Fertility Policies and Social Security Reforms in China

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Abstract

This paper analyzes the impact of relaxing fertility controls and expanding social security in China. We develop an overlapping generations model in which fertility decisions and capital accumulation are endogenously determined in the presence of social security. In our model, children are an alternative savings technology—as they transfer resources to their retired parents. Important feedback links arise between fertility and social security variables: an expansion of social security benefits reduces fertility— partially offsetting the effects of relaxing the one-child policy. The feedback loop between social security variables and fertility suggests that abandoning fertility restrictions may not be as effective in helping to finance China's intended pension reform, especially if children are an important source of old-age support. The sustainability of the pension system is particularly at risk in the event of a growth slowdown. The objective of pension reforms may also be incongruent with other reforms, such as financial liberalization and financial integration.

Key Words: one-child policy, social security, demographics.

JEL Classification: **H55, J11, J13, E21.**

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1 Introduction

The Chinese government is currently undertaking important social security reforms. At present, about half of urban workers are covered by the pension program, while the vast majority of the rural area remains uncovered. Nation-wide, only 31% of China's total workforce, urban and rural combined, is eligible for a public pension of any kind. Coincidentally, intra-family support, on which most Chinese elderly traditionally relied, is weakening as a consequence of families having fewer children. Without imminent reforms, tens of millions of Chinese could arrive in old age without pensions or adequate family support in the next few decades. As a result, the government set out a goal to make the pension program national in scope—reaching full coverage by the year 2020.[1]

However, the challenge of building a national pension system could hardly be more daunting. Demographic aging, due to fertility controls, makes the reform all at once more pressing and more delicate. The 'one-child policy', whose implementation started in the early 1980's,[2] indeed fostered a rapid demographic transition in China and will soon lead to a sharp rise in the ratio of retirees to middle-aged workers: as the share of the elderly in the total population is expected to double between now and 2050, the old age dependency ratio could almost triple and come close to one (Table 1). Concerns over pressure on the social security program are echoed in works such as Feldstein (1999), Barr and Diamond (2010), Nakashima et al. (2009), and Song et al. (2013), among others.

Table 1: Demographic Structure in China

	1970	2010	2050
Share of Young (% of individuals aged 0-30)	65%	42%	28%
Share of Middle-Aged (% of individuals aged 30-60)	28%	44%	39%
Share of Elderly (% of individuals above 60)	7%	14%	33%
Median Age	19.7	34.5	48.7
Fertility (number of children per woman, urban areas)	3.18	1.04	n/a

Note: UN World Population Prospects (2011). The fertility number for the year 1970 (resp. 2010) corresponds to the average fertility over the period 1965-70 (resp. 2004-2009).

In November 2013, the Third Plenary Session announced the much-anticipated relaxation of the one-child policy—partly to slowdown the aging of the society, and partly to help relieve pressure on the financing of social security. But to what extent can the relaxation of fertility controls actually aid pension reforms? The question needs to be addressed in an appropriate framework that takes into account China's particular institutional and social norm. One important aspect of the Chinese society is that children are often the main source of old-age support. Until recently, the absence of

[1] Hu Jintao's report at the 17th National Congress of the Communist Party, October 15, 2007.

[2] The one-child policy aimed at curbing the rapid population growth following Mao's pro-natality program. Household-level data (Urban Household Survey) indicates a strict enforcement of the policy in urban areas in China: over the period 2000-2009, 96% of urban households that had children had only one child. The policy however was significantly less strict in rural areas. According to Census data, the urban fertility rate fell from a bit above 3 in 1970 to close to 1 by 1982 (see Choukhmane et al. (2014)).

social safety net has meant that retirees relied to a large extent on children's support. According to Census data in 2005, family support is the main source of income for more than half of the elderly (65+) urban population (see Choukhmane et al. (2014)). In rural areas where pension coverage is particularly low, family support is even more essential for elderly (see Chapter 3 in Cai et al. (2012)). However, to the extent that there is some substitutability between social security and reliance on children, people may decide to have fewer children when social security benefits increase. This might in turn reduce future contributions necessary to sustain the reform.

In order to take these considerations into account, we develop a framework that allows for endogenous fertility in an overlapping generations model with capital accumulation and intergenerational transfers—in the form of family support and social security.[3] In the model, individuals decide on how many children to bear at the end of their youth period. In middle age, they raise children (at a cost), contribute to the pension program, and transfer resources to their parents. Old retirees consume all available resources, which include social security benefits and offspring's transfers.

This economy is characterized by three key relationships in the steady state. The first is a positive relationship between fertility and interest rates based on optimal savings and investment decisions. A greater number of children increases the marginal productivity of capital and thus investment. Moreover, it is associated with higher expenditures and larger transfers in old age, both of which lead to lower savings. Higher fertility thus leads to higher interest rates. The second condition is a negative relationship based on optimal fertility choices: a higher interest rate lowers the financial benefits of children (relative to capital) and hence discourages fertility. The third key relationship is determined by the sustainability of the social security system, which embeds the impact of fertility and growth on the viability of the program. We show analytically how the interactions of these relationships conjoin to determine the interest rate, fertility and social security contribution rates in the long run. Children in this framework are both 'consumptions goods' and 'investment goods'—in the sense that they transfer resources to support their parents in old age. A change in the social security system alters incentives to have children, both because social security variables (taxes and benefits) alter consumption and thus the demand for children, and because social security affects the investment return on children— via changes in the interest rate, or to the extent that more generous pension benefits can crowd out intra-family transfers to the elderly.

Our framework applied to China shows that relaxing the one-child policy can aid an expansion of pension through an increase in fertility — but that rise is partly offset through three channels. First, insofar as children are considered investment goods, greater reliance on social security and less transfers coming from children means that desired fertility drops. Second, to the extent that children are also consumption goods, social security expansion, financed partly by higher taxes, lowers consumption in middle age and therefore also optimal fertility. Third, the rise in the interest rate occasioned by a fall in national savings also leads to a reduction in desired fertility. Lower fertility requires higher tax rates, which further lowers fertility—and so and so forth. This feedback loop between social security variables and fertility suggests that the effectiveness of abandoning the

[3]Our theory relates to earlier work linking fertility and intergenerational (public or private) transfers such as Nishimura and Zhang (1992), Boldrin and Jones (2002), Boldrin, de Nardi and Jones (2005), Ehrlich and Kim (2005) and Yew and Zhang (2009).

one-child policy in terms of its impact on financing the pension reforms may be overstated —when endogenous fertility considerations are ignored. In the event of a growth slowdown—a likely scenario in China in the coming decades— the circumstances would only become worse. The fertility increase as a consequence of the birth-control relaxation is even more limited because a slower rate of growth reduces expected future transfers—and thus the benefits of having children. Financing the social security expansion thus requires an even more significant rise in taxes.

We also investigate how social security reforms might interact with capital markets reforms. We consider two reforms (1) domestic financial liberalization and (2) lifting capital controls. Those two reforms have qualitatively similar consequences as they both increase repressed interest rates. By raising the return on savings, they reduce the benefits from children and depress further fertility. As a consequence, they exert further pressure on the financing of social security. In our framework, the Chinese government faces a severe tension between expanding the generosity of the pension system and deepening capital markets liberalization—particularly so if growth slows.

The paper is structured as follows. Section 2 describes the basic elements of the framework and the relationships governing the key endogenous variables in the economy. An analytically tractable case of a PAYGO system in the long run is analyzed in Section 3. Section 4 shows the dynamics of the model under various policy experiments. Section 5 concludes. All proofs are relegated in the Appendix.

2 Model

The economy is populated by overlapping generations of agents who live for three periods: youth (y), middle age (m), and old age (o). Agents supply one unit of labor when young and middle-aged, and retire when old. Before entering middle age, individuals decide how many children to have. Let n_t denote the number of children (per young individual) born at the end of period t. Demographics evolves according to $L_{o,t+2} = L_{m,t+1} = L_{y,t} = n_{t-1}L_{y,t-1}$, where $L_{\gamma,t}$ denotes the size of generation $\gamma \in \{y, m, o\}$ in period t.

2.1 Production

Let K_{t-1} denote the aggregate capital stock at the beginning of period t and $e_tL_{y,t} + L_{m,t}$ the total labor input employed in period t, where e_t denotes the relative efficiency of young workers ($e_t < 1$). The gross output is

$$Y_t = K_{t-1}^{\alpha} \left[A_t \left(e_tL_{y,t} + L_{m,t}\right)\right]^{1-\alpha},$$

where A_t is labor-augmenting productivity, and $0 < \alpha < 1$. Productivity grows at an exogenous (gross) rate $g_{A,t}$, so that $A_t = g_{A,t}A_{t-1}$.

Factor markets are competitive. Thus, the wage rates per unit of labor in youth and middle age are

$$w_{y,t} = e_t(1-\alpha)A_tk_{t-1}^{\alpha}, \qquad w_{m,t} = (1-\alpha)A_tk_{t-1}^{\alpha}, \tag{1}$$

where $k_{t-1} \equiv K_{t-1}/[A_t(e_tL_{y,t} + L_{m,t})]$ denotes the capital-effective-labor ratio. The rental rate earned by capital in production equals the marginal product of capital, $r_{K,t} = \alpha k_{t-1}^{\alpha-1}$. We assume full depreciation of capital for simplicity.[4] Therefore the gross rate of return earned between period $t-1$ and t is

$$R_t = \alpha k_{t-1}^{\alpha-1}. \tag{2}$$

2.2 The Social Security System

Our modelling of the social security system is flexible enough to encapsulate a pay-as-you-go system (PAYGO), a fully-funded system, or more generally some combination of the two. Young and middle-aged agents in period t pay a Social Security tax proportional to their labor income, at the contribution rate τ_t. A retiree in period t receives social security benefits in the amount of $\sigma_t w_{m,t-1}$, where σ_t denotes the replacement ratio in that period.

The government runs the social security system and can accumulate assets in a trust fund to finance the system. Let $B_t \geq 0$ denote the government's asset position at the end of period t. The government's flow budget constraint in period $t+1$ is

$$\tau_{t+1}(L_{y,t+1}w_{y,t+1} + L_{m,t+1}w_{m,t+1}) + R_{t+1}B_t = L_{o,t+1}\sigma_{t+1}w_{m,t} + B_{t+1}. \tag{3}$$

The left hand side of the equation represents the sources of funds for the Social security system in period $t+1$, which consist of Social Security taxes plus gross returns on assets. The right hand side represents the uses of funds by the Social Security system in that period, including retirement benefits paid to the elderly and the purchase of capital to hold in the trust fund. Using (1) and (2), the government's flow budget constraint can be rewritten as

$$\frac{n_{t-1}g_{A,t+1}\left(1+n_t e_{t+1}\right)}{1+n_{t-1}e_t}\left(\frac{k_t}{k_{t-1}}\right)^{\alpha}\left[(1-\alpha)\tau_{t+1} - b_{t+1}\right] + \alpha k_t^{\alpha-1}b_t = \frac{1-\alpha}{1+n_{t-1}e_t}\sigma_{t+1}, \tag{4}$$

where $b_t \equiv \frac{B_t}{Y_t}$ denotes government assets as a share of GDP. Equation (4) governs the dynamics of the social security system, defined by the set of variables $\{b_t, \tau_t, \sigma_t\}$, for a given evolution of the endogenous state variables $\{k_t, n_t\}$. This equation makes clear the impact of growth and the interest rate on the financing of the social security system. Higher growth between periods t and $t+1$—driven by productivity, demographics, or capital accumulation— tends to relax the government's budget constraint through its impact on tax revenues relative to benefit spendings — although it also implies higher investment into capital assets for a given $b_{t+1} > 0$. If the government holds a positive asset position ($b_t > 0$), a higher rate of return on capital ($\alpha k_t^{\alpha-1}$) also increases revenues.

2.3 Households

Consider an individual who is young in period t. The individual earns the competitive wage rate $w_{y,t}$ when young, and $w_{m,t+1}$ in the next period. At the end of period t, the agent decides on the number of children n_t to bear. In the next period, the agent pays the cost of raising kids and makes

[4] This assumption is rather innocuous and quantitatively unimportant given that a period corresponds to one generation.

a transfer to his elderly parents, for a total amount of $T_{m,t+1}$. When retired, the individual consumes all available resources, which consist of gross return on accumulated assets, social security transfers ($\sigma_{t+2}w_{m,t+1}$), and transfers received from children $T_{o,t+2}$. The lifetime utility of the agent, which includes the utility derived from children, is

$$U_t = \log(c_{y,t}) + v\log(n_t) + \beta\log(c_{m,t+1}) + \beta^2\log(c_{o,t+2})$$

where $0 < \beta < 1$, and $v > 0$ reflects the preference for children. The agent faces the following sequence of budget constraints:

$$\begin{aligned} c_{y,t} + a_{y,t} &= (1-\tau_t)w_{y,t} \\ c_{m,t+1} + a_{m,t+1} &= (1-\tau_{t+1})w_{m,t+1} + R_{t+1}a_{y,t} - T_{m,t+1} \\ c_{o,t+2} &= R_{t+2}a_{m,t+1} + \sigma_{t+2}w_{m,t+1} + T_{o,t+2}, \end{aligned} \tag{5}$$

where $a_{\gamma,t}$ denotes the net asset holdings at the end of period t by an agent of generation γ.

Intra-Family Transfers. The cost of raising kids falls in the mold of a time cost that is proportional to middle-aged labor income and to the number of children, $\phi n_t w_{m,t+1}$, where $\phi > 0$. These costs can be interpreted as 'mouth-to-feed' costs and education expenses.[5] Transfers from a middle-aged individual to his parents in period $t+1$ amount to a fraction $\psi_{t+1}\left(n_{t-1}^{\varpi-1}/\varpi\right)$ of the wage rate $w_{m,t+1}$, where $\varpi \in (0,1]$. Ascending transfers constitute an important feature of Chinese society (see Choukhmane et al. (2014)). As in Choukhmane et al. (2014), we assume that the fraction of labor income transferred to parents is (weakly) decreasing in the number of siblings—to capture the possibility of free-riding among siblings who share the burden of parent support.[6] The extent of free-riding is measured by $1-\varpi$. The magnitude of transfers towards parents depends on the replacement rate, $\psi_{t+1} = \Psi(\sigma_{t+1})$, with $\Psi(.) > 0$ decreasing and convex. This assumption exogenously captures the possibility that increasing social security benefits might *crowd out* intra-family support.

The combined amount of resources transferred in period $t+1$ by a middle-aged agent to his children and parents thus satisfies

$$T_{m,t+1} = \left(\phi n_t + \psi_{t+1}\frac{n_{t-1}^{\varpi-1}}{\varpi}\right)w_{m,t+1}. \tag{6}$$

When retired, the agent receives transfers from his n_t children, for a total amount

$$T_{o,t+2} = \psi_{t+2}\frac{n_t^{\varpi}}{\varpi}w_{m,t+2}. \tag{7}$$

[5] Implicitly, we assume that transfers from the middle-aged to their children are consumed in an interim childhood period, between the time agents are born and the time they enter the labor market — or equivalently, the cost of raising kids is a pure resource cost. Having the transfers appear in the young's budget constraint would make no substantial difference as long as the credit constraint on the young is binding (see next paragraph).

[6] See Boldrin and Jones (2002) for a model where ascending transfers are decreasing in the number of siblings as the outcome of a strategic game between siblings.

Credit Constraints. Young agents are subject to credit constraints: a young agent in period t can only borrow up to a fraction θ_t of the present value of his future labor income. Tight credit constraints are relevant in the case of China, where credit markets are still underdeveloped (see Coeurdacier, Guibaud and Jin (2014)). Domestic financial liberalization can be interpreted as an increase in θ_t over time. For simplicity, we proceed under the assumption that the credit constraint on the young is binding in all periods, and thus[7]

$$a_{y,t} = -\theta_t \frac{w_{m,t+1}}{R_{t+1}}. \tag{8}$$

Middle-Aged Savings. The assumption of log utility implies that the optimal consumption by a middle-aged agent is a constant fraction of his intertemporal wealth, which consists of disposable income (current wage net of Social Security tax and previous period debt repayment), minus transfers to children and parents, plus the present value of transfers (public and private) to be received in old age:

$$c_{m,t+1} = \frac{1}{1+\beta}\left[\left(1 - \tau_{t+1} - \theta_t - \phi n_t - \frac{\psi_{t+1} n_{t-1}^{\varpi-1}}{\varpi}\right) w_{m,t+1} + \frac{n_t^{\varpi}}{\varpi}\frac{\psi_{t+2} w_{m,t+2}}{R_{t+2}} + \frac{\sigma_{t+2} w_{m,t+1}}{R_{t+2}}\right].$$

It follows from Equation (5) that the optimal asset holding of a middle-aged individual is

$$a_{m,t+1} = \frac{\beta}{1+\beta}\left[\left(1 - \tau_{t+1} - \theta_t - \phi n_t - \frac{\psi_{t+1} n_{t-1}^{\varpi-1}}{\varpi} - \frac{\sigma_{t+2}}{\beta R_{t+2}}\right) w_{m,t+1} - \frac{n_t^{\varpi}}{\beta\varpi}\frac{\psi_{t+2} w_{m,t+2}}{R_{t+2}}\right]. \tag{9}$$

Equation (9) shows the partial equilibrium effects of fertility, the social security system, and the interest rate on the middle-aged's savings. Higher fertility (n_t) tends to reduce savings because of higher spending on children, and because of higher transfers received from children in the future. Similarly, higher social security contributions τ_{t+1} and a higher expected replacement rate σ_{t+2} reduce the amount of savings, by lowering current disposable income and increasing future wealth, respectively. The income and substitution effects associated with changes in the interest rate (R_{t+2}) cancel out under the assumption of log utility — but a lower interest rate, by increasing the present value of future transfers, implies lower savings ('wealth effect').

Fertility. Fertility decisions hinge on equating the marginal utility of bearing an additional child to the net marginal cost of raising the child:

$$\frac{v}{n_t} = \frac{\beta}{c_{m,t+1}}\left(\phi w_{m,t+1} - \frac{\psi_{t+2} n_t^{\varpi-1} w_{m,t+2}}{R_{t+2}}\right). \tag{10}$$

The right hand side is the net marginal cost, in utility terms, of having an additional child. In pecuniary terms, the net marginal cost is equal to the cost of rearing one extra child, $\phi w_{m,t+1}$, less the present value of the impact of one additional child on the amount of transfers received in old age,

[7] For a given value of θ_t, the constraint is binding if the lifetime income profile is steep enough (e small enough and/or g_A high enough). We check in our simulations that the constraint is indeed always binding.

$\partial T_{o,t+2}/\partial n_t$. In this framework, children are at once 'consumption goods' (through the parameter v) and 'investment goods' (through ψ_{t+2}). As of the end of period t, the return on children as a savings vehicle, for fixed parameters ϕ and ψ_{t+2}, depends on the ratio of wage growth to interest rate, $w_{m,t+2}/(w_{m,t+1}R_{t+2})$. A fall in this ratio reduces the returns to investing in children, and thus discourages fertility. Equation (10) can be rewritten as

$$\frac{v}{n_t} = \frac{\beta(1+\beta)\left(\phi k_t^\alpha - \frac{\psi_{t+2}}{\alpha} n_t^{\varpi-1} g_{A,t+2} k_{t+1}\right)}{\left(1-\tau_{t+1}-\theta_t-\phi n_t - \frac{\psi_{t+1}}{\varpi} n_{t-1}^{\varpi-1}\right) k_t^\alpha + \frac{\psi_{t+2}}{\alpha\varpi} n_t^\varpi g_{A,t+2} k_{t+1} + \frac{\sigma_{t+2}}{\alpha} k_{t+1}^{1-\alpha} k_t^\alpha}, \tag{11}$$

and is the first equation describing the dynamics of the two endogenous state variables $\{k_t, n_t\}$, for a given path of social security variables $\{b_t, \tau_t, \sigma_t\}_{t\geq 0}$.

2.4 Capital Market Equilibrium

The market clearing condition for capital equalizes the net asset holdings by households and the government to the aggregate capital stock:

$$L_{y,t+1}a_{y,t+1} + L_{m,t+1}a_{m,t+1} + B_{t+1} = K_{t+1}.$$

Using Equations (8) and (9) along with Equations (1) and (2), the capital market clearing condition can be rewritten as

$$\begin{aligned} & n_t g_{A,t+2} k_{t+1}\left[(1+e_{t+2}n_{t+1}) + \theta_{t+1}\frac{1-\alpha}{\alpha} + \frac{1}{1+\beta}\frac{\psi_{t+2}}{\varpi} n_t^{\varpi-1}\frac{1-\alpha}{\alpha}\right] \qquad (12) \\ = \; & \frac{\beta k_t^\alpha}{(1+\beta)}\left[\left(1-\tau_{t+1}-\theta_t-\phi n_t - \frac{\psi_{t+1}}{\varpi} n_{t-1}^{\varpi-1}\right)(1-\alpha) - \frac{\sigma_{t+2}}{\beta}\frac{1-\alpha}{\alpha} k_{t+1}^{1-\alpha} + \frac{1+\beta}{\beta}(1+e_{t+1}n_t)\, b_{t+1}\right], \end{aligned}$$

which gives the second equation describing the dynamics of the two endogenous state variables $\{k_t, n_t\}$ for a given path of social security variables. The dynamics of the economy is thus defined by the equations for optimal fertility and capital market equilibrium, (11) and (12), along with a path of social security variables $\{b_t, \tau_t, \sigma_t\}_{t\geq 0}$ that satisfies the government's budget constraint (4) in every period. We now turn to the analysis of the model at its steady state.

2.5 Steady State

The steady state is characterized by constant productivity and credit condition parameters ($e_t = e$, $g_{A,t} = g_A$, and $\theta_t = \theta$), constant social security variables ($b_t = b$, $\tau_t = \tau$, $\sigma_t = \sigma$), and constant state variables ($k_t = k$, $n_t = n$). To obtain analytical solutions for the steady state, we make the following additional assumptions.

Assumption 1 *No free-riding between siblings:* $\varpi = 1$.

Assumption 2 *No first-period labor income:* $e_t = 0$ *in every period.*

Assumption 3 $\tau < 1 - \theta - \Psi(0)$.

The first two assumptions are made for analytical convenience. Assumption 3 puts an upper bound on the social security contribution rate and ensures that individuals want a strictly positive number of children. With these assumptions we obtain the two fundamental relationships linking the interest rate and fertility in the steady state, for a given social security system.

KK Curve. The first relationship derives from the capital market equilibrium condition, Equation (12), and captures optimal savings decisions:

$$R_{KK}(n) = \frac{ng_A\left(\Theta + \psi_\sigma\right) + \sigma}{\beta\left(1 - \tau - \theta - \phi n - \psi_\sigma\right) + (1+\beta)\frac{b}{1-\alpha}}, \tag{13}$$

where $\psi_\sigma \equiv \Psi(\sigma)$ and $\Theta \equiv (1+\beta)\left(\frac{\alpha}{1-\alpha} + \theta\right)$. The KK curve is upward sloping with respect to n, encapsulating four different channels through which the rate of return is positively linked to fertility. The first is the standard effect that higher population growth (higher fertility) increases the marginal productivity of capital. The second is that a greater number of children raises total expenditures, which tends to reduce the savings of the middle-aged and drive up the rate of return (term ϕn in the denominator). This channel disappears when the cost of raising kids ϕ goes to zero. The third channel is related to ascending intra-family transfers. With higher fertility, middle-age savers expect to receive larger transfers in old age—which lowers their savings and increases the interest rate (term $ng_A\psi_\sigma$ in the numerator). This channel disappears when ψ_σ goes to zero.[8] Fourth, higher fertility increases the proportion of young borrowers and lowers aggregate savings through a composition effect. This effect is dampened when θ is small. Hence, the steepness of the KK curve with respect to n is larger whenever the cost of children (ϕ) is large, ascending altruism (ψ_σ) is large, or credit constraints are not too tight (large θ).

Partial equilibrium comparative statics on R_{KK}, holding the social security variables (b, τ, σ) and fertility n fixed, yield

$$\frac{\partial R_{KK}}{\partial \theta} > 0, \quad \frac{\partial R_{KK}}{\partial g_A} > 0, \quad \frac{\partial^2 R_{KK}}{\partial \psi_\sigma \partial g_A} > 0.$$

Tighter credit constraints (lower θ) leads to less borrowing by young individuals and thus lowers the interest rate. Higher productivity growth g_A increases the interest rate by raising the marginal productivity of capital. This effect tends to be magnified whenever ascending altruism (ψ_σ) is large. The reason is that higher growth increases the future transfers received from children in old age and therefore lowers incentives to save in middle age.

NN Curve. The second key relationship derives from Equation (11) and captures optimal fertility decisions:

$$R_{NN}(n) = \frac{ng_A\psi_\sigma + \lambda\sigma}{n\phi - \lambda\left(1 - \tau - \theta - \psi_\sigma\right)}, \tag{14}$$

[8]The simplifying assumption $\varpi = 1$ (Assumption 1) matters for the response of the interest rate to changes in fertility. With $\varpi \in (0, 1)$, the third channel ('future transfer' channel) through which higher fertility increases the interest rate is dampened, as transfers received in old age increase *less* than proportionally with the number of children. On the other hand, $\varpi < 1$ also implies that higher fertility reduces ascending transfers from the middle-aged, thereby increasing their savings and lowering the interest rate.

where $\lambda \equiv \frac{v}{v+\beta(1+\beta)}$. Differentiating with respect to n yields

$$\frac{\partial R_{NN}}{\partial n} = -\lambda \frac{(1-\tau-\theta-\psi_\sigma)\, g_A \psi_\sigma + \sigma\phi}{[n\phi - \lambda(1-\tau-\theta-\psi_\sigma)]^2} < 0.$$

Assumption 3 ensures that the NN curve is downward sloping with respect to n. There are essentially two forces driving this result. First, to the extent that children are 'investment goods', a higher interest rate tends to lower the returns from children and thus discourages fertility (term in $g_A\psi_\sigma$). This effect disappears when ψ_σ goes to zero: fertility is more sensitive to interest rate changes when ascending intra-family transfers are important. Second, insofar as children are also 'consumption goods', higher interest rates, by lowering the present value of future social security benefits and thereby intertemporal wealth, lowers consumption—and hence also fertility (term proportional to $\sigma\phi$). This effect disappears in the absence of social security $\sigma = 0$.

Inverting Equation (14), we obtain the following partial equilibrium comparative statics on the steady state level of fertility n, holding the social security system (b, τ, σ) and the interest rate fixed:

$$\frac{\partial n}{\partial \phi} < 0, \quad \frac{\partial n}{\partial v} > 0, \quad \frac{\partial n}{\partial \theta} < 0, \quad \frac{\partial n}{\partial g_A} > 0.$$

These inequalities indicate how the NN curve shifts in response to changes in some of the parameters of the model. For a given R, a higher cost of raising kids or a lower preference for children reduces fertility. A loosening of credit constraints (higher θ) reduces disposable income for consumption when middle-aged and thus the demand for children. Most relevant to our analysis is the effect of growth (g_A): a growth slowdown, holding R fixed, is associated with lower returns on children and thus lower fertility. This effect is absent when $\psi_\sigma = 0$.

Taking the social security system as given, the KK and NN curves together determine a unique steady-state equilibrium.

Proposition 1 *For a given social security scheme (b, τ, σ), a steady-state equilibrium for fertility and the interest rate exists and is unique. The state level of fertility n lies in the interval $(\underline{n}, \overline{n})$, where*

$$\begin{aligned} \underline{n} &\equiv \lambda \frac{1-\tau-\theta-\psi_\sigma}{\phi}, \\ \overline{n} &\equiv \frac{1-\tau-\theta-\psi_\sigma}{\phi} + \frac{1+\beta}{\beta(1-\alpha)} b > \underline{n}. \end{aligned}$$

Steady State under Laissez-Faire. We now characterize fertility and the interest rate (n_0, R_0) in the *laissez-faire* steady state—i.e., in the absence of a social security system ($\tau = \sigma = b = 0$). This benchmark case is useful to build intuition on the mechanisms of the model. Letting $\psi_0 \equiv \Psi(0)$, we obtain from Equations (13) and (14)

$$
\begin{aligned}
R_0 &= \left(\frac{\lambda\Theta + (\beta+\lambda)\,\psi_0}{\beta\phi(1-\lambda)}\right) g_A, \\
n_0 &= \left(\frac{\lambda\Theta + (\beta+\lambda)\,\psi_0}{\Theta + (1+\beta)\,\psi_0}\right) \frac{1-\theta-\psi_0}{\phi}.
\end{aligned}
$$

A fall in productivity growth g_A lowers the equilibrium interest rate as the marginal productivity of capital falls. However, the rate of productivity growth does not affect fertility under *laissez-faire*.[9] A loosening of credit constraints (higher θ, also implying higher Θ) leads to lower fertility and a higher interest rate (through higher borrowing of the young and lower disposable income of the middle-aged-savers). Figure 1 depicts the *laissez-faire* equilibrium under two sets of parameter values. In the top panel, children are mostly 'investment goods' and intra-family transfers are the main motive driving the demand for children ($v \to 0$). The NN curve is almost *horizontal* in that case: any movement of the KK curve will have a very large impact on fertility, and very little impact on the interest rate. To the opposite, when preference for children is the main motive for fertility ($v > 0$ and $\psi_0 \to 0$), the NN curve is almost *vertical* in the equilibrium region (bottom panel): any movement of the KK curve will have a very large impact on the interest rate, and very little impact on fertility. Hence a rise in the replacement rates which shifts the KK curve leftward will engender very different effects on the economy depending on whether intra-family transfers are important or not.

3 Social Security Policies: Steady State Analysis

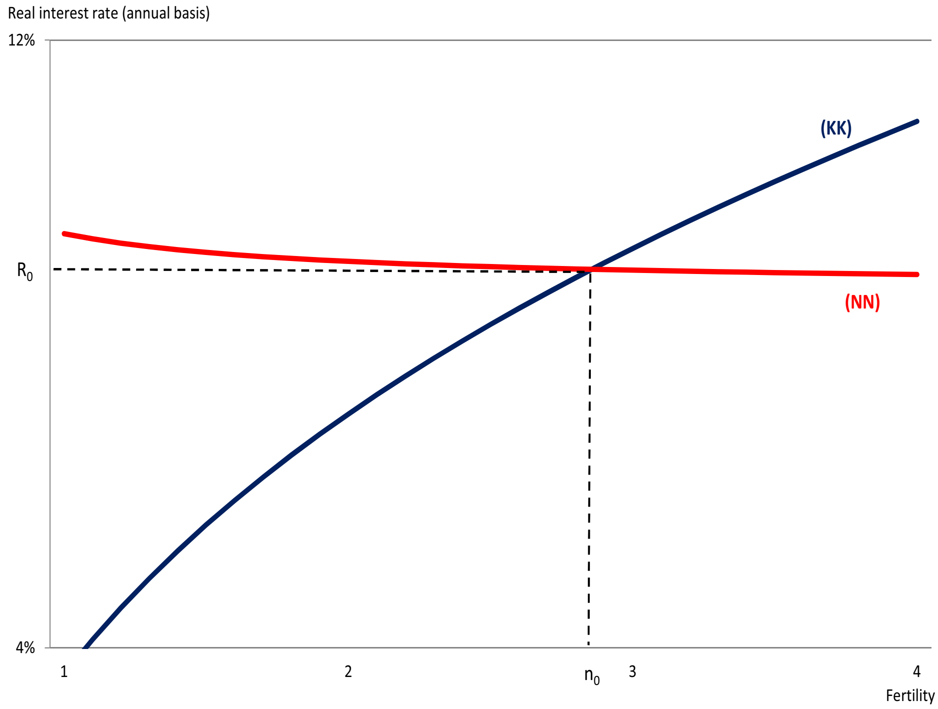
We now proceed to analyze how the introduction of a social security system affects fertility and the interest rate at the steady state in our framework. Under Assumption 2, the government's budget constraint, Equation (11), becomes

$$
\frac{\sigma}{n g_A} - \tau = \left(\frac{R}{n g_A} - 1\right) \frac{b}{1-\alpha} \tag{15}
$$

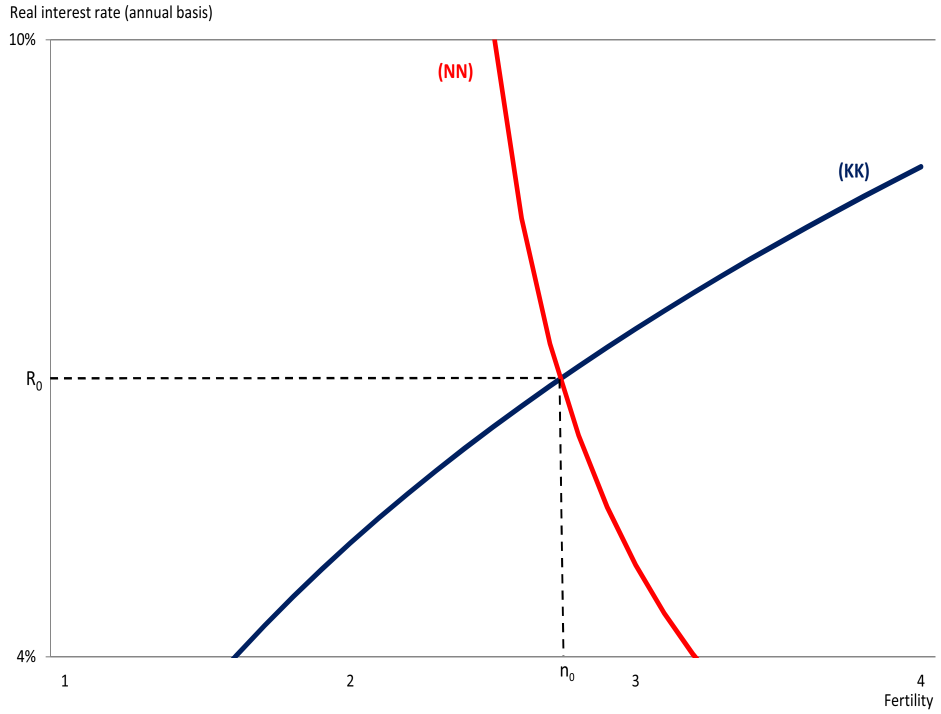
in the steady state. The left hand-side is the (primary) government deficit in a given period, where expenditures are captured by the replacement rate σ divided by the rate of demographic growth (i.e., ratio of contributing workers to retirees) times the rate of productivity growth. Higher economic growth (either through fertility or productivity) relaxes the government's budget constraint. In a PAYGO system ($b = 0$), the contribution and replacement rates must satisfy the condition $\tau = \sigma/(n g_A)$. If the government holds a positive asset position ($b > 0$), the government can use the return on its assets to finance a primary deficit if the interest rate is high compared to economic growth $R > n g_A$. On the contrary, in the case in which $R < n g_A$, the government must run a primary surplus to maintain its level of capital asset holdings to GDP constant.

[9] This result is the outcome of two counteracting effects that exactly offset each other. On the one hand, lower productivity growth reduces incentives to have children (NN curve shifts to the left). On the other hand, it also reduces the marginal productivity of capital and the interest rate (KK curve shifts downwards), which increases fertility. Note that a change in g_A leaves the ratio of wage growth to interest rate (g_A/R), and therefore the incentives to have children, unaffected.

Figure 1: Steady-State Equilibrium under Laissez-Faire.



Notes: Low v and high ψ_0. Parameters values: $\beta = 0.99$ (annual basis), $g_A - 1 = 4.5\%$ (annual basis), $v = 1\%$, $\theta = 1\%$, $\alpha = 30\%$, $\phi = 8\%$, $\psi_0 = 20\%$.



Notes: High v and low ψ_0. Parameters values: $\beta = 0.99$ (annual basis), $g_A - 1 = 4.5\%$ (annual basis), $v = 15\%$, $\theta = 1\%$, $\alpha = 30\%$, $\phi = 8\%$, $\psi_0 = 2\%$.

To determine the steady state equilibrium with social security, we take two of the social security variables (b, τ, σ) as exogenously given and let the third one adjust endogenously. In what follows, we focus on scenarios in which the government targets a given level of asset over GDP (b) and a certain replacement rate (σ), while letting the Social Security tax rate (τ) adjust.[10]

3.1 Fertility and Social Security Taxes in a PAYGO System

To start with, we focus on a PAYGO system ($b = 0$) where the government targets a certain replacement rate $\sigma = \bar{\sigma}$. Equations (13), (14) and (15) together determine the equilibrium fertility, interest rate, and contribution rate in the steady state. Letting $\psi_{\bar{\sigma}} \equiv \Psi(\bar{\sigma})$, the equilibrium $(n_{\bar{\sigma}}, R_{\bar{\sigma}}, \tau_{\bar{\sigma}})$ must satisfy

$$R_{\bar{\sigma}} = \frac{n_{\bar{\sigma}} g_A \left(\Theta + \psi_{\bar{\sigma}}\right) + \bar{\sigma}}{\beta \left(1 - \tau_{\bar{\sigma}} - \theta - \phi n_{\bar{\sigma}} - \psi_{\bar{\sigma}}\right)} \quad \textbf{(KK)}$$

$$R_{\bar{\sigma}} = \frac{n_{\bar{\sigma}} g_A \psi_{\bar{\sigma}} + \lambda \bar{\sigma}}{n_{\bar{\sigma}} \phi - \lambda \left(1 - \tau_{\bar{\sigma}} - \theta - \psi_{\bar{\sigma}}\right)} \quad \textbf{(NN)}$$

$$\tau_{\bar{\sigma}} = \frac{\bar{\sigma}}{n_{\bar{\sigma}} g_A}. \quad \textbf{(SS)}$$

As shown in Lemma A-1 in the Appendix—provided that $\bar{\sigma}$ is below a certain threshold, a steady state equilibrium exists.[11] Next, we characterize how the targeted level of the contribution rate $\bar{\sigma}$ affects the steady state. To this end, we introduce the following assumption.

Assumption 4 $|\Psi'(0)| < M\phi/g_A$, *where* M *(defined in the Appendix) is strictly positive for* $\alpha \geq \frac{\beta}{1+2\beta}$.

This assumption limits the fall in family transfers towards parents when the replacement rate increases —so that the total share of middle-aged wages dedicated to intergenerational transfers $(\tau_{\bar{\sigma}} + \psi_{\bar{\sigma}})$ rises when the replacement rate increases.

Proposition 2 *Under Assumption 4, an increase in the replacement rate* $\bar{\sigma}$ *in the neighborhood of* $\bar{\sigma} = 0$ *causes fertility to fall:* $\frac{dn_{\bar{\sigma}}}{d\bar{\sigma}} < 0$. *The contribution rate increases with an elasticity above unity:* $\frac{d \log \tau_{\bar{\sigma}}}{d \log \bar{\sigma}} > 1$. *The impact on the interest rate* $R_{\bar{\sigma}}$ *is ambiguous.*

As shown in the proof of Proposition 2, a stronger result (global rather than local) holds under the additional restriction that $v/\psi_{\bar{\sigma}} < \beta \left(\frac{\alpha}{1-\alpha} + \theta\right)^{-1}$: in that case, *any* sustainable increase in $\bar{\sigma}$ leads to a fall in fertility. It is useful to illustrate graphically the main forces leading to this result in terms of the KK and NN curves. Holding fertility constant, a rise in the replacement rate lowers aggregate savings: the KK curve shifts to the left, lowering n along the NN curve. Moreover, if more generous pension benefits crowd out intra-family support ($\Psi' < 0$), incentives to have children are

[10] The analysis of the steady state in scenarios where the government lets σ (resp. b) adjust for given (τ, b) (resp. given (τ, σ)) was included in an earlier version of the paper and is available upon request.

[11] Intuitively, the government cannot finance too high a replacement rate and still satisfy its budget constraint. As shown in the proof of Lemma A-1, uniqueness of the steady state is not always guaranteed. In case multiple equilibria exists, the comparative statics discussed in this section apply to the one with the lowest contribution rate.

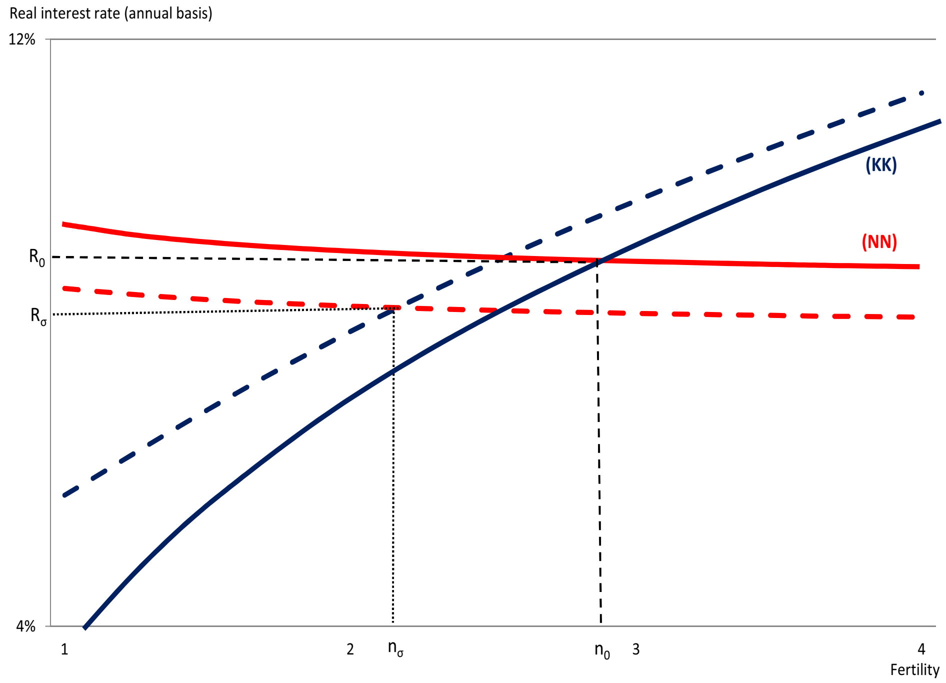
reduced: the NN curve also shifts to the left (downwards), causing a further drop in fertility. As $\bar{\sigma}$ increases and n is lower, social security taxes τ must rise for the viability of the pension program (SS). Higher taxes translate into lower savings (the KK curve shifts further to the left) and lower demand for children (the NN curve shifts further to the left). A feedback loop between endogenous taxes and fertility emerges: higher taxes lowers fertility, and a lower rate of fertility in turn feeds back onto raising taxes further (to keep the pension program sustainable)— and so on and so forth. This feedback loop from higher taxes to lower fertility amplifies the initial drop in fertility.

Figure 2 illustrates how the steady state changes when moving from laissez-faire ($\bar{\sigma} = 0$) to a target replacement rate of $\bar{\sigma} = 30\%$, for two different calibrations of the model. In the absence of intra-family support ($\psi_0 \to 0$ and $v > 0$), the NN curve is almost vertical and fertility adjusts very little to a rise in the replacement rate (bottom panel). In that case, a higher replacement rate increases the interest rate. In the opposite case where intra-family transfers are important, the NN curve is almost horizontal and fertility adjusts much more (top panel). This leads to a larger increase in contribution rates to finance the pensions. The shift of the NN curve is amplified if a higher replacement rate crowds out intra-family transfers more (i.e., when $|\Psi'(.)|$ is higher). Thus, contrary to a standard model where children are 'consumption goods', our theory incorporating intra-family support can generate large changes in fertility and contribution rates when expanding the social security system. Interestingly, more generous pension benefits can cause the interest rate to fall when intra-family transfers are important (as seen in the top panel).

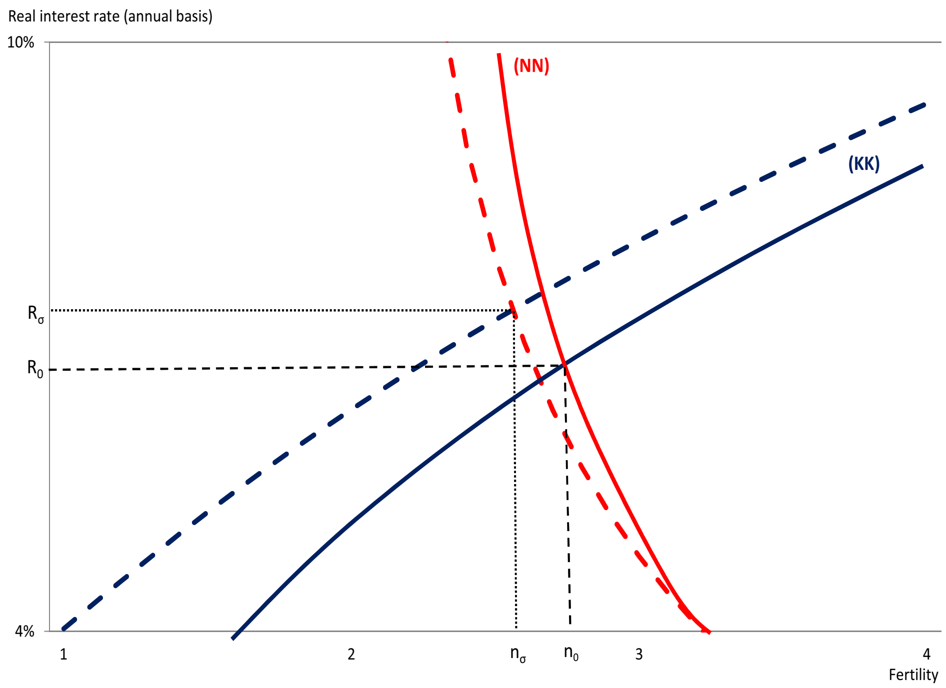
Productivity Growth and Social Security Adjustments. In sharp contrast to the laissez-faire case, fertility is not independent of productivity growth g_A under PAYGO. For a fixed replacement rate $\bar{\sigma}$, a fall in productivity growth requires an increase in the contribution rate τ to balance the budget — thus lowering disposable income, consumption and hence the demand for children. The fall in fertility forces the government to raise taxes further — inducing a further drop in fertility. The long-run equilibrium is the outcome of this feedback loop between falling fertility and rising taxes. The interest rate will be lower under slower productivity growth, due to the fall in the marginal product of capital, even more so when intra-family transfers are important. The reason is that the reduction in fertility exerts upward pressure on savings (net of investment) and thereby pushes down the interest rate further. Most importantly, the impact of a fall in productivity growth on fertility will be magnified if intra-family support is important as lower growth reduces the benefits from children.

Financial Liberalization and Social Security Adjustments. Domestic financial liberalization can be interpreted in our framework as a relaxation of credit constraints (i.e., a rise in θ). The long-run implications of such reforms for fertility and contribution rates are qualitatively similar to the ones induced by a rise in the replacement ratio. A rise in θ lowers savings — the KK curve shifts upwards and fertility falls along the NN curve. Moreover, it also lowers disposable income and middle-aged consumption, causing the NN curve shift to the left, depressing fertility even further. In order to maintain a fixed replacement rate $\bar{\sigma}$, taxes have to rise, feeding back onto lower fertility— and so on an so forth. Moreover, the flatter the NN curve (i.e., intra-family support is important), the larger will be the drop in fertility caused by domestic financial liberalization.

Figure 2: From Laissez-Faire to PAYGO.



Notes: Low υ and high ψ_0. Solid lines are for the Laissez-Faire ($\sigma = \tau = 0$) and dashed lines for PAYGO with $\bar{\sigma} = 30\%$. Parameters values: $\beta = 0.99$ (annual basis), $g_A - 1 = 4.5\%$ (annual basis), $\upsilon = 1\%$, $\theta = 1\%$, $\alpha = 30\%$, $\phi = 8\%$, $\psi_0 = 20\%$, $\psi_{\bar{\sigma}} = \psi_0.e^{-0.5\bar{\sigma}}$.



Notes: High υ and low ψ_0. Solid lines are for the Laissez-Faire ($\sigma = \tau = 0$) and dashed lines for PAYGO with $\bar{\sigma} = 30\%$. Parameters values: $\beta = 0.99$ (annual basis), $g_A - 1 = 4.5\%$ (annual basis), $\upsilon = 15\%$, $\theta = 1\%$, $\alpha = 30\%$, $\phi = 8\%$, $\psi_0 = 2\%$, $\psi_{\bar{\sigma}} = \psi_0.e^{-0.5\bar{\sigma}}$.

3.2 PAYGO in a Small Open Economy

We now briefly examine the situation in which the country is a small open economy, taking the world interest rate as given. While this assumption is arguably questionable in the case of China, it is useful to build intuition regarding some key differences between an economy under autarky and one that is integrated globally. The small open economy example corresponds to the limiting case where the interest rate does not adjust at all. Analyzing this polar case is also helpful to understand the dynamics of fertility and taxes following financial integration.

The dynamics of the model for a small open economy (SOE) are characterized by Equations (4) and (11), with the capital-effective-labor ratio k_t determined by the world interest rate in every period. In the steady state, the KK curve is horizontal, at the level of the world interest rate R^*. Under PAYGO ($b = 0$) and for a fixed replacement rate $\bar{\sigma}$, the equilibrium fertility and contribution rate $(n_{\bar{\sigma}}, \tau_{\bar{\sigma}})$ must satisfy

$$\frac{n_{\bar{\sigma}} g_A \psi_{\bar{\sigma}} + \lambda\bar{\sigma}}{n_{\bar{\sigma}}\phi - \lambda\left(1 - \tau_{\bar{\sigma}} - \theta - \psi_{\bar{\sigma}}\right)} = R^* \quad \textbf{(NN)}$$

$$\tau_{\bar{\sigma}} = \frac{\bar{\sigma}}{n_{\bar{\sigma}} g_A}. \quad \textbf{(SS)}$$

Comparative statics. In the high ψ_0 case, where a rise in the replacement ratio leads to a fall in interest rate in the autarky steady state (top panel of Fig. 2), the impact on fertility of an increase in $\bar{\sigma}$ is *stronger* in the SOE scenario than in a closed economy. Under financial integration, the KK curve is horizontal: any shift to the left of the NN curve due to higher taxes generates a larger fall of fertility as interest rates do not adjust downwards. Thus, if family transfers are important (high ψ_0), a rise in social security benefits will lead to a higher tax rise under financial integration. Fertility also reacts *more* to a fall in productivity growth rate g_A in an open economy. The reason is that in a closed-economy setting, the fall in fertility due to lower growth is dampened by a fall in the interest rate. When this dampening effect is shut off, fertility falls by more and is thus matched by a greater increase in taxes to balance the budget. The impact of a productivity slowdown on fertility is also magnified when intra-family transfers are important.

4 Application: Policy Reforms in China

We next examine the dynamic impact of important policy reforms and economic development in China, with a focus on pending fertility policies and pension reforms. As our theoretical analysis has demonstrated that capital market reforms and an eventual growth slowdown interact with the social security system, we also take these highly relevant scenarios into consideration. Before turning to the experiments, we first provide more details on the main pillars of the recent reform package in China.

4.1 The Chinese Context

As of 2005, most of the Chinese population is not covered by social security—notwithstanding an early pension reform in the late nineties (see Song and Yang (2012) and Dunaway and Arora (2007)). In urban areas, 48% of employees are covered in 2005 (Dunaway and Arora (2007)) and coverage is highly concentrated among workers at state and collectively owned enterprises—who account for a diminishing share of total employment. Most of the fast-growing private sector workforce, including China's population of rural migrants, remains uncovered. In the countryside, where old-age poverty is most severe, formal retirement protection is virtually nonexistent. Rural workers are categorically excluded from the basic pension system.[12] Over the past few years, a series of government reform initiatives were launched to push China's retirement system to extend coverage nation-wide. The government has stepped up efforts to broaden participation in the basic pension system beyond its original base among workers at state and collectively owned enterprises—and has set a goal of achieving universal coverage by 2020. This expansion is under way, though far from reaching the end goal.

Traditionally, in the absence of social security, children supported parents in old age. Not only are intergenerational support a social convention and a family institutional norm, it is also stipulated by Constitutional law: "children who have come of age have the duty to support and assist their parents" (article 49). The importance of intergenerational transfers is fully borne out by the data (see a more detailed description in Choukhmane et al. (2014) and Cai et al. (2012)). According to the Chinese census data in 2005, family support is on average the main source of income for the elderly (65+) population. It is twice as large as pension income and wealth income combined. The on-going pension reform aims at replacing traditional family support, which is under pressure following the implementation of the one-child policy in the early 1980s.[13] The drastic reduction in fertility means that the previous generation of parents are able to rely less on children's support for their retirement. However, the rapid ageing of the Chinese population also hinders the plan to expand the pension system.

Recently, The Third Plenum reform package was announced in November 2013. One of the main pillars was the relaxation of the one-child policy. As a first step, it stipulated that couples in which one member is an only child are allowed to have two children. This rule applies to most of the urban couples currently planning to have children. A second main pillar pertained to financial market reforms. This included accelerated capital account liberalization, and domestic interest rate liberalization. Indeed, despite the rapid economic development in the last few decades, China has seen far slower financial development. For instance, even with recent developments of the mortgage markets in the late nineties, household debt and mortgage debt (as a share of GDP) are still remarkably low—amounting to only 12% and 11% of GDP in 2008, respectively, about 1/8 of the levels observed in the U.S..[14] These imminent reforms—spanning from fertility policies, pension reforms, to finan-

[12] In 2005, only 4.7% of the elderly's income came from pension in rural areas. This is in contrast to the 45.4% in urban areas, with a large heterogeneity among urban workers (Cai et al. (2012)).

[13] The one-child policy is strictly enforced in urban areas and less so in rural areas. According to UHS data, 96% of all urban households who had children had one child. The urban fertility rate fell from a bit above 3 in 1970 to close to 1 by 1982.

[14] In 2008, U.S. household debt as a share of GDP (resp. mortgage debt as a share of GDP) are 95% (resp. 86%).

cial market development—along with a likely growth slowdown— are particularly important in the context of China. Given their impact on social security, we conduct in what follows a comprehensive analysis of their joint impact in the most likely of circumstances.

4.2 Steady State and Calibration

In the policy experiments that follow, the economy always starts from a well-defined initial steady-state under endogenous fertility. The calibration of the initial steady state under autarky is summarized in Table 2, along with values for the structural parameters. The initial period corresponds to 1970, just prior to the implementation of fertility policies in China. While our exercise is not meant to be quantitative, structural parameters are calibrated using reasonable values based on household micro data (Urban Household Survey (UHS) for income and consumption data and China Health and Retirement Longitudinal Study (CHARLS) for data on family transfers; see Choukhmane et al. (2014) for a detailed description of the data). We focus on *urban* households for our calibration because data are mostly available for urban workers and because fertility policies have been essentially binding in urban areas. However, note that implementing social security in rural areas would require an even larger adjustment in contributions since rural workers are currently barely covered by the pension system.

Technology. One period corresponds to a generation, approximately 23 years. We set the capital share to a standard value of 0.3. The growth rate of labour augmenting productivity is set to 4.5% on an annual basis, the average in China over the last thirty years.

Capital Market Frictions. Our model features two types of financial frictions: (1) credit constraints and (2) capital controls (financial autarky). Initial credit constraints need to be quite stringent in order to account for the very low level of household debt in China until 2000 (only 4% of GDP in 2000) and the very low level of borrowing by young households in the early nineties. A value of $\theta = 1\%$ allows us to match the saving rate of individuals under 25, as observed in UHS data in 1992 (see Coeurdacier, Guibaud and Jin (2014)). This low initial value of θ contributes to depress the interest rate in the autarky steady state. In our benchmark experiments, we assume that China remains under financial autarky throughout, but we also perform experiments where China lifts its capital controls ('financial integration'). In the latter scenario, we assume that the exogenous world interest rate R^* is slightly above the interest rate prevailing in the Chinese autarky steady state—an assumption based on the observed direction of capital flows from China to the rest of the world over the last twenty years.[15] The world interest rate faced by China under financial integration is chosen to be 10% (on an annual basis)—which corresponds to a realistic calibration of the model for a financially developed market, such as the U.S..

Intergenerational Transfers. The replacement rate $\bar{\sigma}$ in the initial steady state is set to 30%, which roughly corresponds to the aggregate urban replacement rate in China in the late nineties,

Data on household debt are taken from McKinsey Global Institute and Federal Reserve. Mortgage debt data comes from Hypostat (2010) and the Chinese National Bureau of Statistics.

[15] In reality, since the early nineties, China stands in between financial autarky and complete financial integration, but our framework is not suitable to accommodate partial integration.

Table 2: Benchmark Calibration: Structural Parameters and Steady State Values

Benchmark Calibration		
Parameter	*Calibrated Value*	*Target/Description/Data Source*
β	0.99	Annual basis
α	30%	Capital share
$g_A - 1$	4.5%	Annual basis. Productivity growth rate (1980-2010)
θ	1%	Saving rate of the 20-25 (UHS)
v	0.136	Targeted to match the fertility rate of 3 in 1970 (Census)
ϕ	8%	Average education expenditures over income (UHS)
ψ_0	10%	Choukhmane et al. (2014), Curtis et al. (2011)
ω	0.7	Elasticity of transfers to elderly w.r.t the nb. of siblings (CHARLS)
δ	0.3	Semi-elasticity of transfers to elderly w.r.t replacement rate
$\bar{\sigma}$	30%	Aggregate initial replacement ratio adjusted for coverage (UHS)
b	0	PAYGO simulation
Alternative Calibration (without family transfers towards elderly)		
ψ_0	0.1%	
v	0.20	Targeted to match the fertility rate of 3 in 1970 (Census)
Autarky Steady State under Benchmark Calibration		
$n_{\bar{\sigma}}$	1.50	Fertility of 3
$R_{\bar{\sigma}} - 1$	9.12%	Annual basis
$\tau_{\bar{\sigma}}$	7.27%	
World Interest Rate under Financial Integration		
$R^* - 1$	10%	Annual basis. Based on a reasonable calibration for the U.S.

once adjusting for the extent of social security coverage.[16] Transfers received by elderly parents from their n_t children in period $t+2$ are assumed to have the functional form

$$T_{o,t+2} = \Psi(\sigma_{t+2}) \left(\frac{n_t^{\varpi}}{\varpi} \right) w_{m,t+2} \equiv \psi_0 e^{-\delta \sigma_{t+2}} \left(\frac{n_t^{\varpi}}{\varpi} \right) w_{m,t+2},$$

where ψ_0 measures the overall degree of ascending altruism in China, δ measures the strength of the crowding-out effect from social security benefits on intra-family transfers, and $1-\varpi$ measures the intensity of free-riding among siblings in providing support to their parents. Using data on transfers towards parents from the China Health and Retirement Longitudinal Study (CHARLS), Choukhmane et al. (2014) find that the elasticity of transfers by middle-aged individuals to their parents with respect to the number of siblings (which corresponds to $\varpi - 1$) is between -0.3 and -0.4, leading us to set $\varpi = 0.7$. Direct empirical measures of δ are not available, and thus we perform sensitivity analysis with respect to this parameter in Section 4.6. In our benchmark calibration, we set $\delta = 0.3$—i.e., a 10 percentage points increase in the replacement rate would cause ascending

[16] Song and Yang (2012) document a replacement rate of about 60-70% in state-owned enterprises, covering 44% of the urban population in 1992. In the late nineties, at the time of the first pension reform, the Chinese Statistical Yearbook gives an aggregate replacement (adjusted for coverage) slightly above 30%, while UHS data gives a number close to 25% in 2002. Note that including rural areas would lower our initial replacement rate: any reform to increase benefits/coverage would require an even larger rise in contributions to sustain the pension system.

intra-family transfers to drop by about 3%.[17] Given ϖ, δ and $\bar{\sigma}$, the ascending altruism parameter ψ_0 is set to 10%: a middle-aged individual with two siblings transfers about 11% of his wage income to his parents, in the ballpark of estimates from micro data (see Choukhmane et al. (2014), Curtis et al. (2011)).[18] Finally, based on the Urban Household Survey, transfers to children for education purposes amount to about 8% of household income, therefore ϕ is set to 8% (see Banerjee et al. (2013) for a similar value).

Preferences. We set $\beta = 0.99$ on an annual basis. The parameter v, which governs the preference for children, is set to 0.136 in order match the fertility rate in China before the implementation of fertility controls (roughly 3 children per household in 1970). In the corresponding initial steady state, the contribution rate $\tau_{\bar{\sigma}}$ necessary to finance the social security system under PAYGO ($b = 0$ and $\bar{\sigma} = 0.3$) is equal to 7.3%.[19]

Alternative Calibration. In order to illustrate the role of ascending intra-family transfers, we run some of the experiments under an alternative calibration, in which we set $\psi_0 = 0.1\%$, and adjust the preference for children v accordingly to generate the same fertility rate in the initial steady state—yielding $v = 0.20$. Relative to the benchmark calibration, this corresponds to a situation where children are more consumption goods, and less investment goods.

In the policy experiments that follow, we perform numerical simulations assuming that the government targets a certain path of social security benefits $\{\sigma_t\}$. The objective is to assess how the economy would adjust to shocks if the generosity of the existing system is increased (or at least maintained). The model dynamics are governed by Equations (4), (11) and (12). For most experiments we make the PAYGO assumption, $b_t = 0$. Robustness checks with respect to model parameters and other social security schemes (trust fund) are relegated to Section 4.6.

4.3 Fertility and Social Security Policies

We focus first on the two main pending reforms on the policy agenda in China: (1) the relaxation of the one-child policy and (2) the increase in social security benefits.

The One-Child Policy and its Relaxation. We first investigate the dynamics of the model as the government relaxes the one-child policy for the coming generations of parents. The economy starts of at $t = 1$ (1970) from its autarkic steady-state with endogenous fertility. We assume that fertility is constrained to one child per household for one generation in period $t = 2$ (corresponding

[17] Cai, Giles and Meng (2006) provide an indirect measure of δ for China. In the cross section, they find that when elderly income (before family transfer) increases by 1 yuan, support from children falls by 0.25 yuan (see also Chapter 3 in Cai et al. (2012) and Cox et al. (2004) for similar evidence in the context of the Philippines). Cox et al. (2004) also provide evidence that the presence of retirement benefits crowds out ascending transfers from children.

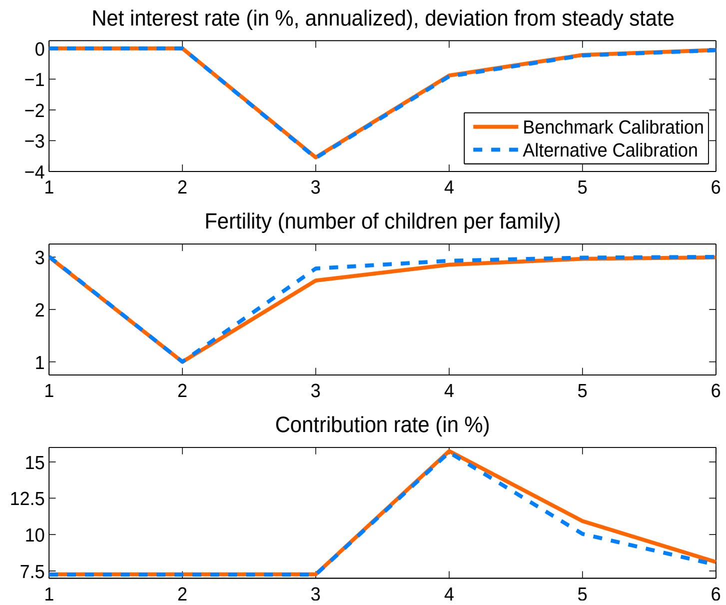
[18] Data on pecuniary transfers from CHARLS would imply a lower ψ_0, around 5-6% as many transfers towards elderly, such as co-residence or healthcare, are non-pecuniary. Restricting our attention to children that are not living with their parents, we find that they transfer about 10% of their wage income, in line with our calibration. Choukhmane et al. (2014) also show that transfers have to be an order of magnitude larger than observed pecuniary ones to match age-saving profiles in China.

[19] This is comparable to the current 13% payroll tax paid by employers given that about half of the urban population is covered.

to the mid-nineties), and then fully relaxed at $t = 3$ (i.e in the very near future). Figure 3 displays the dynamics of the interest rate, fertility and taxes.

The one-child policy leads to a large rise in savings (net of investment) and a required tax hike in $t = 4$ when the one-child generation becomes the main contributor to social security. The large rise in savings occasions a large fall in the interest rate, followed by a gradual rise– as the fertility constraint is relaxed. Required tax hikes are large and long-lasting, as fertility is persistently below steady-state after the relaxation of the policy (in period $t = 4$, fertility is around 2.5 for a tax rate of 15%). These results derive from two feedback loops. As per children being *consumption goods*—higher tax rates feed into lower consumption and lower fertility– and thus back onto higher taxes. As per children being *investment goods*—the relaxation of the policy increases interest rates by raising the marginal productivity of capital, hence restricting the rise in fertility. Without this latter feedback effect, fertility reverts faster to its steady state— as shown in the *alternative* calibration in which intra-family transfers towards the elderly are absent.

Figure 3: The One-Child Policy and its Relaxation.



Notes: The solid (resp. dashed) line corresponds to the *benchmark* (resp. *alternative*) calibration. Parameters values are shown in Table 2. The one-child policy is implemented at t=2 and relaxed at t=3.

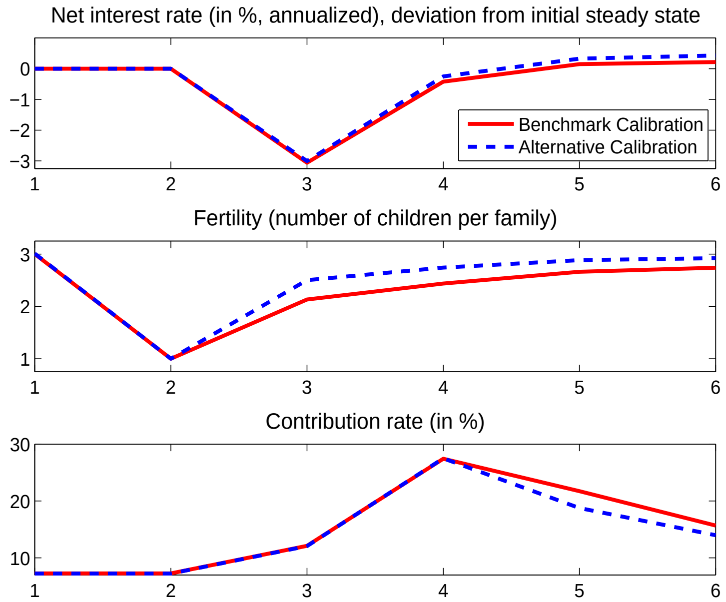
A Rise in the Replacement Ratio. In addition to relaxing the one-child policy, China is currently expanding the coverage of its social security system. While the government's objective is to attain full coverage, the corresponding targeted replacement rate, intended to be lower than the current one, is difficult to assess. We assume that all urban workers will be covered under the reformed system (thus doubling the coverage), at a replacement rate about 15-20 percentage points below its current level.[20] This reform is tantamount to a rise in the *aggregate* replacement ratio in our model.

[20]This is consistent with the early pension reforms of the late nineties. Starting at very high level for covered workers, the replacement ratio recently fell in China by 10 to 15% depending on the region/sector, while coverage has

The experiment we analyze involves a rise in $\bar{\sigma}$ from its initial value of 30% to 50% at $t = 3$, which coincides with the period when the one-child policy is relaxed. Results for the combination of these two policies are shown in Figure 4 and constitutes our *benchmark* experiment.

While the relaxation of fertility constraints may help alleviate the fiscal pressure brought about by the one-child policy, our simulations show that fertility remains permanently lower after the social security reforms—and particularly so if intra-family transfers are important. Fertility is around 2 at $t = 4$ in our benchmark calibration (and only 2.4 one generation after) compared to more than 2.5 at $t = 4$ (and 2.9 at $t = 5$) in the *alternative* calibration.

Figure 4: Benchmark Experiment: Relaxation of the OCP and a Rise in the Replacement Ratio.



Notes: The solid (resp. dashed) line corresponds to the *benchmark* (resp. *alternative*) calibration. Parameters values are shown in Table 2. The one-child policy is implemented at t=2 and relaxed at t=3. The replacement ratio is increased permanently at t=3 from $\bar{\sigma} = 30\%$ to $\bar{\sigma} = 50\%$.

On top of the effects caused by the one-child policy, a rise in replacement rates triggers a rise in interest rates which reduces fertility. Fertility is therefore particularly depressed if intra-family transfers are important—since it becomes more sensitive to the rise in interest rates. Moreover, the rise in replacement rates crowds out intra-family transfers, depressing further fertility. Finally, taxes must rise to stabilize the government budget as replacement rates are higher and fertility lower. The required tax hike exerts additional continual downward pressure on fertility. In the long term, as the generation of only child goes into retirement, taxes fall but remain persistently higher (taxes increase to roughly 27% at $t = 4$ and stay close to 20% one generation after, well above the initial steady state). In sum, despite the relaxation of the one-child policy, taxes remain very high and fertility permanently depressed. This dynamic effects of the policy reform would be absent in a model with

been increasing—such that overall Chinese households are better insured for their old age (see ISSA Report (2013)). Estimates of the World Bank suggest to set a replacement rate of 40% for the sustainability of the system (see Sin (2005)) while the early pension reforms are targeting 60%.

exogenous fertility (or largely mitigated in a model without intra-family support).

We are interested in the dynamic effects of reforms following the one-child policy while the government is at the same time trying to raise social security benefits. Thus, unless stated otherwise, all subsequent simulations include the shocks to fertility (at $t = 2$) and the increase in the replacement rate (at $t = 3$)— as in our *benchmark* experiment.

4.4 Capital Markets Reforms

We now investigate how other plausible policy reforms in China interact with social security policy and the relaxation of the one-child policy. In particular, two reforms of the capital markets are of particular relevance: (1) financial liberalization within China and (2) financial integration with the rest of the world. For these simulations, the benchmark calibration is considered. For comparison purposes, also shown is the path of variables without these additional reforms (*benchmark* experiment).

Financial Liberalization. In our framework, domestic financial liberalization can be understood qualitatively as a rise in θ. It can also be more broadly interpreted as any reform that aims to raise the latently repressed interest rate.[21] The experiment we analyze consists in increasing θ from its initial value of 1% to 10% at $t = 3$ (when the one-child policy is relaxed and social security reforms are implemented).[22] Results are shown in Figure 5.

Compared to the benchmark experiment without financial liberalization, fertility is further depressed (below 2 at $t = 4$ and $t = 5$) and taxes persistently higher. As the rise in θ lowers savings and exerts upward pressure on the interest rate, the rise in fertility is further contained after the relaxation of the one-child policy. This requires the government to set higher taxes to sustain its pension reform, depressing fertility further (and so on and so forth).[23] Thus, the policy reforms face an important trade-off: the persistence of the one-child policy shock makes the government's intentions to liberalize its financial markets and implement its pension reforms potentially problematic. Note that such a trade-off would be absent in a model with exogenous fertility (and largely mitigated in a model where intra-family transfers towards old-age are unimportant).[24]

Financial Integration. In our framework, financial integration can be understood as China lifting its capital controls, so that the return to capital faced by domestic households and the world interest rate are equalized (Small Open Economy case). We assume that when liberalizing its capital account at $t = 3$, China opens up to a world with a higher interest rate $R^* = 10\%$ on an annual basis. Results

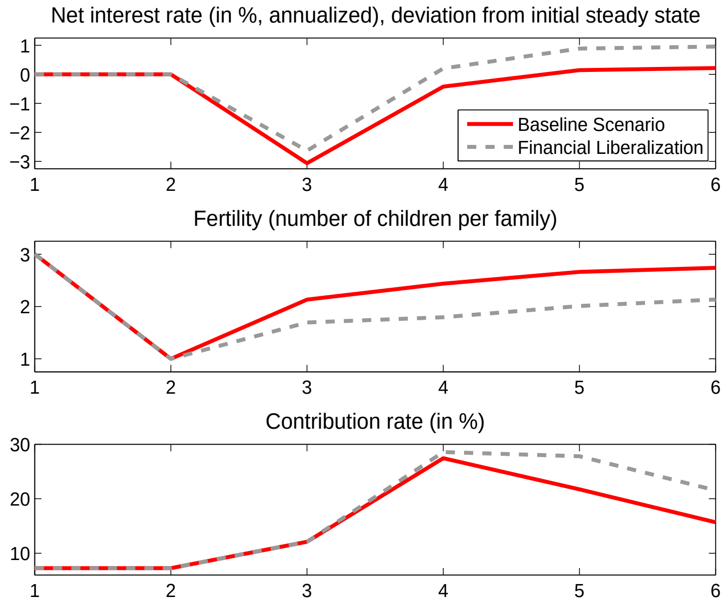
[21] Our financial friction depresses interest rates and drives a wedge between the frictionless interest rate and the equilibrium one. This is qualitatively equivalent to a price distortion due to regulated interest rates. A difference though is that our friction does not generate budget surplus for the government. This implies that, unlike our experiment, a fall in price distortions (interest rate liberalization) would be even more harmful for the government's budget.

[22] Strictly speaking, such a reform implies that households are allowed to borrow ten times more. Such a shock can appear large but note that over the period 2000-2010, household debt (as a share of GDP) in China was multiplied by 3 following, in particular, the mortgage reform in the late nineties. See Coeurdacier, Guibaud and Jin (2014).

[23] Note that the equilibrium interest rate is almost unaffected by financial liberalization, despite the relaxation of credit constraints, due to the countervailing effect of lower fertility and higher taxes.

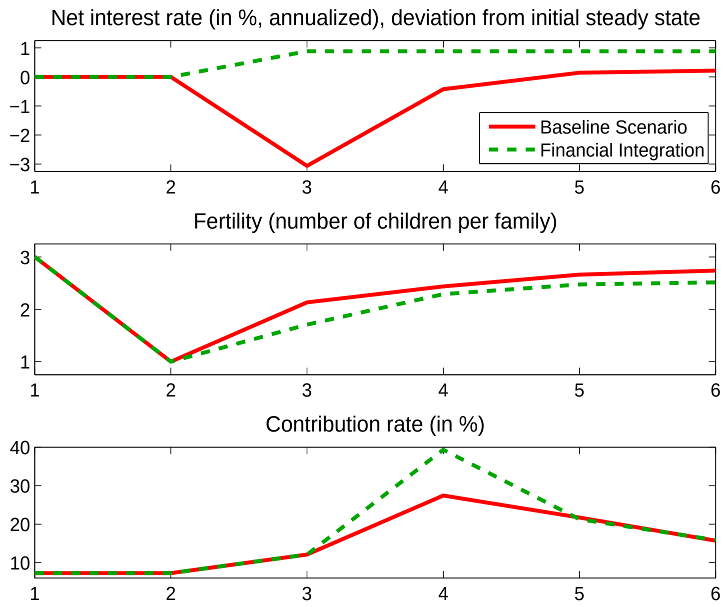
[24] In our *alternative* calibration, fertility reverts back much faster and is less affected by the financial liberalization (not shown). This is so because family-transfers are essential to make fertility interest rate sensitive.

Figure 5: Domestic Financial Liberalization.



Notes: The dashed (resp. solid) line corresponds to the experiment with domestic financial liberalization (resp. the *benchmark* experiment without financial liberalization). Parameters values are shown in Table 2. The one-child policy is implemented at t=2 and relaxed at t=3. The replacement ratio is increased permanently at t=3 from $\bar{\sigma} = 30\%$ to $\bar{\sigma} = 50\%$. Domestic financial liberalization (a rise of θ to 10%) occurs at $t = 3$.

Figure 6: Financial Integration.



Notes: The solid (resp. dashed) line corresponds to the experiment with financial integration (resp. the *benchmark* experiment without financial integration). Parameters values are shown in Table 2. The one-child policy is implemented at t=2 and relaxed at t=3. The replacement ratio is increased permanently at t=3 from $\bar{\sigma} = 30\%$ to $\bar{\sigma} = 50\%$. Financial integration occurs at $t = 3$.

of the simulation under the benchmark calibration are shown in Figure 6, together with the autarky paths in our benchmark experiment.

We observe first that the rise in interest rates following integration triggers a fall in fertility (and higher taxes). Second, the mechanism that tends to ease social security pressure under autarky —whereby lower interest rates feeds into higher fertility—–is now absent. Fertility is therefore even lower in an open economy, which implies substantially larger tax rates (at $t = 4$, taxes reach almost 40%). The impact of the one-child policy shock, together with an increase in pension coverage, is significantly more costly if capital controls are lifted.[25] Thus, China faces another policy trade-off: opening up capital markets will exert pressure on the sustainability of the PAYGO system that the government wishes to expand.

4.5 Alternative Growth Scenarios

Growth has been remarkably high in China over the last thirty years, facilitating the recent expansion of social security coverage. Yet a growth slowdown is a likely scenario. We now examine the impact of the main policy experiments against the background of a permanent growth slowdown.

A Permanent Slowdown in Growth. Our next simulation superimposes to the benchmark experiment a *permanent* fall in productivity growth from an annual rate of 4.5% to 2.5% (occurring at $t = 3$). Results are displayed in Figure 7. The path from the benchmark experiment (without growth slowdown) is shown for comparison purposes.

Compared to the benchmark case, the fall in the growth rate leads to a permanently lower fertility rate and interest rate. As a consequence, higher taxes are required to balance the budget —in turn restraining the rise in fertility ensuing the relaxation of the one-child policy. The combined effects of aging (of the generation of the only child) and the fall in growth exerts a very high pressure on the viability of the pension system — forcing taxes to rise progressively from 19% at $t = 3$ to 42% at $t = 5$, before reverting slightly as the generation of only child ages. Due to the feedback loop from higher taxes to lower fertility, fertility stays permanently depressed, significantly below 2 at $t = 4$ and $t = 5$. The predicament of these circumstances cast serious doubt on whether China is able to pursue its pension reforms.

It is also important to note that the effects of growth are strongly amplified by the presence of intra-family transfers: lower growth implies a lower return from children—thus depressing fertility. When running the same experiment under the alternative calibration (with $\psi_0 = 0.1\%$), we find that fertility converges much more rapidly (close to 2.2 at $t = 3$ and 2.5 at $t = 4$), as it is much less sensitive to growth. This tends to place less pressure on the social security system.[26]

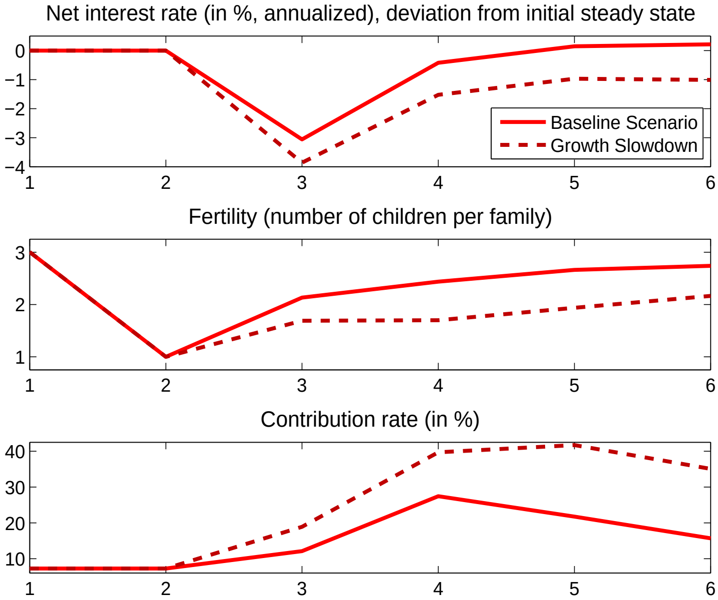
Capital Market Reforms in the Event of a Growth Slowdown. We now investigate the dynamics of the model with domestic financial liberalization (resp. financial integration), occurring at $t = 3$, when the economy concurrently slows down.[27] It turns out that the social security system

[25] Note that our findings constitute an upper-bound of the fiscal costs of financial integration since China, being a fairly large economy, would induce the world interest rate to adjust somewhat.

[26] The key parameter determining the response of fertility to a change in growth over the interest rate (g_A/R) is ψ_σ. When intra-family transfers are important (high ψ_σ), a growth slowdown triggers a larger fall in fertility.

[27] Domestic financial liberalization is modeled as before—a rise in θ from its initial value of 1% to 10%.

Figure 7: A Permanent Slowdown in Growth.



Notes: The dashed (resp. solid) line corresponds to the experiment with a permanent slowdown in growth (resp. the *benchmark* experiment without a permanent slowdown in growth). Parameters values are shown in Table 2. The one-child policy is implemented at t=2 and relaxed at t=3. The replacement ratio is increased permanently at t=3 from $\bar{\sigma} = 30\%$ to $\bar{\sigma} = 50\%$. At $t = 3$, productivity growth falls permanently from 4.5% to 2.5%.

with a high replacement ratio of $\bar{\sigma} = 50\%$ is *no longer sustainable* and cannot be financed. The upward pressure on interest rates driven by capital markets reforms amplifies the fall in fertility, leading to an even larger rise in taxes, feeding back into even lower fertility. The government ends up not being able to finance its increased spending on social security, and the system collapses.[28] The impact of a permanently slower growth is thus very costly for social security sustainability if the government simultaneously liberalizes financial markets.

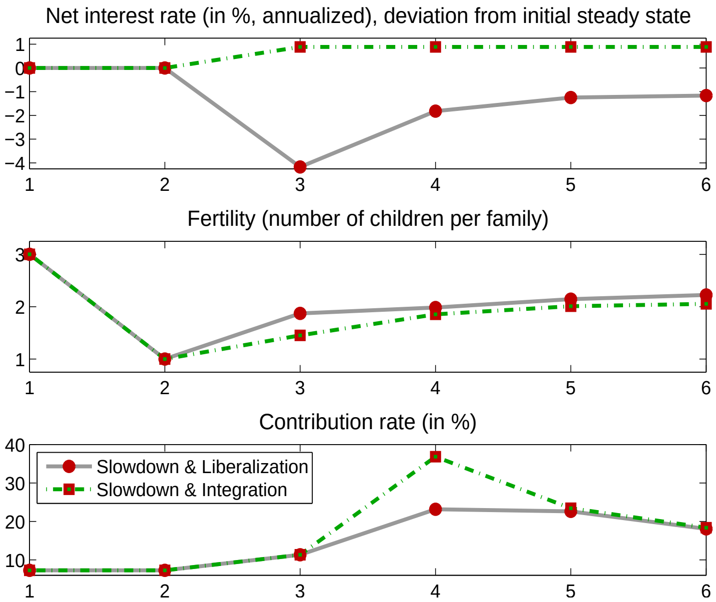
This implies that the Chinese government can face a serious tension between expanding the generosity of the existing pension system and deepening its capital markets. Even maintaining the current social security program becomes difficult: if the replacement ratio remains at its initial value of 30% (Fig. 8), there is still strong downward pressure on fertility and upward pressure on taxes—in both cases of domestic financial liberalization (solid line), or lifting capital controls (dashed line). Financial integration presents greater challenges since the counteracting effect of a fall in interest rates is absent in the open economy and thus the drop in fertility is larger.[29]

Extending the analysis under financial integration to a large open economy model would imply considerable international consequences: the world interest rate could be persistently depressed under this scenario–due to low growth, high taxes and low fertility in China.

[28] This echoes Lemma A-1 which states that social security is sustainable up to certain threshold in the replacement rate. Again, intra-family transfers are essential for this result, such a bankruptcy of the social security system does not happen in the *alternative* calibration.

[29] Alternatively, one can calculate the maximum replacement ratio that China can implement under such a growth scenario *while* implementing domestic financial liberalization (resp. financial integration); we find $\bar{\sigma}$ very close to 45% in both cases.

Figure 8: Capital Markets Reforms in the Event of a Growth Slowdown (Without Pension Reforms).



Notes: The solid (resp. dashed) line corresponds to the experiment with a permanent slowdown in growth and domestic financial liberalization, i.e., a rise of θ to 10% (resp. the experiment with a permanent slowdown in growth and financial integration). Parameters values are shown in Table 2. The one-child policy is implemented at t=2 and relaxed at t=3. The replacement ratio is kept constant throughout at $\bar{\sigma} = 30\%$. Domestic financial liberalization (resp. financial integration) occurs at $t = 3$. At $t = 3$, productivity growth falls permanently from 4.5% to 2.5%.

4.6 Sensitivity Analysis and Discussion

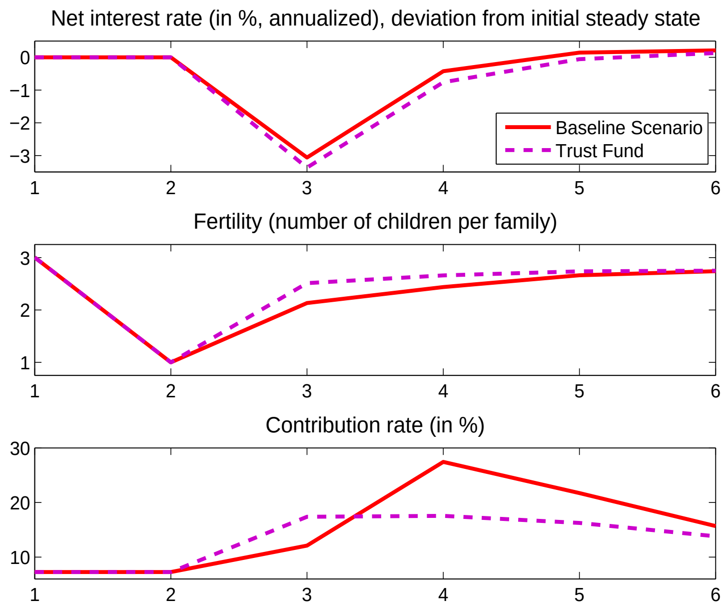
Alternative Social Security Schemes. We investigate how our results depend on the social security scheme under consideration. An alternative scenario moving away from a PAYGO system is the presence of a trust fund, whereby a government accumulates positive assets ($b > 0$) to smooth negative demographic (or productivity) shocks over time, by partly selling its stock of assets. We assume that the government in China starts out initially without assets but increases contribution taxes of the current generation of middle-aged and accumulates assets up to $b = 3.7\%$ at $t = 3$—before the generation of only children ages. Such a value for b allows the government to share the tax burden between the current and future generation of workers just about equally.[30] The government is then assumed to exhaust its trust fund in two generations, reaching $b = 0$ at $t = 6$.

To build intuition, it is helpful to consider the impact of the existence of the trust fund on the autarky steady state. With a higher b, the interest rate is lower (KK shifts upwards) and the fertility rate higher; conversely, reducing the size of the trust fund raises the interest rate and lowers fertility in the steady state. Thus, when building a trust fund to smooth the shocks, the government increases taxes for the current generation of workers but fertility will be higher (and taxes lower) for the coming generations. This is illustrated in Figure 9 together with the benchmark experiment (PAYGO): tax contributions increases significantly for the current generation but less so at $t = 4$,

[30]In our model, b is a small share of GDP due to a low capital output ratio in a model with full depreciation and three periods only while still corresponding to a significant share of the total capital stock.

reaching 17.5% (compared to 27% in our benchmark experimentation). As a consequence, fertility catches up faster and stays persistently higher. Similar findings hold under an experiment involving a permanent growth slowdown.

Figure 9: Tax Smoothing with a Trust Fund.



Notes: Structural parameters are shown in Table 2. The one-child policy is implemented at t=2 and relaxed at t=3. The replacement ratio is increased permanently at t=3 from $\bar{\sigma} = 30\%$ to $\bar{\sigma} = 50\%$. In the trust fund scenario (dashed line), China starts at $b = 0$ but accumulate assets $b = 3.7\%$ at $t = 3$ before running down the fund such that $b = 0.75\%$ at $t = 4$, 0.25% at $t = 5$ before reaching 0 at $t = 6$. The benchmark experiment (solid line) corresponds to $b = 0$ throughout.

If the government undergoes capital market reforms in the presence of a trust fund, the costs of these reforms are also mitigated. If the government accumulates a fund, any increase in the interest rate (due to domestic financial liberalization or financial integration) will tend to relax its budget constraint. Capital markets reforms are thus less costly for the government. A similar simulation (not shown) where the government accumulates $b > 0$ at $t = 3$ shows that taxes are lower in the subsequent periods under financial integration than under autarky (and barely increasing following financial liberalization): the fall in revenues due to the induced drop in fertility is more than compensated by an increase resulting from the higher return on government assets. The feedback loop from higher taxes to lower fertility is thus not in operation, and fertility is higher under financial integration. Taxes (resp. fertility) can also stay at a persistently lower (resp. higher) level than in autarky by running down the trust fund at a slow pace as long as foreign returns are high. This experiment demonstrates that the negative effects of financial integration (and to a lesser extent domestic financial liberalization) can be significantly alleviated if the government can accumulate a trust fund by taxing the current generation.

Sensitivity Analysis. We investigate the sensitivity of our results to parameters δ and ϕ, both of which affect private transfers. The parameter δ measures the intensity of the crowding out of social security on family transfers. We replicate our benchmark experiment with $\delta = 0$ and with $\delta = 0.6$. Differences with our benchmark simulation are very moderate. With $\delta = 0$, fertility is about 0.1 higher all along the transition compared to a simulation with $\delta = 0.6$. Respectively, taxes are at most 1% higher in the latter calibration. Increasing δ has ambiguous effects in our model: on the one hand, a higher δ makes fertility more responsive to an increase in the replacement rate (crowding out effect); on the other, it also increases savings, lowering the interest rate, and thus raises fertility. For our benchmark calibration, δ exerts little effect on the dynamics of the model.

The parameter ϕ measures the cost of raising kids as a fraction of wages and is assumed to be constant over the whole period in our simulations. Recent trends in China suggest that, since 2000, schooling costs and tuition fees have been rising at a fast pace, particularly so in higher education (Dong and Wan (2012)).[31] If this trend were to continue, natural fertility might fall by even more than our experiments predict. In a simulation of the model where ϕ is assumed to increase by 25% at $t = 3$ (not shown), we find that fertility is 0.4 below our benchmark counterpart at $t = 4$, and taxes about 5% higher— amplifying significantly our findings.

Discussion and Caveats. The model is kept parsimonious in order to extract the importance of endogenous fertility in impacting the economy subjected to various shocks—particularly in the presence of intra-family transfers. One could argue though that other margins of adjustment, such as human capital accumulation, labor supply, rural-urban migrations —to name a few, could attenuate some of these effects. We discuss some of these extensions although an in-depth study is left for future studies.

Human Capital Accumulation. If parents can substitute 'quantity' for 'quality'—by raising their only child's human capital through education investment — the rise in the financing costs of the social security system as a result of the policy can be mitigated to a degree. As Choukhmane et al. (2014) show, the one-child policy shock tends to foster rapid human capital accumulation of the only child. However, the argument can be reversed when the policy is relaxed and the social security benefits rise. In particular, increasing social security coverage could depress the return on human capital investment by lowering expected transfers from children—thus exacerbating the fiscal costs of the policy. The interaction of fertility with social security—while incorporating human capital— is left for further research.

Labour Supply. Labor supply responses to various shocks could also mitigate the difficulties in implementing the pension reform. In particular, a persistent fall in fertility could potentially lead to an increase in labor market participation—especially for women—though this effect is not found to be very significant (see He and Zhu (2013)). Older people might also remain active for longer in response to a fall in support from children at retirement. Cai et al. (2012) show that this is particularly relevant for rural areas, because the lack of social safety nets compels individuals in their sixties to rely still heavily on their labor income. In urban areas, according to UHS data, labor income is a negligible fraction of income for individuals above 60, and retirement ages are strictly

[31] According to data from the Bureau of China Statistics (2012), the per capita higher education tuition and fees continued to increase over time. Between 1997-2008, the per capita tuition and fees steadily increased with an average annual growth rate of 14.4%.

observed. Conversely, a rise in replacement rates might trigger a fall in labor market participation of older workers, and thus the overall impact of policy reforms in China on aggregate labour supply remains unclear.

Rural and Urban Migrations. Our framework does not allow for a dual economy with a rural-urban divide, although it would be a natural extension for the case of China. Without harmonization in pension entitlements between rural and urban areas, high migration rates towards cities would help advance the reform in urban areas by increasing the urban working population and contributions (Song et al. (2013)). Naturally, the more universal is the reform, the less helpful internal migrations are. However, a more universal reform might also change incentives for migrations, which might interact with its financing. One particular feature of the current pension system is that most rural migrants (*non-hukou* migrants) are excluded from social security. As the intended reforms plan to remedy this issue, the benefits of migration may become higher and labour supply in cities further boosted. The fact that urban areas pay higher wages means that the required tax adjustment may be smaller. The quantitative importance of such feedback effects remains to be established —particularly in China where migrations are subject to heavy administrative regulations.

5 Conclusion

The Chinese government has ambitious plans to expand its social security system to reach full national coverage. But its recent decision to relax its prolonged one-child policy may not be as helpful in this process as one would expect. In a framework with endogenous fertility where notions of intergenerational support within families are strong, we show that an expansion of social security access/benefits exerts downward pressure on desired fertility—and in turn, offsets some of the positive effect that abandoning the one-child policy has on fertility. This countervailing force is stronger the more important are intra-family transfers in an economy.

Moreover, the government may face implicit policy tradeoffs. Seemingly disparate policy reforms, such as capital market reforms and the pension reform can in fact interact, and in such a way that makes carrying out the social security expansion particularly difficult—in spite of relaxing fertility controls. The rise in interest rates with financial liberalization and financial integration further depresses fertility. Implementing the pension reforms thus becomes more costly—and even more so if China experiences a growth slowdown. These negative effects of growth are amplified by the effects of intergenerational transfers. And in plausible circumstances, the social security reform may be entirely infeasible if the government also chooses to reform its domestic capital market at the same time.

The realistic importance of these feedback loops arising from the interlinkages between interest rates, fertility, and social security can only be illuminated by a more in-depth quantitative investigation that our stylized model is not fully-equipped to undertake. But what our analyses show is that allowing for endogenous fertility responses may be key, particularly in the context of pension reforms. Predictions of the magnitude and the dynamics of the requisite fiscal adjustments for expanding social security are very different in models with exogenous fertility. These adjustments also seem to be distinct in economies in which children are more akin to 'consumption goods' compared to economies in which they are also 'investment goods'.

International spillovers onto other economies —potentially very dissimilar in terms of financial development, pension systems, or the role of children — constitute a natural and relevant extension to consider (as in Fehr et al. (2007) and Attanasio et al. (2007)). As China is becoming ever larger and more important in the world economy, its particular policy choices and growth scenarios will undoubtedly have an important bearing on the world economy.

References

[1] Attanasio, O. and Kitao, S., and Violante, G., 2007. Global demographic trends and social security reform. Journal of Monetary Economics, Elsevier, vol. 54(1), pages 144-198, January.

[2] Banerjee, A., Meng, X., Qian, N. and T. Porzio, 2013, Fertility and Household saving: Evidence from a General Equilibrium Model and Micro Data from Urban China. Mimeo Yale University.

[3] Barr, N. and P. Diamond, 2010. Pension Reform in China: Issues, Options and Recommendations. Mimeo MIT.

[4] Boldrin, M. and L. Jones, 2002. Mortality, Fertility and Saving in a Malthusian Economy. Review of Economic Dynamics 5, 775-814.

[5] Boldrin, M., De Nardi, M., and L. Jones, 2005. Fertility and Social Security. Research Department Staff Report 359, Federal Reserve Bank of Minneapolis.

[6] Cai, F., J. Giles and X. Meng, 2006. How well do children insure parents against low retirement income? An analysis using survey data from urban China, Journal of Public Economics 90 (12) 2229-2255.

[7] Cai, F., J. Giles, P. O'Keefe and D. Wang, 2012. The elderly and old age support in rural China, challenges and prospects. World Bank.

[8] Choukhmane, T., N. Coeurdacier and K. Jin, 2014. The One-Child Policy and Household Savings. Mimeo, London School of Economics and SciencesPo.

[9] Coeurdacier, N., S. Guibaud, and K. Jin, 2014. Credit Constraints and Growth in a Global Economy. Mimeo, London School of Economics and SciencesPo.

[10] Cox, D., Hansen, B.E. and E. Jimenez, 2004. How responsive are private transfers to income? Evidence from a laissez-faire economy. Journal of Public Economics, 88 (910), 2193-2219

[11] Curtis, C., Lugauer, S. and N.C. Mark, 2011. Demographic Patterns and Household Saving in China. Mimeo.

[12] Dong, H. and X. Wan, 2012. Higher Education Tuition and Fees in China: Implications and Impacts on Affordability and Education Equity. Current Issues in Education (15).

[13] Dunaway, S.V., and V. Arora. 2007. Pension Reform in China: The Need for a New Approach. IMF Working Paper WP/07/109.

[14] Ehrlich, I. and J. Kim, 2005. Social security and demographic trends: Theory and evidence from the international experience. Review of Economic Dynamics, 10, 55-77.

[15] Feldstein, Martin. 1999. Social Security Pension Reform in China. China Economic Review, 10(2): 99-107.

[16] Fehr, H., Jokisch, S. and L.J. Kotlikoff, 2007. Will China Eat Our Lunch or Take Us to Dinner? Simulating the Transition Paths of the United States, the European Union, Japan, and China. Fiscal Policy and Management in East Asia, NBER-EASE, Volume 16, University of Chicago Press.

[17] He, X. and Zhu, R., 2013. Fertility and Female Labor Force Participation: Causal Evidence from Urban China. MPRA Paper 44552, University of Munich.

[18] International Social Security Association, 2013. Towards universal social security coverage in China. ISSA report, Social security coverage extension in the BRICs, Chapter 4.

[19] Nakashima, K., Howe, N., and R. Jackson., 2009. China's Long March to Retirement Reform–the Graying of the Middle Kingdom Revisited. CSIS.

[20] Nishimura, K and Zhang, J., 1992. Pay-as-you-go Public Pensions with Endogenous Fertility. Journal of Public Economics, 239-58.

[21] Sin, Y.. 2005. China: Pension Liabilities and Reform Options for Old Age Insurance. World Bank Working Paper No. 20051.

[22] Song, M., K. Storesletten, Y. Wang, and F. Zilibotti, 2013. Sharing high growth across generations: pensions and demographic Transition in China. Mimeo IIES.

[23] Song, M. and D. Yang, 2012. Life Cycle Earnings and Saving in a Fast-Growing Economy. Mimeo Chicago Booth.

[24] Yew, S. L. and Zhang, J., 2009. Optimal social security in a dynastic model with human capital externalities, fertility and endogenous growth. Journal of Public Economics, 93, 605-619.

A Proofs

Proof of Proposition 1: Define

$$\begin{aligned}
\underline{n} &\equiv \lambda \frac{1-\tau-\theta-\psi_{\sigma}}{\phi}, \\
\overline{n} &\equiv \frac{1-\tau-\theta-\psi_{\sigma}}{\phi} + \frac{1+\beta}{\beta(1-\alpha)} b > \underline{n}.
\end{aligned}$$

Note that $R_{KK}(n)$ is strictly positive and finite if and only if $n < \overline{n}$, while $R_{NN}(n)$ is strictly positive and finite if and only if $n > \underline{n}$. Also note that

$$\begin{aligned}
0 < \lim_{n \downarrow \underline{n}} R_{KK}(n) < \lim_{n \downarrow \underline{n}} R_{NN}(n) = \infty, \\
0 < \lim_{n \uparrow \overline{n}} R_{NN}(n) < \lim_{n \uparrow \overline{n}} R_{KK}(n) = \infty.
\end{aligned}$$

The steady state (R, n) is such that $R_{KK}(n) = R_{NN}(n) = R$. The KK and NN curves are increasing and decreasing, respectively, guaranteeing that their intersection is unique. The equilibrium fertility rate lies in $(\underline{n}, \overline{n})$, and $R > 0$. ∎

Lemma A-1 *For any target replacement ratio $\bar{\sigma}$ that satisfies*

$$\bar{\sigma} < \frac{(1-\theta-\psi_{\bar{\sigma}})^2 g_A}{4\phi} \left(\frac{2\lambda\Theta + 2(\lambda+\beta)\psi_{\bar{\sigma}} + \lambda(1+\beta)(1-\theta-\psi_{\bar{\sigma}})}{2\Theta + 2(1+\beta)\psi_{\bar{\sigma}} + (1+\beta\lambda)(1-\theta-\psi_{\bar{\sigma}})} \right), \tag{A-1}$$

there exists a steady state equilibrium under PAYGO. Moreover, if $(1-\lambda)\psi_{\bar{\sigma}} < \lambda\Theta$ and if the contribution rate is constrained to remain below $\tau_{\bar{\sigma}}^{max} \equiv \frac{1-\theta-\psi_{\bar{\sigma}}}{2}$, then the steady state equilibrium is unique.

Equation (A-1) provides implicity an upper bound for $\bar{\sigma}$.

Proof of Lemma A-1: To start with, consider a PAYGO system at steady state with contribution rate $\tau > 0$. Setting $b = 0$ and using (15) to substitute $\sigma = n g_A \tau$ in (13) and (14), we can solve for the equilibrium fertility and interest rate, each expressed as a function of τ:

$$\begin{aligned}
n(\tau) &= \frac{(1-\tau-\theta-\psi_{\bar{\sigma}})}{\phi} \left(\frac{\lambda\Theta + (\lambda+\beta)\psi_{\bar{\sigma}} + \lambda(1+\beta)\tau}{\Theta + (1+\beta)\psi_{\bar{\sigma}} + (1+\beta\lambda)\tau} \right) \\
R(\tau) &= \left(\frac{g_A}{\beta\phi} \right) \left(\frac{\lambda\Theta + (\lambda+\beta)\psi_{\bar{\sigma}} + \lambda(1+\beta)\tau}{1-\lambda} \right).
\end{aligned}$$

Now suppose the government targets a replacement rate $\bar{\sigma} > 0$. The steady state equilibrium triplet $(n_{\bar{\sigma}}, R_{\bar{\sigma}}, \tau_{\bar{\sigma}})$ must satisfy

$$n_{\bar{\sigma}} = n(\tau_{\bar{\sigma}}), \qquad R_{\bar{\sigma}} = R(\tau_{\bar{\sigma}}), \qquad \tau_{\bar{\sigma}} = \frac{\bar{\sigma}}{n_{\bar{\sigma}} g_A}.$$

Noting that $n_{\bar{\sigma}} = \frac{\bar{\sigma}}{g_A \tau_{\bar{\sigma}}}$, the equilibrium contribution rate $\tau_{\bar{\sigma}}$ is pinned down by the equation $n(\tau_{\bar{\sigma}}) = \frac{\bar{\sigma}}{g_A \tau_{\bar{\sigma}}}$, which can also be written $F(\tau_{\bar{\sigma}}) = G(\tau_{\bar{\sigma}})$, where

$$F(\tau) \equiv \tau(1-\tau-\theta-\psi_{\bar{\sigma}}) \qquad G(\tau) \equiv \frac{\phi\bar{\sigma}}{g_A}\left(\frac{\Theta+(1+\beta)\psi_{\bar{\sigma}}+(1+\beta\lambda)\tau}{\lambda\Theta+(\lambda+\beta)\psi_{\bar{\sigma}}+\lambda(1+\beta)\tau}\right).$$

The function F is concave, starts at zero at $\tau = 0$, initially increases to reach a maximum at $\frac{1-\theta-\psi_{\bar{\sigma}}}{2}$

$$F\left(\frac{1-\theta-\psi_{\bar{\sigma}}}{2}\right) = \frac{(1-\theta-\psi_{\bar{\sigma}})^2}{4} \equiv F_{\bar{\sigma}}^{\max},$$

and then decreases, crossing zero again at $\tau = 1-\theta-\psi_{\bar{\sigma}}$. The function G is always strictly positive. Moreover, differentiating G yields

$$G'(\tau) = \frac{\phi\bar{\sigma}}{g_A}\frac{\beta(1-\lambda)\left[(1-\lambda)\psi_{\bar{\sigma}}-\lambda\Theta\right]}{\left[\lambda\Theta+(\lambda+\beta)\psi_{\bar{\sigma}}+\lambda(1+\beta)\tau\right]^2},$$

and

$$G''(\tau) = -2\frac{\phi\bar{\sigma}}{g_A}\frac{\lambda(1+\beta)\beta(1-\lambda)\left[(1-\lambda)\psi_{\bar{\sigma}}-\lambda\Theta\right]}{\left[\lambda\Theta+(\lambda+\beta)\psi_{\bar{\sigma}}+\lambda(1+\beta)\tau\right]^3}.$$

Therefore G is strictly increasing and concave if $(1-\lambda)\psi_{\bar{\sigma}} > \lambda\Theta$, it is constant if $(1-\lambda)\psi_{\bar{\sigma}} = \lambda\Theta$, and it is strictly decreasing and convex if $(1-\lambda)\psi_{\bar{\sigma}} < \lambda\Theta$. In any of these configurations, a sufficient condition for the function F and G to cross at least once is $G\left(\frac{1-\theta-\psi_{\bar{\sigma}}}{2}\right) \leq F_{\bar{\sigma}}^{\max}$, which is equivalent to (A-1). This condition implicitly provides an upper bound on $\bar{\sigma}$. When it is satisfied, there exists at least one value of $\tau \leq \frac{1-\theta-\psi_{\bar{\sigma}}}{2}$ such that $F(\tau) = G(\tau)$. If moreover $(1-\lambda)\psi_{\bar{\sigma}} < \lambda\Theta$, the intersection is unique, which pins down a unique equilibrium contribution rate $\tau_{\bar{\sigma}} < \tau_{\bar{\sigma}}^{\max}$. In the case several equilibria exist, we focus on the one with the lowest contribution rate. The equilibrium fertility and interest rate are determined by $n_{\bar{\sigma}} = n(\tau_{\bar{\sigma}})$ and $R_{\bar{\sigma}} = R(\tau_{\bar{\sigma}})$. ∎

Proof of Proposition 2. From Lemma A-1, an equilibrium must satisfy:

$$\frac{\phi\bar{\sigma}\Gamma(\bar{\sigma})}{g_A\tau_{\bar{\sigma}}} = (1-\tau_{\bar{\sigma}}-\theta-\psi_{\bar{\sigma}}) \tag{A-2}$$

with $\Gamma(\bar{\sigma}) = \left(\frac{\Theta+\psi_{\bar{\sigma}}(1+\beta)+(1+\beta\lambda)\tau_{\bar{\sigma}}}{\lambda\Theta+\psi_{\bar{\sigma}}(\lambda+\beta)+\lambda(1+\beta)\tau_{\bar{\sigma}}}\right)$. We know from Lemma A-1, that there exists (at least) one solution and $\tau_{\bar{\sigma}} \leq \left(\frac{1-\theta-\psi_{\bar{\sigma}}}{2}\right)$. Rewrite $\Gamma(.)$ as follows:

$$\Gamma(\bar{\sigma}) = 1+(1-\lambda)U(\bar{\sigma}) \geq 1$$

with $U(\bar{\sigma}) = \left(\frac{\Theta+\psi_{\bar{\sigma}}+\tau_{\bar{\sigma}}}{\psi_{\bar{\sigma}}\beta+\lambda(\Theta+\psi_{\bar{\sigma}})+\lambda(1+\beta)\tau_{\bar{\sigma}}}\right)$. We denote $\partial x = \frac{\partial x}{\partial\bar{\sigma}}$.

$$\partial U(\bar{\sigma}) = \beta\frac{\partial\tau_{\bar{\sigma}}\left((1-\lambda)\psi_{\bar{\sigma}}-\lambda\Theta\right)-\partial\psi_{\bar{\sigma}}\left[(1-\lambda)\tau_{\bar{\sigma}}+\Theta\right]}{\left(\psi_{\bar{\sigma}}(\beta+\lambda)+\lambda\Theta+\lambda(1+\beta)\tau_{\bar{\sigma}}\right)^2}$$

Let us distinguish two cases: $U(.)$ increasing and $U(.)$ potentially decreasing.

— **Case 1.** $(1-\lambda)\,\psi_{\bar{\sigma}} \geq \lambda\Theta$.
Then $\partial U(\bar{\sigma}) > 0$ if $\partial\tau_{\bar{\sigma}} > 0$ (as $\partial\psi_{\bar{\sigma}} \leq 0$). Eq. A-2 is equivalent to:

$$\phi n_{\bar{\sigma}} = \frac{(1-\tau_{\bar{\sigma}}-\psi_{\bar{\sigma}}-\theta)}{1+(1-\lambda)\,U(\bar{\sigma})} \tag{A-3}$$

Differentiating Eq. A-3:

$$\phi\partial n_{\bar{\sigma}} = -\frac{\partial(\tau_{\bar{\sigma}}+\psi_{\bar{\sigma}})}{1+(1-\lambda)\,U(\bar{\sigma})} - (1-\lambda)\,\partial U(\bar{\sigma})\,\frac{(1-\tau_{\bar{\sigma}}-\psi_{\bar{\sigma}}-\theta)}{(1+(1-\lambda)\,U(\bar{\sigma}))^2} \tag{A-4}$$

Note that the second term is strictly negative if $\partial\tau_{\bar{\sigma}} > 0$ and thus $\partial(\tau_{\bar{\sigma}}+\psi_{\bar{\sigma}}) \geq 0$ implies $\partial n_{\bar{\sigma}} < 0$. Differentiating A-2 leads to:

$$\partial(\tau_{\bar{\sigma}}+\psi_{\bar{\sigma}})(1-2\tau_{\bar{\sigma}}-\psi_{\bar{\sigma}}-\theta) = \partial\psi_{\bar{\sigma}}(1-\tau_{\bar{\sigma}}-\psi_{\bar{\sigma}}-\theta)+\frac{\phi}{g_A}(1+(1-\lambda)\,U(\bar{\sigma}))+\frac{\phi}{g_A}\bar{\sigma}\,(1-\lambda)\,\partial U(\bar{\sigma}) \tag{A-5}$$

$$\partial(\tau_{\bar{\sigma}}+\psi_{\bar{\sigma}}) \geq 0 \Leftrightarrow |\partial\psi_{\bar{\sigma}}| \leq \frac{\phi}{g_A(1-\tau_{\bar{\sigma}}-\psi_{\bar{\sigma}}-\theta)}\left[(1+(1-\lambda)\,U(\bar{\sigma}))+\bar{\sigma}\,(1-\lambda)\,\partial U(\bar{\sigma})\right]$$

The last condition is always verified if $|\partial\psi_{\bar{\sigma}}| \leq \frac{\phi}{g_A(1-\tau_{\bar{\sigma}}-\psi_{\bar{\sigma}}-\theta)}\left[1+(1-\lambda)\,U(0)\right]$ as $U(.)$ is increasing and $\partial U(\bar{\sigma}) \geq 0$. This last condition is a sufficient condition for $\partial n_{\bar{\sigma}} < 0$.
This also gives an upper bound in the neighborhood of $\bar{\sigma} = 0$:

$$|\partial\psi_{\bar{\sigma}}| \leq \frac{\phi}{g_A(1-\psi_0-\theta)}\left[1+(1-\lambda)\,U(0)\right]$$

Note that due to the convexity of $\Psi(.)$, if this condition is satisfied for $\bar{\sigma} = 0$, it is satisfied for all (sustainable) $\bar{\sigma}$.

— **Case 2.** $(1-\lambda)\psi_{\bar{\sigma}} < \lambda\Theta$
In this case, $\partial U(\bar{\sigma})$ can be negative. We still can show in that case that $\partial n_{\bar{\sigma}} < 0$ for $\bar{\sigma} \to 0$.
Take the limit of Eq. A-5 when $\bar{\sigma} \to 0$:

$$\partial\tau_{\bar{\sigma}}(1-\psi_0-\theta) = \frac{\phi}{g_A}(1+(1-\lambda)\,U(0)) + \frac{\phi}{g_A}(1-\lambda)\,(\bar{\sigma}\partial U(\bar{\sigma}))_{\bar{\sigma}=0}$$

Using $(\partial U(\bar{\sigma}))_{\bar{\sigma}=0} = \beta\frac{\partial\tau_{\bar{\sigma}}((1-\lambda)\psi_0-\lambda\Theta)-\partial\psi_{\bar{\sigma}}\Theta}{(\psi_0\beta+\lambda(\Theta+\psi_0))^2}$, we get:

$$\partial\tau_{\bar{\sigma}}\left(1-\psi_0-\theta+(\lambda\Theta-(1-\lambda)\,\psi_0)\,\bar{\sigma}\frac{\phi}{g_A}\frac{(1-\lambda)\,\beta}{(\psi_0\beta+\lambda(\Theta+\psi_0))^2}\right) = \frac{\phi}{g_A}(1+(1-\lambda)\,U(0))$$

Thus, for $\bar{\sigma} \to 0$,

$$\partial\tau_{\bar{\sigma}} = \frac{\phi}{g_A}\left(\frac{1+(1-\lambda)\,U(0)}{1-\psi_0-\theta}\right) \tag{A-6}$$

Using Eq. A-6, Eq. A-4 can be rewritten as follows for $\bar{\sigma} \to 0$:

$$\begin{aligned}(\psi_0(1+\beta)+\Theta)\,\phi\partial n_{\bar{\sigma}} &= -\partial\psi_{\bar{\sigma}}\left[\psi_0\,(\beta+\lambda)+\lambda\Theta-\frac{(1-\lambda)\,\beta\Theta\,(1-\psi_0-\theta)}{\psi_0\,(1+\beta)+\Theta}\right]\\ &\quad -\frac{\phi}{g_A}\left[\frac{\psi_0(1+\beta)+\Theta}{(1-\psi_0-\theta)}+\frac{(1-\lambda)\,\beta\,((1-\lambda)\,\psi_0-\lambda\Theta)}{\psi_0\,(\beta+\lambda)+\lambda\Theta}\right]\end{aligned}$$

Note that $\frac{\psi_0(1+\beta)+\Theta}{(1-\psi_0-\theta)}+\frac{(1-\lambda)\beta((1-\lambda)\psi_0-\lambda\Theta)}{\psi_0(\beta+\lambda)+\lambda\Theta} > 0$ if $\alpha \geq \frac{\beta}{1+2\beta}$. To see that, note that this term is minimal for $\psi_0 = 0$ and equal to: $\frac{\Theta}{1-\theta} - (1-\lambda)\,\beta$. This last term is strictly positive if $\alpha \geq \frac{\beta}{1+2\beta}$. Thus $\partial n_{\bar{\sigma}} < 0$ if:

$$|\partial\psi_{\bar{\sigma}}| \leq \frac{\phi\,[1+(1-\lambda)\,U\,(0)]}{g_A\,(1-\psi_0-\theta)}\left|\frac{1-\frac{(1-\lambda)\beta(\lambda\Theta-(1-\lambda)\psi_0)(1-\psi_0-\theta)}{(\psi_0(\beta+\lambda)+\lambda\Theta)(\psi_0(1+\beta)+\Theta)}}{1-\frac{(1-\lambda)\beta\Theta(1-\psi_0-\theta)}{(\psi_0(1+\beta)+\Theta)((\psi_0\beta+\lambda(\Theta+\psi_0)))}}\right|$$

This inequality is strictly satisfied if:

$$|\partial\psi_{\bar{\sigma}}| \leq \frac{\phi\,[1+(1-\lambda)\,U\,(0)]}{g_A\,(1-\psi_0-\theta)}\left(1-(1-\lambda)\,(1-\theta)\,\frac{1-\alpha}{\alpha}\frac{\beta}{1+\beta}\right)$$

where the right hand-side is strictly positive for $\alpha \geq \frac{\beta}{1+2\beta}$.
To conclude, under the (realistic) assumption that the condition $\alpha \geq \frac{\beta}{1+2\beta}$ is satisfied, let us define

$$M \equiv [1+(1-\lambda)\,U\,(0)]\left(1-(1-\lambda)\,(1-\theta)\,\frac{1-\alpha}{\alpha}\frac{\beta}{1+\beta}\right) > 0. \tag{A-7}$$

If $|\partial\psi_{\bar{\sigma}}| < \frac{\phi}{g_A}M$, we have $\partial n_{\bar{\sigma}} < 0$ in the neighbourhood of $\bar{\sigma} = 0$ for any value of the parameters (ψ_0, v). ∎