity investments), but our friction can also be interpreted more broadly as a reduced form device for capturing the effects of transaction costs, agency, or informational frictions.[4] The asset pricing problem remains non trivial. Indeed, since each investor

[1] Quinn (1997), Bekaert and Harvey (2000) and Kaminsky and Schmuckler (2003) provide direct institutional measures of financial openness. See Lane and Milesi-Ferreti (2003) for the evolution of international equity holdings.

[2] In a standard CAPM world with *perfectly* integrated financial markets and identical preferences, all investors hold the world market portfolio, independently of their nationality. However, even in the absence of frictions in international financial markets, deviations from purchasing power parity or the existence of non-insurable labor income shocks could induce heterogeneous portfolios (for references, see the literature review below).

[3] This friction is by nature different from a *transaction* cost à la Constantinides (1986): it does not bear on transactions but instead reduces cash-flows during the holding period.

[4] Stulz (2005) analyzes the impact of moral hazard on cross-border investment. The role of informational frictions

has a specific "after-tax" investment opportunity set, risk sharing is imperfect, and we cannot use the pricing kernel of a single representative investor holding the world market portfolio and consuming the aggregate endowment at each instant to price assets. In order to characterize the equilibrium, we need to keep track of the time-varying cross-country distribution of wealth. Asset prices can be expressed as functions of three state variables: the world endowment, the relative size of the two economies, and their relative wealth which fluctuates endogenously. Working under the assumptions of logarithmic utility and lognormal endowment processes, we pin down these pricing functions and use them to derive the joint behavior of returns and equity portfolios.

Even though we keep it very parsimonious, the dynamic asset pricing problem that we formulate, with two risky assets and heterogeneous investors, translates into an infinite-horizon coupled forward-backward stochastic differential equations (FBSDE) problem (Ma and Yong (1999)), whose solution cannot be obtained analytically. Instead, we derive approximate analytical formulas for asset prices, the (time-varying) first and second moments of asset returns, and portfolios by taking Taylor expansions around the frictionless case. We first consider an environment with one homogeneous good, and then extend our approach to allow for differentiated goods. We also solve for equilibrium prices and quantities numerically, which enables us to assess the accuracy of the perturbation-based solution method.

Our approximate analytical formulas allow us to characterize the impact of financial integration (which in our model means a decrease in the withholding tax on foreign dividends) through comparative statics. As the size of the friction decreases, we find that asset prices increase, the cross-country correlation of returns and cross-country equity holdings both also increase (the latter being a first-order effect, while the former is a second-order effect) and the volatility of asset returns diminishes (also a second-order effect). The overall impact of financial integration on the cost of funds is not clear-cut, depending on the respective size of the increase in the riskfree rate (due to lower precautionary saving) and of the decrease in the risk premium. The latter effect shows up as an extra term in a modified version of the CCAPM, where the level of the friction interacts with the relative wealth of countries. As a by-product of our analysis, we also derive a gravity equation for international trade in financial assets, thus providing a theoretical foundation for the the use of gravity equations in empirical work on cross-border asset holdings (e.g., Portes and Rey (2005)).

Whether small frictions on cross-border holdings can result in substantial portfolio home bias depends on the elasticity of demand for foreign assets and on the level of substitutability across assets. In our baseline model with homogeneous goods, substitutability between national assets is driven by common shocks affecting national economic fundamentals and by portfolio rebalancing. The portfolio rebalancing mechanism that

is emphasized in, e.g., Gehrig (1993) and Van Nieuweburgh and Veldkamp (2009).

induces the correlation of two assets returns to be higher than their "fundamental" correlation works as in Cochrane et al. (2008). A good shock to domestic dividends drives the price of the domestic asset up and increases its share in investors' portfolios. When financial markets are integrated, investors increase their demand for the foreign asset in order to keep the composition of their portfolios constant, which drives the price of the foreign asset up. In other words, as the share of the domestic asset in the world market portfolio increases, the required return on the foreign asset decreases because its diversification properties become more valuable. In financial autarky, by contrast, a good shock to an asset drives its price up without affecting the price of the other asset and the correlation of asset returns is equal to the correlation of economic fundamentals. In-between complete segmentation and perfect integration, the smaller the frictions between two markets, the higher the comovement of their stock prices, for a given level of fundamental correlation.[5] Quantitatively, we point out that the increase in return correlation induced by portfolio rebalancing, though high for low levels of fundamental correlation and no frictions in financial markets, is small for higher levels of correlation. This result is interesting when one wants to think about home bias from a general equilibrium perspective. Any cost bearing on foreign equity holdings has two opposite effects on portfolios: the direct effect is to reduce cross-border holdings by reducing expected returns on foreign assets; but there is also an indirect effect, which is to reduce the substitutability between national assets by reducing the correlation of their returns, thus increasing the willingness to diversify internationally. The overall quantitative impact of a friction depends on the relative size of the two effects, and the fact that the indirect effect is of small magnitude plays in favor of the result that small frictions can generate a large home bias.

In the extension of our framework with differentiated goods, each country produces one type of good and agents have CES preferences over the two goods. We show that the two-good model is completely isomorphic to the one-good case up to a simple transformation of the state variables. Hence, our perturbation solution approach applies in this extended environment. With non-homogeneous goods, a positive shock to the domestic endowment increases the relative price of foreign goods and therefore translates into higher foreign dividends at market value. Via this Ricardian adjustment of the terms of trade (see also Cole and Obstfeld (1991) and Pavlova and Rigobon (2007)), our model can generate high levels of asset substitutability without requiring a high level of exogenous fundamental correlation, which amplifies the impact of the financial friction on portfolios.

We assess the quantitative performance of the model in a calibration exercise based on data from G7

[5] One might prefer to think in terms of stochastic discount factors (SDFs). The two agents have perfectly correlated SDFs in the perfectly integrated case, so that the two assets are discounted the same way, which increases their correlation compared to the extreme case of complete segmentation where each asset is priced using the corresponding autarkic SDF. As financial integration increases, the discount factors that are applied to national assets become closer to each other, which increases the correlation of their returns.

countries over the period 1978-2008. First, we calculate the level of friction that is necessary to match the degree of home bias in the U.S., given estimated moments for dividend growth and the observed values for the state variables of the model. The estimated level of friction is decreasing over time, which is not surprising given the period we consider. In 2008, we find that frictions akin to a dividend tax of 3 to 7% are necessary to account for the observed equity home bias in the U.S.. Given the implied level of friction, we then proceed to compute asset return moments and the degree of home bias for other G7 countries predicted by the model. The model performs reasonably well when considering quantities (portfolios) but falls short of matching asset return moments, in particular the large increase in international stock returns correlation observed prior to the financial crisis.

Related literature. Basak and Gallmeyer (2003) consider a dynamic asset pricing model with asymmetric dividend taxation and a unique risky asset. Our analysis can be seen as a natural extension of their work to the two-asset case. We follow the same approach to deal with investor heterogeneity through the introduction of a time-varying Pareto-Negishi weight.[6] However our solution method differs from theirs: whereas they only provide a numerical solution (along the lines of Ma, Protter and Yong (1994)), we also derive approximate analytical formulas. Our two-tree specification follows Cochrane et al. (2008), which allows us to use their closed-form pricing formulas to construct our approximations.[7]

A vast strand of the literature in international finance studies portfolios and asset prices in the context of imperfectly integrated financial markets. Early papers by Black (1974), Stulz (1981), Errunza and Losq (1985, 1989), Eun and Jarakiramanan (1986) and Hietala (1989) analyze the impact of international financial barriers on the risk-return tradeoff, taking return second moments as given and characterizing how specific kinds of financial frictions lead to specific deviations from the traditional static CAPM. We derive a modified version of the CCAPM in our dynamic asset pricing model, which is close in spirit to their work. Papers more closely related to ours are Martin and Rey (2004), Bhamra (2009), Evans and Hnatkovska (2012), and Heathcote and Perri (2004). Martin and Rey (2004) consider a static model featuring a transaction cost on international trade in assets. Much as in our paper, this cost induces a home bias and the size of the bias depends on the elasticity of the demand for foreign assets, which is related to investors' risk aversion.[8] The dynamic setup of our model allows us to explore the joint dynamics of asset prices and the wealth distribution, and issues related to portfolio rebalancing and endogenous return correlation.[9] Bhamra (2009)

[6]Stochastic weights have been used in the asset pricing literature to characterize equilibrium under incomplete markets (e.g., Cuoco and He (1995), Basak and Cuoco (1998)). In our setup, like in Basak and Gallmeyer (2003), world markets are dynamically complete but deviation from perfect risk sharing results from differential taxation.

[7]Menzly, Santos and Veronesi (2004), Santos and Veronesi (2006) and Martin (2013) build multi-asset dynamic asset pricing models under alternative assumptions on preferences and cash flows. Our solution method could be generalized to their setups.

[8]This effect shows up in our model through the impact of volatility.

[9]Dumas et al. (2003) and Cochrane et al. (2008) analyze the endogenous determination of asset return correlation

and Evans and Hnatkovska (2012) look at the impact of partial financial integration on the dynamics of asset returns in a full-fledged dynamic equilibrium model but they focus on portfolio constraints, either constraints on the amount of wealth that can be invested abroad (Bhamra (2009)) or on the type of assets that can be traded internationally (Evans and Hnatkovska (2012)).[10] We obtain home bias in a more endogenous way by relating it to small frictions in equity markets. Heathcote and Perri (2004) solve for international portfolio diversification in the presence of a proportional cost on foreign dividends in a standard two-country international real business cycle model. They investigate how increasing portfolio diversification affects business cycle properties and in particular international output comovements. However, they do not allow for portfolio rebalancing (trade in assets only occurs in the initial period), nor do they investigate the asset pricing implications of their model. Instead, we take fundamental (output) correlations as exogenous, but investigate how asset prices and return distributions are endogenously affected by increasing financial globalization.

By focusing on small frictions in financial markets, we depart from the literature which tries to ascribe the observed equity home bias to hedging motives. One strand of this literature focuses on the hedging of real exchange rate fluctuations, which can be induced by the imperfect integration of markets for goods and services (Adler and Dumas (1983), Dumas (1992), Cooper and Kaplanis (1994)). It explores whether portfolio biases can be related to the presence of *trade costs* or home bias in preferences (Uppal (1993), Obstfeld and Rogoff (2000), Kollmann (2006a) Coeurdacier (2009)) or to the presence of *non-tradable goods* (Stockman and Dellas (1989), Baxter, Jermann and King (1998), Serrat (2001), Kollmann (2006b)). Another strand of this literature focuses on the hedging of *non-diversifiable labor income risk* (Baxter and Jermann (1997), Bottazzi, Pesenti and van Wincoop (1996), Heathcote and Perri (2007), Engel and Matsumoto (2008), Coeurdacier and Gourinchas (2011)). In our paper, these hedging motives do not operate. We focus on the implications of frictions in international equity markets.

Our theoretical predictions regarding the impact of financial integration on asset prices relate to some empirical contributions on this subject. Henry (2000) and Chari and Henry (2004) document a positive impact of financial integration on asset prices. These papers however use the lifting of capital account restrictions as a strategy of identification, while financial integration in our model is defined as a fall in the proportional costs of investing abroad. Bekaert and Harvey (2000), Goetzmann et al. (2005) and Quinn and Voth (2008) find evidence of a positive relationship between the level of financial market integration and stock return correlations. Our results are broadly consistent with these findings. Our analysis also sheds light

in the context of perfectly integrated financial markets.

[10] Pavlova and Rigobon (2008) analyze the impact of portfolio constraints on the international propagation of shocks.

on a finding documented in a number of empirical studies on international equity holdings (Portes and Rey (2005), Chan *et al.* (2005) and Lane and Milesi-Ferreti (2008)), which report a puzzling positive relationship between bilateral equity holdings and bilateral stock return correlations. This pattern seems to go against the logic of international portfolio diversification. However our model suggests that the relationship could be spurious, as cross-border holdings and return correlations are both affected by the level of financial integration between country pairs. Coeurdacier and Guibaud (2011) show that, controlling for endogeneity, the puzzle indeed disappears — holding financial frictions constant, investors do tilt their foreign holdings towards countries which offer better diversification opportunities.

From a methodological perspective, our paper relies on the use of perturbation analysis (Judd (1998)) and is related to Devereux and Sutherland (2011), Tille and van Wincoop (2010) and Kogan and Uppal (2001). Building on Judd and Guu (2001), Devereux and Sutherland (2011) and Tille and van Wincoop (2010) independently developed an approximation approach to solve for international portfolios in general equilibrium with heterogeneous agents. Their method also relies on Taylor expansions. While we construct approximations based on the size of the friction (around the frictionless case), they use the variance of shocks (around the non-stochastic steady state). Kogan and Uppal (2001) use Taylor expansions in the degree of risk aversion (around the logarithmic case) to solve for dynamic portfolio choice in a closed economy. Similar perturbation methods have also been used by Kogan (2001) and Janacek and Shreve (2004) to solve for optimal investment decisions.

The rest of the paper proceeds as follows. Section 2 lays out the model. Section 3 characterizes equilibrium asset prices via a system of forward-backward stochastic differential equations, which can be equivalently formulated as a coupled two-dimensional boundary value problem. Section 4 derives the implications of imperfect market integration for asset prices, asset returns and portfolios, using Taylor expansions around the frictionless case. Section 5 extends our results to the two-good model. Section 6 presents our calibration exercise using G7 data and discusses the quantitative performance of the model. Section 7 concludes. The proofs are relegated to the Appendix.

2 Model

We consider a continuous-time economy with an infinite horizon. There are two countries, home (H) and foreign (F), and a single non-storable good. Each country has a representative agent with time-separable expected utility and logarithmic preferences. The utility of agent i at time t is

$$U_{it} = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \log(c_{is}) ds \right], \tag{1}$$

where c_{is} is the consumption rate in country $i = H, F$, and ρ is the common rate of time preference.

Endowments. There is a Lucas tree in each country. We assume the real endowments (dividends) follow geometric Brownian motions:

$$\frac{dD_i(t)}{D_i(t)} = \mu_{D_i} dt + \boldsymbol{\sigma}_{D_i}.d\boldsymbol{W}(t), \qquad i = H, F. \tag{2}$$

All uncertainty is represented by a filtered probability space $(\Omega, \mathbb{P}, \mathcal{F}, \mathcal{F}_t)$ on which is defined a two-dimensional standard Wiener process $\boldsymbol{W}(t)$.[11] We call η the instantaneous correlation of the two dividend growth rates, which we refer to as the "fundamental" correlation.[12] The world endowment $D \equiv D_H + D_F$ follows a diffusion process whose drift and diffusion coefficients are weighted averages of those of D_H and D_F, with a time-varying weight depending on the size of each economy's endowment relative to the world endowment. From (2), we can write

$$\frac{dD(t)}{D(t)} = \underbrace{[\delta(t)\mu_{D_H} + (1-\delta(t))\mu_{D_F}]}_{\equiv \mu_D(t)} dt + \underbrace{[\delta(t)\boldsymbol{\sigma}_{D_H} + (1-\delta(t))\boldsymbol{\sigma}_{D_F}]}_{\equiv \boldsymbol{\sigma}_D(t)}.d\boldsymbol{W}(t), \tag{3}$$

where $\delta(t) \equiv D_H(t)/\left(D_H(t) + D_F(t)\right)$ captures the relative size of the domestic economy. Using the dynamics of D_H and D_F and applying Itô's lemma, one can write

$$\frac{d\delta(t)}{\delta(t)} = \mu_\delta(t) dt + \boldsymbol{\sigma}_\delta(t).d\boldsymbol{W}(t), \tag{4}$$

with

$$\mu_\delta(t) = (1-\delta(t)) \left[\mu_{D_H} - \mu_{D_F} - \delta(t) \left|\boldsymbol{\sigma}_{D_H}\right|^2 + (1-\delta(t))|\boldsymbol{\sigma}_{D_F}|^2 + (2\delta(t)-1)\boldsymbol{\sigma}_{D_H}.\boldsymbol{\sigma}_{D_F}\right], \tag{5}$$

$$\boldsymbol{\sigma}_\delta(t) = (1-\delta(t))(\boldsymbol{\sigma}_{D_H} - \boldsymbol{\sigma}_{D_F}). \tag{6}$$

Menu of assets. The menu of financial assets consists of stocks that are claims on the two Lucas trees (each stock being in constant net supply normalized to one) and an international, instantaneously riskfree asset (in zero net supply and frictionless). We use S_H and S_F to denote the two stock prices and r is the riskfree interest rate. Their processes will be determined as part of the equilibrium.

Frictions on international equity markets. We assume investors have to pay a proportional cost $\tau \in [0, 1)$ on the dividends they earn abroad.[13] For instance, a domestic agent who holds one unit of foreign

[11] Throughout we use bold case for vectors and matrices, $\boldsymbol{a}.\boldsymbol{b}$ for the scalar product of $\boldsymbol{a}$ and $\boldsymbol{b}$, $|\boldsymbol{a}|$ for the Euclidian norm of $\boldsymbol{a}$, and $\boldsymbol{A}^\top$ for the transpose of $\boldsymbol{A}$.

[12] We assume that fundamentals are not affected by the integration process. This assumption may not hold in a model with endogenous production (see Heathcote and Perri (2004)) or if access to new risk sharing opportunities and new sources of finance induced inter-sectoral reallocations (e.g., Obstfeld (1994)).

[13] Our analysis could easily be extended to the case where these costs differ between countries.

stock receives the instantaneous dividend $(1-\tau)D_F(t)dt$. No cost is paid on the domestic dividends. The friction τ could be interpreted literally as capturing differences in the taxation of domestic and foreign dividends. This form of fiscal discrimination is relevant in practice (see Gordon and Hines (2002), Griffith, Hines and Sorensen (2009), and OECD (2010)): it can be due to withholding taxes on foreign dividends (typically in the order of 10-15% in advanced countries),[14] or to tax credits that are extended to domestic residents based on the dividends they receive from their domestic equity holdings.[15] But our friction could be given other interpretations: it could capture for instance higher fees charged by mutual funds investing in international stocks, or it could be micro-founded as an agency cost in a model with moral hazard on cross-border investment. Note that our proportional cost τ on foreign holdings is not meant to capture capital account restrictions and/or regulatory constraints, but should rather be seen as the relevant measure of financial integration once investment restrictions have been lifted. In our view, this is more relevant for advanced economies since many emerging markets still face capital account restrictions (see Lane and Milesi-Feretti (2003)). In what follows, for simplicity, we refer to τ as a tax. When $\tau = 0$, financial markets are perfectly integrated.

We follow Basak and Gallmeyer (2003) and assume that taxes are redistributed in the economy as lump sum transfers, each agent continuously receiving transfers $e_i(t)dt$. This assumption allows us to write the market clearing condition for goods in a simple way, keeping aggregate consumption equal to aggregate dividends at each instant. The particular redistribution scheme under consideration does not matter much for our results. One could assume for instance that each agent receives the taxes paid by the other investor.[16] In that case,

$$
\begin{aligned}
e_H(t) &= \tau a_{FH}(t) D_H(t) && (7)\\
e_F(t) &= \tau a_{HF}(t) D_F(t), && (8)
\end{aligned}
$$

where a_{ij} denotes the number of claims on country j output held by the representative investor in country i.

2.1 Individual optimization

Investor i is endowed with an initial holding $a_{ij}(0)$ of each stock j. At each point in time, given the price processes S_H and S_F, the interest rate process r, her wealth X_i and a transfer process e_i, she chooses

[14] Many bilateral tax treaties lead to exemptions of these withholding taxes (through tax credit schemes), but these exemptions are often subject to some ceilings and do not apply to tax-exempt savings.

[15] "Dividend imputation schemes", meant to avoid the double taxation of dividends at the corporate and at the personal level, are quite common. The "avoir fiscal" in France is an even more extreme case in point: Until a recent reform, a French investor would receive from the French tax authorities an amount equal to 50% of the dividends perceived on stocks held in a *tax-exempt* saving account (Plan d'Epargne en Actions). Since only domestic stocks were eligible in a PEA, this created a powerful incentive to invest domestically.

[16] Since all investors act competitively, the redistribution of taxes does not give rise to any kind of strategic behavior.

consumption c_i and asset holdings $\boldsymbol{a}_i = (a_{iH}, a_{iF})^\top$ in order to maximize her intertemporal utility (1). The induced process for financial wealth X_i is given by

$$dX_i(t) = [r(t)X_i(t) + \boldsymbol{a}_i^\top(t)\boldsymbol{I}_S(t)(\boldsymbol{\mu}_i(t) - \boldsymbol{r}(t)) + e_i(t) - c_i(t)]dt + \boldsymbol{a}_i^\top(t)\boldsymbol{I}_S(t)\boldsymbol{\Sigma}(t)d\boldsymbol{W}(t), \tag{9}$$

with $\boldsymbol{I}_S$ a diagonal matrix that has S_H and S_F as diagonal coefficients, $\boldsymbol{\mu}_i$ the vector of expected returns from the perspective of investor i, and $\boldsymbol{\Sigma} = [\boldsymbol{\sigma}_H \ \boldsymbol{\sigma}_F]^\top$ the matrix of diffusion coefficients for stock prices.[17]

2.2 Definition of equilibrium

Given preferences, initial endowments and a tax reallocation rule, a competitive equilibrium is a set of adapted processes for asset prices, consumption c_i and asset holdings $\boldsymbol{a}_i$ such that $(c_i, \boldsymbol{a}_i)$ is a solution to investor i's optimization problem, and all markets clear at all dates, i.e., for all $t \geq 0$

$$\begin{aligned} c_H(t) + c_F(t) &= D_H(t) + D_F(t) = D(t), \\ \boldsymbol{a}_H(t) + \boldsymbol{a}_F(t) &= \mathbf{1}, \\ X_H(t) + X_F(t) &= S_H(t) + S_F(t). \end{aligned}$$

Imposing that the aggregate financial wealth be equal to the world market capitalization is equivalent to imposing that the aggregate position in the riskfree asset be zero.

3 Equilibrium

In this section, we start with a brief description of the equilibrium in the benchmark case of perfect integration (i.e., $\tau = 0$), recalling the quasi-closed-form expressions obtained for asset prices in that case (Cochrane et al. (2008), Martin (2013)). This is a good starting point to understand, by contrast, the impact of the friction we introduce. Moreover, we later make use of the frictionless solution by deriving Taylor approximations for asset prices around the case of perfect integration.

3.1 Benchmark case: no friction

When $\tau = 0$, all investors face the same opportunity set. Since they have identical preferences, they choose the same portfolio composition – every investor holds the world market portfolio. In this case, one can use the pricing kernel of a logarithmic representative agent consuming the world endowment at every instant to

[17] The adapted processes followed by the instantaneous expected rates of return and by $\boldsymbol{\Sigma}$ are determined in equilibrium.

price each asset as the expected present value of appropriately discounted future dividends:

$$S_{i0}(t) = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(u-t)} \frac{D(t)}{D(u)} D_i(u) du \right], \qquad i = H, F.$$

Using the definition of δ, one can write the price-dividend ratio for each stock as follows:

$$\frac{S_{H0}(t)}{D_H(t)} = \frac{1}{\delta(t)} \mathbb{E}_t \left[\int_t^\infty e^{-\rho(u-t)} \delta(u) du \right] \equiv s_{H0}(\delta(t)), \tag{10}$$

$$\frac{S_{F0}(t)}{D_F(t)} = \frac{1}{1-\delta(t)} \mathbb{E}_t \left[\int_t^\infty e^{-\rho(u-t)} (1-\delta(u)) du \right] \equiv s_{F0}(\delta(t)). \tag{11}$$

Because δ is a Markov process, the price-dividend ratios at time t can be written as functions of $\delta(t)$. Cochrane et al. (2008) show that s_{H0} and s_{F0} can be expressed in terms of the standard hypergeometric function (see details in Appendix A). Moreover, the size-weighted average of the price-dividend ratios in both countries coincides with the level of the price-dividend ratio in the frictionless one-tree economy:

$$\delta s_{H0}(\delta) + (1-\delta) s_{F0}(\delta) = \frac{1}{\rho}. \tag{12}$$

The consumption allocation in the benchmark case is straightforward. The relative consumption ratio is constant over time, both agents consuming a constant fraction of the world endowment according to their relative wealth ratio. There is perfect risk sharing. Besides, due to the logarithmic utility assumption, both agents' consumption-to-wealth ratios are constant, equal to their common rate of time preference ρ.

3.2 Heterogeneity and imperfect risk sharing

When $\tau > 0$, investors effectively face different investment opportunities since they get different "after-tax" returns. In that sense, frictions on international equity markets induce heterogeneity among investors.[18]

Wedge in perceived expected returns. We now pin down precisely the heterogeneity among investors, taking the returns on asset H as an example. The total instantaneous payoff on this asset is $D_H(t)dt + dS_H(t)$ for a domestic investor and $(1-\tau)D_H(t)dt + dS_H(t)$ for a foreign investor. The difference in the payoff on asset H for home and foreign investors comes from the dividend cashflows, which are lower for the foreign investor because of the tax. From this, we can define the total instantaneous expected rates of return on asset H, which we respectively note μ_H for the home investor and $\mu_{F,H}$ for the foreign investor:

$$\mu_H(t)dt = \mathbb{E}_t \left[\frac{D_H(t)dt + dS_H(t)}{S_H(t)} \right], \qquad \mu_{F,H}(t)dt = \mathbb{E}_t \left[\frac{(1-\tau)D_H(t)dt + dS_H(t)}{S_H(t)} \right].$$

[18] The same logic applies in the one-tree model of Basak and Gallmeyer (2003).

Obviously, μ_H is greater than $\mu_{F,H}$, the wedge between the two being equal to the tax rate τ multiplied by the dividend-price ratio of asset H:

$$\mu_H(t) - \mu_{F,H}(t) = \tau \frac{D_H(t)}{S_H(t)}. \tag{13}$$

Analogously, we get

$$\mu_F(t) - \mu_{H,F}(t) = \tau \frac{D_F(t)}{S_F(t)}, \tag{14}$$

where $\mu_{H,F}$ and μ_F respectively denote the total instantaneous expected rates of return on asset F for home and foreign investors. These expressions for the wedges characterize tightly the heterogeneity induced by taxes.

Investor-specific state prices. Investors being heterogeneous, we have to solve their individual optimization problems separately. Since both investors face dynamically complete markets, we use the solution technique of Cox and Huang (1989) and Karatzas et al. (1987). Therefore, we introduce the investor-specific (after-tax) market prices of risk $\boldsymbol{\theta}_H$ and $\boldsymbol{\theta}_F$:

$$\boldsymbol{\theta}_H(t) \equiv \boldsymbol{\Sigma}^{-1}(t)\begin{pmatrix} \mu_H(t) - r(t) \\ \mu_{H,F}(t) - r(t) \end{pmatrix}, \qquad \boldsymbol{\theta}_F(t) \equiv \boldsymbol{\Sigma}^{-1}(t)\begin{pmatrix} \mu_{F,H}(t) - r(t) \\ \mu_F(t) - r(t) \end{pmatrix}. \tag{15}$$

The difference between the market prices of risk relevant for the two representative agents follows directly from (13) and (14). We can write

$$\boldsymbol{\theta}_F(t) - \boldsymbol{\theta}_H(t) = \tau \boldsymbol{\Gamma}(t), \tag{16}$$

where

$$\boldsymbol{\Gamma}(t) \equiv \boldsymbol{\Sigma}^{-1}(t)\begin{pmatrix} -\frac{D_H(t)}{S_H(t)} \\ \frac{D_F(t)}{S_F(t)} \end{pmatrix}. \tag{17}$$

For a given level of friction $\tau > 0$, the degree of investor heterogeneity is captured by $|\boldsymbol{\Gamma}|$, which determines the size of the wedge in investor-specific market prices of risk. Investor i's state-price deflator ξ_i is defined as

$$\xi_i(t) = \exp\left(-\int_0^t r(u)du\right)\exp\left(-\int_0^t \boldsymbol{\theta}_i(u).d\boldsymbol{W}(u) - \frac{1}{2}\int_0^t |\boldsymbol{\theta}_i(u)|^2\, du\right), \qquad i = H, F.$$

Each ξ_i satisfies the following stochastic differential equation:

$$\frac{d\xi_i(t)}{\xi_i(t)} = -r(t)dt - \boldsymbol{\theta}_i(t).d\boldsymbol{W}(t). \tag{18}$$

Using these state prices, each individual dynamic optimization problem can be restated as a static problem, which consists of choosing a vector of contingent consumption rates under a single budget constraint:

$$\max_{\{c_i(t)\}} \quad \mathbb{E}\left[\int_0^\infty e^{-\rho t}\log(c_i(t))dt\right]$$

subject to

$$\mathbb{E}\left[\int_0^\infty \xi_i(t)c_i(t)dt\right] \le X_i(0) + \mathbb{E}\left[\int_0^\infty \xi_i(t)e_i(t)dt\right],$$

where the initial wealth $X_i(0)$ depends on the initial distribution of property rights on the equity claims.

Imperfect risk sharing. Letting Ψ_i denote the Lagrange multiplier on investor i's budget constraint, the first-order conditions of the above problem can be stated as

$$e^{-\rho t}\frac{1}{c_i(t)} = \Psi_i\xi_i(t), \quad \forall t,\ \forall i = H, F. \tag{19}$$

This implies

$$\frac{c_F(t)}{c_H(t)} = \frac{\Psi_H\xi_H(t)}{\Psi_F\xi_F(t)}, \quad \forall t. \tag{20}$$

When $\tau > 0$, ξ_H and ξ_F follow different dynamics. Hence, (20) implies that the consumption ratio c_F/c_H is not constant, i.e., imperfect risk sharing prevails. Using the market clearing condition in the goods market, we can write

$$c_H(t) = \omega(t)D(t), \qquad c_F(t) = (1-\omega(t))D(t), \tag{21}$$

where

$$\omega(t) \equiv \frac{\Psi_F\xi_F(t)}{\Psi_H\xi_H(t) + \Psi_F\xi_F(t)}. \tag{22}$$

The consumption of each agent is a function of the total endowment D and ω, which acts as a time-varying relative Pareto-Negishi weight. This is reminiscent of equilibria under incomplete markets *à la* Cuoco and He (1995). In our model, markets are complete but deviations from perfect risk sharing are induced by imperfect financial integration.

These results have to be contrasted with the case where $\tau = 0$. In a frictionless environment, the two investors face the same state prices, ξ_H/ξ_F is constant, the relative consumption ratio is constant and each agent consumes a constant fraction of the world endowment. In that case, ω is exactly equal to the constant domestic share of wealth $X_H/(X_H+X_F)$. In terms of asset pricing implications, we shall see in Section 4 that imperfect risk sharing, by making stochastic discount factors (SDFs) more volatile and less synchronized, causes an increase in asset return volatility and a fall in return correlation. In particular, whereas the two representative investors have perfectly correlated SDFs in the frictionless benchmark, the discount factors that are applied to dividend flows in the presence of frictions become closer to the autarkic SDFs — which reduces return comovements.

3.3 Asset prices and relative wealth: a forward-backward SDE problem

From the expressions for individual consumption in (21), we obtain the pricing kernels of both agents and use them to price the two assets.

Lemma 1. *Price-dividend ratios at time t are given by*

$$\frac{S_H(t)}{D_H(t)} = \frac{\omega(t)}{\delta(t)}\mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(u-t)}\frac{\delta(u)}{\omega(u)}du\right] \equiv s_H(\delta(t),\omega(t)) \tag{23}$$

$$\frac{S_F(t)}{D_F(t)} = \frac{1-\omega(t)}{1-\delta(t)}\mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(u-t)}\frac{1-\delta(u)}{1-\omega(u)}du\right] \equiv s_F(\delta(t),\omega(t)) \tag{24}$$

When $\tau > 0$, the distribution of wealth captured by the stochastic weight ω acts as a state variable. Stock prices at time t can be written as functions of $D(t)$, $\delta(t)$, and $\omega(t)$: this is enough information to form expectations on the future dividends of both assets and on the pricing kernels of both agents. It is noticeable that, though they do not share the same pricing kernels (because risk sharing is imperfect), the two investors agree on asset prices. They have to do so and what makes it possible is the fact that they do not face exactly the same assets. Indeed, the dividend flows *net of taxes* are different for the two investors. Another way to put it is that investors have different perceptions both of dividends and risk: for a given investor, the bad characteristic of an investment abroad in terms of expected returns is exactly compensated by the good diversification property of that investment.

To complete the characterization of equilibrium, we need to determine the pricing functions s_H and s_F. The conditional expectations in (23)-(24) involve future values of δ and ω. The process for the domestic share of world output δ is exogenous (Eq. (4)). The dynamics of the relative weight ω are determined endogenously. Given the dynamics of ξ_i in (18) and the definition of ω in (21), Itô's lemma implies:

$$\frac{d\omega(t)}{\omega(t)} = \mu_\omega(t)dt + \boldsymbol{\sigma}_\omega(t).d\boldsymbol{W}(t), \tag{25}$$

where

$$\mu_\omega(t) = \boldsymbol{\sigma}_\omega(t).\boldsymbol{\theta}_F(t) + |\boldsymbol{\sigma}_\omega(t)|^2,$$

$$\boldsymbol{\sigma}_\omega(t) = -(1-\omega(t))(\boldsymbol{\theta}_F(t) - \boldsymbol{\theta}_H(t)).$$

The drift and diffusion coefficients driving the dynamics of ω themselves depend on the investor-specific market prices of risk, $\boldsymbol{\theta}_H$ and $\boldsymbol{\theta}_F$. The latter can be shown to verify the following equilibrium restriction.

Lemma 2. *The market prices of risk, as perceived by home and foreign investors, satisfy*

$$\begin{aligned} \boldsymbol{\theta}_H(t) &= \boldsymbol{\sigma}_D(t) - \tau(1-\omega(t))\boldsymbol{\Gamma}(t), \qquad (26) \\ \boldsymbol{\theta}_F(t) &= \boldsymbol{\sigma}_D(t) + \tau\omega(t)\boldsymbol{\Gamma}(t). \qquad (27) \end{aligned}$$

In these expressions, the first term corresponds to the market price of risk in the frictionless case. Indeed, when $\tau = 0$, investors face the same market prices of risk, which are equal to $\boldsymbol{\sigma}_D$, the vector of diffusion coefficients in the world endowment process (Eq. (3)). The second term captures the impact of taxes, their interaction with dividend yields (through $\boldsymbol{\Gamma}$) and the relative wealth of investors.[19] Using Lemma 2, we can rewrite the drift and diffusion of the state variable ω as follows

$$\begin{aligned} \mu_\omega(t) &= \boldsymbol{\sigma}_\omega(t).\boldsymbol{\sigma}_D(t) + \left(1 - \frac{\omega(t)}{1-\omega(t)}\right)|\boldsymbol{\sigma}_\omega(t)|^2, \qquad (28) \\ \boldsymbol{\sigma}_\omega(t) &= -\tau(1-\omega(t))\boldsymbol{\Gamma}(t). \qquad (29) \end{aligned}$$

In our economy, asset prices depend on the current and future distribution of wealth (captured by ω), and the evolution of the latter depends on the dynamics of asset prices. The infinite-horizon backward SDEs defining the valuation functions s_H and s_F are coupled with a forward SDE for ω, so that our pricing problem is a forward-backward SDE problem (Ma and Yong (1999)). The functions s_H and s_F can also be characterized (up to simple transformations) via a system of partial differential equations (PDEs).

Proposition 1. *Let $h(\delta,\omega) = \frac{\delta}{\omega}s_H(\delta,\omega)$ and $f(\delta,\omega) = \frac{1-\delta}{1-\omega}s_F(\delta,\omega)$, where s_H and s_F have been defined in Lemma 1. The functions h and f satisfy the following system of quasilinear elliptic partial differential equations:*

$$\frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2\frac{\partial^2 h}{\partial\delta^2} + \delta\omega\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\omega\frac{\partial^2 h}{\partial\delta\partial\omega} + \frac{1}{2}\omega^2|\boldsymbol{\sigma}_\omega|^2\frac{\partial^2 h}{\partial\omega^2} + \delta\mu_\delta\frac{\partial h}{\partial\delta} + \omega\mu_\omega\frac{\partial h}{\partial\omega} - \rho h + \frac{\delta}{\omega} = 0, \qquad (30)$$

and

$$\frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2\frac{\partial^2 f}{\partial\delta^2} + \delta\omega\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\omega\frac{\partial^2 f}{\partial\delta\partial\omega} + \frac{1}{2}\omega^2|\boldsymbol{\sigma}_\omega|^2\frac{\partial^2 f}{\partial\omega^2} + \delta\mu_\delta\frac{\partial f}{\partial\delta} + \omega\mu_\omega\frac{\partial f}{\partial\omega} - \rho f + \frac{1-\delta}{1-\omega} = 0, \qquad (31)$$

where μ_δ and $\boldsymbol{\sigma}_\delta$ are given in (5)-(6). Explicit expressions for μ_ω and $\boldsymbol{\sigma}_\omega$ as functions of δ and ω, for given functions h and f and their partial derivatives, are given in Appendix C.

[19] Note that $\lim_{\omega\to 1}\boldsymbol{\theta}_H = \lim_{\omega\to 0}\boldsymbol{\theta}_F = \boldsymbol{\sigma}_D$.

Since μ_ω and $\boldsymbol{\sigma}_\omega$ can be expressed in terms of the state variables (δ, ω) given functions h and f and their first partial derivatives, the elliptic PDEs (30) and (31) along with relevant boundary conditions specify a coupled two-dimensional elliptic boundary value problem.[20] This problem can be solved numerically using a finite-difference scheme. However in order to build intuition, we provide in Section 4 approximate analytical solutions for asset prices and portfolios, obtained by perturbation around the case of perfect financial integration. We later make use of the numerical solution to assess the accuracy of our perturbation approach.

3.4 Analytical approximation strategy

Our approach builds on the fact that μ_ω and $\boldsymbol{\sigma}_\omega$ are functions of τ, which we denote by writing $\mu_\omega(\delta, \omega; \tau)$ and $\boldsymbol{\sigma}_\omega(\delta, \omega; \tau)$. In the benchmark frictionless case, ω is constant, so that $\mu_\omega(\delta, \omega; 0) = 0$ and $\boldsymbol{\sigma}_\omega(\delta, \omega; 0) = \mathbf{0}$ for all (δ, ω). From (28) and (29), first-order Taylor expansions in τ for μ_ω and $\boldsymbol{\sigma}_\omega$ are given by

$$\begin{aligned} \mu_\omega(\delta, \omega; \tau) &= -\tau(1-\omega)\boldsymbol{\Gamma}_0(\delta).\boldsymbol{\sigma}_D(\delta) + o(\tau), & (32) \\ \boldsymbol{\sigma}_\omega(\delta, \omega; \tau) &= -\tau(1-\omega)\boldsymbol{\Gamma}_0(\delta) + \boldsymbol{o}(\tau), & (33) \end{aligned}$$

where subscripts 0 refer to values prevailing when $\tau = 0$ (see Section 3.1). In particular,

$$\boldsymbol{\Gamma}_0(\delta) = \boldsymbol{\Sigma}_0^{-1}(\delta) \begin{pmatrix} -\frac{1}{s_{H0}(\delta)} \\ \frac{1}{s_{F0}(\delta)} \end{pmatrix} \tag{34}$$

and the diffusion matrix $\boldsymbol{\Sigma}_0 = [\boldsymbol{\sigma}_{H0} \quad \boldsymbol{\sigma}_{F0}]^\top$ follows from Itô's lemma:

$$\boldsymbol{\sigma}_{i0}(\delta) = \boldsymbol{\sigma}_{Di} + \frac{s'_{i0}(\delta)}{s_{i0}(\delta)} \delta \boldsymbol{\sigma}_\delta(\delta), \qquad i = H, F.$$

In Appendix C, we also derive second-order approximations for μ_ω and $\boldsymbol{\sigma}_\omega$ and use these approximations to derive Taylor expansions in τ for the pricing functions s_H and s_F.

4 Impact of financial integration on asset prices and portfolios

In this section, we provide a full description of the international financial markets equilibrium in the neighborhood of the frictionless case. Section 4.1 gives second order approximations for asset prices. Section 4.2 explores asset return volatility and cross-country return correlations. Section 4.3 gives expressions for risk premia and the riskfree rate. Section 4.4 contains results on the composition of portfolios. Finally, Section 4.5 discusses the accuracy of our approximations.

[20] Boundary conditions for h and f are given in the proof of Proposition 1 in Appendix C. In the symmetric case, i.e., for $\mu_{D_H} = \mu_{D_F}$ and $\boldsymbol{\sigma}_{D_H} = \boldsymbol{\sigma}_{D_F}$, the system of PDEs reduces to a single PDE.

4.1 Asset prices

Proposition 2. *Second-order approximations for s_H and s_F are:*

$$s_H(\delta,\omega;\tau) = \left[1-\tau(1-\omega)+\tau^2\omega(1-\omega)\right] s_{H0}(\delta)+\tau^2\omega(1-\omega)\frac{\varphi_H(\delta)}{\delta}+o(\tau^2), \tag{35}$$

$$s_F(\delta,\omega;\tau) = \left[1-\tau\omega+\tau^2\omega(1-\omega)\right] s_{F0}(\delta)+\tau^2\omega(1-\omega)\frac{\varphi_F(\delta)}{1-\delta}+o(\tau^2), \tag{36}$$

where φ_H and φ_F are solutions of the following ODEs [21]

$$\begin{aligned}
\rho\varphi_H(\delta)-\delta\mu_\delta(\delta)\varphi_H'(\delta)-\frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta(\delta)|^2\varphi_H''(\delta) &= \delta s_{H0}(\delta)|\boldsymbol{\Gamma}_0(\delta)|^2,\\
\rho\varphi_F(\delta)-\delta\mu_\delta(\delta)\varphi_F'(\delta)-\frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta(\delta)|^2\varphi_F''(\delta) &= (1-\delta)s_{F0}(\delta)|\boldsymbol{\Gamma}_0(\delta)|^2,
\end{aligned}$$

with boundary conditions

$$\begin{aligned}
\varphi_H(0) &= 0\\
\varphi_H(1) &= \lim_{\delta\to1}\frac{1}{\rho^2}|\boldsymbol{\Gamma}_0(\delta)|^2\\
\varphi_F(0) &= \lim_{\delta\to0}\frac{1}{\rho^2}|\boldsymbol{\Gamma}_0(\delta)|^2\\
\varphi_F(1) &= 0.
\end{aligned}$$

The first-order effect of imperfect market integration is to reduce equilibrium asset prices. Frictions in financial markets translate into lower prices by reducing expected dividend streams on domestic shares received by foreigners. Note that the decrease in domestic asset prices is more pronounced for a small ω. Indeed, as the relative wealth of domestic investors falls, the relative influence of foreign investors in the pricing of assets becomes stronger, which has a negative impact on the valuation of the domestic asset since foreigners perceive a reduced dividend stream.

The second-order price effect of integration goes through its impact on the riskless rate and on the variance-covariance matrix of returns, discussed in Sections 4.2 and 4.3. Overall the second-order effect of τ is positive, the dominant force being the fall in the riskfree rate induced by stronger precautionary motives.

4.2 Volatility and correlation of asset returns

From Proposition 2, we derive second-order approximations for the diffusion coefficients, $\boldsymbol{\sigma}_H$ and $\boldsymbol{\sigma}_F$, which determine the return covariance matrix $\boldsymbol{\Omega}=\boldsymbol{\Sigma}\boldsymbol{\Sigma}^\top$. Approximate formulas given in Proposition C-1 of

[21] We solve these boundary value problems using Chebychev polynomial approximations.

Appendix C, show that withholding taxes have no first-order impact on asset return second-order moments. To illustrate the effect of financial integration on asset volatility and correlations, we consider two symmetric countries with parameter values: $\rho = 0.03$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H,1} = \sigma_{D_F,2} = 0.097$ and $\sigma_{D_H,2} = \sigma_{D_F,1} = 0.026$.[22] We fix the state variables at $\delta = \omega = 0.5$ (symmetric state) and focus on the impact of τ. These numbers are chosen only for the sake of illustration. In Section 6, we calibrate the model using data for G7 countries and provide a systematic assessment of its quantitative performance.

As depicted in Figures 1 and 2, we find that return volatility decreases with financial integration, while return correlation increases. In order to understand the impact of the degree of market integration on the equilibrium correlation of returns, a good starting point is to contrast the two polar cases of perfect integration and complete segmentation. When markets are completely segmented, a good dividend shock in one country has no impact on the price of assets in another country. However, the story goes differently when investors can hold assets everywhere. The reason is that following the rise in the domestic price induced by a good domestic shock, the share of asset H in the world market portfolio increases, making country F's asset more appealing, because the diversification opportunities it offers are suddenly more valuable. Therefore, the required excess return on asset F decreases and its price increases.[23] For a small but positive τ, the same mechanism is at work but dampened due to investors heterogeneity. Indeed, a good shock to D_H affects each investor differently: the home investor is the most affected since his portfolio is biased towards domestic assets; but because he is reluctant to rebalance his portfolio towards taxed foreign assets, the increase in S_F is attenuated compared to the case of perfect integration. The result that stock return correlations between countries increase when cross-border impediments to foreign equity holdings are relaxed is consistent with the empirical findings of Bekaert and Harvey (2000).[24]

[Figures 1 and 2 here]

Table 1 shows the endogenous asset return correlation η_S as a function of two exogenous parameters: the level of friction τ and the fundamental correlation η. For given η, the correlation of asset returns is monotonically decreasing in τ. It is noticeable that the ratio η_S/η decreases with the exogenous level of the fundamental correlation: the endogenous component of asset return comovements becomes relatively less important. This is because when the fundamental correlation is higher, high dividends in one country are often accompanied by high dividends in the other country, thus reducing the need for portfolio rebalancing.

[22] This corresponds to 10% fundamental volatility and fundamental correlation $\eta = 0.5$.

[23] The increase in S_H is also lower than under full segmentation.

[24] Note that the definition of financial integration in that paper is different from ours since they focus on episodes of stock market opening to foreign investors.

[Table 1 here]

4.3 Risk premia and riskfree rate

We now turn to country equity risk premia, $\mu_i^e = \mu_i - r$, and show how the world CCAPM is modified in our setting with imperfect financial integration.

Proposition 3. *"Before-tax" expected excess returns on assets H and F are*

$$\mu_H^e(\delta,\omega) = \boldsymbol{\sigma}_H(\delta,\omega).\boldsymbol{\sigma}_D(\delta) + \tau(1-\omega)\frac{1}{s_H(\delta,\omega)}, \tag{37}$$

$$\mu_F^e(\delta,\omega) = \boldsymbol{\sigma}_F(\delta,\omega).\boldsymbol{\sigma}_D(\delta) + \tau\omega\frac{1}{s_F(\delta,\omega)}. \tag{38}$$

To the first order, country equity risk premia are

$$\mu_H^e(\delta,\omega) = \boldsymbol{\sigma}_{H0}(\delta).\boldsymbol{\sigma}_D(\delta) + \tau(1-\omega)\frac{1}{s_{H0}(\delta)} + o(\tau), \tag{39}$$

$$\mu_F^e(\delta,\omega) = \boldsymbol{\sigma}_{F0}(\delta).\boldsymbol{\sigma}_D(\delta) + \tau\omega\frac{1}{s_{F0}(\delta)} + o(\tau). \tag{40}$$

In the standard continuous-time consumption-based CAPM with logarithmic utility, the vector of expected excess returns for the two assets is $\boldsymbol{\Sigma}_0\boldsymbol{\sigma}_D$: the risk premia are equal to the covariance of asset returns with aggregate consumption growth. The first-order impact of τ is to drive the risk premia above their benchmark level. This is because both assets are partly held by taxed investors who require a higher pre-tax excess return to compensate for taxation.[25] The prediction that an increase in financial market integration (a decrease in τ) reduces the required excess return is broadly consistent with the empirical evidence (Bekaert and Harvey (2000), Henry (2000), Chari and Henry (2004)). The term in τ that appears in (39)-(40) interacts with the dividend-price ratio and the relative wealth of countries. Our model therefore generates time-varying risk premia both through variations in the relative size of asset cash flows (as in Cochrane et al. (2008)) and through variations in relative wealth. Second-order effects on expected excess returns (shown in Appendix C) operate through the impact of τ on asset valuations and on asset return second moments. Since dividend yields are higher under imperfect integration, the effect of the friction on risk premia gets amplified. The increase in volatility also drives up risk premia, while the decrease in the correlation of returns with aggregate output plays in the opposite direction.

Our next proposition shows that when markets are imperfectly integrated, the riskfree interest rate falls below its benchmark level.

[25] It is straightforward to see that the presence of taxes lowers the "after-tax" risk premia $\mu_{H,F} - r$ and $\mu_{F,H} - r$.

Proposition 4. *The second-order approximation of the riskfree rate is*

$$r(\delta, \omega) = \rho + \mu_D(\delta) - |\boldsymbol{\sigma}_D(\delta)|^2 - \tau^2 \omega(1-\omega)|\boldsymbol{\Gamma}_0(\delta)|^2 + o(\tau^2). \tag{41}$$

In the absence of frictions, (41) reduces to the standard interest rate formula with logarithmic utility: the riskfree rate is determined by the rate of time preference and by the mean and variance of aggregate consumption growth. When $\tau > 0$, higher precautionary saving motives cause a fall in the riskfree rate. The effect is stronger when countries have more symmetric wealth and when the presence of financial frictions causes a greater reduction in risk sharing (as captured by $|\boldsymbol{\Gamma}_0|^2$).[26]

Note that a decrease in τ causes all at once an increase in the riskfree rate and a decrease in the equilibrium excess returns. Therefore, the impact of financial integration on the total cost of capital is not clear-cut, depending on the relative strength of these two effects. There could be non-monotonic effects.

4.4 Portfolios

We now turn to the extent of international portfolio diversification in our setting with imperfectly integrated financial markets. To this end, we let $\pi_{ij} \equiv \frac{a_{ij} S_j}{X_i}$ denote the share of equity j in the wealth of investor i. We also introduce the instantaneous covariance matrix $\boldsymbol{\Omega}_0 = \boldsymbol{\Sigma}_0 \boldsymbol{\Sigma}_0^\top$ and the composition of the world market portfolio $\boldsymbol{\pi}_0 = (\boldsymbol{\Sigma}_0^\top)^{-1} \boldsymbol{\sigma}_D$ under perfect integration.

Proposition 5. *To first order, portfolios $\boldsymbol{\pi}_i = [\pi_{iH}, \pi_{iF}]^\top$ for $i = H, F$ are given by*

$$\boldsymbol{\pi}_H(\delta, \omega) = \boldsymbol{\pi}_0(\delta) + \tau(1-\omega)\boldsymbol{\Omega}_0^{-1}(\delta) \begin{bmatrix} \frac{1}{s_{H0}(\delta)} \\ -\frac{1}{s_{F0}(\delta)} \end{bmatrix} + \boldsymbol{\epsilon}_H(\delta, \omega) + \boldsymbol{o}(\tau), \tag{42}$$

$$\boldsymbol{\pi}_F(\delta, \omega) = \boldsymbol{\pi}_0(\delta) + \tau\omega\boldsymbol{\Omega}_0^{-1}(\delta) \begin{bmatrix} -\frac{1}{s_{H0}(\delta)} \\ \frac{1}{s_{F0}(\delta)} \end{bmatrix} + \boldsymbol{\epsilon}_F(\delta, \omega) + \boldsymbol{o}(\tau). \tag{43}$$

Portfolios can be decomposed into three components. In (42) and (43), the sum of the first two terms corresponds to the first-order approximation of $\boldsymbol{\Omega}^{-1}[\boldsymbol{\mu}_i - \boldsymbol{r}]$, i.e., the standard portfolio composition of a logarithmic investor with financial wealth only. The first term is the world market portfolio held by both investors when $\tau = 0$. The second term shows that for an investor in country H, τ reduces the demand for foreign stocks by reducing after-tax expected returns on these stocks. Symmetrically, due to

[26] The implication of our model that financial integration should coincide with a rise in the world riskfree rate may seem at odds with the fall in interest rates observed over last twenty years (see for instance Caballero, Fahri and Gourinchas (2008)). Our model based on small financial frictions is not meant to capture the effects caused by the integration with emerging markets, viewed as an important explanation for the fall in the world interest rate (e.g., saving glut explanation articulated by Bernanke (2005)). However, we did observe a fall in savings in developed countries, as predicted by the model (reduced precautionary motive due to better risk sharing).

market clearing, τ increases the domestic demand for domestic shares to compensate for the lower demand by foreign investors.[27] The third term $\boldsymbol{\epsilon}_i$ comes from the redistribution of taxes: for instance, if e_H is positively correlated with D_H, this will create a demand for foreign shares in order to hedge against this additional income risk. In the Appendix, we derive a first-order approximation for this term in the case where $e_i = \tau a_{ji} D_i$. However, because this hedging component depends very much on the assumed redistribution scheme and is small when the two countries are not too asymmetric, we neglect the effect of this term in the following expressions.

Comparative statics in a simple symmetric case. In the case of symmetric fundamentals, and under the assumption that the two countries are of equal size so that dividend yields under perfect integration are equal to the rate of time preference ρ, Eq. (42) simplifies to

$$\pi_{HH} \simeq \frac{1}{2} + \tau \frac{\rho(1-\omega)}{\sigma^2(1-\eta_S)} \quad \text{and} \quad \pi_{HF} \simeq \frac{1}{2} - \tau \frac{\rho(1-\omega)}{\sigma^2(1-\eta_S)},$$

where σ denotes the common return volatility and η_S the return correlation. It follows that

$$\frac{\partial \pi_{HF}}{\partial \tau} = -\frac{\rho(1-\omega)}{\sigma^2(1-\eta_S)} < 0, \tag{44}$$

$$\frac{\partial \pi_{HF}}{\partial \eta_S} = -\tau \frac{\rho(1-\omega)}{\sigma^2(1-\eta_S)^2} < 0. \tag{45}$$

These expressions capture the impact of frictions and asset substitutability on the extent of portfolio diversification. The interaction of the two appears in the fact that $\left|\frac{\partial^2 \pi_{HF}}{\partial \eta_S \partial \tau}\right| > 0$, i.e., the effect of the friction on equity holdings is amplified when assets are closer substitutes. When investments are riskier (higher σ), the motive for risk sharing increases and portfolios are more diversified:

$$\frac{\partial \pi_{HF}}{\partial \sigma^2} = \tau \frac{\rho(1-\omega)}{\sigma^4(1-\eta_S)} > 0. \tag{46}$$

Finally, we note that

$$\frac{\partial \pi_{HH}}{\partial \omega} = -\tau \frac{\rho}{\sigma^2(1-\eta_S)} < 0. \tag{47}$$

This is an equilibrium effect. A fall in ω means the relative wealth of foreign investors increases, which strengthens their influence on the pricing of assets and increases the negative impact of the friction on the price of the domestic asset. As a consequence, a smaller ω translates into lower valuation of the domestic asset, which gives an extra incentive for domestic investors to stay invested domestically.[28]

[27] This general equilibrium effect is relevant empirically. Chan et al. (2005) find that countries imposing high withholding taxes to foreign shareholders exhibit a higher home bias.

[28] This prediction of our model that the home bias in portfolios should be larger in countries whose relative wealth is smaller is consistent with some evidence in Chan *et al.* (2005). The lowest three values taken by their measure of home bias are for the US, UK and Japan, and the highest four are for New Zealand, Norway, Portugal and Greece.

Level of home bias. Figure 3 illustrates for the same parameter values as in Section 4.2 and $\delta = \omega = 0.5$, the fraction of total equity holdings invested abroad as a function of τ and as a function of the fundamental correlation η, taking into account the endogeneity of stock returns first and second moments. For $\tau = 10\%$ and $\eta = 0.5$, we obtain a weight on foreign stocks of about 16%, i.e., a reasonable level of friction on cross-border equity holdings, coupled with a high level of return correlation, can generate a realistic level of home bias. In Section 5, we show that the two-good version of our model can generate a substantial level of home bias for lower values of τ and η.

[Figure 3 here]

A gravity equation for bilateral equity holdings. Our model provides some theoretical underpinning to the use of gravity equations in empirical work on bilateral equity holdings. Indeed, when we turn from portfolio shares to the value of equity holdings, we can write (up to a first-order approximation):

$$\log(a_{HF}S_F) = \log X_H + \log\left(\frac{S_F}{S_H+S_F}\right) - \tau\frac{1}{1-\eta_S^2}(1-\omega)\frac{S_H+S_F}{\sigma_F S_F}\left(\frac{1}{\sigma_F}\frac{D_F}{S_F} + \eta_S\frac{1}{\sigma_H}\frac{D_H}{S_H}\right), \tag{48}$$

where the first two terms are the mass terms in the gravity equation. As shown by Portes and Rey (2005), gravity equations give a good description of patterns of international asset holdings.[29] In their work, they use the market capitalizations of origin and destination countries as proxies for the mass terms of the equation. In our model, the relevant size variables are the aggregate wealth of the source country and the relative market capitalization of the destination country. Moreover, Portes and Rey (2005) propose to interact variables that proxy for international frictions with the degree of substitutability between assets, which is captured by $1/\left(1-\eta_S^2\right)$ in the third term of (48). Our model provides a theoretical justification for this procedure.

4.5 Accuracy of approximations

In order to assess the accuracy of the second-order perturbation solution, we solve for the home country price-dividend ratio numerically and compare the numerical and second-order perturbation solutions along various dimensions. We solve the two-dimensional elliptic boundary value problem described in Section 3.3 using a finite difference scheme.[30] We discretize the state space $[0,1]\times[0,1]$ via an equally spaced $n_\delta \times n_\omega$ grid. Along the δ axis, the spacing between grid points is $\Delta_\delta = 1/(n_\delta - 1)$. Along the ω axis, the spacing between grid points is $\Delta_\omega = 1/(n_\omega - 1)$. Thus, we have a general grid point (δ_i, ω_j), where $\delta_i = (i-1)\Delta_\delta$

[29] See Okawa and van Wincoop (2012) for a derivation of a similar equation in a two-period set-up and additional references to empirical papers.

[30] The valuation functions s_H and s_F are obtained from h and f as $s_H(\delta,\omega) = \frac{\omega}{\delta}h(\delta,\omega)$ and $s_F(\delta,\omega) = \frac{1-\omega}{1-\delta}f(\delta,\omega)$.

and $\omega_j = (j-1)\Delta_\omega$. We use a central difference scheme, i.e.

$$\begin{aligned}
\left.\frac{\partial h}{\partial \delta}\right|_{(\delta_i,\omega_j)} &= \frac{h(\delta_{i+1},\omega_j) - h(\delta_{i-1},\omega_j)}{2\Delta_\delta}, \\
\left.\frac{\partial h}{\partial \omega}\right|_{(\delta_i,\omega_j)} &= \frac{h(\delta_i,\omega_{j+1}) - h(\delta_i,\omega_{j-1})}{2\Delta_\omega} \\
\left.\frac{\partial^2 h}{\partial \delta^2}\right|_{(\delta_i,\omega_j)} &= \frac{h(\delta_{i+1},\omega_j) - 2h(\delta_i,\omega_j) + h(\delta_{i-1},\omega_j)}{\Delta_\delta^2} \\
\left.\frac{\partial^2 h}{\partial \delta \partial \omega}\right|_{(\delta_i,\omega_j)} &= \frac{h(\delta_{i+1},\omega_{j+1}) - h(\delta_{i+1},\omega_{j-1}) - (h(\delta_{i-1},\omega_{j+1}) - h(\delta_{i-1},\omega_{j-1}))}{4\Delta_\delta\Delta_\omega}, \\
\left.\frac{\partial^2 h}{\partial \omega^2}\right|_{(\delta_i,\omega_j)} &= \frac{h(\delta_i,\omega_j+1) - 2h(\omega_i,\delta_j) - h(\delta_i,\omega_j-1)}{\Delta_\omega^2},
\end{aligned}$$

and similarly for f. The numerical problem therefore reduces to a system of $2(n_\delta - 2)(n_\omega - 2)$ nonlinear equations, where the unknowns are the values taken by functions h and f on the finite grid.[31]

Appendix E provides a detailed comparison of the results obtained with the second-order approximation and the numerical method along two different metrics, price-dividend ratio and risk premium. Using the the same parameter values as in Section 4.2,[32] and for τ ranging from 5% to 10%, the numerical and second order solutions for the home country price-dividend ratio (s_H) differ by typically less than two percentage points over most of the state space. When looking at the home country risk premium, μ_H^e, the difference between the two solution methods is extremely small (always less than a fifth of a basis point) for $\tau = 0.05$ and remains very small (typically less than one basis point and two basis points at most) for $\tau = 0.1$. Along other metrics not reported here (e.g., return second-moments, portfolios), we also find our second-order approximation to be very close to the numerical solution. The Appendix also provides an assessment of the Euler equation errors for the second-order approximation. These errors are small: around 3 basis points close to $\delta = \omega$ and around 10 basis points elsewhere in the state space.

5 Two-good extension

This section extends our analysis to the case where the goods produced in each country are imperfect substitutes. All the findings of Section 4 go through. Indeed, we show that the two-good model is isomorphic to the case of perfect substitutability. However, this specification has the potential to generate realistic portfolio predictions without requiring very high cross-country correlations between endowments. When

[31] The $2(n_\delta - 2)(n_\omega - 2)$ nonlinear equations have $2(n_\delta - 2)(n_\omega - 2)$ unknowns, which we solve for in *Mathematica* using the built-in solver, `FindRoot`. Boundary conditions for h and f are given in Appendix C.

[32] I.e. $\rho = 0.03$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H,1} = \sigma_{D_F,2} = 0.097$ and $\sigma_{D_H,2} = \sigma_{D_F,1} = 0.026$.

goods are imperfect substitutes, relative prices depend on relative quantities: the relative price of a good is positively related to its relative scarcity, so that a positive output shock in one country is accompanied by a counteracting relative price change. This "terms of trade effect" makes asset cash flows and asset prices evolutions more synchronized. This mechanism is emphasized in Pavlova and Rigobon (2007).

We assume that each country produces one (tradable) good and the representative agent in each country consumes both goods. Endowments in each country, D_H and D_F, follow geometric Brownian motions, as specified in (2). The two representative agents have the same log-CES preferences. Let $c_{ij,t}$ denote agent i's consumption of goods from country j at date t. Agent i's consumption aggregate at time t is given by

$$C_{it} = \left[c_{ii,t}^{\frac{\phi-1}{\phi}} + c_{ij,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}. \tag{49}$$

The parameter $\phi > 0$ denotes the elasticity of substitution between Home and Foreign goods ($\phi = \infty$ corresponds to the one-good case). Agent i's utility at time t is

$$U_{it} = \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(s-t)} \log(C_{is}) ds \right]. \tag{50}$$

Let $p_{H,t}$ and $p_{F,t}$ denote the prices of the two goods, normalized by taking the consumption index as numeraire:

$$\left[p_{H,t}^{1-\phi} + p_{F,t}^{1-\phi} \right]^{\frac{1}{1-\phi}} = 1. \tag{51}$$

Optimal (intra-temporal) consumption allocation implies

$$c_{ij} = p_j^{-\phi} C_i, \qquad \text{for all } (i,j). \tag{52}$$

Resource constraints for the two goods are:

$$c_{Hj} + c_{Fj} = D_j, \quad j = H, F. \tag{53}$$

Eqs. (52) and (53) pin down the terms of trade as a function of relative quantities. The ratio of outputs evaluated at market prices (i.e., the ratio of dividend cash-flows paid by the two risky assets) is

$$\frac{p_H(t) D_H(t)}{p_F(t) D_F(t)} = \left(\frac{D_H(t)}{D_F(t)} \right)^{\frac{\phi-1}{\phi}}. \tag{54}$$

The strength of the terms of trade effect increases as goods become less substitutable. For an elasticity of substitution below one, the effect is so strong that, following a good domestic shock, the cash-flows of domestic assets are lower than the ones of foreign assets. In the special case of an elasticity of substitution equal to one (i.e., Cobb-Douglas preferences), the change in relative prices exactly compensates the change in relative quantities, so that cash flows on domestic and foreign assets are perfectly correlated. In that case, portfolios are indeterminate (Cole and Obstfeld (1991)). We assume $\phi \neq 1$ in our analysis.

We redefine the state variable D as the world endowment in the composite good:

$$D(t) \equiv \left[D_{H,t}^{(\phi-1)/\phi} + D_{F,t}^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)}. \tag{55}$$

Eqs. (52) and (53) imply that in equilibrium, $C_H(t) + C_F(t) = D(t)$. Let $\omega(t)$ denote the domestic share of world consumption, so that

$$C_H(t) = \omega(t) D(t) \qquad \text{and} \qquad C_F(t) = (1 - \omega(t)) D(t). \tag{56}$$

We redefine the state variable δ as

$$\delta(t) \equiv \frac{p_H(t) D_H(t)}{D(t)} = \frac{1}{1 + (D_F(t)/D_H(t))^{\frac{\phi-1}{\phi}}}, \tag{57}$$

where the second equality follows from (54), along with the fact that $p_H(t)D_H(t) + p_F(t)D_F(t) = D(t)$.

Lemma 3. *In the case of perfect integration with differentiated goods, price-dividend ratios can be written*

$$\frac{S_i(t)}{p_i(t) D_i(t)} = s_{i0}(\delta(t)), \quad i = H, F, \tag{58}$$

for newly defined functions s_{H0} and s_{F0} which can be expressed in terms of the standard hypergeometric function, as in Cochrane, Longstaff and Santa-Clara (2008).

We show in Appendix C how the elasticity of substitution coefficient ϕ enters in the construction of the new valuation functions s_{H0} and s_{F0}. From Lemma 3, we can derive the stochastic properties of asset returns for $\tau = 0$. Figures 4 and 5 depict return volatility and correlation in the symmetric case as functions of the elasticity of substitution ϕ (for orthogonal fundamentals, keeping the same volatility as before). For ϕ close enough to 1, return correlations can be arbitrarily high despite zero fundamental correlation, through the terms of trade adjustment.

[Figures 4 and 5 here]

Proposition 6. *For $\tau > 0$, the approximate expressions of Section 4 for asset prices and portfolios remain valid, for the newly defined state variable δ and pricing functions s_{H0} and s_{F0}.*

Due to the isomorphism with the one-good case, portfolios can be described as in Proposition 5. The key difference is that a reasonable level of home bias can be obtained for small positive values of τ without requiring an unrealistically high level of fundamental correlation. Figure 6 shows the level of foreign exposure increasing as a function of the elasticity of substitution ϕ, for $\tau = 5\%$ and under the assumption that fundamentals are uncorrelated. For reasonable values of ϕ, portfolios exhibit a strong home bias.

[Figure 6 here]

6 Empirical implications

In this section, we explore the quantitative performance of the model using data for G7 countries over the period 1978-2008.[33] A detailed description of the data is provided in Appendix F.

6.1 Time series implications: United States vs. rest of G7 countries

We first apply the model to the U.S. versus an aggregate of the other G7 countries [Rest of the World (ROW)]. We proceed in two steps. First, we determine the model-implied level of financial frictions needed to account for the level of home bias in the U.S. and its evolution over time. We then confront the asset pricing predictions of the model to the data.

Calibration. Table 2 summarizes our baseline calibration. We calibrate the fundamentals of the model using quarterly data on annual real dividend growth.[34] For the U.S., we use real dividend growth computed for a value-weighted combination of all stocks in CRSP. Real dividend growth for the rest of the world is constructed as a GDP-weighted average of real dividend growth for the remaining six G7 countries, obtained from Global Financial Data.[35] Series are deflated using national CPI indices from the OECD Main Economic Indicators. We compute U.S. real dividend growth volatility, ROW real dividend growth volatility and the correlation of real dividend growth between the U.S. and ROW (η) based on these data, and we set $\mu = 2.2\%$, the average annual real GDP growth rate of the U.S. economy for the same thirty-year period. The discount factor ρ is set to 0.03 to match the average level of the aggregate dividend yield in the U.S. over the period 1978-2008.[36]

[33] G7 countries account for 73% of world market capitalization on average over this period.

[34] While real dividend growth and real GDP growth coincide in the model, they differ substantially in the data, in terms of volatility in particular. We use real dividends as they are the closest empirical counterpart to the equity cash flows considered in the model.

[35] Constant GDP-weights over 1978-2008 are respectively: 7.6% for Canada, 14.4% for France, 20.0% for Germany, 13.9% for Italy, 29.9% for Japan and 14.2% for UK.

[36] This is equal to 2.98% (data obtained from Robert Shiller's website).

We employ different values for the elasticity of substitution across goods ϕ. While the time-series macro literature estimating the response of trade to exchange rate fluctuations suggests a low elasticity of substitution between 0.5 and 2 (see Hooper and Marques (1995), Backus, Kehoe and Kydland (1994) and Heathcote and Perri (2002)), bilateral trade data suggest a larger elasticity of above 5 for most sectors (see Harrigan (1993), Hummels (2001) and Baier and Bergstrand (2001) among others). Recent work by Imbs and Mejean (2010) reconciles both sets of findings and suggests a value between 4 and 6. Hence, we set $\phi = 5$ in our baseline calibration, but also present results for a wide range of values for the elasticity of substitution.

Finally, we set the state variables (δ, ω) in a given year equal to their observed counterparts. These are computed using output and consumption data for G7 countries. The variable δ is defined as the share of U.S. output in world output at market prices, and is computed empirically by taking U.S. output as a fraction of total G7 output in US dollar. In the same way, ω is the share of U.S. consumption in world consumption at market prices, so we use U.S. consumption as a fraction of total G7 consumption in US dollar.

[Table 2 here.]

Estimating financial frictions to match the U.S. equity home bias. We back out the level of friction τ required to match the share of foreign equity in total U.S. equity holdings as observed in 1988, 1998 and 2008, given our calibrated parameter values and the values taken by the state variables of the model. To compute the observed share of foreign equities in the U.S. equity portfolio, whose model counterpart is given by $\pi_{HF}/(\pi_{HH} + \pi_{HF})$, we use annual data on foreign equity holdings and market capitalizations (see Appendix F for details). The targeted portfolio shares are shown in Table 3.

Table 3 shows the results of our estimation of the degree of financial frictions, for different values of the elasticity parameter ϕ. The estimated friction τ is, not surprisingly, decreasing over time during the period of financial globalization we consider. Quantitatively, we find that reasonably small frictions can explain the large degree of home bias. In order to explain the observed decline in the U.S. equity home bias, the friction must have roughly decreased by 30% over two decades. In our baseline case ($\phi = 5$), the friction is found to have fallen from 10.5% to 6.8%. The level of the estimated friction is increasing in the elasticity of substitution ϕ. A higher ϕ reduces endogenous co-movements of dividends and asset prices through relative price movements, increasing the benefits from diversification. As a result it takes larger frictions to account for the observed level of home bias.

Asset-pricing implications. Given our estimated level of financial friction, our values for parameters and state-variables, we compute the time-varying asset pricing moments implied by the model and compare them to the data in 1988, 1998 and 2008. To estimate the time-varying moments of the joint distribution of equity returns empirically, we use quarterly data on real stock returns and compute sample moments over ten-year time windows. A value-weighted stock index is used for every country, and real stock returns for non-U.S. G7 countries are computed as a GDP-weighted average of real stock returns for the corresponding six countries. The volatility of the riskfree rate in each observation year is computed using quarterly data over the previous ten years on three-month T-bill rates deflated by annual CPI inflation.

Results are shown in Table 3. Confronting model-predicted return correlations with the data, we find that the model cannot simultaneously account for the level of stock return correlation in 1988 and match the observed increase in correlation over the period of financial globalization. For very high values of ϕ, the model can generate a significant increase in return correlation given the estimated fall in τ, but at the expense of not being able to match the level. When matching the level (for ϕ around 2), the endogenous increase in correlation induced by financial globalization becomes very small; indeed, since stock markets tend to comove more strongly due to the adjustment of relative prices, investors have less incentive to rebalance their portfolios following asymmetric shocks to dividends, thus muting the channel through which financial globalization affects asset price comovement. This finding echoes the results of Cole and Obstfeld (1991): the more risk sharing operates through the terms-of-trade adjustment, the less portfolio adjustments matter. This is clearly a limitation of our framework and points to an avenue for further research. Regarding stock return volatility, our model generates stock returns that are less volatile than fundamentals, which is at odds with the data. The same result holds in the frictionless model of Cochrane, Longstaff and Santa-Clara (2008), and even though the presence of frictions induces higher volatility, the effect is not strong enough to overturn their finding. Finally, our model predicts a fall in the volatility of the riskfree rate with financial integration: with a lower τ, agents engage in less borrowing and lending since more risk sharing occurs through equity trade. Thus, with a lower τ, endowment shocks generate less bond trading and the interest rate becomes less sensitive to the shocks. This prediction is in line with the fall in bond trading and the fall in the volatility of bond returns in the nineties in G7 countries documented by Evans and Hnatkovska (2012).[37] Note however that the model generates too little volatility of the riskfree rate, despite the presence of frictions on equity markets (see Martin (2013) for a similar observation in a frictionless model).

[Table 3 here]

[37]In the U.S., the volatility of the riskfree rate falls markedly before increasing slightly in the last years of our sample (see Table 3). Note that the initial fall remains if one considers the riskfree rate volatility in the eighties after the Volcker disinflation shock at the beginning of the sample.

6.2 Cross-sectional implications

We now assess the ability of the model at explaining heterogeneity in equity home bias and return correlations across G7 countries over the recent past.

Calibration. For each G7 country, we recalibrate the model to the fundamentals of the country versus the remaining G7 countries, as we did for the U.S.. We also measure the state variables (δ, ω) as observed in 2008 from the perspective of each country. We set $\phi = 5$ and we use the level of friction previously estimated for the U.S. in 2008, $\tau = 6.8\%$.

Home bias and stock return correlation across G7 countries. For each country, we compute the model predictions for the share of foreign equity in total equity holdings and for the level of stock return correlation with other G7 countries. We then compare these predictions to their empirical counterparts (Table 4).

[Table 4 here]

The model does fairly well at explaining the cross-section of equity portfolios for G7 countries, as depicted in Figure 7. Note that the only portfolio data used in the calibration are for the U.S.. The model explains 30% of the cross-sectional variance in the composition of equity portfolios for G7 countries. But, like for the U.S., even though model-predicted return correlations are always significantly above the fundamental ones,[38] the model fails to account for the very high level of stock market comovements observed in the decade prior to the financial crisis.

[Figure 7 here]

7 Conclusion

This paper employs a perturbation approach to provide a complete description of equilibrium asset prices and portfolio holdings in a dynamic model of imperfectly integrated stock markets. We analyze how frictions on cross-border equity holdings affect asset prices and portfolios, and characterize the effects of financial integration (i.e., a reduction in the size of the frictions). Our setup is very parsimonious, and yet able to account for various dimensions of the data and to shed light on different facets of financial integration. Frictions in international equity markets depress asset valuations. The predictions of the fully-integrated

[38] Estimated fundamental correlations range from 0.16 for Italy to 0.43 for Canada, with a mean of 0.26.

world CCAPM are modified and the impact of integration on the cost of capital depends on the respective size of opposing effects on the riskfree rate and on the risk premium. Integration has a second-order effect on asset return second moments, driven by the fact that frictions on international portfolio holdings impair risk sharing and make pricing kernels more volatile and less synchronized. Small frictions can generate large portfolio home bias when the endogenous asset return correlation is reasonably high. In the two-good extension of the model, terms of trade fluctuations can generate higher return correlation for a given level of fundamental correlation, leading to a more pronounced home bias for a given level of financial friction. In a quantitative assessment of the model using G7 data, we find that the model can generate a level of home bias similar to the data with fairly small frictions. The model calibrated to US data can also account for the cross-section of home bias in G7 countries. However, it falls short of explaining quantitatively asset return second moments. In particular, while our model predicts a rise in stock return correlation caused by financial integration, it cannot match the level of correlation observed prior to the financial crisis.

We maintain the assumption of log utility throughout the paper for tractability. Having power utility with relative risk aversion higher than one would likely affect our results in several ways. For a given level of financial integration and correlations, higher risk aversion would imply more diversification, thus reducing home bias. However, a fall in the elasticity of intertemporal substitution would generate an increase in stock return correlation, as pointed out by Dumas, Harvey and Ruiz (2003) in the context of frictionless markets. The latter effect would dampen the direct effect of higher risk aversion on the extent of portfolio diversification. Moreover, as assets become better substitutes, the model would generate less portfolio rebalancing, thus reducing the effect of financial frictions on return correlation — whereas one would rather like to strengthen their effect to better fit the data. One could increase the degree of risk aversion while maintaining a high elasticity of intertemporal substitution by assuming Epstein-Zin preferences. This extension of our model could potentially generate realistic market prices of risk, riskfree rate and portfolios, while leaving room for financial integration to affect significantly second moments. We leave the quantitative investigation of our mechanisms under more general preferences for further research.

TABLES

Table 1: Stock return correlation η_S (%) as a function of fundamental correlation η and τ.

	$\tau = 0$	$\tau = 0.025$	$\tau = 0.05$	$\tau = 0.1$
$\eta = 0$	11.8	11.7	11.3	9.7
$\eta = 0.3$	37.9	37.8	37.3	35.6
$\eta = 0.6$	63.5	63.3	62.8	61.0

Table 2: U.S. vs. rest of G7, baseline calibration.

Parameters					
μ	σ_H	σ_F	η	ϕ	ρ
0.022	0.083	0.081	0.23	5	0.03

State variables			
	1988	1998	2008
US output share (δ)	0.40	0.45	0.43
US consumption share (ω)	0.44	0.49	0.47

Table 3: US vs. rest of G7, results.

		1988	1998	2008
Data	Share of foreign equity $\left(\frac{\pi_{HF}}{\pi_{HH}+\pi_{HF}}\right)$	0.05	0.11	0.23
	Return correlation	0.64	0.77	0.85
	Return volatility (%)	14.8	10.8	17.8
	Riskless rate volatility (%)	2.75	1.11	1.64
$\phi = 5$	Implied level of τ (%)	10.5	9.1	6.8
	Return correlation	0.42	0.44	0.45
	Return volatility (%)	7.6	7.6	7.6
	Riskless rate volatility (%)	0.49	0.38	0.21
$\phi = 0.6$	Implied level of τ (%)	6.4	6.0	4.5
	Return correlation	0.59	0.58	0.59
	Return volatility (%)	7.9	7.6	7.7
	Riskless rate volatility (%)	0.32	0.29	0.16
$\phi = 2$	Implied level of τ (%)	4.2	3.7	2.8
	Return correlation	0.73	0.73	0.74
	Return volatility (%)	6.9	6.8	6.9
	Riskless rate volatility (%)	0.32	0.25	0.14
$\phi = 1,000$	Implied level of τ (%)	15.8	13.8	10.4
	Return correlation	0.22	0.24	0.26
	Return volatility (%)	8.3	8.2	8.1
	Riskless rate volatility (%)	0.59	0.47	0.27

Table 4: Cross-sectional results for G7 countries in 2008.

		Share of foreign equity	Stock return correlation
Canada	Data	0.20	0.86
	Model	0.18	0.55
France	Data	0.34	0.93
	Model	0.13	0.55
Germany	Data	0.47	0.91
	Model	0.49	0.42
Italy	Data	0.48	0.82
	Model	0.73	0.32
Japan	Data	0.26	0.61
	Model	0.49	0.36
United Kingdom	Data	0.46	0.89
	Model	0.27	0.40
United States	Data	0.23	0.85
	Model	0.23	0.45

FIGURES

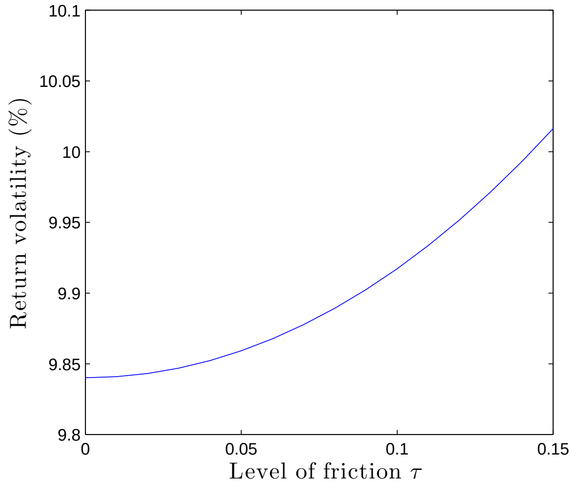

Figure 1: Stock return volatility as a function of τ when $\delta = \omega = 0.5$ (calibration : $\rho = 0.03$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H} = \sigma_{D_F} = 0.1$, $\eta = 0.5$).

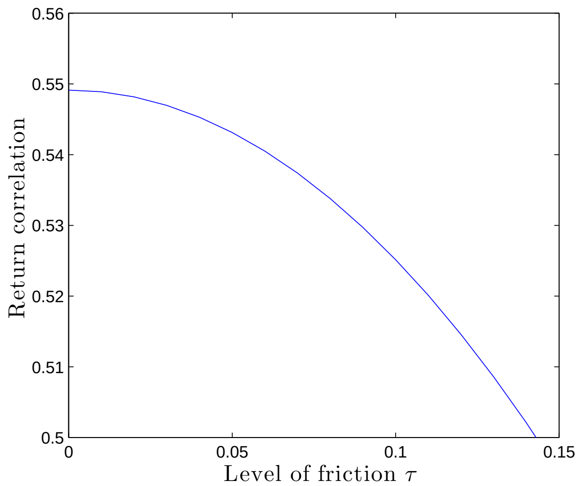

Figure 2: Stock return correlation as a function of τ when $\delta = \omega = 0.5$ (calibration : $\rho = 0.03$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H} = \sigma_{D_F} = 0.1$, $\eta = 0.5$).

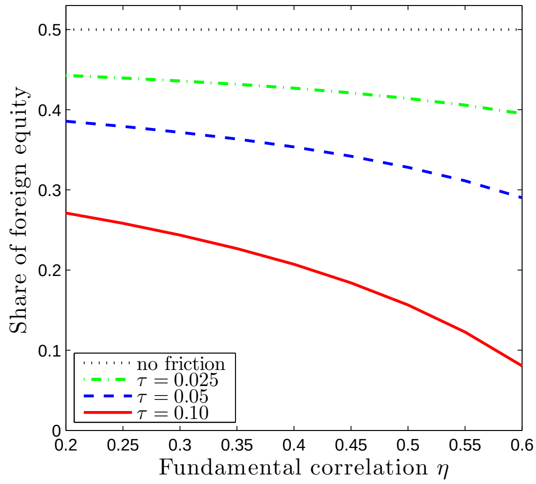

Figure 3: Share of foreign equities in total equity holdings as a function of fundamental correlation and τ, when $\delta = \omega = 0.5$ (calibration: $\rho = 0.03$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H} = \sigma_{D_F} = 0.1$, $\eta = 0.5$).

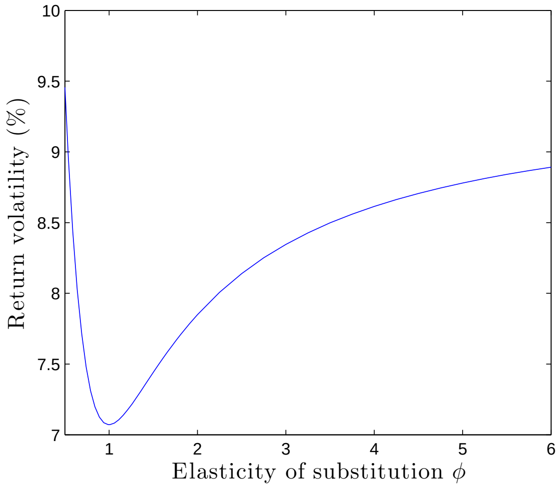

Figure 4: Stock return volatility as a function of the elasticity of substitution ϕ when $\tau = 0$ and $\delta = 0.5$ (calibration: $\rho = 0.03$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H} = \sigma_{D_F} = 0.1$, $\eta = 0$).

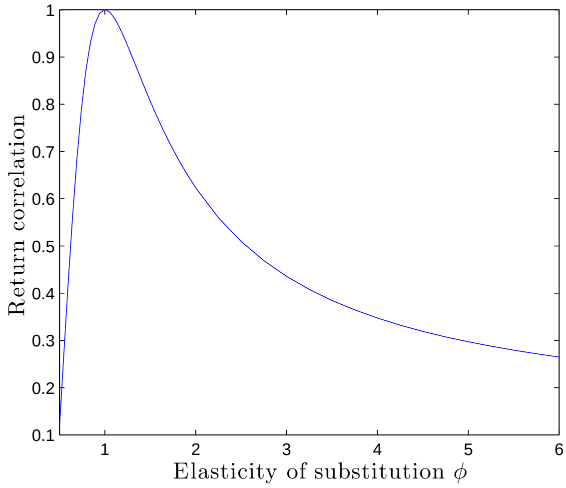

Figure 5: Stock return correlation as a function of the elasticity of substitution ϕ when $\tau = 0$ and $\delta = 0.5$ (calibration: $\rho = 0.03$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H} = \sigma_{D_F} = 0.1$, $\eta = 0$).

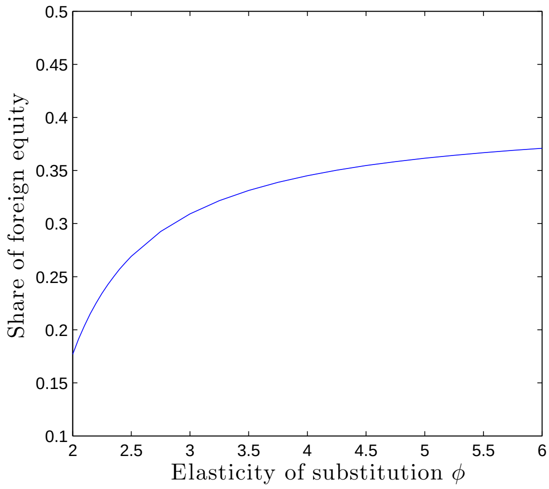

Figure 6: Portfolio share of foreign equities as a function of the elasticity of substitution ϕ, when $\delta = \omega = 0.5$ and $\tau = 0.05$ (calibration: $\rho = 0.03$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H} = \sigma_{D_F} = 0.1$, $\eta = 0$).

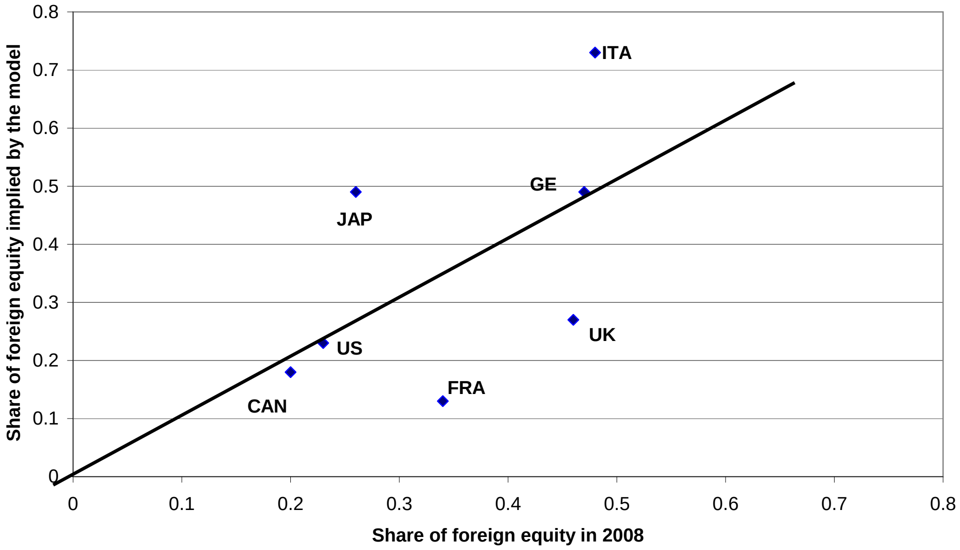

Figure 7: Home bias across G7 countries in 2008, predicted vs. observed.

APPENDICES

A Hypergeometric function

Cochrane et al. (2008) show that

$$\begin{aligned} s_{H0}(\delta) &\equiv \frac{1}{\delta}\mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\delta(s)ds \middle| \delta(t)=\delta\right] \\ &= \frac{1}{(1-\delta)\psi(1-\gamma)}\mathcal{F}\left(1, 1-\gamma; 2-\gamma; \frac{\delta}{\delta-1}\right) + \frac{1}{\delta\psi\theta}\mathcal{F}\left(1, \theta; 1+\theta; \frac{\delta-1}{\delta}\right), \end{aligned}$$

with $\mathcal{F}$ the standard (2,1)-hypergeometric function and

$$\begin{aligned} \psi &= \sqrt{\nu^2 + 2\rho\chi^2}, \\ \gamma &= \frac{\nu-\psi}{\chi^2}, \\ \theta &= \frac{\nu+\psi}{\chi^2}, \end{aligned}$$

where

$$\nu = \mu_{D_F} - \mu_{D_H} - \frac{\sigma^2_{D_{F,1}} + \sigma^2_{D_{F,2}}}{2} + \frac{\sigma^2_{D_{H,1}} + \sigma^2_{D_{H,2}}}{2}, \tag{A-1}$$

$$\chi^2 = \left(\sigma^2_{D_{H,1}} + \sigma^2_{D_{H,2}}\right) + \left(\sigma^2_{D_{F,1}} + \sigma^2_{D_{F,2}}\right) - 2(\sigma_{D_{H,1}}\sigma_{D_{F,1}} + \sigma_{D_{H,2}}\sigma_{D_{F,2}}). \tag{A-2}$$

In the same way

$$\begin{aligned} s_{F0}(\delta) &\equiv \frac{1}{1-\delta}\mathbb{E}\left[\int_0^\infty e^{-\rho(s-t)}\left(1-\delta(s)\right)ds \middle| \delta(t)=\delta\right] \\ &= \frac{1}{\delta\psi(1+\theta)}\mathcal{F}\left(1, 1+\theta; 2+\theta; \frac{\delta-1}{\delta}\right) - \frac{1}{(1-\delta)\psi\gamma}\mathcal{F}\left(1, -\gamma; 1-\gamma; \frac{\delta}{\delta-1}\right). \end{aligned}$$

In Appendix C, we refer to the functions $y_H(\delta) \equiv \delta s_{H0}(\delta)$ and $y_F(\delta) \equiv (1-\delta)s_{F0}(\delta)$. The Feynman-Kac theorem implies that

$$\rho y_H(\delta) = \delta + \delta\mu_\delta y'_H(\delta) + \frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2 y''_H(\delta), \tag{A-3}$$

$$\rho y_F(\delta) = (1-\delta) + \delta\mu_\delta y'_F(\delta) + \frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2 y''_F(\delta). \tag{A-4}$$

B Useful change of variable

In some of the derivations of Appendix C, we use the modified state variable $\lambda \equiv \omega^{-1} - 1$, which captures the relative consumption of country F, $\lambda(t) = \frac{c_F(t)}{c_H(t)}$. From (20), we can write $\lambda(t) = \frac{\Psi_H \xi_H(t)}{\Psi_F \xi_F(t)}$. The dynamics of λ are given by

$$\frac{d\lambda(t)}{\lambda(t)} = \mu_\lambda(t)dt + \boldsymbol{\sigma}_\lambda(t).d\boldsymbol{W}(t), \tag{B-1}$$

where

$$\mu_\lambda = \boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_D + \frac{1}{1+\lambda}|\boldsymbol{\sigma}_\lambda|^2, \tag{B-2}$$

$$\boldsymbol{\sigma}_\lambda = \tau\boldsymbol{\Gamma}. \tag{B-3}$$

Given a function $g(\delta,\omega)$ on $(0,1)^2$, we let $\tilde{g}(\delta,\lambda)$ denote the function on $(0,1)\times\mathbb{R}^+$ defined by

$$\tilde{g}(\delta,\lambda) = g(\delta,\omega(\lambda)),$$

where $\omega(\lambda) = (1+\lambda)^{-1}$. In the same way, given a function $\tilde{g}(\delta,\lambda)$, we can define $g(\delta,\omega) = \tilde{g}(\delta,\lambda(\omega))$. In particular, given the functions h and f defined in Proposition 1, we have

$$\tilde{h}(\delta,\lambda) \equiv \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\left[1+\lambda(s)\right]\delta(s)ds \,\middle|\, \delta(t)=\delta, \lambda(t)=\lambda\right], \tag{B-4}$$

$$\tilde{f}(\delta,\lambda) \equiv \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\left(1+\frac{1}{\lambda(s)}\right)(1-\delta(s))ds \,\middle|\, \delta(t)=\delta, \lambda(t)=\lambda\right]. \tag{B-5}$$

The Feynman-Kac theorem implies that $\tilde{h}$ and $\tilde{f}$ are solutions to the following PDE's

$$\rho\tilde{h} = (1+\lambda)\,\delta + \delta\mu_\delta\tilde{h}_\delta + \lambda\mu_\lambda\tilde{h}_\lambda + \tfrac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2\tilde{h}_{\delta\delta} + \tfrac{1}{2}\lambda^2|\boldsymbol{\sigma}_\lambda|^2\tilde{h}_{\lambda\lambda} + \delta\lambda(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\lambda)\tilde{h}_{\delta\lambda}, \tag{B-6}$$

$$\rho\tilde{f} = \tfrac{1+\lambda}{\lambda}(1-\delta) + \delta\mu_\delta\tilde{f}_\delta + \lambda\mu_\lambda\tilde{f}_\lambda + \tfrac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2\tilde{f}_{\delta\delta} + \tfrac{1}{2}\lambda^2|\boldsymbol{\sigma}_\lambda|^2\tilde{f}_{\lambda\lambda} + \delta\lambda(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\lambda)\tilde{f}_{\delta\lambda}, \tag{B-7}$$

C Proofs

Proof of Lemma 1: Using the stochastic discount factor of the representative agent of country i to price the stock market of country i for $i = H, F$, we obtain:

$$\begin{aligned} S_H(t) &= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{c_H(t)}{c_H(s)}D_H(s)ds\right] = \omega(t)D(t)\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{\delta(s)}{\omega(s)}ds\right], \\ S_F(t) &= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{c_F(t)}{c_F(s)}D_F(s)ds\right] = (1-\omega(t))D(t)\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{1-\delta(s)}{1-\omega(s)}ds\right]. \end{aligned}$$

The expressions for price-dividend ratios follow immediately. These can be expressed as functions of $\delta(t)$ and $\omega(t)$ since δ and ω are jointly Markov. ∎

Proof of Lemma 2: The outline of the proof is as follows: start from first-order condition (19) and apply Itô's lemma to identify individual consumption volatility. Then use market clearing for goods to identify aggregate consumption volatility and derive an equilibrium condition on the market prices of risk.

Applying Itô's lemma on both sides of (19) implies

$$-\rho e^{-\rho t}\frac{1}{c_i(t)}dt - e^{-\rho t}\frac{1}{(c_i(t))^2}dc_i + e^{-\rho t}\frac{1}{(c_i(t))^3}(dc_i)^2 = -\Psi_i\xi_i(t)[r(t)dt + \boldsymbol{\theta}_i.d\boldsymbol{W}(t)]. \quad \text{(C-1)}$$

We define μ_{c_i} and $\boldsymbol{\sigma}_{c_i}$ such that

$$\frac{dc_i(t)}{c_i(t)} = \mu_{c_i}(t)dt + \boldsymbol{\sigma}_{c_i}(t).d\boldsymbol{W}(t), \qquad i = H, F. \quad \text{(C-2)}$$

Identifying diffusion terms in (C-1) implies that

$$\boldsymbol{\sigma}_{c_i}(t) = \boldsymbol{\theta}_i(t), \qquad i = H, F. \quad \text{(C-3)}$$

The market clearing condition $c_H + c_F = D$ then implies

$$c_H(t)\boldsymbol{\theta}_H(t) + c_F(t)\boldsymbol{\theta}_F(t) = c_H(t)\boldsymbol{\sigma}_{c_H}(t) + c_F(t)\boldsymbol{\sigma}_{c_F}(t) = D(t)\boldsymbol{\sigma}_D(t). \quad \text{(C-4)}$$

Solving Eqs. (16) and (C-4) for $\boldsymbol{\theta}_H$ and $\boldsymbol{\theta}_F$, we obtain the expressions given in Lemma 2. ∎

Proof of Proposition 1: By definition,

$$\begin{aligned} h(\delta(t),\omega(t)) &= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{\delta(s)}{\omega(s)}ds\right], & \text{(C-5)} \\ f(\delta(t),\omega(t)) &= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{1-\delta(s)}{1-\omega(s)}ds\right]. & \text{(C-6)} \end{aligned}$$

A straightforward application of the Feynman-Kac Theorem implies that functions h and f satisfy the PDEs (30) and (31). We now express $\boldsymbol{\sigma}_\omega$ in terms of δ, ω, h, f and their partial derivatives. Using the definition of $\mathbf{\Gamma}$ in (17), the expression for $\boldsymbol{\sigma}_\omega$ in (29) implies

$$\mathbf{\Sigma}\boldsymbol{\sigma}_\omega = -\tau(1-\omega)\begin{pmatrix} -\frac{1}{s_H} & \frac{1}{s_F} \end{pmatrix}^\top. \tag{C-7}$$

Recall that $\mathbf{\Sigma} = [\boldsymbol{\sigma}_H \ \boldsymbol{\sigma}_F]^\top$. Applying Itô's Lemma to $S_i = D_i s_i(\delta, \omega)$ gives

$$\boldsymbol{\sigma}_i = \boldsymbol{\sigma}_{D_i} + \frac{\delta}{s_i}\frac{\partial s_i}{\partial \delta}\boldsymbol{\sigma}_\delta + \frac{\omega}{s_i}\frac{\partial s_i}{\partial \omega}\boldsymbol{\sigma}_\omega, \qquad i = H, F. \tag{C-8}$$

Hence, (C-7) implies

$$s_H\boldsymbol{\sigma}_{D_H}.\boldsymbol{\sigma}_\omega + \delta\frac{\partial s_H}{\partial \delta}\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\omega + \omega\frac{\partial s_H}{\partial \omega}|\boldsymbol{\sigma}_\omega|^2 \quad = \quad \tau(1-\omega), \tag{C-9}$$

$$s_F\boldsymbol{\sigma}_{D_F}.\boldsymbol{\sigma}_\omega + \delta\frac{\partial s_F}{\partial \delta}\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\omega + \omega\frac{\partial s_F}{\partial \omega}|\boldsymbol{\sigma}_\omega|^2 \quad = \quad -\tau(1-\omega). \tag{C-10}$$

Combining (C-9) and (C-10), it follows that

$$\boldsymbol{\nu}.\boldsymbol{\sigma}_\omega = \tau(1-\omega)\frac{\partial(s_H + s_F)}{\partial \omega}, \tag{C-11}$$

where

$$\boldsymbol{\nu} \equiv \frac{\partial s_F}{\partial \omega}\left(s_H\boldsymbol{\sigma}_{D_H} + \delta\frac{\partial s_H}{\partial \delta}\boldsymbol{\sigma}_\delta\right) - \frac{\partial s_H}{\partial \omega}\left(s_F\boldsymbol{\sigma}_{D_F} + \delta\frac{\partial s_F}{\partial \delta}\boldsymbol{\sigma}_\delta\right) = [\nu_1 \quad \nu_2]^\top. \tag{C-12}$$

From (C-11), we solve for $\sigma_{\omega,2}$ as a function of $\sigma_{\omega,1}$

$$\sigma_{\omega,2} = A + B\sigma_{\omega,1}, \tag{C-13}$$

where

$$A \quad = \quad \tau\frac{(1-\omega)}{\nu_2}\frac{\partial(s_H + s_F)}{\partial \omega}, \tag{C-14}$$

$$B \quad = \quad -\nu_1/\nu_2. \tag{C-15}$$

Substituting (C-13) into (C-9) gives

$$\alpha\sigma_{\omega,1}^2 + \beta\sigma_{\omega,1} + \gamma = 0, \tag{C-16}$$

where

$$\alpha = \omega\frac{\partial s_H}{\partial\omega}(1+B^2) \tag{C-17}$$

$$\beta = s_H\sigma_{D_H,1} + B\sigma_{D_H,2} + 2\omega\frac{\partial s_H}{\partial\omega}AB + \delta\frac{\partial s_H}{\partial\delta}(\sigma_{\delta,1} + B\sigma_{\delta,2}), \tag{C-18}$$

$$\gamma = s_H\sigma_{D_H,2}A + \delta\frac{\partial s_H}{\partial\delta}A\sigma_{\delta,2} + \omega\frac{\partial s_H}{\partial\omega}A^2 - \tau(1-\omega). \tag{C-19}$$

Therefore

$$\sigma_{\omega,1} = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}.$$

Since $\lim_{\tau\to 0}\gamma = 0$, we have $\lim_{\tau\to 0}\sigma_{\omega,1} = -\frac{\beta}{\alpha}$ or 0. To ensure that $\lim_{\tau\to 0}\sigma_{\omega,1} = 0$, we take the positive root, and so

$$\sigma_{\omega,1} = \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}. \tag{C-20}$$

Given the relationship between s_H (resp. s_F) and h (resp. f), we therefore have expressed $\boldsymbol{\sigma}_\omega$ as a function of δ, ω, and functions h and f and their partial derivatives. The expression for μ_ω follows from (28).

We now derive the boundary conditions for h and f. Observe that when $\omega = 0$, prices are determined by Agent F, and so $\lim_{\omega\to 0} s_H = (1-\tau)s_{H0}$ and $\lim_{\omega\to 0} s_F = s_{F0}$, where s_{H0} and s_{F0} are the price-dividend ratios in countries H and F when $\tau = 0$ (see Section 3.1). Similarly, when $\omega = 1$, prices are determined by Agent H, and so $\lim_{\omega\to 1} s_H = s_{H0}$ and $\lim_{\omega\to 1} s_F = (1-\tau)s_{F0}$.

The boundary at $\delta = 0$ is absorbing. Therefore, when $\delta = 0$, s_F is the same as in the one-tree model analyzed in Appendix D, where the only dividend is the foreign country dividend, and Agent H (resp. F) receives the dividend flows $(1-\tau)D_F$ (resp D_F). Therefore, $\lim_{\delta\to 0} s_F$ can be obtained by considering the one-tree limit. We cannot pin down $\lim_{\delta\to 0} s_H$. However, we do know that $\lim_{\delta\to 0} h = 0$. Similarly, $\lim_{\delta\to 1} s_H$ can be obtained by considering the one-tree limit, and $\lim_{\delta\to 1} f = 0$.

To summarize, we have

$$\begin{aligned}
\lim_{\omega\to 0} s_H(\delta,\omega) &= (1-\tau)s_{H0}(\delta),\\
\lim_{\omega\to 1} s_H(\delta,\omega) &= s_{H0}(\delta),\\
\lim_{\delta\to 0} h(\delta,\omega) &= 0,\\
\lim_{\delta\to 1} s_H(\delta,\omega) &= s(\omega),
\end{aligned}$$

where $s(.)$ is the valuation function in the one-tree model introduced in Proposition D-1, and

$$\begin{aligned}\lim_{\omega\to 0} s_F(\delta,\omega) &= s_{F0}(\delta),\\ \lim_{\omega\to 1} s_F(\delta,\omega) &= (1-\tau)s_{F0}(\delta),\\ \lim_{\delta\to 0} s_F(\delta,\omega) &= s(1-\omega),\\ \lim_{\delta\to 1} f(\delta,\omega) &= 0,\end{aligned}$$

Since $h = \frac{\delta}{\omega}s_H$, and $f = \frac{1-\delta}{1-\omega}s_F$, we thus obtain

$$\lim_{\omega\to 0} h(\delta,\omega) = \lim_{\omega\to 0}\frac{\delta}{\omega}(1-\tau)s_{H0}(\delta) = \infty \tag{C-21}$$

$$\lim_{\omega\to 1} h(\delta,\omega) = \delta s_{H0}(\delta), \tag{C-22}$$

$$\lim_{\delta\to 0} h(\delta,\omega) = 0, \tag{C-23}$$

$$\lim_{\delta\to 1} h(\delta,\omega) = \frac{1}{\omega}s(\omega), \tag{C-24}$$

and

$$\lim_{\omega\to 0} f(\delta,\omega) = (1-\delta)s_{F0}(\delta), \tag{C-25}$$

$$\lim_{\omega\to 1} f(\delta,\omega) = \lim_{\omega\to 1}\frac{1-\delta}{1-\omega}(1-\tau)s_{F0}(\delta) = \infty, \tag{C-26}$$

$$\lim_{\delta\to 0} f(\delta,\omega) = \frac{1}{1-\omega}s(1-\omega), \tag{C-27}$$

$$\lim_{\delta\to 1} f(\delta,\omega) = 0. \tag{C-28}$$

∎

Proof of Proposition 2: We start with the derivation of first-order approximations for the functions $\tilde{h}$ and $\tilde{f}$ introduced in Appendix B, using the modified state variable λ. For $s > t$, we can write

$$\lambda(s) = \lambda(t)\exp\left\{\int_t^s \left[\mu_\lambda(u) - \frac{1}{2}|\boldsymbol{\sigma}_\lambda(u)|^2\right]du + \int_t^s \boldsymbol{\sigma}_\lambda(u).d\boldsymbol{W}(u)\right\}. \tag{C-29}$$

Given (B-2) and (B-3), first-order approximations for μ_λ and $\boldsymbol{\sigma}_\lambda$ are

$$\mu_\lambda = \tau\boldsymbol{\Gamma}_0(\delta).\boldsymbol{\sigma}_D(\delta) + o(\tau), \tag{C-30}$$

$$\boldsymbol{\sigma}_\lambda = \tau\boldsymbol{\Gamma}_0(\delta) + o(\tau). \tag{C-31}$$

We now simplify the first-order approximation for μ_λ. Using the definition of $\mathbf{\Gamma}_0$, we have

$$\mathbf{\Gamma}_0.\boldsymbol{\sigma}_D = \left[-\frac{1}{s_{H0}} \quad \frac{1}{s_{F0}}\right]\left(\mathbf{\Sigma}_0^\top\right)^{-1}\boldsymbol{\sigma}_D.$$

The vector $\left(\mathbf{\Sigma}_0^\top\right)^{-1}\boldsymbol{\sigma}_D$ corresponds to the composition of the world market portfolio in a frictionless economy. Using the fact that $\delta s_{H0} + (1-\delta)s_{F0} = 1/\rho$, it is easy to show that

$$\left(\mathbf{\Sigma}_0^\top\right)^{-1}\boldsymbol{\sigma}_D = [\rho\delta s_{H0} \quad \rho(1-\delta)s_{F0}]^\top.$$

It is then immediate that $\mathbf{\Gamma}_0.\boldsymbol{\sigma}_D = \rho(1-2\delta)$. Hence we can write $\mu_\lambda = \tau g(\delta) + o(\tau)$, where $g(\delta) = \rho(1-2\delta)$. Plugging first-order approximations for μ_λ and $\boldsymbol{\sigma}_\lambda$ in (C-29), we obtain

$$\begin{aligned}\lambda(s) &= \lambda(t)\exp\left\{\tau\left[\int_t^s g(\delta_u)du + \int_t^s \mathbf{\Gamma}_0(\delta_u).d\boldsymbol{W}_u\right] + o(\tau)\right\} \\ &= \lambda(t)\left[1 + \tau\int_t^s g(\delta_u)du + \tau\int_t^s \mathbf{\Gamma}_0(\delta_u).d\boldsymbol{W}_u\right] + o(\tau).\end{aligned}$$

Using the definition of $\tilde{h}$ in (B-4), we can write

$$\begin{aligned}\tilde{h}(\delta,\lambda) &= \mathbb{E}\left\{\int_t^\infty e^{-\rho(s-t)}\left[1 + \lambda_t + \tau\lambda_t\int_t^s g(\delta_u)du + \tau\lambda_t\int_t^s \mathbf{\Gamma}(\delta_u).d\boldsymbol{W}_u + o(\tau)\right]\delta_s ds\middle|\,\delta_t = \delta, \lambda_t = \lambda\right\} \\ &= (1+\lambda)y_H(\delta) - \tau\lambda H(\delta) + o(\tau), \qquad \text{(C-32)}\end{aligned}$$

where we used the definition of y_H introduced in Appendix A and

$$H(\delta_t) \equiv -\mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\left(\int_t^s g(\delta_u)du + \int_t^s \mathbf{\Gamma}_0(\delta_u).d\boldsymbol{W}_u\right)\delta_s ds\middle|\,\delta_t = \delta\right]. \qquad \text{(C-33)}$$

Therefore, we can write

$$S_H = D\frac{1}{1+\lambda}\tilde{h}(\delta,\lambda) = D\left[y_H(\delta) - \tau\frac{\lambda}{1+\lambda}H(\delta)\right] + o(\tau).$$

In the same way, we show that we can write

$$S_F = D\frac{\lambda}{1+\lambda}\tilde{f}(\delta,\lambda) = D\left[y_F(\delta) - \tau\frac{1}{1+\lambda}F(\delta)\right] + o(\tau).$$

Lemma C-1. *The functions H and F must satisfy the following boundary value problems*

$$\begin{aligned}\rho H - \delta\mu_\delta H' - \frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2 H'' &= \delta \\ H(0) &= 0 \\ H(1) &= 1/\rho \\ \rho F - \delta\mu_\delta F' - \frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2 F'' &= 1-\delta \\ F(0) &= 1/\rho \\ F(1) &= 0.\end{aligned}$$

Proof: We rewrite (B-6) using the expression for $\tilde{h}$ given in (C-32). Then by subtracting (A-3), we obtain:

$$\rho H(\delta) - \delta\mu_\delta H'(\delta) - \frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2 H''(\delta) = -\rho(1-2\delta)y_H(\delta) - \delta\left(\boldsymbol{\sigma}_\delta(\delta).\boldsymbol{\Gamma}_0(\delta)\right)y_H'(\delta).$$

The first boundary condition follows from the fact that given the nature of the dividend process, S_H needs to go to zero as δ goes to zero. The second boundary condition follows from the fact that as δ goes to 1 and λ goes to infinity, S_H should go to $(1-\tau)D/\rho$. Indeed in that limit, the economy tends to an economy with one tree only ($D = D_H$) and one investor located in the foreign country, thus facing an after-tax dividend stream $(1-\tau)D$. In the same way, we show that the function F entering in the valuation of the foreign tree satisfies the following ODE

$$\rho F - \delta\mu_\delta F' - \frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2 F'' = \rho(1-2\delta)y_F + \delta\left(\boldsymbol{\sigma}_\delta.\boldsymbol{\Gamma}_0\right)y_F',$$

with analogous boundary conditions.

We can simplify the non-homogeneous term in the ODE for H as follows

$$\begin{aligned}-\rho(1-2\delta)y_H - \delta\left(\boldsymbol{\sigma}_\delta.\boldsymbol{\Gamma}_0\right)y_H' &= -(\boldsymbol{\Gamma}_0.\boldsymbol{\sigma}_D)y_H - \delta\left(\boldsymbol{\sigma}_\delta.\boldsymbol{\Gamma}_0\right)y_H' \\ &= -y_H\boldsymbol{\Gamma}_0.\left(\boldsymbol{\sigma}_D + \frac{\delta}{y_H}y_H'\boldsymbol{\sigma}_\delta\right) \\ &= -y_H\boldsymbol{\Gamma}_0.\boldsymbol{\sigma}_{H0} \\ &= -y_H\boldsymbol{\sigma}_{H0}^\top\boldsymbol{\Sigma}_0^{-1}\begin{pmatrix}-\frac{1}{s_{H0}} & \frac{1}{s_{F0}}\end{pmatrix}^\top \\ &= \delta,\end{aligned}$$

where the final result obtains by noting that $\boldsymbol{\sigma}_{H0}^\top\boldsymbol{\Sigma}_0^{-1} = (1 \quad 0)$ and using the fact that $y_H = \delta s_{H0}$. We prove in the same way that $\rho(1-2\delta)y_F + \delta\left(\boldsymbol{\sigma}_\delta.\ \boldsymbol{\Gamma}_0\right)y_F' = 1-\delta$. □

It is immediate that functions y_H and y_F are solutions to the boundary value problems stated in Lemma C-1. Therefore,

$$
\begin{aligned}
S_H &= D\left[1-\tau\frac{\lambda}{1+\lambda}\right]y_H(\delta)+o(\tau),\\
S_F &= D\left[1-\tau\frac{1}{1+\lambda}\right]y_F(\delta)+o(\tau).
\end{aligned}
$$

First-order approximations for the pricing functions follow by a simple change of variable. We get

$$
\begin{aligned}
s_H(\delta,\omega;\tau) &= [1-\tau(1-\omega)]\,s_{H0}(\delta)+o(\tau), && \text{(C-34)}\\
s_F(\delta,\omega;\tau) &= [1-\tau\omega]\,s_{F0}(\delta)+o(\tau). && \text{(C-35)}
\end{aligned}
$$

In order to obtain the second-order effect of τ on pricing functions, we now derive the second-order approximation for $\boldsymbol{\sigma}_\lambda$. Eqs. (B-3) and (17) imply that

$$
\boldsymbol{\sigma}_\lambda = \tau\boldsymbol{\Sigma}^{-1}\begin{pmatrix}-\frac{1}{s_H} & \frac{1}{s_F}\end{pmatrix}^\top .
$$

The first-order approximations for price-dividend ratios s_H and s_F, (C-34) and (C-35), translate into the following approximations for dividend yields

$$
\begin{aligned}
\frac{1}{s_H} &= \left[1+\tau\frac{\lambda}{1+\lambda}\right]\frac{1}{s_{H0}}+o(\tau),\\
\frac{1}{s_F} &= \left[1+\tau\frac{1}{1+\lambda}\right]\frac{1}{s_{F0}}+o(\tau).
\end{aligned}
$$

It follows that

$$
\boldsymbol{\sigma}_\lambda = \tau\boldsymbol{\Gamma}_0+\tau^2\boldsymbol{\Lambda}\boldsymbol{\Gamma}_0+o(\tau^2), \tag{C-36}
$$

where $\boldsymbol{\Lambda}$ denotes a diagonal matrix with coefficients $\lambda/(1+\lambda)$ and $1/(1+\lambda)$. Then the second-order approximation for μ_λ follows from (B-3):

$$
\begin{aligned}
\mu_\lambda &= \tau\boldsymbol{\Gamma}_0.\boldsymbol{\sigma}_D+\tau^2\left(\boldsymbol{\Lambda}\boldsymbol{\Gamma}_0\right).\boldsymbol{\sigma}_D+\tau^2\frac{1}{1+\lambda}|\boldsymbol{\Gamma}_0|^2+o(\tau^2)\\
&= \tau\rho(1-2\delta)+\tau^2\left(\frac{\rho}{1+\lambda}-\rho\delta+\frac{1}{1+\lambda}|\boldsymbol{\Gamma}_0|^2\right)+o(\tau^2) && \text{(C-37)}
\end{aligned}
$$

where the last equation obtains by substituting $\boldsymbol{\Gamma}_0.\boldsymbol{\sigma}_D=\rho(1-2\delta)$ and $\boldsymbol{\Gamma}_0^\top\boldsymbol{\Lambda}\boldsymbol{\sigma}_D=\frac{\rho}{1+\lambda}-\rho\delta$.

Along the same logic as in the derivation of (C-32), we can use (C-36) and (C-37) to write the second-order approximation of $\tilde{h}$ in the following form, for some function Φ_H to be determined:

$$
\tilde{h}(\delta,\lambda;\tau)=(1+\lambda)\,y_H(\delta)-\tau\lambda y_H(\delta)+\tau^2\lambda\Phi_H(\delta,\lambda)+o(\tau^2). \tag{C-38}
$$

We now substitute (C-38) in (B-6). Equalizing terms in τ^2 gives the following PDE for Φ_H

$$\rho\Phi_H - \delta\mu_\delta \frac{\partial \Phi_H}{\partial \delta} - \frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2 \frac{\partial^2 \Phi_H}{\partial \delta^2} = \frac{1}{1+\lambda}\delta + \frac{1}{1+\lambda}|\boldsymbol{\Gamma}_0|^2 y_H, \tag{C-39}$$

where the non-homogeneous term on the right-hand side of (C-39) follows from the fact that

$$\begin{aligned} -\rho(1-2\delta)y_H - \delta(\boldsymbol{\sigma}_\delta.\boldsymbol{\Gamma}_0)y'_H &= \delta, \\ \left(\frac{\rho}{1+\lambda} - \rho\delta\right) y_H + \delta(\boldsymbol{\Gamma}_0^\top \boldsymbol{\Lambda}\boldsymbol{\sigma}_\delta)y'_H &= -\frac{\lambda}{1+\lambda}\delta. \end{aligned}$$

(C-39) and (A-3) together imply that there exists a function φ_H such that $\Phi_H(\delta,\lambda) = \frac{1}{1+\lambda}\left[y_H(\delta) + \varphi_H(\delta)\right]$, where φ_H satisfies

$$\rho\varphi_H - \delta\mu_\delta\varphi'_H - \frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2\varphi''_H = |\boldsymbol{\Gamma}_0|^2 y_H.$$

Respectively for $\tilde{f}$, we can show that

$$\tilde{f}(\delta,\lambda;\tau) = \frac{1+\lambda}{\lambda}y_F(\delta) - \frac{\tau}{\lambda}y_F(\delta) + \frac{\tau^2}{\lambda}\Phi_F(\delta,\lambda) + o(\tau^2), \tag{C-40}$$

with $\Phi_F(\delta,\lambda) = \frac{\lambda}{1+\lambda}\left[y_F(\delta) + \varphi_F(\delta)\right]$, where φ_F is solution to the ODE

$$\rho\varphi_F - \delta\mu_\delta\varphi'_F - \frac{1}{2}\delta^2|\boldsymbol{\sigma}_\delta|^2\varphi''_F = |\boldsymbol{\Gamma}_0|^2 y_F.$$

At this stage, we can therefore write

$$S_H = D\left[y_H(\delta)\left(1 - \tau\frac{\lambda}{1+\lambda} + \tau^2\frac{\lambda}{(1+\lambda)^2}\right) + \tau^2\frac{\lambda}{(1+\lambda)^2}\varphi_H(\delta)\right] + o(\tau^2), \tag{C-41}$$

$$S_F = D\left[y_F(\delta)\left(1 - \tau\frac{1}{1+\lambda} + \tau^2\frac{\lambda}{(1+\lambda)^2}\right) + \tau^2\frac{\lambda}{(1+\lambda)^2}\varphi_F(\delta)\right] + o(\tau^2). \tag{C-42}$$

We now turn to the determination of boundary conditions for φ_H and φ_F. The conditions $\varphi_H(0) = \varphi_F(1) = 0$ are required since the price of assets yielding zero payoff must be zero. The derivation of the other two boundary conditions, for $\varphi_H(1)$ and $\varphi_F(0)$, is more subtle. When δ goes to 1, S_H tends to

$$\frac{D}{1+\lambda}\mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\left[1+\lambda_s\right]ds \middle| \lambda_t = \lambda\right].$$

Let $V(\lambda) \equiv \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\left[1+\lambda_s\right]ds\,|\lambda_t = \lambda\right]$, so that $\lim_{\delta\to 1} S_H = \frac{D}{1+\lambda}V(\lambda)$. Applying the Feynman-Kac formula to V gives

$$\rho V = (1+\lambda) + \lambda\bar{\mu}_\lambda V' + \frac{1}{2}\lambda^2|\bar{\boldsymbol{\sigma}}_\lambda|^2 V'', \tag{C-43}$$

where

$$\bar{\mu}_\lambda = \lim_{\delta\to 1} \mu_\lambda = -\tau\rho + \tau^2\left[-\frac{\lambda\rho}{1+\lambda} + \frac{1}{1+\lambda}|\mathbf{\Gamma}_0(1)|^2\right],$$
$$\bar{\boldsymbol{\sigma}}_\lambda = \lim_{\delta\to 1} \boldsymbol{\sigma}_\lambda = \tau\mathbf{\Gamma}_0(1) + \tau^2\mathbf{\Lambda}\mathbf{\Gamma}_0(1).$$

We know that up to the second order in τ:

$$\tilde{h}(\delta,\lambda) = (1+\lambda)\, y_H(\delta) - \tau\lambda y_H(\delta) + \tau^2\frac{\lambda}{1+\lambda}\left[y_H(\delta) + \varphi_H(\delta)\right]. \tag{C-44}$$

Taking the limit as δ goes to 1, we get

$$\lim_{\delta\to 1}\tilde{h}(\delta,\lambda) = V(\lambda) = \frac{1}{\rho}\left[1+\lambda-\tau\lambda+\tau^2\frac{\lambda}{(1+\lambda)} + \tau^2\frac{\lambda}{(1+\lambda)}\rho\varphi_H(1)\right].$$

From this, we can compute $V'(\lambda)$ and $V''(\lambda)$ and plug the expressions for V and its derivatives in (C-43). Then, identifying terms in τ^2, we get:

$$\varphi_H(1) = \frac{1}{\rho^2}|\mathbf{\Gamma}_0(1)|^2.$$

In the same way, we obtain the boundary condition $\varphi_F(0) = \frac{1}{\rho} + \frac{1}{\rho^2}|\mathbf{\Gamma}_0(0)|^2$. The expressions for s_H and s_F in Proposition 2 follow from (C-41) and (C-42) by a simple change of variable. ∎

Proposition C-1. *Second-order approximations of asset prices diffusion coefficients $\boldsymbol{\sigma}_H$ and $\boldsymbol{\sigma}_F$ are:*

$$\boldsymbol{\sigma}_H(\delta,\omega) = \boldsymbol{\sigma}_{H0}(\delta) + \tau^2\omega(1-\omega)\left\{-\mathbf{\Gamma}_0(\delta) + \left[-\left(\frac{1-\omega}{\omega} + \frac{\varphi_H}{\delta s_{H0}}\right)\frac{\partial s_{H0}}{\partial\delta} + \frac{\delta\varphi_H' + \varphi_H}{\delta^2}\right]\frac{\delta}{s_{H0}(\delta)}\boldsymbol{\sigma}_\delta(\delta)\right\} + \boldsymbol{o}(\tau^2),$$
$$\boldsymbol{\sigma}_F(\delta,\omega) = \boldsymbol{\sigma}_{F0}(\delta) + \tau^2\omega(1-\omega)\left\{\mathbf{\Gamma}_0(\delta) + \left[-\left(\frac{\omega}{1-\omega} + \frac{\varphi_F}{(1-\delta)s_{F0}}\right)\frac{\partial s_{F0}}{\partial\delta} + \frac{(1-\delta)\varphi_F' + \varphi_F}{(1-\delta)^2}\right]\frac{\delta}{s_{F0}(\delta)}\boldsymbol{\sigma}_\delta(\delta)\right\} + \boldsymbol{o}(\tau^2).$$

Proof: To obtain $\boldsymbol{\sigma}_H$, we start from (C-8) and use the following approximations derived from (35):

$$\frac{1}{s_H(\delta,\omega)} = \frac{1}{s_{H0}(\delta)}\left[1+\tau(1-\omega) - \tau^2\omega(1-\omega)\left(1 + \frac{\varphi_H(\delta)}{\delta s_{H0}(\delta)}\right)\right] + o(\tau^2),$$
$$\frac{\partial s_H}{\partial\delta}(\delta,\omega) = \left[1-\tau(1-\omega)+\tau^2\omega(1-\omega)\right]\frac{\partial s_{H0}(\delta)}{\partial\delta} + \tau^2\omega(1-\omega)\frac{\delta\varphi_H'(\delta) - \varphi_H(\delta)}{\delta^2} + o(\tau^2),$$
$$\frac{\partial s_H}{\partial\omega}(\delta,\omega) = [\tau+\tau^2(1-2\omega)]s_{H0}(\delta) + \tau^2(1-2\omega)\frac{\varphi_H(\delta)}{\delta} + o(\tau^2).$$

The second-order approximation for $\boldsymbol{\sigma}_F$ can be derived in the same way. ∎

Proof of Proposition 3: Let $\boldsymbol{\mu}_i - \boldsymbol{r}$ denote the vector of after-tax expected returns from the perspective of investor i. By definition of $\boldsymbol{\theta}_i$ in (15), we have $\boldsymbol{\mu}_i - \boldsymbol{r} = \boldsymbol{\Sigma}\boldsymbol{\theta}_i$. Then, Lemma 2 implies

$$\begin{aligned}
\boldsymbol{\mu}_H - \boldsymbol{r} &= \boldsymbol{\Sigma}\boldsymbol{\sigma}_D + \tau(1-\omega)\begin{pmatrix} \frac{D_H}{S_H}, \\ -\frac{D_F}{S_F} \end{pmatrix} \\
\boldsymbol{\mu}_F - \boldsymbol{r} &= \boldsymbol{\Sigma}\boldsymbol{\sigma}_D + \tau\omega\begin{pmatrix} -\frac{D_H}{S_H} \\ \frac{D_F}{S_F} \end{pmatrix}.
\end{aligned}$$

The before-tax risk premia are given by the upper element of $\boldsymbol{\mu}_H - \boldsymbol{r}$ and by the lower element of $\boldsymbol{\mu}_F - \boldsymbol{r}$. First-order Taylor expansions (39)-(40) follow immediately. Second-order expansions are given by

$$\mu_H^e(\delta,\omega) = \boldsymbol{\sigma}_H^{(2)}(\delta,\omega).\boldsymbol{\sigma}_D(\delta) + \tau(1-\omega)\frac{1}{s_H^{(1)}(\delta,\omega)} + o(\tau^2), \tag{C-45}$$

$$\mu_F^e(\delta,\omega) = \boldsymbol{\sigma}_F^{(2)}(\delta,\omega).\boldsymbol{\sigma}_D(\delta) + \tau\omega\frac{1}{s_F^{(1)}(\delta,\omega)} + o(\tau^2), \tag{C-46}$$

where approximate formulas for price-dividend ratios and diffusion loadings, $s_i^{(1)}(\delta,\omega)$ and $\boldsymbol{\sigma}_i^{(2)}(\delta,\omega)$, are given in Proposition 2 and Proposition C-1, respectively. ∎

Proof of Proposition 4: We apply Itô's lemma to both sides of (19) and equalize the drift coefficients. Using the notations μ_{c_i} and $\boldsymbol{\sigma}_{c_i}$ previously introduced in (C-2), we obtain

$$-\rho\frac{1}{c_i(t)} - \frac{1}{c_i(t)}\mu_{c_i}(t) + \frac{1}{c_i(t)}|\boldsymbol{\sigma}_{c_i}(t)|^2 = -\frac{1}{c_i(t)}r(t).$$

Using (C-3), this gives

$$r(t) = \rho + \mu_{c_i}(t) - |\boldsymbol{\theta}_i(t)|^2, \quad i = H, F. \tag{C-47}$$

Applying Itô's lemma to $c_H = \omega D$, we get:

$$\begin{aligned}
\mu_{c_H} &= \mu_D + \mu_\omega + \boldsymbol{\sigma}_D.\boldsymbol{\sigma}_\omega \\
&= \mu_D - 2\tau(1-\omega)\boldsymbol{\Gamma}.\boldsymbol{\sigma}_D + \tau^2(1-\omega)(1-2\omega)|\boldsymbol{\Gamma}|^2,
\end{aligned}$$

where the second equality follows from (28)-(29). Using (26), we can also write

$$|\boldsymbol{\theta}_H(t)|^2 = |\boldsymbol{\sigma}_D|^2 - 2\tau(1-\omega)\boldsymbol{\Gamma}.\boldsymbol{\sigma}_D + \tau^2(1-\omega)^2|\boldsymbol{\Gamma}|^2.$$

The riskfree rate is therefore given by[39]

$$\begin{aligned}
r(t) &= \rho + \mu_{c_H}(t) - |\boldsymbol{\theta}_H(t)|^2 \\
&= \rho + \mu_D - |\boldsymbol{\sigma}_D|^2 - \tau^2\omega(1-\omega)|\boldsymbol{\Gamma}|^2.
\end{aligned} \tag{C-48}$$

[39]One would obtain the same result using instead μ_{c_F} and $|\boldsymbol{\theta}_F|^2$.

To find the conditional volatility of the risk-free rate, applying Itô's Lemma and comparing with

$$dr(t) = \mu_r(t)dt + \boldsymbol{\sigma}_r(t).d\boldsymbol{W}(t)$$

gives

$$\begin{aligned}
\boldsymbol{\sigma}_r(t) &= \delta\frac{\partial r(t)}{\partial \delta}\boldsymbol{\sigma}_\delta(t) + \omega\frac{\partial r(t)}{\partial \omega}\boldsymbol{\sigma}_\omega(t) \\
&= \delta\frac{\partial r(t)}{\partial \delta}\boldsymbol{\sigma}_\delta(t) + o(\tau^2) \\
&= \delta\left[\mu_D'(\delta) - 2\boldsymbol{\sigma}_D(\delta).\boldsymbol{\sigma}_D'(\delta) - 2\tau^2\omega(1-\omega)\boldsymbol{\Gamma}_0(\delta).\boldsymbol{\Gamma}_0'(\delta)\right]\boldsymbol{\sigma}_\delta(t) + o(\tau^2),
\end{aligned}$$

where $\mu_D'(\delta) = \frac{\partial \mu_D(\delta)}{\partial \delta}$, $\boldsymbol{\sigma}_D'(\delta) = \frac{\partial \boldsymbol{\sigma}_D(\delta)}{\partial \delta}$, and $\boldsymbol{\Gamma}_0'(\delta) = \frac{\partial \boldsymbol{\Gamma}_0(\delta)}{\partial \delta}$.

∎

Proof of Proposition 5: We start from the intertemporal budget constraint of agent i at time t

$$\begin{aligned}
\xi_i(t)X_i(t) &= \mathbb{E}_t\left[\int_t^\infty \xi_i(s)(c_i(s) - e_i(s))ds\right] \\
\Rightarrow X_i(t) &= \mathbb{E}_t\left[\int_t^\infty \frac{\xi_i(s)}{\xi_i(t)}(c_i(s) - e_i(s))ds\right] \\
&= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{c_i(t)}{c_i(s)}(c_i(s) - e_i(s))ds\right] \\
&= c_i(t)\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\left(1 - \frac{e_i(s)}{c_i(s)}\right)ds\right] \\
&= c_i(t)\left[\frac{1}{\rho} - \mathbb{E}_t\int_t^\infty e^{-\rho(s-t)}\frac{e_i(s)}{c_i(s)}ds\right], \quad i = H, F. \qquad \text{(C-49)}
\end{aligned}$$

Since lump-sum transfers are proportional to τ, we can introduce u_H such that

$$\begin{aligned}
X_H(t) &= c_H(t)\left[\frac{1}{\rho} - \tau u_H(t)\right] \\
&= \omega(t)D(t)\left[\frac{1}{\rho} - \tau u_H(t)\right].
\end{aligned}$$

Itô's lemma implies that, if we write $dX_H/X_H = \mu_{X_H}dt + \boldsymbol{\sigma}_{X_H}.d\boldsymbol{W}$, we have

$$\boldsymbol{\sigma}_{X_H} = \boldsymbol{\sigma}_D + \boldsymbol{\sigma}_\omega + \tau\boldsymbol{\sigma}_e = \boldsymbol{\theta}_H + \tau\boldsymbol{\sigma}_e, \qquad \text{(C-50)}$$

where $\boldsymbol{\sigma}_e$ is related to the redistribution term u_H. This allows us to identify the diffusion term in (9) and to deduce the composition of investor H's portfolio:

$$\boldsymbol{\pi}_H = (\boldsymbol{\Sigma}^\top)^{-1}\boldsymbol{\theta}_H + \boldsymbol{\epsilon}_H = \boldsymbol{\Omega}^{-1}[\boldsymbol{\mu}_H - \boldsymbol{r}] + \boldsymbol{\epsilon}_H,$$

where $\boldsymbol{\epsilon}_H \equiv \tau(\boldsymbol{\Sigma}^\top)^{-1}\boldsymbol{\sigma}_e$. We obtain investor F's portfolio in the same way and the Taylor approximations follow immediately.

When $e_H = \tau a_{FH} D_H$, we can approximate the hedging component $\boldsymbol{\epsilon}_H$ as follows. We write

$$\begin{aligned} e_H = \tau a_{FH} D_H &= \tau \frac{D_H}{S_H} a_{FH} S_H \\ &= \tau \frac{D_H}{S_{H0}} \frac{S_{H0}}{S_{H0}+S_{F0}} X_F + o(\tau) \\ &= \tau \frac{D_H}{S_{H0}+S_{F0}} X_F + o(\tau) \\ &= \tau \frac{D_H}{D} \frac{D}{S_{H0}+S_{F0}} X_F + o(\tau) \\ &= \tau \delta \rho X_F + o(\tau). \end{aligned}$$

Therefore, (C-49) implies

$$X_H(t) = c_H(t)\left[\frac{1}{\rho} - \tau \mathbb{E}_t \int_t^\infty e^{-\rho(s-t)} \delta(s) \frac{\rho X_F(s)}{c_H(s)} ds\right] + o(\tau)$$

Since $X_F = \frac{1-\omega}{\omega} X_H + o(1)$ and $c_H = \rho X_H + o(1)$, we have $\frac{\rho X_F}{c_H} = \frac{1-\omega}{\omega} + o(1)$. Besides, for $s > t$, $\frac{1-\omega(s)}{\omega(s)} = \frac{1-\omega(t)}{\omega(t)} + o(1)$. Therefore, we get

$$\begin{aligned} X_H(t) &= c_H(t)\left[\frac{1}{\rho} - \tau \frac{1-\omega(t)}{\omega(t)} \mathbb{E}_t \int_t^\infty e^{-\rho(s-t)} \delta(s) ds\right] + o(\tau) \\ &= \omega(t) D(t) \left[\frac{1}{\rho} - \tau \frac{1-\omega(t)}{\omega(t)} y_H(\delta(t))\right] + o(\tau), \end{aligned}$$

where y_H was introduced in Appendix A. Therefore in (C-50) we have

$$\boldsymbol{\sigma}_e = \frac{D y_H}{X_H} \boldsymbol{\sigma}_\omega - (1-\omega) \frac{D}{X_H} y_H' \delta \boldsymbol{\sigma}_\delta$$

The first-order approximation of the hedging portfolio $\boldsymbol{\epsilon}_H = \tau(\boldsymbol{\Sigma}^\top)^{-1}\boldsymbol{\sigma}_e$ follows:

$$\boldsymbol{\epsilon}_H = -\tau(1-\omega) \frac{D}{X_H} y_H' \delta (\boldsymbol{\Sigma}_0^\top)^{-1} \boldsymbol{\sigma}_\delta + o(\tau).$$

This expression simplifies by noting that

$$\begin{aligned}
\boldsymbol{\sigma}_{H0} &= \boldsymbol{\sigma}_D + \frac{y_H'}{y_H}\delta\boldsymbol{\sigma}_\delta \\
(\boldsymbol{\Sigma}_0^\top)^{-1}\boldsymbol{\sigma}_{H0} &= (\boldsymbol{\Sigma}_0^\top)^{-1}\boldsymbol{\sigma}_D + \frac{y_H'}{y_H}\delta(\boldsymbol{\Sigma}_0^\top)^{-1}\boldsymbol{\sigma}_\delta \\
y_H'\delta(\boldsymbol{\Sigma}_0^\top)^{-1}\boldsymbol{\sigma}_\delta &= y_H\left[(\boldsymbol{\Sigma}_0^\top)^{-1}\boldsymbol{\sigma}_{H0} - (\boldsymbol{\Sigma}_0^\top)^{-1}\boldsymbol{\sigma}_D\right] \\
&= y_H\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} S_{H0}/(S_{H0}+S_{F0}) \\ S_{F0}/(S_{H0}+S_{F0}) \end{pmatrix}\right].
\end{aligned}$$

Simple algebra finally gives

$$\boldsymbol{\epsilon}_H = \tau\frac{1-\omega}{\omega}\frac{y_H y_F}{(y_H+y_F)^2}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + o(\tau).$$

One can prove in the same way that

$$\boldsymbol{\epsilon}_F = \tau\frac{\omega}{1-\omega}\frac{y_H y_F}{(y_H+y_F)^2}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + o(\tau).$$

∎

Proof of Lemma 3: The domestic asset price is

$$\begin{aligned}
S_H(t) &= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{D(t)}{D(s)}p_H(s)D_H(s)ds\right] &\text{(C-51)} \\
&= D(t)\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\delta(s)ds\right]. &\text{(C-52)}
\end{aligned}$$

As recalled in Appendix A, the above expectation can be written as $y_H(\delta(t))$, for some hypergeometric function y_H. However, because the drift and diffusion of δ as defined in Section 5 differ from those given in (4), the parameters of the hypergeometric function have to be modified accordingly. This is done by setting

$$\begin{aligned}
\nu &= \frac{\phi-1}{\phi}\left[\mu_{D_F} - \mu_{D_H} - \frac{\sigma_{D_{F,1}}^2 + \sigma_{D_{F,2}}^2}{2} + \frac{\sigma_{D_{H,1}}^2 + \sigma_{D_{H,2}}^2}{2}\right], \\
\chi^2 &= \left(\frac{\phi-1}{\phi}\right)^2\left[\left(\sigma_{D_{H,1}}^2 + \sigma_{D_{H,2}}^2\right) + \left(\sigma_{D_{F,1}}^2 + \sigma_{D_{F,2}}^2\right) - 2(\sigma_{D_{H,1}}\sigma_{D_{F,1}} + \sigma_{D_{H,2}}\sigma_{D_{F,2}})\right].
\end{aligned}$$

The price of the foreign asset can be written as $S_F(t) = D(t)y_F(\delta(t))$, where $y_F(\delta) = \frac{1}{\rho} - y_H(\delta)$. ∎

Proof of Proposition 6: Asset prices are

$$
\begin{aligned}
S_H(t) &= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{C_H(t)}{C_H(s)}p_H(s)D_H(s)ds\right] \\
&= \omega(t)D(t)\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{\delta(s)}{\omega(s)}ds\right], \qquad \text{(C-53)} \\
S_F(t) &= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{C_F(t)}{C_F(s)}p_F(s)D_F(s)ds\right] \\
&= (1-\omega(t))D(t)\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{1-\delta(s)}{1-\omega(s)}ds\right]. \qquad \text{(C-54)}
\end{aligned}
$$

Therefore, our approximations of the valuation functions s_H and s_F (defined in Lemma 1) allow us to characterize the equilibrium of the two-good version of the model. ∎

D One-tree model

In this section of the Appendix, we analyze the one-tree model, which serves in Appendix C as a limiting case of the two-tree economy. In this economy, there is a unique dividend stream and two agents, one who does not pay taxes on dividends and another who pays taxes on dividends at the rate τ.[40] We denote variables pertaining to the taxed agent by the symbol $\hat{\cdot}$.

The dividend process is given by

$$\frac{dD(t)}{D(t)} = \mu_D dt + \sigma_D dW(t),$$

where W is a one-dimensional standard Brownian motion. The equity claim paying the dividend in perpetuity (referred to as "stock"), has date-t value $S(t)$. The stock has expected return $\mu(t)$ from the perspective of the untaxed agent, and $\hat{\mu}(t)$ from the perspective of the taxed agent, where

$$\mu(t)dt = \mathbb{E}_t\left[\frac{D(t)dt + dS(t)}{S(t)}\right], \qquad \hat{\mu}(t)dt = \mathbb{E}_t\left[\frac{(1-\tau)D(t)dt + dS(t)}{S(t)}\right],$$

and so

$$\mu(t) - \hat{\mu}(t) = \tau\frac{D(t)}{S(t)},$$

showing that differential taxation drives a wedge between agents' expected returns. We define the price-dividend ratio, $s(t) = \frac{S(t)}{D(t)}$, and so

$$\mu(t) - \hat{\mu}(t) = \frac{\tau}{s(t)}.$$

We also define agent-specific market prices of risk, $\theta(t)$ and $\hat{\theta}(t)$, i.e.

$$\theta(t) = \frac{\mu(t) - r(t)}{\sigma_R(t)}, \qquad \hat{\theta}(t) = \frac{\hat{\mu}(t) - r(t)}{\sigma_R(t)},$$

where $\sigma_R(t)$ is the volatility of stock returns. Hence, the difference between agents' prices of risk is given by

$$\theta(t) - \hat{\theta}(t) = \frac{\mu(t) - \hat{\mu}(t)}{\sigma_R(t)} = \frac{\tau}{\sigma_R(t)\, s(t)}. \tag{D-1}$$

The following proposition shows how the price-dividend ratio can be obtained as function of the state variable $\omega(t)$, which denotes the proportion of aggregate dividends consumed by the untaxed agent.

[40] Observe that this economy is the infinite horizon version of Basak & Gallmeyer (2003).

Proposition D-1. *Let $\omega(t) = c(t)/D(t)$. The time-t price-dividend ratio can be written $s(t) = s(\omega(t))$ and the valuation function $s(.)$ satisfies the ordinary differential equation*

$$\mu_D + \omega\mu_\omega(\omega)\frac{s'(\omega)}{s(\omega)} + \frac{1}{2}\omega^2(\sigma_\omega(\omega))^2\frac{s''(\omega)}{s(\omega)} + \frac{1-(1-\omega)\tau}{s(\omega)} - r(\omega) = \sigma_D^2, \tag{D-2}$$

where

$$\sigma_\omega(\omega) = \frac{1}{2}\frac{\sigma_D}{\frac{\omega s'(\omega)}{s(\omega)}}\left(\sqrt{1+\tau\frac{4s'(\omega)\omega(1-\omega)}{(s(\omega)\sigma_D)^2}} - 1\right), \tag{D-3}$$

$$\mu_\omega(\omega) = \frac{(\sigma_\omega(\omega))^2}{1-\omega} - \sigma_D\sigma_\omega(\omega)$$

$$r(\omega) = \rho + \mu_D - \left[(\sigma_D + \sigma_\omega(\omega))^2 - \frac{(\sigma_\omega(\omega))^2}{1-\omega}\right], \tag{D-4}$$

subject to the boundary conditions $s(0) = (1-\tau)\frac{1}{\rho}$ and $s(1) = \frac{1}{\rho}$.

Proof: The equilibrium consumption allocations, c and $\hat{c}$ are given by

$$c = \omega D, \qquad \hat{c} = (1-\omega)D,$$

where ω is the consumption share of the untaxed agent. First, we derive expressions for μ_ω and σ_ω. We will use the fact that the equilibrium prices of risk θ and $\hat{\theta}$ satisfy

$$\theta = \sigma_D + (1-\omega)\frac{\tau}{\sigma_R s}, \tag{D-5}$$

$$\hat{\theta} = \sigma_D - \omega\frac{\tau}{\sigma_R s}. \tag{D-6}$$

Indeed, market clearing in the consumption good implies that

$$c + \hat{c} = D.$$

Hence

$$\omega\frac{dc}{c} + (1-\omega)\frac{d\hat{c}}{\hat{c}} = \frac{dD}{D}.$$

We can write

$$\frac{dc}{c} = \mu_c dt + \sigma_c dW, \quad \frac{d\hat{c}}{\hat{c}} = \hat{\mu}_c dt + \hat{\sigma}_c dW,$$

and note that (since preferences are logarithmic)

$$\sigma_c = \theta, \;\; \hat{\sigma}_c = \hat{\theta}.$$

Hence

$$\omega\mu_c + (1-\omega)\hat{\mu}_c = \mu_D, \quad \omega\theta + (1-\omega)\hat{\theta} = \sigma_D.$$

The expressions for θ and $\hat{\theta}$ in (D-5)-(D-6) follow from the last equation along with (D-1).

The dynamics of $\lambda = \frac{\hat{c}}{c}$ are given by

$$\frac{d\lambda}{\lambda} = \mu_\lambda dt + \sigma_\lambda dW,$$

where

$$\begin{aligned} \mu_\lambda &= (\theta - \hat{\theta})\hat{\theta}, \\ \sigma_\lambda &= \hat{\theta} - \theta. \end{aligned}$$

Since $\omega = \frac{1}{1+\lambda}$ it follows from Itô's Lemma that

$$\begin{aligned} d\omega &= \left[-(1+\lambda)^{-2}\lambda\mu_\lambda + (1+\lambda)^{-3}\lambda^2\sigma_\lambda^2\right] dt - \lambda(1+\lambda)^{-2}\sigma_\lambda dW \\ &= \omega(1-\omega)(\hat{\theta}-\theta)\left[\omega\theta + (1-\omega)\hat{\theta} - (\theta - \hat{\theta})\right] dt - \omega(1-\omega)(\hat{\theta}-\theta)dW \\ &= \omega(1-\omega)(\theta-\hat{\theta})(\theta - \hat{\theta} - \sigma_D)dt + \omega(1-\omega)(\theta-\hat{\theta})dW. \end{aligned}$$

Therefore

$$\frac{d\omega}{\omega} = \mu_\omega dt + \sigma_\omega dW,$$

where

$$\begin{aligned} \mu_\omega &= (1-\omega)(\theta-\hat{\theta})(\theta-\hat{\theta}-\sigma_D) = \frac{\sigma_\omega^2}{1-\omega} - \sigma_D\sigma_\omega & \text{(D-7)} \\ \sigma_\omega &= (1-\omega)(\theta-\hat{\theta}) = (1-\omega)\frac{\tau}{\sigma_R s}. & \text{(D-8)} \end{aligned}$$

Since $S = sD$ it follows from Itô's Lemma that

$$\sigma_R = \sigma_D + \frac{s'}{s}\omega\sigma_\omega. \tag{D-9}$$

where $s' = \frac{ds}{d\omega}$. From the latter two equations it follows that

$$\frac{\tau\omega(1-\omega)}{s\sigma_D + s'\omega\sigma_\omega} = \omega\sigma_\omega.$$

We thus obtain the following quadratic equation for σ_ω

$$s'\omega^2\sigma_\omega^2 + s\sigma_D\omega\sigma_\omega - \tau\omega(1-\omega) = 0.$$

Solving the above quadratic gives

$$\sigma_\omega = \frac{1}{2}\frac{\sigma_D}{\frac{\omega s'}{s}}\left(\pm\sqrt{1+\tau\frac{4s'\omega(1-\omega)}{(s\sigma_D)^2}} - 1\right).$$

To ensure that $\lim_{\tau\to 0}\sigma_R = \sigma_D$, we choose the positive root, and so Equation (D-3) holds.

We now derive an expression for the riskfree rate. Since the untaxed agent has logarithmic preferences, the riskfree rate is given by

$$r = \rho + \mu_c - \sigma_c^2,$$

where μ_c and σ_c have been defined earlier. Applying Itô's Lemma to $c = \omega D$ gives

$$\begin{aligned}\mu_c &= \mu_D + \mu_\omega + \sigma_\omega\sigma_D,\\ \sigma_c &= \sigma_D + \sigma_\omega,\end{aligned}$$

and so

$$\begin{aligned} r &= \rho + \mu_D + \mu_\omega - (\sigma_D^2 + \sigma_D\sigma_\omega + \sigma_\omega^2)\\ &= \rho + \mu_D + \frac{\sigma_\omega^2}{1-\omega} - \sigma_D\sigma_\omega - (\sigma_D + \sigma_D\sigma_\omega + \sigma_\omega^2).\end{aligned}$$

Equation (D-4) follows.

We now derive an ordinary differential equation for the price-dividend ratio, s. The date-t value of the stock is given by

$$S(t) = \mathbb{E}_t\int_t^\infty e^{-\rho(u-t)}\frac{c(t)}{c(u)}D(u)du.$$

Hence,

$$s(t) = \omega(t)\mathbb{E}_t\int_t^\infty e^{-\rho(u-t)}\omega(u)^{-1}du = s(\omega(t)).$$

We have the boundary conditions $s|_{\omega=1} = \frac{1}{\rho}$, and $s|_{\omega=0} = (1-\tau)\frac{1}{\rho}$. Starting from the fundamental asset pricing equation:

$$\mathbb{E}_t\left[\frac{dS(t)+D(t)dt}{S(t)} - r(t)dt\right] = -\mathbb{E}_t\left[\frac{d\xi(t)}{\xi(t)}\frac{dS(t)}{S(t)}\right],$$

where

$$\frac{d\xi}{\xi} = -rdt - \theta dW.$$

Applying Itô's Lemma to $S = sD$ where $s = s(\omega)$ gives

$$\begin{aligned}
\frac{dS}{S} &= \frac{dD}{D} + \frac{ds}{s} + \frac{dD}{D}\frac{ds}{s} \\
&= \frac{dD}{D} + \frac{s'}{s}d\omega + \frac{1}{2}\frac{s''}{s}(d\omega)^2 + \sigma_D\sigma_\omega\frac{s'}{s} \\
&= \mu_D dt + \sigma_D dW + \omega\frac{s'}{s}(\mu_\omega dt + \sigma_\omega dW) + \frac{1}{2}\omega^2\sigma_\omega^2\frac{s''}{s}dt + \sigma_D\sigma_\omega\omega\frac{s'}{s}dt \\
&= \left[\mu_D + \omega\left(\mu_\omega + \sigma_D\sigma_\omega\right)\frac{s'}{s} + \frac{1}{2}\omega^2\sigma_\omega^2\frac{s''}{s}\right]dt + \left(\sigma_D + \omega\sigma_\omega\frac{s'}{s}\right)dW.
\end{aligned}$$

Hence the fundamental asset pricing equation can be written

$$\mu_D + \omega\left(\mu_\omega + \sigma_D\sigma_\omega\right)\frac{s'}{s} + \frac{1}{2}\omega^2\sigma_\omega^2\frac{s''}{s} + \frac{1}{s} - r = \theta\left(\sigma_D + \omega\sigma_\omega\frac{s'}{s}\right).$$

From (D-5) and (D-9), we obtain

$$\begin{aligned}
\mu_D + \omega\left(\mu_\omega + \sigma_D\sigma_\omega\right)\frac{s'}{s} + \frac{1}{2}\omega^2\sigma_\omega^2\frac{s''}{s} + \frac{1}{s} - r &= \left(\sigma_D + (1-\omega)\frac{\tau}{\sigma_R s}\right)\sigma_R \\
\mu_D + \omega\left(\mu_\omega + \sigma_D\sigma_\omega\right)\frac{s'}{s} + \frac{1}{2}\omega^2\sigma_\omega^2\frac{s''}{s} + \frac{1}{s} - r &= \sigma_D\sigma_R + (1-\omega)\frac{\tau}{s} \\
\mu_D + \omega\left(\mu_\omega + \sigma_D\sigma_\omega\right)\frac{s'}{s} + \frac{1}{2}\omega^2\sigma_\omega^2\frac{s''}{s} + \frac{1}{s} - r &= \sigma_D^2 + \sigma_D\omega\sigma_\omega\frac{s'}{s} + (1-\omega)\frac{\tau}{s} \\
\mu_D + \omega\mu_\omega\frac{s'}{s} + \frac{1}{2}\omega^2\sigma_\omega^2\frac{s''}{s} + \frac{1}{s} - r &= \sigma_D^2 + (1-\omega)\frac{\tau}{s}.
\end{aligned}$$

Rewriting the above equation gives (D-2). ∎

E Additional results on the accuracy of approximations

This appendix provides more detailed results on the accuracy of our second-order perturbation solution. First, we compare the second order approximation with the numerical solution. Second, we compute Euler equation errors for the second order approximation. We use the parameter values $\rho = 0.03$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H,1} = \sigma_{D_F,2} = 0.097$ and $\sigma_{D_H,2} = \sigma_{D_F,1} = 0.026$.[41]

E.1 Comparison of the second-order approximation with the numerical solution

We first look at the size of the difference between the second-order perturbation solution and the numerical solution for the domestic price-dividend ratio, expressed as a percentage of the numerical solution, i.e. $|s_H^{(2)} - s_H^{\text{num}}|/s_H^{\text{num}}$, where $s_H^{(2)}$ is the second-order solution and s_H^{num} is the numerical solution. Figure E-1 shows that for $\tau = 0.05$, the percentage difference between the second-order and numerical solutions is typically less than 2%, most often less than 1%, and never greater than 3%. Figure E-2 shows that even when $\tau = 0.10$, the percentage difference remains under 2% over most of the state space, is often less than 1%, and never gets higher than 5%.

We also assess the quality of our perturbation approach by computing the size of the difference between the domestic expected excess returns obtained using the second-order expansion and the numerical solution, $|\mu_H^{e,(2)} - \mu_H^{e,\text{num}}|$. When $\tau = 0.05$, the difference between the second-order and numerical solutions is extremely small over the entire state space, always remaining less than a fifth of a basis point. When $\tau = 0.10$, the difference between the two solutions is still very small, remaining below 1 basis point (bp) over most of the state space, and never getting larger than 2.3 bp (which occurs in a part of the state space that is not relevant empirically). Table E-1 summarizes the comparison between the two methods when using the domestic risk premium as a metric.

[41] This corresponds to 10% fundamental volatility and fundamental correlation $\eta = 0.5$.

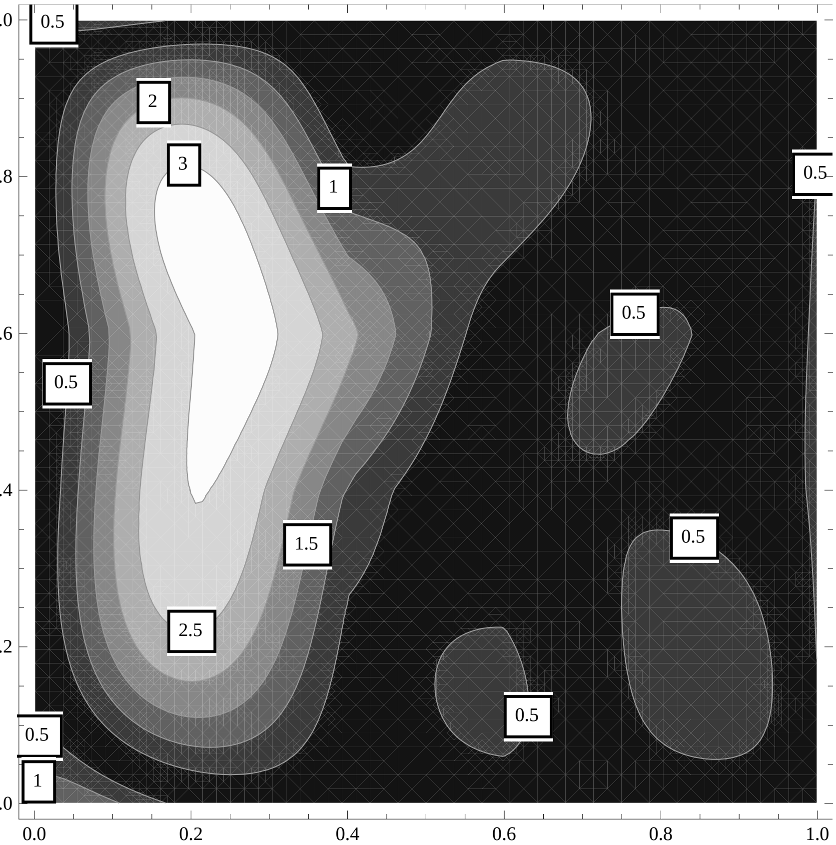

Figure E-1: $|s_H^{(2)} - s_H^{\text{num}}|/s_H^{\text{num}}$, $\tau = 0.05$.

This contour plot shows the size of the difference between the second-order perturbation and the numerical solution for the domestic price-dividend ratio s_H over the entire state space, expressed as a percentage of the numerical solution, for $\tau = 0.05$. The horizontal axis is $\delta \in [0, 1]$ and the vertical axis $\omega \in [0, 1]$. Numbers indicate the level of the contour lines, expressed in percentage points — e.g., in the black region, the difference is less than 0.5% of the numerical solution, while in the darker grey region, the difference is between 0.5% and 1% of the numerical solution.

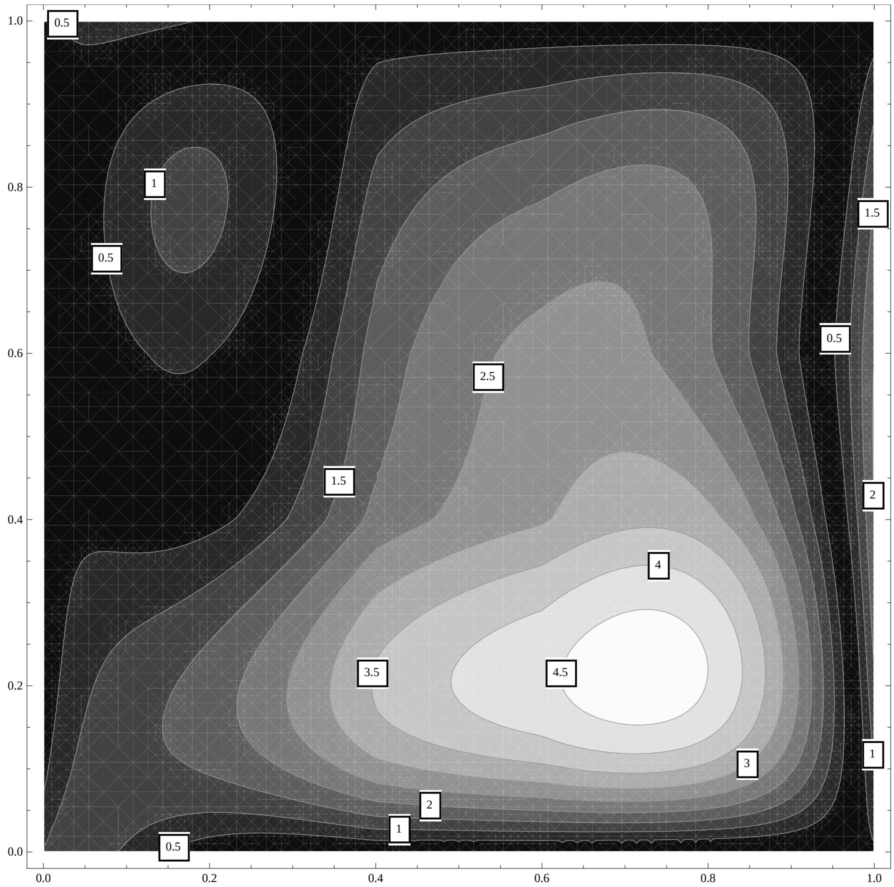

Figure E-2: $|s_H^{(2)} - s_H^{\mathrm{num}}|/s_H^{\mathrm{num}}$, $\tau = 0.10$.

This contour plot shows the size of the difference between the second-order perturbation and the numerical solution for the domestic price-dividend ratio s_H over the entire state space, expressed as a percentage of the numerical solution, for $\tau = 0.10$. The horizontal axis is $\delta \in [0, 1]$ and the vertical axis $\omega \in [0, 1]$. Numbers indicate the level of the contour lines, expressed in percentage points — e.g., in the black region, the difference is less than 0.5% of the numerical solution, while in the darker grey region, the difference is between 0.5% and 1% of the numerical solution.

Table E-1: $|\mu_H^{e,(2)} - \mu_H^{e,\text{num}}|$, summary results.

	Min	Max	Average	$\delta = \omega$	$\delta = \omega = .5$
$\tau = 0.05$	0.004	0.20	0.07	0.11	0.08
$\tau = 0.1$	0.45	2.26	0.93	0.81	0.49

This table summarizes the comparison between the second-order perturbation solution and the numerical solution, using the absolute value of the difference in domestic expected excess return as a metric. For a given value of τ, we report the minimum difference over the finite state space grid, the maximum difference, the arithmetic average over the entire grid, the arithmetic average on the 45-degree line (i.e., for $\delta = \omega$), and the difference at the centre point $\delta = \omega = .5$. All numbers are expressed in basis points.

E.2 Euler equation errors for the second-order approximation

We now use the Euler equation errors (see, for e.g. Den Haan and Marcet (1994) and Judd (1998)) to assess internally the accuracy of the second- order approximation without reference to the numerical solution.

We start from the basic continuous-time Euler equation

$$\mathbb{E}_t \left[\frac{\int_t^T \xi_H(u) D_H(u) du + \xi_H(T) S_H(T)}{\xi_H(t) S_H(t)} \right] = 1. \tag{E-1}$$

Setting $T = t + dt$ gives

$$\mathbb{E}_t \left[\frac{\int_t^{t+dt} \xi_H(u) D_H(u) du + \xi_H(t+dt) S_H(t+dt)}{\xi_H(t) S_H(t)} \right] = 1, \tag{E-2}$$

which is the continuous-time counterpart of the discrete-time Euler equation

$$\mathbb{E}_t \left[\frac{\xi_H(t+1)}{\xi_H(t)} \frac{D_H(t) + S_H(t+1)}{S_H(t)} \right] = 1. \tag{E-3}$$

We could use the errors in the Euler equation (E-2) to assess accuracy, but the errors are not easily interpretable from an economic viewpoint. To overcome this problem, we rewrite the Euler equation in the following form (see e.g. Cochrane (2001))

$$\mathbb{E}_t \left[\frac{dS_H(t) + D_H(t) dt}{S_H(t)} - r(t) dt \right] = -\mathbb{E}_t \left[\frac{d\xi_H(t)}{\xi_H(t)} \frac{dS_H(t)}{S_H(t)} \right]. \tag{E-4}$$

For a given second-order approximate solution $S_H(t)$, we can compute the left and right-hand sides of to obtain two values for the domestic excess return. The closer these two values are, the more accurate the solution. Consequently, the difference between these two values for the the domestic excess return is an easily interpretable metric for the accuracy of the second-order solution and is also an Euler equation error.

For $\tau = 0.05$, the mean error over the state space is 7.5 bp, and along the diagonal where $\delta = \omega$, the mean error is 7.4 bp. Even a larger taxer rate of $\tau = 0.10$, the errors remain small: the mean error over the state space is 8.8 bp and 8.7 bp along the diagonal.

F Data sources

Data covers the period 1978-2008. Except portfolio data, data are available on a quarterly basis.

- Quarterly stock returns with and without dividends from CRSP for the US. US Dividend price ratio is computed using the difference between gross returns with and without dividends. Stock market indices and dividend price ratios are from Global Financial Data for other G7 countries (main stock market index for each country).

- Quarterly U.S. 3 months T-bill interest rate from Global Financial Data.

- Deflator: Quarterly CPI from OECD Main Economic Indicators.

- Values for the state variables (δ, ω) computed using output and consumption data for G7 countries in USD from OECD Main Economic Indicators.

- 'Fundamental' moments for real dividend growth are computed using raw standard deviations of (quarterly) real dividend growth and raw correlations with the Rest of the World over the period 1978-2008. Rest of the world's real dividend growth is computed for each country using constant GDP-weights from OECD Main Economic Indicators.

- Observed portfolio weights for the U.S. in 1988, 1998, 2008 are computed using total U.S. market capitalization from the World Federation of Exchanges (WFE) in a given year and total U.S. equity foreign assets and liabilities from the International Investment Position (IMF). Data for 2008 for other G7 countries are computed accordingly using the same data sources. See Coeurdacier and Rey (2011) for more details.

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