A Growth Perspective on Foreign Reserve Accumulation*

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Abstract

Based on a dynamic open-economy macroeconomic model, this paper analyzes the relationship between productivity growth, financial underdevelopment and foreign reserve accumulation in emerging market economies. The demand for foreign reserves stems from the interaction between positive productivity shocks, borrowing constraints and the lack of domestic financial assets. Foreign reserve accumulation can thus be regarded as part of a catching-up strategy in an economy with underdeveloped financial market. In fact, if domestic firms are credit-constrained, domestic saving instruments are necessary to increase their retained earnings so as to invest in capital. The central bank plays the role of financial intermediary and provides domestic firms with liquid public bonds while investing the bond proceeds abroad in the form of foreign reserves. I also show that during the economic transition, the social welfare in an economy where the central bank accumulates foreign reserves and imposes capital controls is higher than in a financially liberalized economy. By controlling private capital flows, the central bank can not only provide sufficient domestic liquid assets by investing abroad in foreign reserves, it can also adjust the domestic interest rate to cope with the positive productivity shocks.

JEL Classification: E22, F31, F41, F43

Keywords: Foreign reserves, capital controls, credit constraints, domestic savings, capital investment, economic growth, Chinese economy

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1 Introduction

Since the last decade, foreign reserve accumulation has regained attention both in policy debates and academic research. Facing the spectacular growth of foreign reserves in emerging and developing countries, which exceeds a half of the world total reserves in 2005 (Figure 1), there has been a growing literature examining the underlying motives of the demand for reserve assets.

![Figure 1: Evolution of the world reserves](image)

A country may accumulate foreign reserves for reasons of different natures: to constitute a buffer stock in order to smooth output and consumption in times of crisis \(^1\), to insure market confidence and reduce sovereign borrowing costs \(^2\), to undervalue the domestic currency for the export-led growth strategy \(^3\), or to hold foreign reserves due to domestic financial underdevelopment and fragility \(^4\). In fact, as a few recent empirical papers and policy reports \(^5\) underline, foreign reserve accumulation is a multi-faceted phenomenon; the underlying motives can be time-varying and specific to a country or a country group.

As a complement to the existing literature on motives of foreign reserve accumulation, this paper delves into the relationship between fast productivity growth, financial frictions

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\(^1\)See Aizenman and Lee (2007), Aizenman and Hutchison (2010), Jeanne and Rancière (2011), Benigno and Fornaro (2012), Bussière et al. (2013) and Calvo et al. (2013)

\(^2\)See Alfaro and Kanazuk (2009), Bianchi et al. (2013)

\(^3\)See Dooley et al. (2003), Korinek and Serven (2010) and Jeanne (2012)

\(^4\)See Caballero et al. (2008), Dominguez (2010), Obstfeld et al. (2010), Wen (2011) and Bacchetta et al. (2013)

\(^5\)See Delatte and Fouquau (2012), Ghosh et al. (2012) and IEO (2012)
and foreign reserve accumulation, three features commonly observed in a number of emerging market economies. Despite the extensive academic and policy debates regarding the demand for international reserves, none has jointly examined these ingredients in a growth model.

I shall develop a new model and argue that foreign reserve accumulation stems from - at least partially - the interaction between positive productivity growth and stringent financial frictions. Three sets of interesting results will be derived from the model I develop. First, the demand for foreign assets (either in the form of foreign reserves held by the central bank or private foreign assets) is motivated by a strong demand for domestic liquid assets in an economy with an underdeveloped domestic financial market. The demand for liquidity is in its turn triggered by a positive productivity shock which strengthens the private sector’s incentives to invest, albeit the latter’s borrowing ability being limited by credit constraints (Section 2). Second, I justify the central bank’s intervention by showing that the domestic interest rate is too low in a financial autarky due to credit constraints and this is sub-optimal as firms partially rely on retained earnings to invest (Section 3). Finally and most importantly, I answer the question why it is better for the central bank to accumulate foreign reserves (imposing capital controls on private flows) instead of letting the private sector get a direct access to the foreign capital market (financial liberalization). In fact, I demonstrate in Section 4 that the central bank’s reserve policy generates welfare gains during the economic transition in an economy with capital controls. This is because the joint policy of reserve accumulation and capital controls allows the central bank to channel sufficient funding from abroad to the domestic economy while keeping the domestic interest rate under control; it can thus adjust the domestic interest rate to fit the needs of savers and investors.

My paper is closely related to the literature on credit constraints, domestic imbalances between savings and investment as well as demand for foreign assets. There is first a seminal paper by Woodford (1990) which demonstrates the welfare-improving effects of public debts in promoting domestic capital investment in an economy facing credit

\footnote{It will be clearly showed in Section 4 that capital controls are not frictions but a policy choice of the central bank.}
constraints. Woodford’s paper is however based on a financial autarky setting. In an open economy setting, the presence of credit constraints triggers precautionary savings which generate a wedge between aggregate savings and investment, source of capital outflows. Wen (2011), Song et al. (2011) and Bacchetta and Benhima (2012) are the most relevant references regarding these aspects. Bacchetta and Benhima (2012) show that the demand for foreign bonds is a complement to domestic investment rather than a substitute. The underlying reason is that foreign bonds constitute supplementary corporate saving assets for credit-constrained firms. The retained earnings can be used to invest in capital even if firms’ borrowing capacity is constrained. In the same vein, Song et al. (2011) shed light on how a constrained access to bank loans of private firms compared to state-owned enterprises can lead to a large level of corporate precautionary savings, and thus a persistent current account surplus. As a result of privatization, the overall precautionary savings increase, so do foreign reserves. Wen (2011) tries to ‘make sense of China’s excessive foreign reserves’ by looking at household savings. He argues that a large uninsured risk (e.g. the lack of a sound social safety net), stringent borrowing constraints, and rapid income growth can jointly generate high household saving rates and large current account surpluses in emerging economies. In this three models, capital outflows are a residual of excess domestic savings, resulting thus from the private sector’s behavior in a small open economy. One cannot distinguish foreign reserve accumulation by the monetary authorities from private foreign asset purchasing as public and private capital flows are perfect substitutes. My current work differs from theirs by introducing the central bank’s policy and argues that public and private flows are imperfect substitutes. The accumulation of foreign reserves by the central bank can be welfare improving.

This paper is however not the first one which analyzes the central bank’s optimal policy of reserve accumulation as well as its relationship with capital account policy. Benigno and Fornaro (2012) and Bacchetta et al. (2013) are among the precursors. My paper differs from Bacchetta et al. (2013) as I nest foreign reserve accumulation in a

\[7\] For additional references, see also Bénassy-Quéré et al. (2011) and Coeurdacier et al. (2012)

\[8\] Benigno and Fornaro (2012) also look at the imperfect substitutability between public and private flows. Instead of imposing capital controls as my work does, they choose to impose an external borrowing constraint on private firms. Also, they study different motives of reserve accumulation from what I do.
growth model and focus on the contribution of reserves to domestic capital formation, driver of economic catch-up in emerging market economies. The introduction of capital is very important for two reasons. First, this enables me to examine production efficiency gains instead of redistributive effects and consumption smoothing gains. According to the literature on growth theory, the gains from production efficiency have a much more persistent and stronger impact on growth than from consumption smoothing (focus of Bacchetta et al. 2013). Secondly, the introduction of capital formation makes my model more relevant for fast-growing countries with large stock of foreign reserves, such as China. The growth in these countries is largely driven by positive technology shocks and resultant capital accumulation (Nelson and Pack 1999, Bond et al. 2010 and Ahuja and Nabar 2012) 9.

Notice that in the scope of this paper, I rule out the aspect of international trade and exchange rate policies, as my objective is to provide an alternative explanation of large foreign reserve accumulation in emerging economies, which is related to domestic financial conditions. Obviously, having an exchange rate target might strengthen the demand for foreign reserves; however, the question about the relative weight of different motives of foreign reserve accumulation is out of the scrutiny of this paper.

This paper is organized as follows. Section 2 describes the model setting. Section 3 analyzes reserve accumulation and capital formation in a decentralized economy. Section 4 introduces the optimal policy of the central bank and presents numerical results. Section 5 provides some descriptive observations about domestic financial conditions, foreign reserve accumulation and capital formation in China. Section 6 concludes.

2 Model setting

The model I develop is inspired by Bacchetta and Benhima (2012) and here I explicitly model the central bank’s behavior. It is an open-economy macroeconomic model with

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9Notice that this view challenges somehow the widespread view on Chinese export-led growth. However, one can easily calculate the contribution to the Chinese GDP growth. Investment is by far the most important contributor (more than 40% since 2000).
heterogeneous agents and a central bank \(^{10}\).

2.1 Policy regimes

The benchmark economy is however different from a standard small open economy setting, because capital controls can be imposed by the central bank. Namely, the central bank can decide to forbid the private sector to get access to the international financial markets; it plays the role of financial intermediary between the domestic economy and foreign markets. This situation is dubbed ‘semi-open economy’ à la Bacchetta et al. (2013). The model thus allows me to compare three different policy regimes: a financial autarky, an economy with financial liberalization (open economy) and an economy with stringent capital controls (semi-open economy). Table 1 \(^{11}\) summarizes the main features of these three regimes. It can be seen in Table 1 that an economy with capital controls are modeled through two main variables \(B^* \in \mathbb{R}^+\) and \(r \in \mathbb{R}^+\). \(B^* \in \mathbb{R}^+\) means that the economy as a whole has a full access to foreign assets; \(r \in \mathbb{R}^+\) means that the domestic interest rate is significantly different from the world interest rate; domestic assets and foreign assets are thus imperfect substitutes (public capital outflows cannot be fully offset by private inflows). On the contrary, in an open economy, domestic assets and foreign assets are perfect substitutes.

Table 1: Policy regimes

<table>
<thead>
<tr>
<th>Policy regime</th>
<th>Characteristics</th>
<th>Policy Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>No external lending/borrowing</td>
<td>(B^* = 0) (\quad r \in \mathbb{R}^+)</td>
</tr>
<tr>
<td>Open economy</td>
<td>Financial liberalization</td>
<td>(B^* \in \mathbb{R}^+) (\quad r = r^*)</td>
</tr>
<tr>
<td>Semi-open economy</td>
<td>Capital controls on private flows</td>
<td>(B^* \in \mathbb{R}^+) (\quad r \in \mathbb{R}^+)</td>
</tr>
</tbody>
</table>

\(^{10}\)More standard models with a representative agent can be found in Korinek (2011), Jeanne (2012) etc.

\(^{11}\)\(B^*\) stands for foreign reserves; \(r\) and \(r^*\) designate respectively domestic and world interest rate.
2.2 The private sector

The private sector in this paper is modeled by two symmetric family businesses. Each of them is made up of a continuum of individuals of measure one. Family members in each family business are either a worker or an entrepreneur. Within the family business, the worker provides the labor force to the entrepreneur who in turn pays the worker at the marginal product of labor. Importantly, I assume that the family business pools together the incomes of both types of family members and optimally make consumption and investment decisions at the family level. This is a parsimonious way to model households and firms all combined in contrast with Bacchetta and Benhima (2012). The advantage of doing so is twofold: it simplifies the program of the private sector and renders the Ramsey program neater; it also allows both the worker and the entrepreneur to save and thus to contribute to physical capital investment.

I assume that each family business is infinitely lived and capital is invested every two periods. As a result, any family produces in one period and invests in the other and so on so forth. That is, each of the two families changes its status every two periods, alternating between a ‘producing-saving’ period (denoted $S$) and an ‘investing-borrowing’ period (denoted $I$). The assumption of two symmetric family businesses is to guarantee that in each period there is always one family in its ‘producing-saving’ period and the other one in its ‘investing-borrowing’ period, so that the credit constraint facing the ‘investing-borrowing’ family is present and may be binding every period.

Family businesses’ program

As the family businesses are symmetric, it is sufficient to look at the program of one of them. Let’s consider the family who starts at time $t$ in a ‘producing-saving’ period. It faces a standard intertemporal utility function with a discount factor $\beta$:

\[\text{12 There are other papers which adopt this strategy of modeling a family business (or ‘representative family’) composed of two types of members, such as Merz (1995) or Ljungqvist and Sargent (2007).}\n\[\text{13 Bacchetta and Benhima (2012) only allow the corporate sector to save as the worker in their model is ‘hand-to-month’ and consumes all the labor income every period.}\]
\[
\sum_{t=0}^{\infty} \beta^t \left( U(c_t^S) + \beta U(c_{t+1}^I) \right)
\] (1)

It has the following budget constraints every two periods:

At \( t \):
\[
F(A_t, K_t, N_t) - r_t L_t = c_t^S + S_{t+1} + \frac{T_t}{2}
\] (2)

At \( t+1 \):
\[
r_{t+1} S_{t+1} + L_{t+2} = c_{t+1}^I + K_{t+2} + \frac{T_{t+1}}{2}
\] (3)

A typical family business in its ‘producing-saving’ period, here at time \( t \), harvests an output \( F(A_t, K_t, N_t) \) (produced with inputs \( K_t \) and \( N_t \) previously chosen), and makes the decision between current consumption \( c_t^S \) and savings \( S_{t+1} \). The ‘producing-saving’ family is able to save because its counterpart is in the ‘investing-borrowing’ period and needs funding \(^{14}\). The willingness to save is explained by the fact that the output is only harvested every two periods. Namely, at \( t+1 \) the family will be in its ‘investing-borrowing’ period and will rely on retained earnings \( r_{t+1} S_{t+1} \) as well as domestic borrowing \( L_{t+2} \) to invest in physical capital \( K_{t+2} \) and to consume \( c_{t+1}^I \). \( r_t \) denotes the sequence of the domestic gross interest rate (\( r_t > 1 \), for all \( t \)). \( T_t \) denotes lump-sum taxes (transfers) to (from) an implicit government.

The production function \( F(A_t, K_t, N_t) \) is a standard neoclassical production function: increasing in all arguments, concave and homogeneous of degree one. I use \( F_{K,t} \) and \( F_{N,t} \) to denote the marginal product of capital and of labor respectively. \( A_t \) stands for production technology. As we will see later on, the only shock in this model is a productivity shock so as to mimic the fast economic catch-up in emerging market economies. The wage payment does not appear in the above budget constraints. This is because I assume that the wage payment between the worker and the entrepreneur is carried out internally with \( w_t = F_{N,t} \) while the allocation decision is made by the head of the family at the family

\(^{14}\)In the current setting, families lend to each other directly and the banking sector is absent. However, the results that I derive in the subsequent sections will not change even if a competitive banking sector is introduced. Therefore, to keep the model tractable, I decide to leave the financial sector aside.
level. It is further assumed that the labor supply is inelastic with \( N_t = 1 \).

Similarly, the family business which starts at \( t \) in an ‘investing-producing’ period has the following budget constraints:

At \( t \):
\[
 r_t S_t + L_{t+1} = c_t^I + K_{t+1} + \frac{T_t}{2}
\]

At \( t+1 \):
\[
 F(A_{t+1}, K_{t+1}, N_{t+1}) - r_{t+1} L_{t+1} = c_{t+1}^S + S_{t+2} + \frac{T_{t+1}}{2}
\]

Credit constraint and demand for liquid assets

Most importantly, there is a credit constraint facing the family in its ‘investing-borrowing’ period:

\[
 L_{t+1} \leq \frac{\psi F(A_{t+1}, K_{t+1})}{r_{t+1}} \tag{4}
\]

The maximum loan that an investing family can get is conditional on the discounted value of its next period output, thus negatively correlated with the domestic interest rate and positively related to the production technology and the contemporaneous capital investment. \( \psi \) denotes the tightness of the credit constraint with \( \psi \in [0, 1] \). The smaller the value of \( \psi \), the tighter the constraint 15.

Whenever the credit constraint is binding, the supply of loans (savings of the ‘producing-saving’ family) exceeds the demand for loans of the ‘investing-producing’ family, namely \( S_{t+1} - L_{t+1} > 0 \). This leads either to a lower supply of savings and a repressed domestic interest rate (e.g. in an autarkic competitive market economy) - as savings \( S_{t+1} \) are pinned down by a constrained borrowing \( L_{t+1} \) - or to the demand for supplementary liquid assets for saving. The supplementary liquid assets can be offered on the international financial market when the economy is financially open \( B_{t+1}^* = S_{t+1} - L_{t+1} \) or by the central bank when the latter imposes controls on private capital flows \( B_{t+1} = S_{t+1} - L_{t+1} \).

The demand for supplementary liquid assets is thus motivated in this paper by the

15The form of the credit constraint follows the standard setting in the literature of financial accelerator (e.g. Bernanke et al. 1999).
interaction between a fast productivity growth which generates strong incentives to invest and a borrowing constraint which confines both domestic savings and borrowing in a suboptimal level. The motive of the demand for liquid assets is thus different from Bacchetta and Benhima (2012) where the liquidity demand is induced by the need to pay the labor force in advance. Although in theory the supply of liquid assets can either comes from abroad ($B^*$) or from the central bank ($B$), I will show in Section 4 that the supply of liquid bonds by the central bank (i.e. capital controls are imposed) dominates financial liberalization during the economic transition.

Finally, I argue that the credit constraint is institutional and cannot be removed in the short-run. It is very common to observe obstacles to domestic financing in various emerging and developing countries: direct lending is costly or banks have preference biases in selecting firms to which they grant loans, etc. For example, in China, households have a large level of savings that they are willing to lend to firms in need but fail to do so short of a developed capital (Chamon and Prasad 2010). In addition, being mostly state-owned, commercial banks in China prefer (and sometimes are obliged) to lend to public firms; domestic private firms, notwithstanding more productive, are unable to get enough loans from commercial banks (see Song et al. 2011). To remove the credit frictions, many structural reforms need to be implemented and this takes time.

### 2.3 The central bank’s behavior

**General setting**

In a semi-open economy, the central bank in our model issues domestic saving bonds, $B_{t+1}$, which aim at filling out the gap between domestic savings and borrowing. Due to the limited scope of domestic financial market, the central bank then invests domestic bond proceeds in foreign assets, $B^*_{t+1}$. The central bank’s flow budget constraint is described as follows:

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16One may especially think of fiscal reforms (reinforcing domestic firms' financial situation) or standardization of accounting principles, etc.
\[ r^*_t B^*_t + B_{t+1} + T_t = r_t B_t + B^*_{t+1} \] (5)

\( r^*_t \) is the world interest rate; it is assumed that the world interest rate is constant and equals to the inverse of the world time preference discount factor \( \beta \), namely \( r^* \beta = 1 \) (assuming the same time preference in the domestic economy and abroad). \( T_t \) is a lump-sum transfer from the government which taxes households to pay this transfer (this can be a transfer to the government too if \( T < 0 \)). To focus on the central bank’s policy, it is assumed that the transfer from (to) the government is exogenous, used only to balance the central bank’s flow budget constraint each period; it captures the financial gains (losses) from the central bank’s investment, which are transferred to the government as the latter is assumed to be the stakeholder of the central bank.

The supply of liquid bonds by the central bank is determined by the demand on the domestic market (difference between domestic savings and borrowing):

\[ B_{t+1} = \underbrace{S_{t+1}}_{\text{supply of liquid bonds}} - \underbrace{L_{t+1}}_{\text{demand for liquid bonds}} \] (6)

\( B^* \) and \( B \) are perfect substitutes when the economy is fully open; they are no longer substitutes in a semi-open economy where the central bank excludes the private sector from foreign financial markets. Figure 2 shows the two different mechanisms to supply domestic liquid assets depending on the policy regime of the economy. The supplementary liquid saving instruments can provided by the international financial market; in this case the central bank plays no role (dashed blue line). On the contrary, the central bank can play the role of financial intermediary between the domestic economy and the international financial market should it accumulates foreign reserves and impose capital controls on private capital flows (solid blue line). I will show in Section 4 that the latter strategy dominates the first option during the economic transition.
Central bank’s purchases of foreign reserves

The use of public debt, e.g., government or central bank bonds, has proved to be welfare-improving in the presence of market frictions, as Woodford (1990) demonstrates. In this paper, I argue that in a semi-open economy, in order to provide this liquid bond the central bank needs to invest domestic bond proceeds abroad. One reason is that in a financially underdeveloped economy, domestic assets which can absorb the central bank’s bond proceeds are scare. This assumption is plausible if one looks at the features of the domestic financial market in China: domestic private bond and equity markets are dwarfed by the large public bond issuance (both government and central bank). Another reason is that foreign assets generate higher returns than domestic assets as the presence of credit constraints drives down the domestic interest rate. Figure 3 illustrates the gap between the interest rate on Chinese central bank bills and that on US Treasury bills from 2003 (when the interest rates on the PBOC’s bills started to be available) to the onset of the global financial crisis in 2008. It is observed that during this period which corresponds to a period of fast reserve accumulation, China paid lower interest rates on its central bank bills than that earned on U.S. Treasury bills. Furthermore, the job of liquidity provision cannot be undertaken by the government as government financing is
usually more closely regulated by law than central bank bonds.

In sum, from Equation (4) and (6), as well as Figure 2, I show my first result: foreign reserve accumulation (in the semi-open economy situation) is motivated by a strong demand for domestic liquid assets in an economy with a rapid technology growth rate and an underdeveloped domestic financial market.

To close the model, I present below the aggregate budget constraint by combining the private sector’s and the central bank’s budget constraints:

\[
F(A_t, K_t) = c_t^S + c_t^I + K_{t+1} - r^*tB_t^* + B_{t+1}^*
\]  

Output produced by the family in its ‘producing-saving’ period is consumed by both families, invested in capital, and used to buy foreign assets.

Figure 3: Interest rates in China and in the US (%)

3 Competitive market equilibrium

3.1 Definition

In this section, I present fundamental features of the model in a competitive market equilibrium and emphasize on how the credit constraint motivates either a financial liberalization (open economy) or an active intervention of the central bank (semi-open
An analysis on domestic capital formation will also be given a special attention. In order to derive analytical results, I use log-utility and Cobb-Douglas production function henceforth, namely \( U(c_i^t) = \log(c_i^t), i \in (S,I) \) and \( F(K_t, A_t) = K_t^\alpha A_t^{1-\alpha} \). For simplicity, I assume \( T = 0 \), that is the central bank balances its balance sheet with domestic bonds and foreign assets every period.

**Definition 1** A perfect foresight market equilibrium in a decentralized economy is a sequence of allocation \( \{c^S_t, c^I_t, K_{t+1}, S_{t+1}, L_{t+1}\} \), for a given sequence of price \( \{r_t\} \), policy set \( \{B^*_t, B_t, T_t\} \), and initial conditions \( \{K_0, L_0, S_0, B_0, B^*_0, T_0\} \), such that for all \( t > 0 \):

1. The utility function (1) is maximized subject to the family business’ budget constraints (2) and (3) 

2. The borrowing constraint (4) is verified 

3. The central bank’s budget constraint (5), financial and good markets clearing conditions (6) and (7) are all verified

The first order conditions for the family business in its ‘producing-saving’ stage at time \( t \) are presented below (the first order conditions of the other family can be derived by symmetry):

\[
(1 + \psi \lambda_t) F_{K,t} = r_t (1 + \lambda_t) \tag{8}
\]
\[
u'(c^S_t) = \beta r_{t+1} u'(c^I_{t+1}) \tag{9}
\]
\[
u'(c^I_{t+1}) = \beta r_{t+2} u'(c^S_{t+2})(1 + \lambda_{t+2}) \tag{10}
\]

\( \lambda_t u'(c^S_t) \) denotes the Lagrange multiplier associated with the credit constraint at time \( t \). Equation (8) tells us that the marginal product of capital is equal to the cost of capital \( r_t \) augmented by a coefficient \( \frac{1 + \lambda_t}{1 + \psi \lambda_t} \), which is related to the credit constraint. Equations (9) and (10) are Euler equations. The Euler equation (10), which relates the

\[17\] Log-utility is only required to derive the uniqueness of the constrained steady state. The results do not change with a more general form of the utility function (e.g. constant relative risk aversion (CRRA).
marginal utility of consumption of an investing family to that of the same family in its ‘producing-saving’ stage a period later, depends clearly on the credit constraint, \( \lambda_{t+2} \).

According to the Kuhn-Tucker theorem, the Lagrange multiplier with respect to the credit constraint verifies \( \lambda_t u'(c_t^S) [r_t L_t - \psi F(A_t, K_t)] = 0 \). Therefore, the credit constraint may be binding \( r_t L_t = \psi F(A_t, K_t) \) or non-binding \( \lambda_t = 0 \).

Using the log-utility and Equation (2) and (3), I can rewrite the consumptions as a share of the wealth: \( c_t^S = (1 - \beta) [F(A_t, K_t) - r_t L_t] \) and \( c_{t+1}^I = (1 - \beta) r_{t+1} S_{t+1} \).

This gives:

\[
S_{t+1} = \beta [F(A_t, K_t) - r_t L_t] \quad (11)
\]
\[
K_{t+2} - L_{t+2} = \beta r_{t+1} S_{t+1} \quad (12)
\]

### 3.2 Uniqueness in steady state

The steady state is uniquely determined when the credit constraint is binding. This result is important to analyze capital formation and other features of the model later on.

For this purpose, I normalize all endogenous variables by the output \( F(A_t, K_t) \), namely I define \( b^* = \frac{B^*}{F(A,K)}, b = \frac{B}{F(A,K)} \) and \( \tau = \frac{T}{F(A,K)} \).

The central bank’s budget constraint (5) becomes:

\[
(r^* - 1)b^* + \tau = (r - 1)b \quad (13)
\]

Given international and domestic interest rates as well as the level of government transfer, one can observe from Equation (13) that if the central bank wants to provide liquid assets in the domestic economy, it can achieve this objective by purchasing foreign reserves.

**Proposition 1** Considering the simplest case where \( \tau = 0 \) (central bank do not have financial gains/losses in the long-run), the financially constrained steady state is uniquely determined if \( 0 < b < \beta (1 - 2\psi) \).

**Proof.** See Appendix A □
Proposition 1 implies that the credit constraint is not binding when $b \geq \beta(1-2\psi)$; the central bank can achieve this goal by raising $b^*$. When the credit constraint is unbinding, the domestic interest rate is equal to the world interest rate $r = r^* = \frac{1}{\beta}$; when the credit constraint is binding, the domestic interest rate is repressed and lower than its international counterpart. Moreover, when the central bank provides liquid bonds which are scarce in the economy, it lowers the price of the liquid bonds and pushes the domestic interest rate up to the world interest rate level. $r$ is indeed increasing in $b$: $r = \frac{\psi}{\beta(1-\psi) - b}$.

It can be further proved that the coefficient of the tightness of the credit constraint is important for whether the credit constraint is binding in the steady state. When $\psi \geq \frac{1}{2}$, the credit constraint is unbinding in the steady state. When $\psi < \frac{1}{2}$, the credit constraint is strictly binding in the steady state (See Appendix B for details).

We can further look at different steady states in different policy regimes described in Table 1.

**Financial autarky**  In a closed economy where $B^* = 0$, if $\psi \geq 1/2$, the credit constraint does not bind in the steady state. In this case, $\beta r = 1$, namely, the domestic interest rate is equal to the world interest rate. $K = A(\alpha\beta)^{\frac{1}{1-\alpha}}$, the capital stock achieves its first-best level\(^1\). And consumptions are perfectly smoothed, namely the consumption in the saving stage and that in the investing stage are equalized $c^I = c^S$.

If $\psi < 1/2$, the credit constraint binds in the steady state. The Lagrange multiplier associated with the credit constraint is thus positive, $\lambda = \frac{1}{\beta(1-\psi)} - 1 > 0$. The domestic interest rate is lower than the world interest rate, namely $\beta r = \frac{\psi}{1-\psi} < 1$. The capital stock, $K = A\left[\alpha((1-\psi)\beta^2 r + \frac{\psi}{1-\psi})\right]^{\frac{1}{1-\alpha}}$, cannot achieve the first-best level. As for consumptions, that of the investing stage is always smaller than that of the saving stage, $c^I < c^S$.

In fact, in a financial autarky, with a positive productivity shock, the more capital the better. There is thus a strong demand for capital investment. However, investing families are financially constrained and the aggregate economy does not have sufficient resources to finance investment as no external financing is possible. The left panel in Figure 4

\(^1\)The first-best is defined as the long-run steady state with an unbinding credit constraint, namely without any wedge between domestic savings and investment.
shows that in case of a decrease in $\psi$, the credit constraint becomes more binding and the demand for domestic assets increases while the supply remains fixed (due to the financial autarky). This results in a sharp decrease in the domestic interest rate. This is undesirable both for capital investment and domestic consumptions (as we can see in Section 3.3).

**Open economy**  One big difference here compared to the financial autarky is that the economy has access to external financing. It results in the equalization between the domestic interest rate and the world interest rate, $r = r^* = \frac{1}{\beta}$. There is an unlimited supply of external financing. In this case the credit constraint is never binding even without any central bank intervention, leading to $b = b^* = \beta(1-2\psi)$. In an open economy, both the central bank and the private sector get access to the international financial market and therefore central bank bonds and foreign assets are perfect substitutes. The saving family can choose to save with either financial assets. In aggregate, there are sufficient assets with which the ‘producing-saving’ family can save at the world interest rate $r^*$ (See the mid-panel in Figure 4). Therefore, in steady state, there is enough funding to support capital investment; and the economy reaches its first-best level. Clearly, in the steady state, the open economy dominates the financial autarky.

**Semi-open economy**  Another possibility to amend the shortcoming presented in a financial autarky is to allow the central bank to intervene by providing extra liquid assets. The semi-open economy is indeed a nested situation of the financial autarky and open economy. In the steady state, the central bank can mimic the open economy by purchasing the same amount of foreign assets as in the open economy case. The only difference is that central bank bonds and foreign assets are no longer perfect substitutes (the demand for and the supply of domestic liquid assets have standard shapes, as illustrates the right panel in Figure 4).

The steady state with an unbinding credit constraint being defined as the first-best benchmark, a fully open economy or a semi-open economy can always achieve the first-best

---

19Theoretically speaking, in an open economy, it is equivalent to solve the model in a decentralized economy or to solve the model with the central bank’s program.
best in the long-run while the financial autarky with a strong credit constraint is clearly dominated. This is the second result I show in this paper (Table 2 provides a summary). How to discriminate the open economy and semi-open economy during the economic transition will be the subject of Section 4.

Table 2: Decentralized economy

<table>
<thead>
<tr>
<th>Economic Structure</th>
<th>Steady State</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>constrained</td>
<td>$\psi &lt; \frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>unconstrained</td>
<td>$\psi \geq \frac{1}{2}$</td>
</tr>
<tr>
<td>Open economy</td>
<td>unconstrained</td>
<td>$\forall \psi$</td>
</tr>
<tr>
<td>Semi-open economy</td>
<td>unconstrained</td>
<td>$\forall \psi$</td>
</tr>
</tbody>
</table>

3.3 Capital formation

How does the capital stock evolve over time? From (11) and (12), capital accumulation which is consistent with optimal consumptions in a financially constrained economy satisfies:

$$K_{t+2} = \beta r_{t+1} S_{t+1} + L_{t+2} \tag{14}$$

$$= \beta^2 r_{t+1} (1 - \psi) F(A_t, K_t) + \frac{\psi F(A_{t+2}, K_{t+2})}{r_{t+2}} \tag{15}$$

Notice that retained earnings and loans are reversely affected by the interest rate. An increase in the domestic interest rate raises the retained earnings of the saving family.
(revenue effect) but reduces the loans that the same family may make next period during its investing period (borrowing effect). Whether capital stock increases or decreases depends on which effect dominates.

A feedback loop effect can be observed in Equation (15). Namely, more capital invested provides a stronger collateral (as the capital enters in the production function) allowing the investing family to borrow more during the investing stage and relaxing the credit constraint. For example, an initial positive shock on retained earnings (either an increase in $r_{t+1}$ or an increase in initial capital stock $K_t$) will lead to an increase in capital formation at $t + 2$, which in turn drives loans up. Accordingly, capital formation $K_{t+2}$ further increases, generating a virtuous cycle.

From (15), one can easily show that, ceteris paribus, $\frac{\partial K_{t+2}}{\partial r_{t+1}} > 0$ and $\frac{\partial K_{t+2}}{\partial r_{t+2}} < 0$.

**Proposition 2** Capital stock represents a U-shape curve as a function of $\beta r$ in the steady state. Purchasing foreign reserves, which leads to a higher supply of domestic liquid assets and thus a higher interest rate, helps increase capital stock and raise production in the economy.

**Proof.** See Appendix C ■

![Figure 5: U-shape capital stock](image)

From Figure 5, the capital stock achieves the first-best level if $\beta r = 1$ (i.e. in an open economy steady state) or $\beta r = \frac{\psi}{1-\psi}$. However, the latter case is not feasible as long as the gross interest rate $r > 1$, which is the case here (see Appendix A for details).
When $\frac{\psi}{1-\psi} < \beta r \leq \sqrt{\frac{\psi}{1-\psi}}$, the curve is decreasing. That is, when financial constraints are strong, the initial domestic interest rate is very low, any increase in interest rate will decrease the value of the collateral, leading to a decrease in borrowing. The initial domestic savings are also very low short of saving instruments, an increase in retained earnings (due to a raise of interest rate) cannot offset the decrease in borrowing. Therefore, on this part of the curve, the borrowing effect of the interest rate dominates the revenue effect, leading to a decrease in capital stock. I argue that the initial steady state (before the realization of any technology shock) should be located on the lower part of this decreasing curve as the upper left part assumes an extremely stringent credit constraint with no domestic bond supply and an interest rate near zero. This is hardly the case in emerging market economies.

When $\sqrt{\frac{\psi}{1-\psi}} < \beta r \leq 1$, the curve is increasing. This is because when the central bank provides more liquid assets, families save more. The supply of central bank bonds also decreases the price of liquidity previously overpriced and accordingly increases the interest rate. Families’ retained earnings increase due to both a quantity effect and a price effect. The retained earnings are used for capital investment a period later when the family faces investment opportunities. The revenue effect thus dominates the borrowing effect on this part of the curve.

The equation below presents a more precise insight on how capital formation evolves with the interest rate (derivation in Appendix C):

$$F_K = \frac{r}{(\beta r)^2 + \psi[1 - (\beta r)^2]}$$  \hspace{1cm} (16)

Three effects which affect capital stock can be identified from Equation (16):

- Opportunity cost of capital, $r$
- Revenue effect, $(\beta r)^2 = \frac{1}{1+\lambda}$
- Borrowing effect, $\psi[1 - (\beta r)^2]$

The first effect, the opportunity cost of capital, is standard: the higher $r$, the higher
the cost of investing in capital stock, the less capital is demanded. The other two effects are specific to this model.

The second term \((\beta r)^2\) is related to families’ saving revenues. As families are occasionally credit-constrained, they have incentives to partly rely on retained earnings to invest in capital. As the retained earnings are increasing in interest rate, the revenue effect has a positive impact on capital stock. Notice that the square appears as the capital is invested every two periods. In an extreme case, when \(\psi = 0\) (i.e. no borrowing is possible), families rely entirely on retained earnings for capital investment. The revenue effect clearly dominates the opportunity cost. As a result, a higher supply of liquid bonds leads to a higher domestic interest rate and higher retained earnings. This unambiguously increases the capital stock.

The third effect stems from the fact that the loans that an investing family can ask are conditional on the discounted value of the future production. An increase in \(r\) decreases the borrowing capacity of the investing family, as it lowers the discounted value of the collateral. The interest rate has thus a negative borrowing effect on capital stock. As described above, the borrowing effect dominates the revenue effect only when the supply of liquidity is very low. With a credit constraint becoming less stringent, the borrowing effect cancels out with the revenue effect, leading to a competitive equilibrium without financial frictions (e.g. extreme case: \(\psi = 1\)).

From simple comparative statics in Figure 6, one can see that as long as the central bank provides liquid bonds \(B\) financed by foreign reserves \(B^*\), it raises the domestic interest rate to the world interest rate level and accordingly pushes domestic capital formation to its first-best level (Figure 5).

4 The central bank’s optimal policy

In Section 3, it is showed that an open economy and a semi-open economy are equivalent in the steady state as both policy regimes help the economy to achieve the first-best unconstrained steady state. Using a Ramsey problem, I demonstrate in this section the
difference between a financially open economy and semi-open economy (namely with capital controls) during the economic transition. This is the third important result of this paper: an economy where the central bank imposes capital controls and accumulates foreign reserves (semi-open economy) dominates a financially open economy on the convergence path. This is because the incentives to invest are very strong during a fast-growing economic transition; in addition to the supply of sufficient funding to domestic firms, the central bank can effectively control the domestic interest rate to cope with unanticipated productivity shocks and the dynamics of domestic savers and investors should it imposes capital controls on private flows.

4.1 General setting

The central bank optimizes the consumptions of contemporaneous families in their different stages according to the following objective function.

\[
\max_{\{S_{t+1}, L_{t+1}, K_{t+1}, c_i^S, c_i^I, B_{t+1}, r_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left\{ u(c_i^S) + u(c_i^I) \right\}
\]  

(17)

Definition 2 (Ramsey problem) The central bank’s optimal policy consists of choosing a sequence of policy variables \(\{B_{t+1}, B_{t+1}, r_{t+1}\}\) and a sequence of endogenous private sector’s variables \(\{c_i^S, c_i^I, K_{t+1}, L_{t+1}, S_{t+1}\}\) for all \(t > 0\) such that:

- The corresponding competitive equilibrium allocation ((8) to (10)) maximizes the welfare function (17)
• Individual families’ budget constraints, (2) and (3), are verified

• Central bank’s budget constraint (5), bond market clearing (6) and resource con-
  straint (7) are all respected

The way to solve the Ramsey problem follows the ‘Primal approach’ described by

The full Ramsey problem is presented below and the details about the first order
conditions derived from the program can be found in Appendix D.

\[
\max_{\{S_{t+1}, L_{t+1}, K_{t+1}, c^S_t, c^I_t, B^*_{t+1}, r_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left\{ u(c^S_t) + u(c^I_t) \\
+ \eta^S_t \left[ F(A_t, K_t) - r_t L_t - c^S_t - S_{t+1} - \frac{T_t}{2} \right] \\
+ \eta^I_t \left[ r_t S_t + L_{t+1} - c^I_t - K_{t+1} - \frac{T_t}{2} \right] \\
+ \eta^G_t \left[ F(A_t, K_t) - c^S_t - c^I_t - K_{t+1} - B^*_{t+1} + r^*_t B^*_t \right] \\
+ \theta^S_t \left[ \frac{1}{c^S_t} - \beta r_{t+1} \frac{1}{c^S_{t+1}} \right] \\
+ \theta^I_t \left[ \frac{1}{c^I_t} - \beta r_{t+1} (1 + \lambda_{t+1}) \frac{1}{c^S_{t+1}} \right] \\
+ \rho_t \left[ (1 + \psi \lambda_t) F_{K,t} - r_t (1 + \lambda_t) \right] \\
+ \Lambda_t [\psi F(A_t, K_t) - r_t L_t] \right\}
\]

As it is presented in Section 2, the way to model capital controls in this paper is
through two key variables \( B^*_t \) and \( r_{t+1} \). In a semi-open economy with capital controls,
one needs to derive the first order conditions with respect to both \( B^*_t \) and \( r_{t+1} \), as they
are all policy variables controlled by the central bank. In an open economy, however,
only the first order condition with respect to \( B^*_t \) is derived as the central bank does not
control the domestic interest rate which is always equal to the world interest rate. I show
in the following section the superiority of the semi-open economy over the open economy,
precisely by manipulating the first order condition with respect to the domestic interest
rate \( r_{t+1} \).
4.2 Semi-open economy vs. open economy in the economic transition

By imposing capital controls on private capital flows in a semi-open economy, the central bank has indeed two policy instruments in hand: foreign reserves and domestic interest rates. It has thus sufficient funding to provide liquid assets in the country and at the same time it is able to adjust the domestic interest rate to follow the savers/investors dynamics. In this section, I show analytically how a financially liberalized economy is sub-optimal than an economy where foreign reserve accumulation and capital controls are jointly used.

\[ FOC(r_{t+1}) : \beta \eta_{t+1}^I S_{t+1} - \beta \left( \eta_{t+1}^S + \Lambda_{t+1} \right) L_{t+1} - \beta \frac{\theta_t^S}{c_{t+1}} - \beta \frac{\theta_t^I (1 + \lambda_{t+1})}{c_{t+1}^I} - \beta \rho_{t+1} (1 + \lambda_{t+1}) = 0 \]

Based on the above first order condition derived from the Ramsey problem, it is showed that when the central bank increases the domestic interest rate, it is beneficial for the saver of the current period as the saving revenues from the next period will be higher (first term). However, this is harmful for the investor as it has to repay higher interests next period on loans it asks during the current investment stage (second term). It also changes the intertemporal behavior of family businesses, leading to more savings and less consumptions (third and fourth terms). Finally, a higher domestic interest rate implies a higher opportunity cost (the last term). The first two terms translate a redistributive trade-off that the central bank has to face. This redistributive effects also imply welfare costs to the economy. Indeed, if the current investors are not very constrained and the central bank thinks the long-run steady state can be achieved soon, it has incentives to decrease the interest rate to alleviate the interest burden of future investing families. If, on the contrary, the central bank believes that the economy will be constrained for a long period, it has incentives to rise the domestic interest rate, so that the savers may transfer a larger amount of interest revenue to the future when they will be constrained.
Depending on the tightness of the credit constraint and the speed of convergence to the steady state, the central bank can decide to favor saving or investing families in order to enhance the social welfare. This is only possible in a semi-open economy as capital controls guarantee the free adjustment of the domestic interest rate.

To see why an open economy is not optimal during transition, I rearrange the first order condition with respect to $r_{t+1}$ and denote $H_t$ the left hand side of this equation. In an open economy with a binding credit constraint I can prove (see Appendix E):

$$H_t = \left( \sum_{i=1}^{\infty} \Lambda_{t+2i} \right) S_{t+1} - \left( \sum_{i=0}^{\infty} \Lambda_{t+2i+1} \right) L_{t+1} \quad (18)$$

Whenever $H_t$ is different from 0, the central bank has incentives to intervene and the open economy is not optimal. If $H_t > 0$, the central bank has incentives to raise the interest rate to favor savers. If $H_t < 0$, the central bank has incentives to lower the interest rate to favor investors. In general, as Table 3 summarizes, with financial frictions an open economy is suboptimal during transition while an economy with closed capital account and sufficient foreign reserves can achieve higher a social welfare during the transition. Therefore, imposing capital controls is a policy choice of the central bank in order to generate a higher welfare than in an open economy.20

<table>
<thead>
<tr>
<th>Economic Structure</th>
<th>Central Bank Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open Economy</td>
<td>unconstrained suboptimal</td>
</tr>
<tr>
<td>Semi-open</td>
<td>unconstrained optimal</td>
</tr>
</tbody>
</table>

4.3 Numerical results

In order to gain a more direct insight of the superiority of the semi-open economy over an open economy, I provide some numerical simulations21 in this section.

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20 For empirical studies on the joint use of foreign reserves and capital controls, see Aizenman et al. (2011) and Bussière et al. (2013).

21 The simulations are obtained using Dynare 4.3.2 (see Adjemian et al. 2011).
Calibration

The productivity growth is assumed to follow an AR(1) process: $A_t = (1 + g_t)A_{t-1}$ and the growth rate of the productivity is determined by $g_t = \sigma g_{t-1} + \epsilon$. $\sigma$ stands for the autoregressive coefficient of the productivity growth which is less than 1 and $\epsilon$ is an unanticipated shock. I simulate the model by setting a positive shock of 10% on the productivity growth rate at the beginning of the period 1. The productivity process and the shock are used to mimic the fast economic catch-up in emerging economies.

I calibrate the model based on the recent literature on economic growth and foreign reserve accumulation in China. The discount factor $\beta$ is calibrated to match the average yields of the one-year US Treasury bills. This gives $\beta = \frac{1}{R^*} = \frac{1}{1.05} = 0.95$. The capital share is set to $\alpha = 0.5$, consistent with Bai et al. (2006) and also used by Song et al. (2011) which is the most recent and influential quantitative work on the Chinese economic growth in recent years. I choose the value of the key parameter $\psi$, tightness of the credit constraint, so as to match the savings-to-output ratio in the initial steady state of simulations with China’s average gross saving rate (as a percentage of the national income). This gives $\psi = 0.12$.

Notice that due to the parsimony of the model, the numerical results I present below aim to illustrate the qualitative results derived from the model instead of giving quantitative interpretations.

Table 4: Benchmark calibration

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>share of capital in the production</td>
<td>0.50</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\psi$</td>
<td>credit constraint tightness</td>
<td>0.12</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>initial shock on technology growth</td>
<td>0.10</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>technology growth autoregressive coefficient</td>
<td>0.50</td>
</tr>
</tbody>
</table>

---

22 Song et al. (2011), Wen (2011) and Bacchetta et al. (2013) are the major references.
23 The average rate of return on one-year US Treasury bills amounts to of 5% between 1998 and 2005. This rate can be regarded as a proxy to measure the world interest rate in this paper. Data source: Datastream.
24 This amounts to 42% between 1998 and 2005. Data source: World Bank WDI database.
Superiority of the semi-open economy in the economic transition

It has been demonstrated in Section 3 that a semi-open economy and an open-economy can both achieve the long-run steady state with an unbinding credit constraint (first-best situation). In fact, Figure 7 and 8 show that all variables achieve the same final steady state regardless of the policy regime. Recall that a semi-open economy is an economy where only the central bank can access to foreign financial market while private capital flows are strictly controlled; an open economy is on the contrary an economy with a fully liberalized capital account. In steady state, accumulating foreign reserves while keeping the capital account closed do not allow the central bank to do better than in a fully open economy. This is because the central bank in the steady state will accumulate sufficient foreign assets up to the level of total foreign assets in an open economy, driving up the domestic interest rate to the level of the world interest rate.

However, during the economic transition, especially when the productivity growth becomes suddenly higher, the combined use of foreign reserves and capital controls allows the semi-open economy to achieve the final steady state more rapidly than the open economy. The red line which represents the semi-open economy is always higher than the black dashed line representing the open economy. Why a semi-economy is better than an open economy on the convergence path?

Let’s look at Figure 7. Notice first that unless otherwise notified (i.e. for the current account balance which is expressed in absolute change), all impulse response variables are expressed in percentage deviation from the initial steady state (period 0). Facing the same productivity shock at the beginning of the period 1, the central bank raises immediately the domestic interest rate in a semi-open economy while the interest rate is fixed in an open economy (Panel (a)). To achieve this, the central bank provides more saving instruments (Panel (b)) with which the ‘producing-saving’ family can save to overcome the credit constraint. The total savings increase thus more rapidly in a semi-open economy (Panel (c)). On the one hand, this is because it is financially more interesting for the private sector to save with a higher interest rate; on the other hand, a positive productivity shock triggers a stronger demand for investment as the marginal
product of capital raises. In the presence of future binding credit constraints, the private can increase the investment only by saving more. The reserve accumulation accelerates accordingly (Panel (d)) as the central bank invests its bond proceeds abroad. As for the loans (Panel (e)), in a semi-open economy, the demand for loans first declines with the raise of the domestic interest rate, as a higher interest rate reduces the discounted value of the collateral, making the contemporaneous credit constraint more binding. However, I showed in Section 2.2 that there is a positive feedback loop effect with the capital which serves as a collateral for borrowing. With an aggregate increase in domestic capital formation, the demand for loans will ultimately increase through a positive collateral effect. From Panel (f), one can indeed observe that capital decreases in the first period due to the decline of loans after a raise of the domestic interest rate, but increases afterwards with a more rapid pace to achieve the steady state in a semi-open economy, thanks to the higher level of the private sector’s saving revenues.

Turning to Figure 8, we can see that output increases steadily to reach a 20% higher new steady state but with an accelerated pace in the semi-open economy, so do consumptions in the ‘producing-saving’ period and in the ‘investing-borrowing’ period. I also present in Panel (j) the deviation from the initial steady state of the period welfare defined as the sum of two types of families’ utilities in each period. The social welfare increases faster in a semi-open economy due to an accelerated growth of production. Finally, Panel (k) shows that the economy runs a current account surplus during the economic transition as the central bank constantly purchases foreign assets to finance domestic bond issuing. The current account becomes balanced in the steady state.

In sum, one can see that in a semi-open economy, the central bank can not only provide sufficient domestic funding by investing in foreign reserves, it can also tackle with unanticipated productivity shocks by adjusting the domestic interest rate. The combined policy of reserve accumulation and capital controls makes the semi-open economy better off and allows it to more rapidly reach the higher final steady state.
Financial frictions and capital controls

How does the welfare gains in a semi-open economy depend on the assumption of a binding credit constraint? After all, the financial frictions, along with a positive productivity shock, are the key to understand foreign reserve accumulation in my model.

I calculate first the social welfare according to Equation (17) separately in an open economy and a semi-open economy, for different values of the credit constraint tightness $\psi$. Then, I calculate the welfare gains in a semi-open economy compared to an open economy and present it as a function of the parameter $\psi$. The result is shown in Figure 9.

One can observe that the bigger $\psi$ - the credit constraint less binding - the lower the welfare gains of a semi-open economy over an open economy. Moreover, the closer $\psi$ gets to the threshold value 0.5, the sharper the decline in the welfare gains. This is because when $\psi$ goes bigger and close to 0.5, the demand for loans of the ‘investing-borrowing’ family is close to the unbinding level and almost absorbs the maximum savings that the ‘producing-saving’ family is willing to provide. In the extreme case where there is no more credit constraint, a semi-open economy can do no better than an open economy; the welfare gains associated with capital controls and foreign reserve accumulation diminish. Put it in another way, in an economy with little financial frictions, liberalizing the capital account is the first-best strategy.

This result contributes to the debate on the optimal sequencing between opening-up capital account and financial reforms. Capital controls are welfare improving and need to be maintained only if there are strong imperfections on the domestic financial market. Once appropriate financial reforms eliminate financial frictions, there is no more reason for imposing capital controls.
Figure 7: Simulation: shock and policy responses

Figure 8: Simulation: welfare effects
5 Observations of foreign reserve accumulation in China

The model I present in this paper captures well some key features of the Chinese economy. In this section I provide some descriptive observations about how the model fits in the Chinese situation. This section, however, does not aim to empirically test the validity of the model in this section.

5.1 Financial market failure in China

First of all, the model captures several important features of the Chinese economy. This is a fast-growing economy where stringent market frictions, especially on the domestic financial market, still exist. China has also accumulated a very large stock of foreign reserves since 2000s; its foreign reserves represent more than a third of the world total reserve assets. Moreover, in addition to foreign reserve accumulation, Chinese authorities closely control its capital account (both on capital inflows and outflows (Jeanne 2012)).

Let’s focus on the financial market frictions in China. The financial intermediation in
China is inefficient and underdeveloped. First, one can observe a gap between domestic deposits and domestic credit. Using a database on development of financial institutions constructed by Beck et al. (2009), I show in Figure 10 that the bank credit to deposits ratio keeps decreasing since the end 1980s when the Chinese financial market was relatively more market-oriented. Beck et al. (2009) use this ratio to indicate the inefficiency in financial intermediaries’ fund allocation. The decline of bank credits relative to bank deposits also coincides with the period of massive reserve accumulation, as one can see from Figure 10. Following Song et al. (2011), the wedge between bank deposits and bank credit can be largely explained by capital misallocation between state-owned enterprises and private firms. The latter mainly rely on corporate savings for investment. Moreover, this credit misallocation seems to be an institutional constraint which can be hardly removed in the short-run. As Walter and Howie (2012) argue, the preference of granting loans to public enterprises is inherent to the political regime in China. Many bank loans have been made by administrative orders to public firms for non-economic reasons (e.g. social stability). The lack of uniform accounting standards for small private firms and fiscal fraud also accentuate the difficulties of small firms to get funding from banks.

![Figure 10: Inefficient fund allocation in China](image)

On the other hand, as documented by Walter and Howie (2012), domestic private financial market (bond and equity) in China is very limited in scope and thus domestic investment opportunities are scarce. As a result, if the central bank supplies domestic
liquid saving assets, the proceeds cannot be fully reinvested domestically. Figure 11 captures the close relationship between foreign reserve accumulation, the supply of central bank bills and the gap between domestic savings and loans. If one looks at the supply of central bank bills by decomposing the Chinese bond market, it can be observed that the proportion of the central bank bonds has steadily increased from 2002 to 2010, a period when Chinese foreign reserves increased rapidly and the government’s capacity of issuing new debts was limited.  

25The government bond issuance is closely regulated by law in China. The Chinese government can only issue bonds for infrastructure construction or reduction of the government’s deficit.
5.2 Chinese foreign reserve accumulation and capital formation

I provide here some descriptive evidence on the bivariate relationship between foreign reserve accumulation and capital formation in China. Capital accumulation is proxied by a macroeconomic variable *gross fixed capital formation* (GFCF).

Figure 13 shows a positive correlation between foreign reserves to GDP ratio and GFCF to GDP ratio in China from 1980 to 2010. Figure 14 illustrates that the year-on-year growth rate of foreign reserves and that of gross fixed capital formation coincide very closely since the late 1990s.

Obviously, these simple results only show correlations between foreign reserve accumulation and domestic capital formation. But they give us some hints about this bivariate relationship which merits further exploration using advanced time-series econometrics.

![Figure 13: Foreign reserves vs. GFCF (%GDP)](image1.png)

![Figure 14: Annual growth rate of foreign reserves and of GFCF](image2.png)

6 Conclusion

Based on the model presented in this paper, I can partly attribute the rapid accumulation of foreign exchange reserves in a number of emerging economies, such as in China, to the joint force of a positive productivity shock and domestic financial market frictions. In fact, facing a rapid economic catch-up, the private sector in these countries has a strong demand for liquid assets to support capital investment. In the absence of a sound financial market with sufficient domestic assets, the central bank has to serve as a financial intermediary to provide domestic liquid assets and thus relax the credit...
constraint. The central bank’s bond proceeds are invested abroad in the form of foreign exchange reserves short of domestic investment opportunities.

This paper also proves that during the economic transition characterized by a positive productivity shock, it is welfare-improving to let the central bank accumulate foreign reserves and supply domestic liquid assets instead of allowing the private sector itself hold foreign reserves. That is, a combined used of foreign reserves and capital controls leads to a faster economic transition than in a fully open economy. This is because when private capital flows are controlled, the central bank can adjust the domestic interest rate to cope with any positive productivity shock. The welfare gains in a semi-open economy with capital controls diminish, however, with the development of the domestic financial market; welfare gains vanish when the domestic credit constraint hardly binds. This suggests that capital controls are only useful when the domestic financial market is underdeveloped. In the long-run, a country needs to implement structural reforms and opens its capital account when the financial market is ready.

Finally, there are several possibilities to address other interesting issues by extending my current theoretical framework. On the one hand, one may think about endogenizing the credit constraint as it should be gradually relaxed with capital deepening in the economy. On the other hand, Schmitt-Grohe and Uribe (2003) and Bénassy-Quéré et al. (2011) provide a framework to make capital controls a continuous variable; this would be useful to study foreign reserve accumulation under the perspective of the optimal sequencing between financial reforms and capital account liberalization. Finally, the current model finds it optimal for the domestic economy to accumulate reserves and set capital controls, it might not be the case if cross-country spill-over effects are taken into account. It would be interesting to study the same question in an international context by extending the model into a two-country framework.

26There are a couple of new papers studying the impact of foreign reserves using a two-country model in the context of bilateral relations between China and the United States (e.g. Bonatti and Fracasso 2013); but the motives of foreign reserve accumulation that these papers have examined are different from the vision I presented in this paper.
References


A Proof of Proposition 1

Proof. The central bank has to have a balanced budget (5) and the financial market has to be cleared (6). The steady state of the model with a binding credit constraint is determined by these two conditions:

- Central Bank’s budget balance
- Financial market clearing

Using the binding credit constraint (4), (6), (11) as well as normalized policy variable, one can derive:

\[ B = S - L \]
\[ \Rightarrow B = \beta[F(A, K) - rL] - L \]
\[ \Rightarrow B = \beta(1 - \psi)F(A, K) - \frac{\psi F(A, K)}{r} \]
\[ \Rightarrow b = \beta(1 - \psi) - \frac{\psi}{r} \]

The financial market clearing shows a negative linear relationship between \( b \) and \( \frac{1}{r} \).

Moreover, in the simplest case without any government transfer, using (13) gives,

\[ (r - 1)b = (r^* - 1)b^* \]
\[ \Rightarrow b = \frac{1}{r - 1}(r^* - 1)b^* \]
\[ \Rightarrow b = \left( \frac{1}{1 - \frac{1}{r}} - 1 \right)(r^* - 1)b^* \]

The balanced central bank’s budget shows a non-linear and positive relationship between \( b \) and \( \frac{1}{r} \).

The uniqueness of the steady state can be thus demonstrated graphically (Figure 15).

Figure 15 shows that there are two solutions of \( \frac{1}{r} \), corresponding respectively to \( \frac{1}{r} < 1 \) (equivalent to \( b > 0 \)) and \( \frac{1}{r} > 1 \) (equivalent to \( b < 0 \)). I focus here on the first case;
$r$ being a *gross* interest rate should be greater than 1. This equilibrium corresponds to $b > 0$ (central bank issues bonds). The interest rate is uniquely determined, $\beta < \frac{1}{r} < \min(1, \frac{\beta(1-\psi)}{\psi})$. For $\psi < \frac{1}{2}$, $\frac{\beta}{\beta(1-\psi)} < r < \frac{1}{\beta}$.

Therefore, when $0 < b < \beta(1-2\psi)$, the economy has a unique binding steady state, provided that $\psi < \frac{1}{2}$ (I will show in Appendix B that $\psi < \frac{1}{2}$ is the condition for the credit constraint to bind.).

![Figure 15: Uniqueness of binding steady state](image)

### B Deriving the threshold value of $\psi$

**Proof.** In a steady state when the credit constraint is not binding, $S = L$ and $\beta r = 1$.

From (11) and (12):

\[
S = \beta [F(A, K) - rL] \\
\Rightarrow S = L = \frac{1}{2} \beta F(A, K)
\]
When the credit constraint is not binding, \( rL \leq \psi F(A, K) \), namely \( \frac{1}{2} r \beta F(A, K) \leq \psi F(A, K) \).

Therefore, in a steady state with an unbinding credit constraint, I obtain:

\[
\psi \geq \frac{1}{2}
\]

Accordingly, the steady state is binding if \( \psi < \frac{1}{2} \). 

\[ \blacksquare \]

C Proof of Proposition 2

Proof. In a constrained state steady, (9) and (10) generate:

\[
1 + \lambda = \frac{1}{\beta^2 r^2}
\]

Using this result, along with the first order condition of capital (8), gives Equation (16):

\[
F_K = r \frac{1 + \lambda}{1 + \psi \lambda}
= \frac{r}{(\beta r)^2 + \psi[1 - (\beta r)^2]}
\]

Graphically, \( K \) first decreases in \( \beta r \) then increases to go back to the first-best level (Figure 5). 

\[ \blacksquare \]
D Details on the Ramsey program

I derive first order conditions with respect to all endogenous variables from the Ramsey program described in Section 4.1.

\[ FOC(S_{t+1}) : \eta^S_t = \beta r_{t+1} \eta^I_{t+1} \quad (20) \]
\[ FOC(L_{t+1}) : \eta^I_t = \beta r_{t+1} (\eta^S_{t+1} + \Lambda_{t+1}) \quad (21) \]
\[ FOC(B^*_{t+1}) : \eta^G_t = \beta r^*_t \eta^I_{t+1} \quad (22) \]
\[ FOC(K_{t+1}) : \beta \eta^S_{t+1} F_{K,t+1} - \eta^I_t - \eta^G_{t+1} F_{K,t+1} + \beta \psi \Lambda_{t+1} F_{K,t+1} = 0 \quad (23) \]
\[ FOC(c^S_t) : \frac{1}{c^S_t} - \eta^S_t - \eta^G_t = \frac{\theta^S_t}{(c^S_t)^2} + \frac{\theta^I_{t-1}}{(c^S_t)^2} = 0 \quad (24) \]
\[ FOC(c^I_t) : \frac{1}{c^I_t} - \eta^I_t - \eta^G_t = \frac{\theta^S_t}{(c^I_t)^2} + \frac{\theta^I_{t-1}}{(c^I_t)^2} = 0 \quad (25) \]
\[ FOC(r_{t+1}) : \beta \eta^I_{t+1} S_{t+1} - \beta (\eta^S_{t+1} + \Lambda_{t+1} L_{t+1} - \beta \frac{\theta^S_t}{c^I_{t+1}}) - \beta \rho_{t+1} (1 + \lambda_{t+1}) = 0 \quad (26) \]
\[ FOC(\lambda_{t+1}) : -\theta^I_t \beta r_{t+1} c^S_{t+1} + \beta \rho_{t+1} (\psi F_{k,t+1} - r_{t+1}) = 0 \quad (27) \]

E Deriving Equation (18)

Proof. As long as the Ramsey first order condition on the credit constraint (27) is verified, \( \theta^I_t = 0 \), as \( F_{K,t+1} = r_{t+1} \) in an open economy.

In a fully open economy, \( r = r^* \beta = 1 \) and \( u'(c^S_t) = u'(c^I_{t+1}) \). From (20), (21) and (22) one can derive:

\[ \eta^S_t = \eta^I_{t+1} \]
\[ \eta^I_t = \eta^S_{t+1} + \Lambda_{t+1} \]
\[ \eta^G_t = \eta^G_{t+1} \]

From (24) valued at \( t \) and (25) valued at \( t+1 \):

\[ \theta^I_t u''(c^S_t)(1 + r^*) = 0 \]
As \( u''(c_i^S)(1 + r^*) < 0 \), \( \theta_i^S = 0 \).

Substitute \( \theta_i^S = \theta_i^I = 0 \) in (26) and solve it forward, one obtains Equation (18).