Push-Me Pull-You: Comparative Advertising in the OTC Analgesics Industry*

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Abstract

We investigate how firms strategically use self-promoting and comparative advertising to push up own brand perception along with pulling down the brand images of targeted rivals. We watched video files of all TV advertisements in the US OTC analgesics industry for 2001-2005 to code the content of each ad and merge these data with spending per ad to derive an Attack Matrix of how much each brand spent targeting each rival. We develop a model for which we predict oligopoly equilibrium advertising. We do not specify demand beyond assuming a discrete choice model. Still we are able to get estimates of some diversion ratios for this market. These in turn are used to compute measures of how much a firm’s profit is hurt by one more dollar of attacks against it by one of its rivals. We find: i) higher market shares are associated with higher self-promotion advertising; ii) outgoing attacks are half as powerful as direct self-promotion ads in raising own perceived quality; iii) every dollar spent by its competitors on incoming attacks has a statistically and economically strong effect on the

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profit of the attacked brand. Measures of the profit loss range from $1.92 to $9.80 for an additional dollar of attack.

Keywords: Comparative Advertising, persuasive advertising, targeted advertising, diversion ratios, analgesics.

1 Introduction

This paper investigates how firms strategically use self-promoting and comparative advertising to push up own brand perception along with pulling down the brand images of targeted rivals and the extent to which the profits of the targeted firms are affected by the comparative advertising.\(^1\) While self-promotion advertising involves only positive promotion, a comparative advertisement, by comparing one’s own product favorably relative to a rival, has both a positive promotion component (in common with self-promotion advertising) and an indirect effect through denigrating a rival. Denigration is advantageous insofar as consumers who switch from the demeaned product are picked up by the denigrating firm.

THE FOLLOWING SEEMS NOT CRUCIAL TO BE PUT HERE... However, they may also be picked up by other rival firms. This logic indicates a possible free-rider situation in the provision of comparative advertising against any particular rival, but it also indicates an equilibrium at which each firm’s positive promotion (through both comparative and self-promotion channels) is devalued by others’ comparative advertising.

We propose a simple model of targeting advertising to determine which firms engage in what kind of advertising against whom, and then use a novel dataset from the Over-The-Counter (OTC) analgesics industry in the US to look for whether those relationships are actually there and how large they are. In addition, we uncover demand-side relations from analyzing the supply side (advertising) decisions because of the special nature of comparative advertising as being targeted against specific rivals. This would not be possible with, say, purely self-promotional advertising or simple pricing relations that conflate all effects into a single variable (advertising or price). This feature underscores the importance of cod-

\(^1\)The Pushmi-Pullyu is a fictitious two-headed llama befriended by Dr Doolittle. The heads are pointed in different directions. When one pushes forward, it pulls the other end back from its preferred direction.
ing the advertising data by attack subject (as opposed to just using aggregate advertising expenditures).

Our push-pull model is based on a discrete choice approach to demand, in which firms’ perceived qualities are shifted by advertising. The way in which advertising enters the model is most simply thought of as persuasive advertising that shifts demand up. Promoting one’s own product increases demand directly, whether through self-promotion advertising or comparative advertising, while denigrating a rival helps a firm indirectly by decreasing perceived rival quality. By hurting the rival product directly, some consumers are diverted, and the comparative advertiser succeeds in attracting some portion of those consumers.

We use the model to derive the advertising first order conditions that predict oligopoly equilibrium relations between advertising levels (for different types of advertising) and market shares. In particular, we use the equilibrium pricing (first-order) conditions to eliminate prices from the relation between advertising and sales. Then, we relate ad levels of the different ad types to other observable market variables, like market shares.

We show that key ingredients of the firms’ comparative advertising behavior are diversion ratios. The diversion ratio from one product to another measures what percentage of demand lost by the first product when its price is increased, goes to the second product. We show that they can be estimated from the comparative advertising first order conditions without having to specify demand. Furthermore, they can be used to derive measures of how much an additional dollar of comparative advertising by a firm hurts the target’s profit.

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2This is consistent with “hype” in the Johnson and Myatt (2004) taxonomy of demand shifts and with complementary advertising of the type propounded by Stigler-Becker (1977) and Becker and Murphy (1993).

3A somewhat similar approach is expounded in Harrington and Hess (1996). These authors treat positive and negative advertising by 2 politicians with given locations in a policy space. Negative advertising shifts a rival candidate away from the median voter, while positive advertising shifts a candidate closer. This framework would provide an interesting base to develop a product market model.

4One advantage of this approach is that we bypass dealing with price data, which involve multiple price points for multiple variants of the same brand. See the Appendix for more on this.

5Firms with a lot of advertising are typically those with large market shares. They also tend to set high prices. This does not say that high prices drive high market shares, nor that advertising creates high prices, nor indeed is it the high prices that create the desire to advertise. All of these variables are jointly determined in equilibrium, and are driven by intrinsic brand “qualities” (fixed effects) NOT SURE WHAT IS MEANT HERE and the marginal efficiency of advertising types across firms.
may be written as the weighted average of a "pure push" effect and a "pure pull" effect. Pure push measures how much the target would be hurt if advertising had been purely non-comparative and pure push measures the hurt of some purely comparative advertising that would have no impact on the perceived quality of the advertising firm.

To estimate the model we need to find out how much is spent on comparative advertising. For coding reasons discussed below, a cross section study across industries is clearly infeasible, and so we need to analyze a particular industry. Advertising spending by firms, even when the data are available (which is already rare), is not broken down into comparative and self-promotion advertising. Ideally, we should analyze an industry for which comparative advertising is prevalent and represents a large fraction of industry sales, for which data on spending on ads is available for a full sample of firms and for a reasonably long period of time. Video files (or audio files for radio ads or photographic files for newspaper/magazine ads) need to be available and their content readily coded to determine targets. Fortunately, all these criteria are met with the US OTC industry (basically, medicine for minor pain relief, involving as major brands Advil, Aleve, Bayer Aspirin, and Tylenol). We watched over four thousand individual video files of all TV advertisements in the US OTC analgesics industry for 2001-2005 and recorded which brand(s) (or class of drugs) were compared (e.g. to Advil or Aleve; or to Ibuprofen-based drugs).

There are two main concerns to address when estimating the advertising first order conditions: left-censoring of advertising (in some periods some brands do not engage in some types of advertising - there are corner solutions) and endogeneity of market shares and advertising expenditures. We control for the left-censoring by running Tobit regressions. We use brand fixed effects and two sources of exogenous variation to control for endogeneity. First, we construct a dataset of news shocks that hit the OTC analgesic markets in the time period. Second, we use data on the prices of the generic products as instrumental variables.

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6 See Liaukonyte (2009) for a paper that estimates demand-side parameters using this same dataset.
7 While explicit comparative advertising has flourished in the US over the past 20 years (with the blessing of the FTC), it varies widely across industries. The US OTC analgesics industry exhibits high advertising levels in general, and extraordinary levels of explicit comparative claims. Most spending is for TV ads.
8 The idea of using a natural experiment to study the effect of advertising (on prices) is the crucial insight
of the shares of the branded products.

First, we find that higher shares, ceteris paribus, are associated with higher self-promotion advertising. In particular, the results in Table zzz imply that for an increase of 1 units of pain leads to an increase of 46.3 cents in self-promotion advertising expenditures. Second, outgoing attacks are approximately half as powerful as direct self-promotion ads in raising perceived quality. The marginal effect of a one dollar increase in incoming attacks would lead to an increase of 21.7 cents in self-promotion advertising, conditional on shares remaining constant. Finally, we show that endogeneity concerns are of crucial importance and are mostly addressed by including a Top Brand fixed effect, which is estimated to have a negative sign, implying that the larger firms, Aleve, Tylenol and Advil have inherently higher advertising base allure than the other brands.

Bagwell (2009) provides a comprehensive survey of the literature on advertising and discusses three roles for advertising: persuasive, informative, and complementary. Our approach is broadly consistent with advertising as a demand shifter (as in Dixit and Norman, 1978) and the complementary view of Stigler and Becker (1977) and Becker and Murphy (1993).

The theoretical economics literature on comparative advertising is quite scarce. Anderson and Renault (2009) model it as directly informative revelation of horizontal match characteristics of products. Barigozzi, Garella, and Peitz (2006) and Emons and Fluet (2008) apply the signaling model of advertising (which goes back to insights in Nelson, 1970 and 1974, and was formalized in Milgrom and Roberts, 1986). Our theory engages the complementary view with the added element of pulling down the rival.

There are three key ways in which we contribute to the empirical literature on advertising. First, previous papers have used total ad expenditures as the sole advertising explanatory variable (see Nevo, 2000 and 2001, and Goeree, 2008). Here, because we have data on content, we break down the ad expenditures into comparative and self-promotion expenditures, and the comparative expenditures are further broken down into attacker-target pairs. Second,

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in Milyo and Waldfogel (1999).

9Anderson and Renault (2006) show that a firm may be hurt by information disclosure about its own product, so there might be incentives for competitors to provide that information through comparative ads.
we estimate a full equilibrium static model of firm behavior, where firms jointly choose product prices and advertising levels. The choice of a static model is driven by practical considerations. One could model only one side of the market in a dynamic setting and must relinquish analyzing a full equilibrium model (Hendel and Nevo, 2006, and Gowrisankaran and Rysman, 2009, Roberts and Samuelson, 1988, and Dube, Hitsch, Manchanda, 2005, Erdem and Keane, 1996, Ackerberg, 2001 and 2003) of analyzing a full equilibrium static model (Gasmi, Laffont, and Vuong, 1992, and Goeree, 2008). Our demand (like Goeree, 2008) is derived from a discrete choice model, while Gasmi, Laffont, and Vuong (1992) postulate a set of residual demand functions. Third, we use a combination of exogenous shocks and firm-specific generic prices to construct sources of exogenous variation in the data. By contrast, Gasmi, Laffont, and Vuong (1992) use aggregate variables (e.g. the price of sugar), while Goeree (2008) uses the type of instrumental variables introduced by Bresnahan (1987).

2 The Model

The theoretical model suggests certain regularities between market shares and both self-promotion (non-comparative) and comparative advertising. We first describe the demand side assumptions. We assume that each product is associated to a quality index and demand depends on the quality indices of all firms, in a manner familiar from, and standard in discrete choice analysis. These quality indices are influenced positively by own advertising (both self-promotion and comparative) and negatively by competitors’ comparative advertising. They are also influenced by medical news shocks which unexpectedly indicate good news or bad news about the health effects of the product(s).

We then derive the equilibrium relations from the model. These take the form of advertising intensities as a function of market shares, and they form the basis of the estimation which follows. Taking these relations to the data enables us to identify how a firm’s perceived qual-

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10 The problem is both computational complexity and multiplicity of solutions. One would have to solve for rational expectations that consumers and producers have on the future values of the state variables, which means finding a fixed point. There might be multiple future values of the state values for which such consistency requirements hold.
ity is affected by various types of advertising, and it enables us to uncover substitution patterns in demand from equilibrium ad choices. Targeting of comparative advertising enables us to identify diversion ratios between products (because the incentive to attack depends on how much of the target’s lost demand will be picked up by the attacker), even without direct demand estimation. It is noteworthy from a methodological standpoint that we can uncover demand-side relations from analyzing the supply side (advertising) decisions. It is because of the special nature of comparative advertising as being targeted against specific rivals that we can find diversion ratios. This would not be possible with, say, purely self-promotional advertising or simple pricing relations that conflate all effects into a single variable (advertising or price). This feature underscores the importance of coding the advertising data by attack subject (as opposed to just using aggregate advertising expenditures).

2.1 Core Concepts

We use the first-order conditions for self-promotion to identify and estimate the relation between a firm’s self-promotion and its brand size and incoming and outgoing attacks. Doing so enables us to determine key parameters of the quality function. One such parameter here measures the marginal rate of substitution between outgoing comparative advertising and self-promotion by that firm. We assume this to be constant, at rate $\lambda$. It measures how much self-promotion a firm gets out of spending on comparative advertising.

We can also measure the harm to others of the two types of advertising. We define $Push_{jk}$ as the reduction in Firm $k$’s profit from each additional dollar of self-promotion by Firm $j$. Similarly, $Pull_{jk}$ is the reduction in Firm $k$’s profit from each additional dollar of "pure attack" by Firm $j$ (to be more specifically defined below). We then seek a measure of the harm to the target of comparative advertising attacks against it. It turns out to be a weighted sum $\lambda Push_{jk} + (1 - \lambda) Pull_{jk}$, where $\lambda$ is the marginal rate of substitution between outgoing comparative advertising and self-promotion defined above.

We show in our theoretical analysis that the push and pure pull effects may be expressed using market shares along with diversion ratios. The diversion ratio between goods $j$ and
has been proposed as a useful statistic for analyzing the price effects of mergers (see for example Shapiro, 19xx) insofar as it tells us what fraction of the demand lost by product $j$ due to a price increase is picked up by product $k$. It is defined as

$$d_{jk} = -\frac{ds_k}{ds_j} \cdot \frac{1}{d\delta_j} \in (0, 1).$$

One useful way to think of it is in terms of consumers’ second preferences because consumers switch to their next preferred option when the price of the erstwhile first choice rises. For substitute differentiated products, $d_{jk}$ is positive. In our discrete choice context, $\sum_k d_{jk} < 1$ because some customers no longer purchase at all when $j$ gets less attractive. We estimate the diversion ratios from the comparative advertising first-order conditions.

Using first order conditions we show that in equilibrium $Push_{jk} = -\frac{s_k}{s_j} d_{kj}$ and $Pull_{kj} = -\frac{s_k}{s_j} \frac{1}{d_{jk}}$. The pure pull effect is to be interpreted with care. It is the hurt on Firm $k$’s profit of one more dollar of comparative advertising by Firm $j$ against Firm $k$, assuming that comparative advertising yields no self promotion benefit for Firm $j$ (i.e. $\lambda = 0$) and holding market shares at their equilibrium level with the true value of $\lambda$.

Being able to estimate the diversion ratios, we can provide measures of the marginal harm on a firm of an additional dollar of comparative advertising against it. We may also attribute this decrease in the target’s profit to a push effect and a pull effect. This illustrates that comparative advertising is inherently different from some nontargeted advertising for which it would not be possible to identify whether its impact is due to an increase in the advertising firm’s perceived quality or a decrease in the perceived quality of its rivals.

Finally, it is noteworthy that by using analogous methods, we may use estimates of diversion ratios to provide measures of other marginal harm or benefit for firms of changes in advertising levels or even prices.

### 2.2 Demand

Suppose that Firm $j = 1, \ldots, n$ charges price $p_j$ and has perceived quality $Q_j(\cdot)$, $j = 1, \ldots, n$. We retain the subscript $j$ on $Q_j(\cdot)$ because when we get to the econometrics, exogenous
variables such as medical news shocks and random variables summarizing the unobserved determinants of perceived quality will enter the errors in the equations to be estimated.

Firms can increase own perceived quality through both types of advertising, and degrade competitors’ quality through comparative advertising. Comparative advertising, by its very nature of comparing, both raises own perceived quality and reduces the perceived quality of rival products. The corresponding arguments of $Q_j(.)$ are advertising expenditure by Firm $j$ which directly promotes its own product, denoted by $A_{jj}$; “outgoing” advertising by Firm $j$ targeted against Firm $k$, $A_{jk}$, $k \neq j$, which has a direct positive effect; and “incoming” comparative advertising by Firm $k$ targeting Firm $j$, $A_{kj}$, $k \neq j$, which has a negative (detraction) effect on Firm $j$’s perceived quality. Thus, we write $j$’s perceived quality as $Q_j(A_{jj}, \{A_{jk}\}_{k \neq j}, \{A_{kj}\}_{k \neq j})$, $j = 1, ..., n$, which is increasing in the first argument, increasing in each component of the second (outgoing) group, and decreasing in each component of the third (incoming) group, with $\frac{\partial^2 Q_j}{\partial A_{jj}^2} < 0$ and $\frac{\partial^2 Q_j}{\partial A_{kj}^2} > 0$ for $k \neq j$.

The demand side is generated by a discrete choice model of individual behavior where each consumer buys one unit of her most preferred good. We will not estimate this demand model from (aggregate) choice data; we simply use it to frame the structure of the demand system. Preferences are described by a (conditional indirect) utility function:

$$U_j = \delta_j + \mu \varepsilon_j, \quad j = 0, 1, ..., n, \quad (2)$$

in standard fashion, where $\varepsilon_j$ is a product-idiosyncratic match value and

$$\delta_j = Q_j(.) - p_j \quad (3)$$

is the “objective” utility, and where we let the “outside option” (of not buying a painkiller) be associated to an objective utility $\delta_0 = V_0$. The parameter $\mu$ expresses the degree of horizontal consumer/product heterogeneity.

The distribution of the random terms determines the form of the market shares, $s_j$, $j = 0, ..., n$, and each $s_j$ is increasing in its own objective utility, and decreasing in rivals’

\footnote{Throughout, we assume sufficient concavity that the relevant second order conditions hold.}
objective utilities. Assume that there are $M$ consumers in the market, so that the total demand for product $j$ is $M_s j, j = 0,...,n$.

2.3 Equilibrium Relations

Assume that product $j$ is produced by Firm $j$ at constant marginal cost, $c_j$. Firm $j$’s profit-maximizing problem is:

$$\max_{\{p_j, A_j\}} \pi_j = M(p_j - c_j)s_j - A_{jj} - \sum_{k \neq j} A_{jk} \quad j = 1,...n.$$  \hspace{1cm} (4)

where the advertising quantities (the A’s) are dollar expenditures.

Prices and advertising levels are determined simultaneously in a Nash equilibrium.

The price condition is determined in the standard manner by:

$$\frac{d\pi_j}{dp_j} = M s_j - M(p_j - c_j)\frac{ds_j}{d\delta_j} = 0, \quad j = 1,...n,$$ \hspace{1cm} (5)

which yields a solution $p_j > c_j$: firms always select strictly positive mark-ups.

Self-promotion advertising expenditures are determined by:

$$\frac{d\pi_j}{dA_{jj}} = \frac{d\pi_j}{d\delta_j}\frac{\partial Q_j}{\partial A_{jj}} - 1 = M(p_j - c_j)\frac{ds_j}{d\delta_j}\frac{\partial Q_j}{\partial A_{jj}} - 1 \leq 0, \quad \text{with equality if } A_{jj} > 0 \quad j = 1,...,n,$$ \hspace{1cm} (6)

where the partial derivative function $\frac{\partial Q_j}{\partial A_{jj}}$ may depend on any or all of the arguments of $Q_j$.

Substituting the pricing first-order condition (5) into the advertising one (6) gives\footnote{The advertising-size relation is also consistent with a representative consumer model with $\delta_j = Q_j - p_j$ replacing $-p_j$ in the corresponding indirect utility function.}

$$M s_j \frac{\partial Q_j}{\partial A_{jj}} \leq 1, \quad \text{with equality if } A_{jj} > 0, \quad j = 1,...,n.$$ \hspace{1cm} (7)

The interpretation is that raising $A_{jj}$ by $1$ and raising price by $\frac{\partial Q_j}{\partial A_{jj}}$ too leaves $\delta_j$ unchanged.

This change therefore increases revenue by $\frac{\partial Q_j}{\partial A_{jj}}$ on the existing consumer base (i.e., $M s_j$)

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\footnote{For example, in the standard multinomial logit model, we have $s_j = \frac{\exp(\delta_j/\mu)}{\sum_{k=0}^{n} \exp(\delta_k/\mu)} \quad j = 0,...,n.$}

\footnote{Below we (implicitly) invoke sufficient concavity of $Q_j$ for interior solutions to (7): if $\frac{\partial Q_j}{\partial A_{jj}}$ were constant (if ads entered perceived quality linearly), then this is unlikely.}
consumers). This extra revenue is equated to the $1 marginal cost of the change, the RHS of (7). The relation in (7) implicitly determines self-promotion as a function of whatever advertising variables are in $Q_j$ (these all involve firm $j$ as either sender or target), along with $j$’s share.

Recalling our assumption that $\frac{\partial^2 Q_j}{\partial A_{jj}^2} > 0$, from (7), firms for which $s_j$ is larger will advertise more (choose a higher value of $A_{jj}$) than those with smaller market shares. The intuition is that the advertising cost per customer is lower for larger firms (this is a useful characterization result for advertising in general). The other relations in the following proposition follow similarly.

**Proposition 1 (Self-promotion Advertising levels)** The choice of self-promotion advertising level is determined by $M s_j \frac{\partial Q_j}{\partial A_{jj}} \leq 1$, with equality if $A_{jj} > 0$. For $A_{jj} > 0$, this relationship defines $A_{jj}$ as an implicit function of own market share, $s_j$: $j$’s outgoing attacks, $\{A_{jk}\}_{k \neq j}$; and the incoming attacks on $j$, $\{A_{kj}\}_{k \neq j}$. then $A_{jj}$ is an increasing function of $s_j$. $A_{jj}$ is a decreasing (increasing) function of $A_{jk}$ if and only if $\frac{\partial^2 Q_j}{\partial A_{jj} \partial A_{jk}} < 0$ ($> 0$). $A_{jj}$ is an increasing (decreasing) function of $A_{kj}$ if and only if $\frac{\partial Q_j}{\partial A_{jj} \partial A_{kj}} > 0$ ($< 0$).

The advertising relationships in the Proposition hold for firms with large enough market shares (otherwise, from (5) the term $(p_j - c_j) \frac{ds_j}{d\delta_j}$ is small enough that the derivative $\frac{d\pi_j}{dA_{jk}}$ in (6) is negative when $\frac{\partial Q_j}{\partial A_{jj}}$ is evaluated at $A_{jj} = 0$). They will be estimated below using a simple specification in which $A_{jj}$ is written as a linear function of $s_j$ and the other relevant advertising quantities.

We now turn to comparative advertising levels. Since the perceived quality is $Q_j(A_{jj}, \{A_{jk}\}_{k \neq j}, \{A_{kj}\}_{k \neq j})$, we can determine the advertising spending against rivals by differentiating (4) to get (for $k \neq j$):

$$
\frac{d\pi_j}{dA_{jk}} = \frac{d\pi_j}{d\delta_j} \cdot \frac{\partial Q_j}{\partial A_{jk}} + \frac{d\pi_j}{d\delta_k} \cdot \frac{\partial Q_k}{\partial A_{jk}}
$$

This can be split into two terms: one representing an increase in $Q_j$ due to $A_{jj}$'s own advertising spending, and the other representing a decrease in $Q_k$ due to $A_{jj}$'s advertising spending on another firm:

$$
= M(p_j - c_j) \frac{ds_j}{d\delta_j} \frac{\partial Q_j}{\partial A_{jj}} + M(p_j - c_j) \frac{ds_k}{d\delta_k} \frac{\partial Q_k}{\partial A_{kk}} - 1 \leq 0,
$$

- **own Q enhancement**
- **competitor's Q denigration**
with equality if $A_{jk} > 0$. We proceed by substituting the attackers’ pricing condition and its self-promotion condition to rewrite this comparative advertising condition in a form to be estimated.

Inserting the price first-order conditions (5) gives (for $k \neq j$):

$$\frac{d\pi_j}{dA_{jk}} = Ms_j \frac{\partial Q_j}{\partial A_{jk}} - Ms_j d_{jk} \frac{\partial Q_k}{\partial A_{jk}} \leq 1,$$

(8)

where $d_{jk} = -\frac{ds_j}{\delta k} \frac{ds_j}{\delta j} > 0^{15}$ is the diversion ratio discussed above. Loosely, the diversion ratio measures how much custom is picked up from a rival per customer it sheds. The restriction on the diversion ratios ($d_{jk} \in [0, 1]$) motivates restrictions below in the estimation.

The comparative advertising derivative, (8), can be used to derive a bound on the size of the marginal rate of substitution between outgoing comparative advertising and self-promotion ($\frac{\partial Q_j}{\partial A_{jk}} / \frac{\partial Q_j}{\partial A_{jj}}$). Assume for the present argument that the solution for self-promotion spending (see (7)) is interior. Then, substituting the self-promotion condition ($Ms_j \frac{\partial Q_j}{\partial A_{jj}} = 1$) into (8) implies that

$$\frac{\partial Q_j}{\partial A_{jk}} / \frac{\partial Q_j}{\partial A_{jj}} \leq 1 + Md_{jk} \frac{\partial Q_k}{\partial A_{jk}} < 1$$

(9)

where the last inequality follows because $\frac{\partial Q_k}{\partial A_{jk}} < 0$. In summary:

**Proposition 2 (Self-promotion and outgoing comparative advertising)** If Firm $j$ uses a strictly positive amount of self-promotion, then the marginal rate of substitution between outgoing comparative advertising against Firm $k$ and self-promotion ($\frac{\partial Q_j}{\partial A_{jk}} / \frac{\partial Q_j}{\partial A_{jj}}$) is strictly below 1.

If this were not the case, then comparative advertising would drive out self-promotion since it would give a direct own-quality benefit per dollar greater than self-promotion, while additionally helping the attacker by denigrating a rival. We will assume in the estimation that the marginal rate of substitution between outgoing comparative advertising and self-promotion in (9) is constant, at rate $\lambda$, so that the testable implication of Proposition 2 is

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15For discrete choice models we have $\frac{ds_j}{\delta k} = \frac{ds_j}{\delta j}$ (see Anderson, de Palma, and Thisse, 1992, Ch. 3, p. 67), and hence $d_{jk} = -\frac{ds_k}{\delta j} / \frac{ds_j}{\delta j}$. This substitution is used in deriving the first-order condition as written.
that $\lambda < 1$. Then we can write:

\[
(0 <) - M s_j d_{jk} \frac{\partial Q_k}{\partial A_{jk}} \leq 1 - \lambda.
\]  

(10)

The intuition is as follows. Raising $A_{jk}$ by $1$ is equivalent to brand $k$ raising its price by $\$s_jd_{jk}$ (since the same $\delta_k$ is attained). Such a rival price change (which $j$ thus effectuates through comparative advertising) causes $j$’s market share to rise by $\frac{ds_j}{\delta_k}$. This increment is valued at $M(p_j - c_j)$. By the price first-order condition, $p_j - c_j = \frac{s_j}{\delta_j}$, and (10) follows.

To determine predictions for how $A_{jk}$ depends on the other relevant advertising levels, recall that $\frac{\partial^2 Q_k}{\partial A_{jk}^2} > 0$.

**Proposition 3 (Comparative Advertising levels)** The choice of comparative advertising level by Firm $j$ against Firm $k$ is determined by $-M\psi_{jk} \frac{\partial Q_k}{\partial A_{jk}} \leq 1 - \lambda$, with equality if $A_{jk} > 0$. For $A_{jk} > 0$, this relationship defines $A_{jk}$ as an implicit function of $\psi_{jk} = s_jd_{jk}$, the target’s outgoing advertising, and the incoming attacks upon it. Then $A_{jk}$ is an increasing function of $\psi_{jk}$. $A_{jk}$ is an increasing (decreasing) function of $A_{kk}$ if and only if $\frac{\partial^2 Q_k}{\partial A_{kk}^2} < 0 (> 0)$. $A_{jk}$ is an increasing (decreasing) function of $A_{kl}$ if and only if $\frac{\partial^2 Q_k}{\partial A_{jk} \partial A_{lk}} < 0 (> 0)$. $A_{jk}$ is a decreasing (increasing) function of $A_{lk}$ if and only if $\frac{\partial^2 Q_k}{\partial A_{jk} \partial A_{lk}} > 0 (< 0)$.

We deal first with the comparative statics of the advertising levels in $Q_k$. The sign of the cross-partial $\frac{\partial^2 Q_k}{\partial A_{kk} \partial A_{jk}}$ is already accomplished from the estimation of the Self-Promotion equation. That is, from Proposition 1, if self-promotion increases with incoming comparative advertising, then comparative advertising decreases with target self-promotion.

The cross-partial $\frac{\partial^2 Q_k}{\partial A_{kk} \partial A_{jk}}$ has the same sign as that of the cross partial $\frac{\partial^2 Q_k}{\partial A_{kk} \partial A_{jk}}$ because we assume that $\lambda$ is constant and $\frac{\partial Q_k}{\partial A_{kk}} = \lambda \frac{\partial Q_k}{\partial A_{kk}}$, where both $Q_k$ derivatives are positive by assumption (hence $\lambda > 0$, which implies the two cross-partials have the same sign). Hence estimating the self-promotion equation will also indicate that comparative advertising decreases with target out-going comparative ads if and only if self-promotion increases with incoming comparative advertising.

The intuition for the two comparative static results above is that a firm is attacked less when it advertises more if the increase in outgoing ads decreases the negative impact of
attacks (i.e., \( \frac{\partial^2 Q_k}{\partial A_{jk} \partial A_{kj}} > 0 \)). Notice too that for both of them it is not the choice of a specific functional form for \( Q \) that restricts cross comparative static properties. Rather, these are implied by the model.

Finally, attacks against \( k \) by \( j \) increase with attacks on \( k \) by others if \( \frac{\partial^2 Q_k}{\partial A_{jk} \partial A_{lk}} < 0 \). This cross-partial sign implies that more hurt is inflicted with a marginal attack by \( j \) because the others’ attacks have rendered \( k \)’s quality more susceptible. Notice that in our empirical specification below the sign of \( \frac{\partial^2 Q_k}{\partial A_{jk} \partial A_{lk}} \) is tied down to be negative so we have effectively imposed this result. !!

There are several ways to approach the result that there are more attacks on a given link, ceteris paribus, the higher is \( \psi_{jk} \), because there are different alternative ways to write \( \psi_{jk} \). Basically, the term compounds a diversion measure and market shares. Writing \( \psi_{jk} = s_j d_{jk} \), the result says that there are more attacks for given diversion ratio \( d_{jk} \) the higher the attacker market share. This is roughly borne out in the raw data insofar as Advil and Aleve are the largest attackers of Tylenol. Likewise, for a given attacker share, attacks are larger for a bigger diversion ratio. Alternatively, we can write \( \psi_{jk} = s_k D_{jk} \) where \( D_{jk} = \frac{s_j}{s_k} d_{jk} \) is the ratio of cross elasticity of demand to own elasticity. In this case, for a given value of \( D_{jk} \), a bigger target is attacked more. This roughly concurs with the data that the largest firm, Tylenol, is attacked most. Or indeed we can write \( \psi_{jk} = d_{kj} \frac{s_k}{s_j} \frac{\partial Q_k}{\partial Q_k} s_k \) where the \( \varepsilon \) are own-price demand elasticities. This last version has too the property that it increases in \( s_k \) (target share). The other interesting feature of this formulation is that \( \psi_{jk} \) depends on \( d_{kj} \) directly. That is, the incentive to attack depends on how much of the demand shift emanating from an attack goes to the attacker. By contrast, the other formulations involve the reverse diversion ratio, \( d_{jk} \), which is that from attacker to target (this expression results from the substitution of the attacker’s pricing condition (5) and the symmetry property \( \frac{ds_j}{ds_k} = \frac{ds_k}{ds_j} \)). We shall proceed for the estimation by taking \( \psi_{jk} = s_j d_{jk} \), and we will estimate \( d_{jk} \) for each pair. Thus we are implicitly constraining the diversion ratios to be constant over time.

We next show how the measure of the damage of an extra dollar of comparative advertising from Firm \( j \) against target \( k \) is a weighted average of push and pull effects, both of
which can be written in terms of diversion ratios.

The full effect (of a marginal dollar of comparative advertising from $j$) on $k$’s profits, 

$$
\pi_k^* = M (p_k^* - c_k) s_k^* - A_{kk}^* - \sum_{l \neq k} A_{kl}^* \tag{10}
$$

(where the stars denote equilibrium values), holding constant all other actions except the best reply of $k$ is determined by the envelope theorem as

$$
\frac{d\pi_k^*}{dA_{jk}} = M (p_k^* - c_k) \left( \frac{ds_k}{d\delta_k} \frac{\partial Q_k}{\partial A_{jk}} + \frac{ds_k}{d\delta_j} \frac{\partial Q_j}{\partial A_{jk}} \right).
$$

Substituting in $k$’s pricing condition (see (5)) implies

$$
\frac{d\pi_k^*}{dA_{jk}} = M s_k \left( \frac{\partial Q_k}{\partial A_{jk}} - d_{kj} \frac{\partial Q_j}{\partial A_{jk}} \right)
= -\frac{s_k}{s_j} \left( \frac{(1 - \lambda)}{d_{jk}} + \lambda d_{kj} \right) \tag{11}
$$

where we have substituted in the equality versions of conditions (10) and (7) at the second step.\footnote{Equivalently, we can write this as \( \frac{d\pi_k^*}{dA_{jk}} = (1 - \lambda) \text{Pull}_{jk} + \lambda \text{Push}_{jk} = \frac{(1 - \lambda)}{D_{jk}} + \lambda D_{kj}. \)}

The first term here is the amount of self-promotion required to restore $Q_k$ and the second term is the hurt inflicted by the rival’s increased self-promotion component of the comparative advertising (hence the $\lambda$ weight corresponding to the push effect). Note that the effect on profit here and below is measured in dollars: equivalently (by the target’s optimality condition that the $\$1$ marginal cost of an extra dollar’s advertising equals its marginal benefit), it is the amount of self-promotion advertising that would have to be spent to offset the hurt. Our empirical analysis will provide estimates of relevant parameters so the marginal hurt (or compensating advertising) to the target can be estimated.

**Proposition 4 (Damage Measure)** Assume that the target, $k$, engages in a strictly positive level of self-promotion, and assume that outgoing comparative ads are perfectly substitutable with self-promotion at rate $\lambda$. Then the profit lost by target $k$ after an additional dollar of comparative advertising attack by Firm $j$ is a weighted average of a pull effect, $\frac{1}{d_{jk} s_j}$, and a push effect, $\frac{s_k}{s_j} d_{kj}$, with respective weights $(1 - \lambda)$ and $\lambda$ respectively.

In like manner we can determine the spill-over benefit (related to free riding in compar-
ative advertising) to $l$ of an attack by $j$ on $k$ as
\[
\frac{d\pi_l^*}{dA_{jk}} = \frac{s_l}{s_j} \left( d_{lk} \left(1 - \lambda\right) - \lambda d_{lj} \right).
\]
The first term here is the direct benefit to $l$ from the hurt inflicted on $k$ (pull); the second is (as above) the damage incurred by $l$ from $j$ improving its quality through the comparative advertising channel (push).

[we can write this and the others below in terms of the Push and Pull variables we defined above, or indeed simply in terms of $D'$s.]

From these expressions we can determine the total externality on firms other than $k$ of an attack on $k$ as:
\[
\sum_{l \neq k, j} \frac{s_l}{s_j} \left( d_{lk} \left(1 - \lambda\right) - \lambda d_{lj} \right)
\]
so that if this is positive there is insufficient comparative advertising by $j$ against $k$ from the perspective of all the attackers. The expression is ambiguous: $\lambda$ close to zero means not enough comparative advertising (from this group perspective), while $\lambda$ near 1 means too much (because it is almost all push-up effect). To benchmark, if $\lambda$ is close to $1/2$, and the $d$'s are not too dissimilar, there is not enough: loosely, an attack pulls down the target so much that all benefit despite the push effect.

We can also look at the impact of switching the marginal self-promotional ad dollar into comparative ads. There is no first-order effect on the attacker because its portfolio was optimally arranged. The effect on the others is given by noting that $\frac{d\pi_l^*}{dA_{jj}} = -d_{lj} \frac{s_l}{s_j}$, so that the effect of a switch is to increase $l$’s profit by
\[
\frac{d\pi_l^*}{dA_{jk}} - \frac{d\pi_l^*}{dA_{jj}} = \left(1 - \lambda\right) \frac{s_l}{s_j} \left( \frac{d_{lk}}{d_{jk}} + d_{lj} \right).
\]
Now consider the effects on the target. If $j$ switches a self-promotion ad to an attack on $k$, the net gain to $k$ is
\[
\frac{d\pi_k^*}{dA_{jk}} - \frac{d\pi_k^*}{dA_{jj}} = \left(1 - \lambda\right) \frac{s_k}{s_j} \left( -\frac{1}{d_{jk}} + d_{kj} \right).
\]
Since the $d$'s are between zero and one, this is negative as expected (an attack is more harmful than self-promotion).

Summing over all firms, then the total profit benefit from a switch is:
\[
\frac{1 - \lambda}{s_j} \left( \sum_{l \neq k, j} s_l d_{lk} - s_k \frac{d_{lk}}{d_{jk}} + \sum_{l \neq j} s_l d_{lj} \right).
\]
Then a switch from self-promotion to attacking \( k \) by Firm \( j \) decreases total industry profits if and only if
\[
\sum_{l \neq k, j} s_l d_{lk} (1 + d_{jk}) + s_k d_{kj} - s_k < 0. \tag{12}
\]

As a point of comparison, suppose all the diversion ratios and shares are the same. Then this condition becomes \((n - 2) d (1 + d) + d - 1 < 0\): taking \( d \) close to \( \frac{1}{n} \) verifies the condition, which suggests the conclusion that comparative advertising is a mutually harmful undertaking relative to self-promotion (although it is an empirical matter whether (12) is satisfied). This result in turn suggests the observation that comparative ads are rare in most industries, perhaps because of a tacit mutually awareness that the practice is damaging for all.

3 Description of Industry and Data\(^{17}\)

The OTC analgesics market is worth approximately $2 billion in retail sales per year (including generics) and covers pain-relief medications with four major active chemical ingredients. These are Aspirin, ACT, Ibuprofen, and Naproxen Sodium. The nationally advertised brands are such familiar brand names as Tylenol (ACT), Advil and Motrin (ibuprofen), Aleve (naproxen sodium), Bayer (aspirin or combination), and Excedrin (ACT or combination).

Table 1 summarizes market shares, ownership, prices and advertising levels in this industry.

3.1 Sales Data

The sales data, collected by AC Nielsen, consist of average prices, dollar sales, and dollar market shares (excluding Wal-Mart sales) of all OTC oral analgesics products sold in the U.S. national market from March of 2001 through December of 2005 (58 monthly observations).

We construct a measure of a *serving* of pain medication, or a *pain episode*, so that we can aggregate across different package sizes and across different medication strengths. In particular, to each analgesic product in the sales dataset we assign the strength of its active ingredient in milligrams, and hence we derived the maximum number of pills that a consumer

\(^{17}\)A detailed description of how we construct the dataset is provided in Appendix ZZZ.
can take for OTC consumption over 24 hours as defined and required by the FDA (e.g. 3 in the case of Aleve, and from 6 to 12 for Tylenol, depending on the ACT formula). Then, an episode of pain is given by the maximum number of pill times the average number of pain days per month in the population, which is three.\textsuperscript{18}

<table>
<thead>
<tr>
<th>Brand</th>
<th>Active Ing.</th>
<th>Price per serving</th>
<th>Sales Share</th>
<th>Brand Vol. Share</th>
<th>Weighted Share</th>
<th>Max Pills</th>
<th>TA/CA/CA Pills</th>
<th>TA/CA/CA Sales</th>
<th>TA/CA/CA Ship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tylenol</td>
<td>ACT</td>
<td>$2.15</td>
<td>29.16%</td>
<td>38.90%</td>
<td>30.51%</td>
<td>7.22</td>
<td>17.35%</td>
<td>3.34%</td>
<td>19.26%</td>
</tr>
<tr>
<td>Advil</td>
<td>IB</td>
<td>$1.60</td>
<td>17.15%</td>
<td>22.87%</td>
<td>24.21%</td>
<td>5.9</td>
<td>19.96%</td>
<td>13.26%</td>
<td>66.43%</td>
</tr>
<tr>
<td>Aleve</td>
<td>NS</td>
<td>$0.83</td>
<td>8.25%</td>
<td>11.00%</td>
<td>22.40%</td>
<td>3</td>
<td>25.97%</td>
<td>20.04%</td>
<td>75.65%</td>
</tr>
<tr>
<td>Excedrin</td>
<td>ACT</td>
<td>$2.40</td>
<td>8.80%</td>
<td>11.74%</td>
<td>8.28%</td>
<td>9.22</td>
<td>26.42%</td>
<td>3.43%</td>
<td>13.19%</td>
</tr>
<tr>
<td>Bayer</td>
<td>ASP</td>
<td>$1.85</td>
<td>5.73%</td>
<td>7.65%</td>
<td>6.98%</td>
<td>10.07</td>
<td>28.80%</td>
<td>6.44%</td>
<td>22.36%</td>
</tr>
<tr>
<td>Motrin</td>
<td>IB</td>
<td>$1.71</td>
<td>5.83%</td>
<td>7.78%</td>
<td>7.68%</td>
<td>5.86</td>
<td>20.37%</td>
<td>8.07%</td>
<td>39.63%</td>
</tr>
<tr>
<td>Generic</td>
<td>ACT</td>
<td>$1.17</td>
<td>8.00%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>IB</td>
<td>$0.66</td>
<td>9.25%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>ASP</td>
<td>$0.82</td>
<td>6.08%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generic</td>
<td>NS</td>
<td>$0.57</td>
<td>1.66%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The market size for OTC analgesic products is the US population 18 years or older minus the number of people who shop at Wal-Mart, a store that does not provide information on the sales of products. Then, we compute each brand’s market share as the fraction of total number of episodes of pain sold by the brand over market size. The average price of a pain episode is computed as the ratio of the total sales by a brand divided by the total number of episodes of pain sold in a month. We do likewise for the generic products, which differ only by their active ingredient. The resulting output is the time series of average prices of episodes of pain relief for each of the four active ingredients for the generic products. We maintain that the generic products are provided by a competitive fringe and that the generic prices are set equal to the marginal cost.

\textsuperscript{18}This information is from the Morbidity and Mortality Weekly Report, Centers for Disease Control and Prevention, Feb 27, 1998/47(07);134-140.
3.2 Advertising Data

Our advertising dataset is from TNS-Media Intelligence and data is reported on monthly basis. The advertising data contain monthly advertising expenditures on each ad, and video files of all TV advertisements for the 2001-2005 time period for each brand advertised in the OTC analgesics category. The unit of observation in the raw dataset is a single ad. There are 4,506 unique ads (346 of which have missing advertising videos). For each ad, we know the amount spent in each month and the number of times that creative was shown during the specific month. Each ad is also associated with a video file.

We watched all the ads and coded according to their content. We recorded whether the product was explicitly compared to any other products. If a commercial was comparative, we recorded which brand (or class of drugs) it was compared to (e.g. to Advil or Aleve; or to Ibuprofen-based drugs). If an ad had multiple targets, the ad is assigned equally among them proportionately.

If an ad had no comparative claims, it was classified as a self-promotion ad. In the data we observe situations when a firms carry indirect attacks on their competitors. An indirect attack occurs when one brand makes a claim against “all other regular” brands. We code such indirect attacks as self-promotion ads. We look at other coding options in Appendix ZZZ.

Table 2 presents the complete picture of cross targeting and the advertising expenditure on each of the rival brand targeting. This table shows every nationally advertised brand used comparative advertising during the sample period. However, the brands against which comparisons were made are only a subset of the nationally advertised brands. The targets are Tylenol, Advil, Aleve, and Excedrin. Notice that these data provide some informal support for the larger firms both using more comparative advertising and being targeted more. The entries on the diagonal are zeroes through not attacking oneself.

 Motrin does not attack Tylenol because the parent company is the same; likewise, Bayer does not attack Aleve for the same reason.
### TABLE 2. Advertising and Comparative Advertising Target Pairs

<table>
<thead>
<tr>
<th>Advertiser ↓</th>
<th>TARGET:</th>
<th>Advil</th>
<th>Aleve</th>
<th>Bayer</th>
<th>Excedrin</th>
<th>Motrin</th>
<th>Tylenol</th>
<th>Total CA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advil</td>
<td>92.09</td>
<td>17.80</td>
<td>-</td>
<td>4.26</td>
<td>-</td>
<td>160.20</td>
<td>182.26</td>
<td>274.35</td>
<td></td>
</tr>
<tr>
<td>Aleve</td>
<td>-</td>
<td>42.54</td>
<td>0.0003</td>
<td>0.48</td>
<td>-</td>
<td>131.66</td>
<td>132.14</td>
<td>174.68</td>
<td></td>
</tr>
<tr>
<td>Bayer</td>
<td>13.77</td>
<td>104.98</td>
<td>-</td>
<td>-</td>
<td>15.69</td>
<td>29.47</td>
<td>131.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excedrin</td>
<td>-</td>
<td>1.96</td>
<td>2.15</td>
<td>158.38</td>
<td>-</td>
<td>19.96</td>
<td>24.08</td>
<td>182.46</td>
<td></td>
</tr>
<tr>
<td>Motrin</td>
<td>18.84</td>
<td>18.79</td>
<td>-</td>
<td>-</td>
<td>57.32</td>
<td>-</td>
<td>37.63</td>
<td>94.95</td>
<td></td>
</tr>
<tr>
<td>Tylenol</td>
<td>9.60</td>
<td>31.64</td>
<td>36.56</td>
<td>-</td>
<td>-</td>
<td>359.02</td>
<td>77.80</td>
<td>404.03</td>
<td></td>
</tr>
<tr>
<td>Total CA</td>
<td>42.61</td>
<td>70.19</td>
<td>38.71</td>
<td>4.74</td>
<td>-</td>
<td>327.51</td>
<td>483.38</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Row \((i)\) indicates the advertiser brand and Column \((j)\) indicates the comparative advertising target by brand \(i\). The upper part of cell \(ij\) indicates comparative ad expenditure in $mln; whereas the bottom part denotes how many time periods (out of 58) the attack pair \(ij\) happened; The diagonal entries indicate expenditures on positive advertising. The totals on the right are presented separately for both comparative only and overall advertising levels.

### 3.3 News Shocks

The OTC analgesics market endured several major medical news shocks over the time period. Following Chintagunta, Jiang and Jin (2007) we used Lexis-Nexis to search over all articles published between 2001 and 2005 on relevant topics. We recorded the article name, source and date to construct a data-set of news shocks. Multiple articles reporting the same news were assigned to a unique shock ID. Second, we checked whether a news shock was associated with any new medical findings that were published in major scientific journals. Finally, we checked that the news shock was reported in a major national newspaper (USA Today, Washington Post, Wall Street Journal, New York Times) As a result of this data cleaning, our news shock data-set includes 8 news shocks between March of 2001 and December of 2005. Table 3 reports the news shocks, by their title, date, scientific publication.

For each shock that happened during period \(t\) we construct a dummy variable which is
equal to 1 for three months after \( t \) and at time \( t \): (i.e., \( t \) through \( t+3 \)).

### TABLE 3. Medical News Shocks

<table>
<thead>
<tr>
<th>No</th>
<th>News Shock Description</th>
<th>Date</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Risk of Cardiovascular Events Associated With Selective COX-2 Inhibitors</td>
<td>8/21/2001</td>
<td>Journal of the American Medical Association (JAMA), 2001;286:954-959</td>
</tr>
<tr>
<td>3</td>
<td>FDA Panel Calls for Stronger Warnings on Aspirin and Related Painkillers</td>
<td>9/21/2002</td>
<td>FDA Public Health Advisory</td>
</tr>
<tr>
<td>6</td>
<td>Vioxx Withdrawn From the Market</td>
<td>9/30/2004</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Long Term Use of Naproxen (Aleve) May be Associated with an Increased Cardiovascular Risk</td>
<td>12/23/2004</td>
<td>FDA Public Health Advisory</td>
</tr>
<tr>
<td>8</td>
<td>Bextra Withdrawn</td>
<td>4/7/2005</td>
<td></td>
</tr>
</tbody>
</table>

### 4 The Econometric Model

#### 4.1 A Quality Function

After extensive experimentation, we chose the following functional form for the quality function, which combines the push and pull effects of advertising into a net persuasion effect and an additional term for incoming attacks:

\[
Q_j(.) = \alpha \ln \left( A_{jj} + \lambda \sum_{k \neq j} A_{jk} - \beta \sum_{k \neq j} A_{kj} + \bar{A}_{jj} \right) - \varphi \sum_{k \neq j} \ln \left( \bar{A}_{kj} + A_{kj} \right). \tag{13}
\]

Here, \( Q_j(.) \) depends on incoming and outgoing advertising levels. Other variables pertaining to \( j \) enter through \( \bar{A}_{jj} \) and \( \bar{A}_{kj} \) along with unobserved factors captured by random errors. Variables (such as product characteristics) that are specific to \( j \)'s quality with no interaction with \( j \)'s advertising are omitted since they would not enter into the advertising first order conditions.

The push effect is incorporated through the weighted sum of self-promotion and outgoing comparative ads \( A_{jj} + \lambda \sum_{k \neq j} A_{jk} \), where \( \lambda \) is the marginal rate of substitution.
between outgoing comparative and self-promotion ads, which is assumed constant. Recall from Proposition 2 that we should expect $\lambda < 1$.

The pull effect from incoming comparative ads ($A_{kj}$) impacts the quality function in two ways. First, in the "net persuasion" term, the sign of $\beta$ gives the sign of the cross effect between incoming ads and outgoing ones, as determines the comparative static properties in Propositions 1 and 3. The value of $\beta$ is found from estimating the self-promotion relation (7) and may *a priori* be positive or negative. $\beta$ measures the intensity of the cross effects between attacks on a given target, $j$. The sign of the effect is necessarily positive for our specification: from Proposition 3, comparative ads from $j$ on $k$ are increasing in attacks by others.

Second, incoming ads enter in a separable way with associated parameter $\phi$. This extra term ensures - if $\phi > 0$ - that the overall pull effect is negative (with $\phi$ large enough, it also ensures that $\frac{\partial^2 Q_k}{\partial A_{jk}^2} > 0$ locally). The inclusion of the parameter $\phi$ also ensures that the full pull effect is not pre-determined from the cross effect. Through this separable term, we also allow the $A_{kj}$ to not be perfect substitutes: without this, if $\varphi = 0$, then each firm would be attacked by at most one of its competitors in any period. The data suggest that this is not the case.

$\bar{A}_{jj}$ and $\bar{A}_{kj}$ determine the marginal efficiency of self-promotion and comparative advertising. The higher is $\bar{A}_{jj}$ (the inherent quality perception of a brand), the lower is the marginal efficiency of self-promotion advertising. The larger $\bar{A}_{kj}$ (the inherent quality perception advantage of firm $k$ on $j$) the less firm $k$ has to attack brand $j$ to push its quality down a given amount.

### 4.2 The Equations To Be Estimated

The first order condition for self-promotion ads, corresponding to equation (7) above may be written as

$$
\begin{align*}
A_{jjt}^* &= -\alpha M s_{jt} - \lambda \sum_{k\neq j} A_{jkt} + \beta \sum_{k\neq j} A_{kjt} - Z_{SP,jt}^\prime \Phi - \xi_{SP,jt}, \\
\xi_{SP,jt} &\sim N(0, \sigma_{SP}^2), \quad A_{jjt} = \max(A_{jjt}^*, 0), \quad j = 1, \ldots, n.
\end{align*}
$$

(14)
Here, \( \tilde{A}_{jkt} = Z'_{SP,jt}\Phi + \xi_{SP,jt} \), where \( Z_{SP,jt} \) are exogenous variables that shift the inherent quality perception of Firm \( j \). \( \xi_{SP,jt} \) is the structural unobservable component of perceived quality that interacts with \( A_{jkt} \). The structural error is modeled as entering linearly in the term \( \tilde{A}_{jkt} \). The equation above is a Tobit regression that is linear in the parameters.

The first order condition for comparative ads follows from applying the quality function (13) in (10) above and inserting the target’s self-promotion condition (7). This gives first

\[
-Ms_{j}d_{jk} \left( \frac{-\alpha\beta}{A_{kk} + \lambda\sum_{l\neq k} A_{kl} - \beta\sum_{l\neq k} A_{lk} + A_{kk}} - \frac{\phi}{(A_{jk} + A_{jk})} \right) \leq 1 - \lambda,
\]

with equality if \( A_{jk} > 0 \), and, second, using the target \( k \)'s self-promotion equation when \( A_{kk} > 0 \) (namely \( A_{kk} + \lambda\sum_{l\neq k} A_{kl} - \beta\sum_{l\neq k} A_{lk} + \tilde{A}_{kk} = \alpha M_{s_{k}} \)), this becomes

\[
\frac{\phi M_{s_{k}} d_{jk}}{(1-\lambda)A_{kk} - \beta d_{jk}} \leq (\tilde{A}_{jk} + A_{jk}),
\]

which leads to the econometric specification below:

\[
\begin{align*}
A_{jkt} = \max \left\{ \frac{s_{k}d_{jk}}{A_{kk} + \lambda\sum_{l\neq k} A_{kl} - \beta\sum_{l\neq k} A_{lk} + A_{kk}} - Z'_{CA,jkt}\Phi - \xi_{CA,jkt}, 0 \right\}, \\
\xi_{CA,jkt} \sim N(0, \sigma_{CA}^2), \quad A_{jkt} = \max (A_{jkt}', 0), \quad j = 1, \ldots, n.
\end{align*}
\]

(15)
as long as \( A_{kk} > 0 \). Here, \( \tilde{A}_{jk} = Z'_{CA,jk}\Phi + \xi_{CA,jk} \), where \( \xi_{CA,jk} \) is the unobservable component that is not observed by the econometrician. We will assume that diversion ratios are constant over time, and given by \( d_{jkt} = d_{jk} \). Equation (15) is a Tobit regression that is nonlinear in the parameters.

4.3 Identification Strategy

In both the Tobit regressions above, the unobservables are correlated with the explanatory advertising and share variables. The first step to address the endogeneity is to exploit the panel structure of our data to account for time-constant differences across brands. Essentially, for the self-promotion equation, we set \( \xi_{SP,jt} = \tilde{\xi}_{SP,j} + \Delta \xi_{SP,jt} \), where \( \tilde{\xi}_{SP,j} \) is a brand fixed effect, while \( \Delta \xi_{SP,jt} \) are time specific idiosyncratic shocks. In a similar vein, for the comparative ad equation, we set \( \xi_{CA,jkt} = \tilde{\xi}_{CA,jk} + \Delta \xi_{CA,jkt} \), where \( \tilde{\xi}_{CA,jk} \) is a fixed effects that might be brand or brand-pair specific, while \( \Delta \xi_{CA,jkt} \) are time-specific idiosyncratic shocks. The dummy variables control for the brands’ advertising base allure advantage, so that it picks up any persistent component of such advantage. [CITE OF SAMUELSON HERE?]
The remaining source of endogeneity in our regressions then comes from any potential correlation between temporary shocks, here picked up by $\Delta \xi_{jt}$, and $A_{jkt}$ and $A_{kjt}$. It turns out that the fixed effects control for most of endogeneity bias affecting the effect of shares on the advertising decisions.

The second step is to use instrumental variables. We consider unexpected news shocks and the prices of the generic products. Clearly, the news shocks are exogenous since they require new medical discoveries, which ‘surprise’ both the consumers and the firms, and alter the perception of the health properties of the products. Before using the news shocks as instrumental variables we will experiment whether they should enter into the quality function directly.

Generic prices can be used as instrumental variables as long as the marginal cost of production of a generic product does not depend on the quantity produced. Pricing at constant marginal cost for mature generic pharmaceutical products seems reasonable (JLE CITE). We will discuss the instrumental variables in detail when we present the results of our empirical analysis.

To implement our estimation in our non-linear models, we use control functions (Heckman and Robb [1985,1986]). Our methodology follows Blundell and Smith (1986) and Rivers and Vuong (1988). The only econometric difficulty in the application of that methodology is created by the fact that two of the explanatory variables in the self-promotion equation, $\sum_{k \neq j} A_{jkt}$ and $\sum_{k \neq j} A_{kjt}$, are left-censored, and thus the residuals that are required to construct the control functions are not well defined. To address this econometric problem, we construct the generalized residuals, as proposed by Gourierou et al. We describe the econometric approach in detail in Appendix ZZZ.

---

20 Notice that we can allow generic brands to charge prices that are higher than marginal costs as long as this is explained by local conditions that national brands do not take into account when they set their prices.

21 [GOEREE HERE] - she does GMM, we do two-steps. no overidentification of parameters in our case.
5 Empirical Analysis

5.1 Self-Promotion and Net Persuasion

Each column in Table 4 presents the results for the parameters $\alpha$, $\beta$, and $\lambda$ for various specifications of Equation (14).

Across all specifications $\alpha$, $\beta$, and $\lambda$ are positive and statistically significant. $\alpha > 0$ is consistent with Proposition 1 that larger shares are associated with more self-promotion advertising. $\lambda > 0$ means that self-promotion and outgoing attacks are substitutes, and $\lambda < 1$ is consistent with Proposition 2 that comparative advertising does not drive out self-promotion since the direct own quality benefit per dollar is smaller than the benefit from self-promotion. $\beta > 0$ implies that self-promotion increases with incoming advertising, which, by Proposition 3, implies that comparative advertising decreases with target self-promotion. None of these empirical results rejects the theoretical model. Next, we investigate the economic significance of the results in Table 4.

Column 1 of Table 4 shows the results from a straight Tobit regression, where self-promotion ad expenditures are regressed on sales, outgoing attacks and incoming attacks. We estimate $\alpha = 0.123$, which means that for any additional consumer a firm would spend 12.3 cents in self-promotion advertising.
<table>
<thead>
<tr>
<th>Version</th>
<th>Baseline</th>
<th>Brand Dummy</th>
<th>News Shocks</th>
<th>IV (News Shocks)</th>
<th>IV (Generics Linear &amp; Shocks)</th>
<th>IV (Generics Non Linear &amp; News Shocks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>0.123 (0.027)</td>
<td>0.432 (0.076)</td>
<td>0.524 (0.079)</td>
<td>0.551 (0.045)</td>
<td>0.520 (0.044)</td>
<td>0.520 (0.032)</td>
</tr>
<tr>
<td>Lambda</td>
<td>0.768 (0.072)</td>
<td>0.660 (0.074)</td>
<td>0.641 (0.073)</td>
<td>0.616 (0.087)</td>
<td>0.652 (0.074)</td>
<td>0.652 (0.045)</td>
</tr>
<tr>
<td>Beta</td>
<td>0.429 (0.063)</td>
<td>0.297 (0.068)</td>
<td>0.248 (0.070)</td>
<td>0.447 (0.037)</td>
<td>0.451 (0.029)</td>
<td>0.451 (0.028)</td>
</tr>
<tr>
<td>Control: Outgoing Ads</td>
<td>-0.039 (0.043)</td>
<td>-0.283 (0.040)</td>
<td>-0.284 (0.037)</td>
<td>0.018 (0.071)</td>
<td>0.003 (0.059)</td>
<td>-0.003 (0.041)</td>
</tr>
<tr>
<td>Control: Incoming Ads</td>
<td>-0.164 (0.035)</td>
<td>-0.170 (0.029)</td>
<td>-0.170 (0.029)</td>
<td>-0.164 (0.035)</td>
<td>-0.170 (0.029)</td>
<td>-0.170 (0.029)</td>
</tr>
<tr>
<td>Control: Shares</td>
<td>-0.309 (0.043)</td>
<td>-0.486 (0.052)</td>
<td>-0.487 (0.034)</td>
<td>-0.353 (0.081)</td>
<td>-0.440 (0.085)</td>
<td>-0.525 (0.054)</td>
</tr>
<tr>
<td>Brand dummy</td>
<td>0.137 (0.023)</td>
<td>-0.059 (0.024)</td>
<td>-0.043 (0.021)</td>
<td>0.137 (0.023)</td>
<td>-0.001 (0.038)</td>
<td>0.194 (0.008)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.194 (0.008)</td>
<td>0.185 (0.003)</td>
<td>0.185 (0.002)</td>
<td>0.194 (0.008)</td>
<td>0.189 (0.008)</td>
<td>0.180 (0.007)</td>
</tr>
<tr>
<td>/sigma</td>
<td>47.955</td>
<td>75.089</td>
<td>63.059</td>
<td>63.059</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F-test: Outgoing Ads
F(6,341)=6.27 F(22,325)=2.90 F(30,317)=3.12
Prob>F=0.000 Prob>F=0.000 Prob>F=0.000

F-test: Incoming Ads
F(6,341)=22.58 F(21,326)=7.91 F(29,318)=13.77
Prob>F=0.000 Prob>F=0.000 Prob>F=0.000

F-test: Shares
F(6,340)=40.09 F(22,324)=12.36 F(30,316)=21.25
Prob>F=0.000 Prob>F=0.000 Prob>F=0.000

With Shocks       | No | No | Yes | No | No | No |
IV: Generic Linear| Yes | Yes |     | Yes |     |     |
IV: Shocks        | No | Yes |     | Yes |     |     |
IV: Generic Quadratic| No | No | Yes |       |     |     |
The marginal rate of substitution between outgoing attacks and self-promotion ads, \( \lambda \), is equal to 0.768.\(^{22}\) \( \beta = 0.429 \) means from Proposition 1 that firms respond to incoming attacks by increasing their self-promotion and outgoing attacks. In particular, for each dollar spent attacking target \( j \), Firm \( j \) would react by spending 42.9 cents in self-promotion advertising, conditional on shares remaining constant.\(^{23}\)

We now investigate how the results change when we address the endogeneity of the explanatory variables.

In Column 2 we run the Tobit regression including one dummy variable that is equal to 1 if the observation is for one for the top brands (Advil, Aleve, Tylenol), and zero otherwise. Thus, we have \( \tilde{\xi}_{SP,j} = \tilde{\xi}^{TB}_{SP,j} \) for a top brand and \( \tilde{\xi}_{SP,j} = \tilde{\xi}^{OB}_{SP,j} \) otherwise. Using this specification, the coefficient estimate of \( \lambda \) drops from 0.710 to 0.660 and the coefficient estimate of \( \beta \) drops from 0.429 to 0.297. In contrast, the coefficient estimate of \( \alpha \) increases from 0.123 to 0.432. The contrasting direction of the bias between the advertising explanatory variables and the shares is useful to understand the relationship between the omitted variables, the unobserved component of perceived quality, and the explanatory variables. In particular, it is reasonable to think that products with a higher unobserved component of perceived quality will have a larger market share, \textit{ceteris paribus}. Then, this means that products with a stronger unobserved component of perceived quality do less self-promotion advertising than the other firms, \textit{ceteris paribus}. This, in turn, implies that firms with a higher perceived quality are attacked less and do attack less than firms with a lower perceived quality. This discussion is mirrored by the result on the coefficient estimate of the Top Brand dummy. The Top Brand fixed effect, \( \tilde{\xi}^{TB}_{SP,j} \) is equal to −0.353. It has a negative sign, which means that the larger firms, Aleve, Tylenol and Advil have inherently higher advertising base allure than the other brands.

In Column 3 we add the variables that measure the occurrence of a news shock. The results in Column 3 are remarkably similar to those in Column 2, suggesting that adding the

\(^{22}\)Notice that comparative advertising also pulls down the rivals, but we need to wait for the estimation results of the comparative ad equation, and of \( \varphi \) in particular, to get a measure of the marginal rate of substitution between incoming attacks and outgoing attacks.

\(^{23}\)0.427 is the marginal effect on unconditional [CHECK ON THIS].
news shocks as control variables is not changing the way that model fits the data.

In Column 4 and 5 we run two versions of an instrumental variable regression. The three endogenous variables are shares, outgoing comparative ads, and incoming attacks. In Column 4 the estimation with We find that the instrumental variables do a fair job at explaining the first stage variation in outgoing comparative advertising and in incoming attacks. The control functions for incoming attacks and market shares are statistically significant, suggesting that those two variables are endogenous in the Tobit regression (Blundell and Smith (1986)). On the contrary, the control function for the outgoing comparative ads is not statistically significant. While the variables are shown to be endogenous, the empirical significance of such endogeneity is minimal, given how similar the results are in Column 2 and 3. e report the coefficient estimates of the control functions associated with the endogenous variables. Moreover, the first-stage $F$ tests lead to the rejection of the null hypotheses that generic prices do not explain any of the first stage variation.

The instrumental variables are the news shocks; the own generic prices; the sum of the generic prices over all the competitors; second and third order terms for these variables that are created using generic prices; and the interactions of the own generic price and of the sum of the generic prices of the competitors with the news shocks. Overall, we have approximately thirty instrumental variables.

5.2 Comparative Advertising and Diversion Ratios

Table 5 presents the estimation results for the parameter $\varphi$ and for the diversion ratios $d_{jk}$. The diversion ratios are treated as parameters to be estimated from the data and are restricted to be between 0 and 1. The attractive feature of treating the diversion ratios as parameters is that we do not impose any distributional form assumption on the demand. The disadvantage of this approach is that the diversion ratios might change over time, and here we only estimate their mean value. This limitation is largely offset by the fact that in the industry that we study the market shares for all firms were remarkably constant over

\[^{24}\text{One could address this limitation by allowing the mean diversion ratio to change by year, but we do not do that because our data on market shares do not change much over time.}\]
time, and thus the diversion ratios were also mostly unchanged over the period of study. Only Aleve suffered a loss of market share in 2005, which Aleve was able to recover in few months. Because we know that the diversion ratios are functions of the market shares, then we conclude that our approach is appropriate in this industry.\footnote{Aleve suffered a large \textit{[QUANTIFY HERE EXACTLY]} loss of market share in 2005, which Aleve was able to recover in few months.}

Each column in Table 5 corresponds to a different specification of Equation (\ref{eq:diversion}). All specifications use the same number of observations, 601. There are 12 (CHECK) diversion ratios estimated. There are three reasons for a diversion ratio to be missing. First, there were too few attack months so the variable was omitted, for example Aleve attacked Aleve only ZZZ times and the total advertising expenditures were equal to ZZZ (see Table 2). Second, we do not observe direct attacks on siblings, for example Bayer does not attack Aleve. Third, we do not estimate attack for which the target did no self-promotion, since the first order condition for comparative advertising would not hold with equality.

Because our approach consists of a two-step approach, we match the specifications in Table 5 with those in Table 4 so that the exogenous variables are the same in the first and second steps. In particular, in column 2 we treat the market shares that enter in equation (\ref{eq:diversion}) as exogenous, so this specification is consistent with the specification in column 2 of Table 4. In column 3 we include a brand dummy that is equal to 1 if the attacking brand is one of the top brands (Advil, Aleve, or Tylenol) and another brand dummy that is equal to 1 if the attacked brand is one of the top brands. Thus, the specification in column 3 of Table 5 is consistent with the one in column column 2 of Table 4. Finally, the specification in column 4 of Table 5 uses both the news shocks and the generic prices as instrumental variables and thus it is consistent with the specification in column 6 of Table 4.
TABLE 5. Comparative Advertising Equation and Diversion Ratios

<table>
<thead>
<tr>
<th></th>
<th>With Brand Dummies</th>
<th>IV Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Using Parameters Values from Column 2 of Table 4)</td>
<td>(Using Parameters Values from Column 6 of Table 4)</td>
</tr>
<tr>
<td><strong>ALEVE ON:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tylenol, (d_{At,Ty})</td>
<td>0.229 (0.056)</td>
<td>0.110 (0.046)</td>
</tr>
<tr>
<td><strong>ADVIL ON:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tylenol, (d_{Ty,Ad})</td>
<td>0.230 (0.057)</td>
<td>0.111 (0.045)</td>
</tr>
<tr>
<td>Aleve, (d_{Ty,Al})</td>
<td>0.060 (0.024)</td>
<td>0.022 (0.016)</td>
</tr>
<tr>
<td>Excedrin, (d_{Ad,Ex})</td>
<td>0.016 (0.018)</td>
<td>0.000 (0.006)</td>
</tr>
<tr>
<td><strong>TYLENOL ON:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advil, (d_{Ty,Ad})</td>
<td>0.037 (0.017)</td>
<td>0.028 (0.020)</td>
</tr>
<tr>
<td>Aleve, (d_{Ty,Al})</td>
<td>0.072 (0.020)</td>
<td>0.047 (0.028)</td>
</tr>
<tr>
<td>Bayer, (d_{Ty,Ba})</td>
<td>0.062 (0.015)</td>
<td>0.041 (0.019)</td>
</tr>
<tr>
<td><strong>BAYER ON:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advil, (d_{Ba,Ad})</td>
<td>0.223 (0.073)</td>
<td>0.147 (0.090)</td>
</tr>
<tr>
<td>Tylenol, (d_{Ba,Ty})</td>
<td>0.312 (0.076)</td>
<td>0.185 (0.099)</td>
</tr>
<tr>
<td><strong>MOTRIN ON:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advil, (d_{Mo,Ad})</td>
<td>0.231 (0.077)</td>
<td>0.161 (0.096)</td>
</tr>
<tr>
<td>Aleve, (d_{Mo,Al})</td>
<td>0.235 (0.069)</td>
<td>0.155 (0.094)</td>
</tr>
<tr>
<td><strong>EXCEDRIN ON:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tylenol, (d_{Ex,Ty})</td>
<td>0.120 (0.085)</td>
<td>0.094 (0.082)</td>
</tr>
<tr>
<td><strong>Top Brand Attacker Dummy</strong></td>
<td>0.278 (0.068)</td>
<td>-0.039 (0.085)</td>
</tr>
<tr>
<td>Control Function for (s_j)</td>
<td></td>
<td>0.023 (0.071)</td>
</tr>
<tr>
<td>Control Function for (s_k)</td>
<td></td>
<td>0.638 (0.192)</td>
</tr>
<tr>
<td>(\phi)</td>
<td>0.329 (0.102)</td>
<td>0.693 (0.154)</td>
</tr>
<tr>
<td>Constant Term</td>
<td>-0.145 (0.034)</td>
<td>-0.392 (0.078)</td>
</tr>
<tr>
<td>Variance Unobservable</td>
<td>0.138 (0.007)</td>
<td>0.138 (0.008)</td>
</tr>
<tr>
<td>Log-Likelihood Function</td>
<td>10.967</td>
<td>15.616</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>601</td>
<td>601</td>
</tr>
</tbody>
</table>

30
Here \( j \) is a line and \( k \) is a column, so that the diversion ratio from Aleve to Tylenol is 0.276, meaning that if Aleve sheds 100 consumers through a price rise (say), then 27.6 of them go to Tylenol. The fairly large numbers in the Tylenol column suggest that it is the major gainer from other brands, consistent with its high market share (and, to a lesser extent, Advil).

The columns all sum to less than 1, as the theory hopes for (we imposed that they are all below one, but we did not restrict the sum). These numbers seem not unreasonable!

There are 4 pairs for which we estimate the diversion ratios in both directions. Comparing them indicates relative own demand derivatives. In particular, \( \frac{d_{jk}}{d_{kj}} = \frac{ds_k}{ds_j} / \frac{ds_j}{ds_k} \). Take for example \( j = \text{Tylenol} \) and \( k = \text{Bayer} \), so \( d_{TB} = 0.1 \) and \( d_{BT} = 0.4 \) and \( \frac{dT_B}{dB_T} \) is around \( \frac{1}{4} \) \((ds_T/d\delta_T \approx 4ds_B/d\delta_B)\). This means the demand derivative is much more price sensitive for Tylenol. At first blush, this may seem to presage a poor prospect for the estimates, given that Tylenol has a much higher price than Bayer aspirin (suggesting a more inelastic demand). However, a rough calibration brings this into perspective. The price of a "serving" (here roughly 3 days of pain relief) of Tylenol is roughly $2.15; taking the generic price of $1.17 as representing marginal cost gives a mark-up of approximately $1. [should we do this exercise for all, and present a Table? and check numbers, should we have the table of these prices in our text] A similar mark-up is found for Bayer, with a brand price of $1.85 and a generic price of about $0.8. Now, the pricing equation sets mark-up equal to demand over own demand derivative (in absolute value). Using the market shares of .3 for Tylenol and .06 for Bayer [these are rough market shares as a fraction of total market including generics, but without outside good, from earlier Table], then the pricing formula would predict roughly a demand derivative for Tylenol of .3 and for Bayer of .06, which 5-to-1 ratio compares favorably to the 4-to-1 ratio we get from the ratio of diversion ratios.

The patterns in the structure of demand relations revealed in the diversion ratio matrix are as follows. [can we go deeper?] For example, looking at the first row, which is how Advil’s lost consumers are split up, then it loses 27% to Tylenol, and only 8% and 6% to Aleve and Excedrin respectively. The rest are lost to generics, the outside good, and the
other (unestimated) brands. Advil loses a similar large fraction to Tylenol, Bayer even more (41%), which might be expected since Bayer is the oldest brand and Tylenol came next (really?). The figure for Excedrin is surprisingly low (at 13%) since it shares ACT as active ingredient in many of its variants. More convincing in the latter regard is the high loss (38%) of Motrin to Advil: they share Ibuprofen as active ingredient (though the equally high number for Aleve is a bit surprising). The diversion ratios are decidedly asymmetric (as illustrated above with the Bayer-Tylenol example) in the reverse directions. Most such pairs involve Tylenol: it loses just 27% to its 3 main attackers, but picks up at least that amount from each of them.

It is useful to think about the diversion ratios in relation to relative market shares. First note that whenever we have both diversion ratios, we see that the diversion from small to large is larger than vice versa. This is a property that would hold with a logit demand (recall for logit $d_{jk} = \frac{s_k}{1-s_j}$). For logit, $d_{jk}$ is increasing in $s_k$ (as customers are shed, they go to other brands in proportion to those brands’ shares). This works well: the only violation is from Tylenol to Advil and Aleve. However, another properties of the logit do not hold. For logit, $d_{jk}$ is increasing in $s_j$ (why does this make sense anyway?) but we see no clear relation in the table of diversion ratios on this count.

Need to check other estimates in the literature for diversion ratios in differentiated products industries. Are these differences commensurate? Are there estimates for merger cases? check Weyl on First-Order Mergers for possible references. BLP found semi-elasticities, it seems, can we then calculate D’s and hence d’s? Nevo may have something, are their estimates sensible, can they estimate the cross-effects?

A Table of D’s might be useful for when we can interpret it better.

5.3 Damage Measures

....
### TABLE 6. check mid points

<table>
<thead>
<tr>
<th></th>
<th>Advil</th>
<th>Aleve</th>
<th>Bayer</th>
<th>Excedrin</th>
<th>Motrin</th>
<th>TYLENOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advil</td>
<td>7.442</td>
<td>N/A</td>
<td>6.667</td>
<td>3.243</td>
<td>0.023</td>
<td>3.267</td>
</tr>
<tr>
<td></td>
<td>[2.541], [8.524]</td>
<td></td>
<td>[1.945], [3.773]</td>
<td>[1.192], [3.693]</td>
<td>[0.015], [0.069]</td>
<td>[1.265], [3.709]</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>0.042</td>
<td>3.524</td>
<td></td>
</tr>
<tr>
<td>Aleve</td>
<td>3.482</td>
<td></td>
<td></td>
<td>8.160</td>
<td>0.120</td>
<td>8.280</td>
</tr>
<tr>
<td></td>
<td>[1.316], [3.937]</td>
<td></td>
<td>[2.743], [8.409]</td>
<td>[0.097], [0.286]</td>
<td>[3.010], [8.511]</td>
<td></td>
</tr>
<tr>
<td>Bayer</td>
<td>8.106</td>
<td></td>
<td>8.160</td>
<td>0.120</td>
<td>8.280</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.655], [8.721]</td>
<td></td>
<td>[2.743], [8.409]</td>
<td>[0.097], [0.286]</td>
<td>[3.010], [8.511]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excedrin</td>
<td></td>
<td></td>
<td></td>
<td>13.455</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[3.926], [3.817]</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Motrin</td>
<td>6.708</td>
<td>6.512</td>
<td>N/A</td>
<td>1.887</td>
<td>0.028</td>
<td>1.915</td>
</tr>
<tr>
<td></td>
<td>[2.310], [7.526]</td>
<td>[2.064], [7.398]</td>
<td>N/A</td>
<td>[0.788], [2.513]</td>
<td>[0.028], [0.079]</td>
<td>[0.843], [2.544]</td>
</tr>
<tr>
<td>Tylenol</td>
<td>9.727</td>
<td>5.402</td>
<td>N/A</td>
<td>1.887</td>
<td>0.028</td>
<td>1.915</td>
</tr>
<tr>
<td></td>
<td>[3.083], [16.234]</td>
<td>[1.871], [6.405]</td>
<td>N/A</td>
<td>[0.788], [2.513]</td>
<td>[0.028], [0.079]</td>
<td>[0.843], [2.544]</td>
</tr>
<tr>
<td></td>
<td>0.069</td>
<td>0.065</td>
<td>5.467</td>
<td>1.887</td>
<td>0.028</td>
<td>1.915</td>
</tr>
<tr>
<td></td>
<td>[0.067], [0.187]</td>
<td>[0.061], [0.172]</td>
<td>[2.017], [6.467]</td>
<td>[0.788], [2.513]</td>
<td>[0.028], [0.079]</td>
<td>[0.843], [2.544]</td>
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<td>9.797</td>
<td>5.467</td>
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<td>[0.028], [0.079]</td>
<td>[0.843], [2.544]</td>
</tr>
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</table>

Note: and $MRS_{jk}$ (2) dropped out; Possible Mirrors: $d_{jk}$ (1) and $dQ_k/dA_{jk}$ (3), and $dQ_j/dA_{jk}$ (4), and $dP_t$ (5)

### References


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This measure is the marginal rate of substitution, $MRS_{k,j}$, between incoming attacks on Firm $k$ from Firm $j$ and self-promotion by $k$ in the production of its quality. It measures how much additional self-promotion $k$ should do in order to offset the quality reduction following an extra dollar of attack. We show below that it can be determined from the parameter $\lambda$ and the diversion ratio between target and attacker.

The marginal rate of substitution between self-promotion and incoming attacks tells us how much self-promotion by $k$ will compensate in $k$’s quality function for an extra dollar’s worth of incoming comparative advertising from Firm $j$ targeting Firm $k$. This is a measure of the direct damage caused by $j$, although it understates the profit loss because the comparative advertising also has an effect in raising the quality of rival $j$. The comparative advertising first order condition (derived below) implies

$$\left(0 < \right) MRS_{k,j} \leq \frac{1 - \lambda s_k}{d_{jk} s_j},$$

with equality when $A_{jk} > 0$ and where $d_{jk}$ is the diversion ratio from $j$ to $k$ and $s_k$ and $s_j$ are firms’ market shares.

Note from the expression for $MRS_{k,j}$ that (because $d_{jk} \in (0, 1)$), if two firms of equal size are observed to target each other ($s_j = s_k$), their marginal rates of substitution through their quality functions must both exceed $(1 - \lambda)$.\(^{26}\) This property highlights the strength of the damage inflicted by comparative advertising as compared to self-promotion.

The latter effects may be directly the marginal rate of substitution between incoming attacks and target self-promotion, can be written as a function of the diversion ratio. Indeed, if Firm $k$ undertakes some strictly positive amount of self-promotion (so that its first-order condition holds with equality), then the corresponding first order condition, $M_k \frac{\partial Q_k}{\partial A_{jk}} = 1$, may be substituted into (10) to yield (16).\(^{27}\)

**Proposition 5 (Damage Measure)** Assume that the target, $k$, engages in a strictly positive level of self-promotion, and assume that outgoing comparative ads are perfectly substitutable.\(^{26}\) If they are of different sizes and one MRS is below $(1 - \lambda)$, the other must be (significantly) above $(1 - \lambda)$.\(^{26}\)

\(^{27}\)Alternatively, we can write the comparative advertising condition as $MRS_k \leq \frac{1 - \lambda}{D_{jk}}$.\(^{27}\)
utable with self-promotion at rate $\lambda$. Then the amount of self-promotion that target $k$ needs to spend to restore its quality after an additional attack by Firm $j$ is $MRS_{k,j} \leq \frac{1 - \lambda s_k}{d_{jk} s_j}$, with equality if $A_{jk} > 0$. Here $MRS_{k,j} > 0$, and $d_{jk} > 0$ with $\sum_k d_{jk} < 1$.

As noted above, the marginal rate of substitution, $MRS_{k,j}$, understates the damage to Firm $k$ of a marginal dollar of comparative advertising from $j$, because the $MRS$ measures only the amount of self-promotion that $k$ needs to restore its perceived quality, but the comparative advertising also improves the rival $j$’s perceived quality.